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RECENT ADVANCES IN ADAPTIVE VARIABLE NEIGHBORHOOD SEARCH METHODS

Abstract. This paper investigates the development and application of adaptive Variable Neighborhood Search (VNS) heuristics for solving complex optimization problems. Through a systematic literature review of studies from the past decade, the research aims to identify how adaptive mechanisms have been integrated into VNS methods, the learning strategies employed, and the specific VNS variants enhanced by these adaptations. The review reveals that most studies have focused on improving the intensification phase of the Basic VNS (BVNS) using probabilistic-based learning strategies. However, there is a noticeable gap in understanding the overall impact of these adaptive mechanisms on the performance of VNS algorithms. Future research should explore alternative VNS variants and develop more comprehensive adaptive strategies that simultaneously address multiple phases of the VNS process. This approach could lead to more balanced and effective optimization methods. The findings suggest that, while significant progress has been made, further investigation is needed to fully understand and leverage the potential of adaptive VNS heuristics.

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1. Introduction

Metaheuristics encompass a class of heuristic optimization techniques that are distinct from traditional analytical optimization methods [67,68]. Unlike exact optimization algorithms, metaheuristics do not provide mathematical guarantees of identifying an optimal solution within a finite amount of computational time or a predetermined number of iterations. This empirical nature allows them to effectively address complex and large-scale optimization problems where exact methods may be infeasible or inefficient. Metaheuristics represent general and high-level optimization frameworks that can be suitably adapted and customized to develop problem-specific heuristic strategies [12]. These strategies are capable of generating high-quality solutions within relatively short computational times [72]. Recently, researchers within the metaheuristics community have expressed concerns regarding the recurrent practice of repackaging existing ideas and methods under new names [9, 67, 78]. This issue stems from the reliance on metaphorical reasoning associated with already established approaches [2, 67]. Therefore, it is imperative for the scientific community to shift its focus on advancing and refining well-established metaheuristics, ensuring genuine innovation and progress in the field [67]. A particularly promising research avenue is the development of adaptive heuristic algorithms [60,73]. This approach focuses on designing and incorporating low- or high-level learning mechanisms to dynamically reconfigure the components of a metaheuristic during its execution, leveraging the data generated throughout the process [33].

Variable Neighborhood Search (VNS) is a classic, trajectory-based, metaheuristic introduced by [47]. VNS is predicated on the fundamental principle of employing multiple neighborhood structures and systematically alternating between them to effectively escape local optima [17]. This methodological flexibility improves the search process, promoting the discovery of high-quality solutions in complex optimization landscapes [7, 19]. VNS is a well-established and sufficiently documented

powerful metaheuristic following the standard optimization terminology [7,19]. The initial framework, known as Basic VNS (BVNS), comprises three primary procedures [7,18,19]. The first one, termed the shaking procedure, functions as the main diversification mechanism of the method, utilizing a predetermined set of search operators. In the context of BVNS, the classic shaking procedure is intensified by incorporating a parameter that specifies the maximum number of iterations for each shaking operator. Consequently, during each iteration of the BVNS, a shaking operator is randomly selected and applied a number of times as dictated by the value of this parameter. The next critical component of BVNS is the improvement procedure, which constitutes the intensification phase. This phase involves the application of local search operators. Specifically, during each iteration of BVNS, a local search operator is selected from a predefined set of operators based on a selection criterion. The chosen operator is then applied according to a specific search strategy with the objective of improving the incumbent solution. The final essential component of BVNS is the neighborhood change step. This step dictates the manner in which search operators are alternated, contingent upon whether an improvement has been achieved.

Due to its high efficacy in solving hard optimization problems, and through modifications to the aforementioned three critical procedures, several variants of VNS have been developed. Variable Neighborhood Descent (VND) is a prominent variant of BVNS that eliminates the shaking phase [62], [13]. Various VND schemes have been proposed, each incorporating different neighborhood change procedures. The basic VND (bVND) employs the classic sequential neighborhood change procedure [19]. Cyclic VND (cVND) uses a cyclic neighborhood change method [76], while pipe VND (pVND) implements the pipe neighborhood change step [77]. Additionally, union VND (uVND), also known as the multiple neighborhood search, explores the union of all the neighborhoods considered as a single entity [19]. In contrast, Reduced Variable Neighborhood Search (RVNS) simplifies the BVNS framework by eliminating the improvement phase. Instead, RVNS uses only a shaking procedure within the neighborhood change step [7]. Another generalization of BVNS is the Skewed Variable Neighborhood Search (SVNS) [19]. SVNS introduces a predefined threshold that allows acceptance of not only improved solutions but also some inferior ones within a specified distance during the neighborhood change step. This approach facilitates the exploration of broader regions of the search space relative to the incumbent solution, potentially improving the overall effectiveness of search [16]. A powerful variant of BVNS is General Variable Neighborhood Search (GVNS), which enhances the improvement phase by incorporating a VND variant to intensify the search [19], [63]. The underlying principle of GVNS is based on the notion that a globally optimal solution is also locally optimal with respect to all neighborhood structures. Therefore, employing a VND that considers multiple neighborhood structures increases the likelihood of discovering a globally optimal solution [7].

These classic variants of VNS have been effectively applied to solve hard optimization problems in a wide range of domains [3, 5, 6, 14, 30, 31, 39, 43–46, 49, 51,

53–55, 58, 64, 75]. However, several other efficient variants of the VNS methodology have also emerged in the literature [66], [48], and [4].

To this end, the present work aims to investigate the potential benefits of developing adaptive VNS-based heuristics for solving optimization problems. To achieve this objective, a systematic review of the literature of relevant research conducted over the past decade has been designed. Section 2 outlines the methodology used for this review of the literature. The analysis of the reviewed works and the corresponding results are presented in Section 3. Finally, Section 4 provides an overview of the findings and offers insight into promising directions for future research.

2. Methodology

This section presents a comprehensive methodology for conducting a systematic literature review on the development of adaptive VNS heuristics for efficiently solving challenging optimization problems. The methodology consists of four main steps. The first step involves defining the research questions. The second step establishes the criteria for identifying relevant research works from the literature. The third step specifies the selection criteria for these works, and the final step outlines the criteria for their analysis.

Definition of research questions. The following research questions were formulated to conduct a comprehensive literature review on the scientific contributions to the development of efficient adaptive VNS methods for the solution of hard optimization problems.

(1) In which phase of the VNS algorithm have adaptive mechanisms been applied?
(2) What types of learning formulas or strategies have been utilized in the adaptive mechanisms?
(3) Which specific variants of VNS have been selected and enhanced by the incorporation of adaptive mechanisms?

Locating research works. An advanced search utilizing Boolean operators was conducted in the Scopus database to identify relevant research works, using the following search keywords: *("Adaptive VNS" OR "Adaptive Variable Neighborhood Search" OR "Adaptive VND" OR "Adaptive Variable Neighborhood Descent" OR "Adaptive GVNS" OR "Adaptive General Variable Neighborhood Search" OR "Adaptive Skewed VNS" OR "Adaptive Skewed Variable Neighborhood Search" OR "Adaptive Reduced VNS" OR "Adaptive Reduced Variable Neighborhood Search").* To avoid retrieving irrelevant results, the search was restricted to the title, abstract, and keywords of the documents. Additionally, the search was confined to the last decade (2014-2024 [up to 12/7/24]) and included only scientific journal articles written in English.

Screening & selection. The subsequent step in the designed methodology involved applying specific exclusion criteria to concentrate on the most pertinent research contributions. The initial search, utilizing the previously defined filters,

yielded 50 articles. These articles were then further refined using the following exclusion criteria.

- Works that did not sufficiently describe the adaptive mechanism(s) employed.
- Works in which VNS was not the primary (or a leading) solution approach.
- Works that addressed hyper-heuristics and did not consider adaptive mechanisms as inner-VNS procedures.

The application of these exclusion criteria ultimately resulted in a selection of 40 articles for detailed analysis.

Analysis of the literature. In the final step of the methodology, the selected research works were examined in relation to the formulated research questions. The findings from this analysis were presented in a tabular format, accompanied by a thorough discussion.

3. Results and discussion

This section presents a tabular overview of the analysis conducted on the selected research works in (Table 1), accompanied by a detailed discussion of the findings.

To clarify, the abbreviation *GVNDS, mentioned in Table 1, denotes a hybrid approach that integrates GVNS with an advanced variant known as Variable Neighborhood Decomposition Search (VNDS).

At this point, illustrative summaries are presented in Figures 1 through 3 to improve understanding of the review findings. Specifically, Figure 1 displays a bar plot that depicts the distribution of various VNS variants across the studies. Figure 2 features a heatmap that illustrates the relationship between learning strategies and adaptive phases, with the intensity of the color in each cell representing the frequency of specific combinations. Lastly, Figure 3 shows a stacked bar chart that visualizes the distribution of different adaptive phases across VNS variants.

The analysis carried out reveals that most of the research efforts have focused on enhancing BVNS by developing score-based probabilistic learning mechanisms, in alignment with the principles of Adaptive Large Neighborhood Search [56]. Predominantly, these adaptive mechanisms have been incorporated into the VNS improvement phase. In particular, as highlighted by [7], there has been limited emphasis on simultaneously integrating adaptive mechanisms into both the improvement and shaking phases. Moreover, it is clear that the majority of adaptive VNS algorithms predominantly focus on integrating adaptive operator selection mechanisms within the improvement phase, as well as the shaking phase. Typically, the selection procedures for adaptive shaking operators occur within the intensified shaking method. Specifically, conventional random selection of the shaking operators is replaced by adaptive selection, which bases its choices on the operators' performance in previous iterations. Although research in this area is less extensive compared to improvement-focused adaptive VNS schemes, the advantages of incorporating adaptiveness in the shaking phase are evident. Furthermore, these benefits are amplified when adaptive mechanisms are simultaneously implemented in both the improvement and shaking phases. This approach has been validated through extended

TABLE 1. Overview of the literature

Study	VNS variant	Learning strategy	Adaptive phase
[80]	BVNS	Score-based probability	Shaking
[36]	BVNS	Weight-based roulette wheel rule	Improvement
[1]	Multi-objective BVNS	Score-based probability	Shaking
[59]	BVNS	Score-based roulette wheel rule	Shaking
[20]	BVNS	Score-based roulette wheel rule	Shaking
[74]	Multi-objective BVNS	Score-based probability	General
[35]	BVNS	Score-based probability	General
[77]	GVNS	Success-based weights	Improvement
[8]	BVNS	Score-based roulette wheel rule	Improvement
[69]	BVNS	Score-based probability	Improvement
[21]	BVNS	Score-based roulette wheel rule	Shaking
[70]	BVNS	Score-based probability	Improvement
[23]	BVNS	Score-based roulette wheel	Shaking
[38]	BVNS	Ratio-based probability	Phases selection
[32]	GVNS	Improvements frequency counter	Shaking
[40]	(Hybrid) BVNS	Weight-based probability	Improvement
[81]	GVNS	Weight-based ratio	Improvement
[11]	(Hybrid) VND	Weight-based probability	Improvement
[57]	(Hybrid) BVNS	Weight-based roulette wheel rule	Improvement, shaking
[84]	BVNS	Score-based roulette wheel rule	Shaking
[28]	GVNS	Improvements frequency counter	Improvement, shaking
[41]	BVNS	Weight-based probability	Improvement
[25]	BVNS	Weight-based roulette wheel rule	Improvement
[83]	GVNDS*	Weight-based probability	Parameter tuning
[82]	BVNS	Probabilistic	Improvement
[15]	GVNS	Weight-based roulette wheel rule	Shaking
[50]	BVNS	Score-based probability	Improvement
[85]	GVNS	Weight-based roulette wheel rule	Shaking
[22]	BVNS	Ratio-based probability	Improvement
[65]	BVNS	Ratio-based probability	Improvement
[24]	Multi-objective BVNS	Score-based probability	General
[37]	GVNS	Weight-based roulette wheel rule	Improvement, shaking
[52]	BVNS	Score- & Improvements frequency-based probability	Improvement
[61]	BVNS	Improvements frequency score-based probability	Improvement
[10]	BVNS	Weight-based probability	Improvement
[34]	GVNS	Extreme learning machine algorithm	Improvement
[71]	BVNS	Score-based probability	Improvement
[29]	GVNS	Improvements frequency counter	Improvement, shaking
[42]	BVNS	Score-based probability	Improvement
[79]	GVNS	Score-based probability	Shaking

computational experiments reported in recent research [28,29,37]. A discussion for each research work follows.

[80] investigated the heterogeneous fleet vehicle routing problem with three-dimensional loading constraints. The authors proposed a hybridization of the BVNS within an adaptive diversification procedure, effectively rendering the approach as an adaptive BVNS due to the dynamically reshaped search space facilitated by the adaptive shaking mechanism. Technically, the authors employed a score-based probabilistic formula that considered the number of improvements achieved by each shaking operator and the frequency of their application. The selection probability for each shaking operator was calculated as the ratio of its individual score to the total scores of the available shaking operators. This adaptive

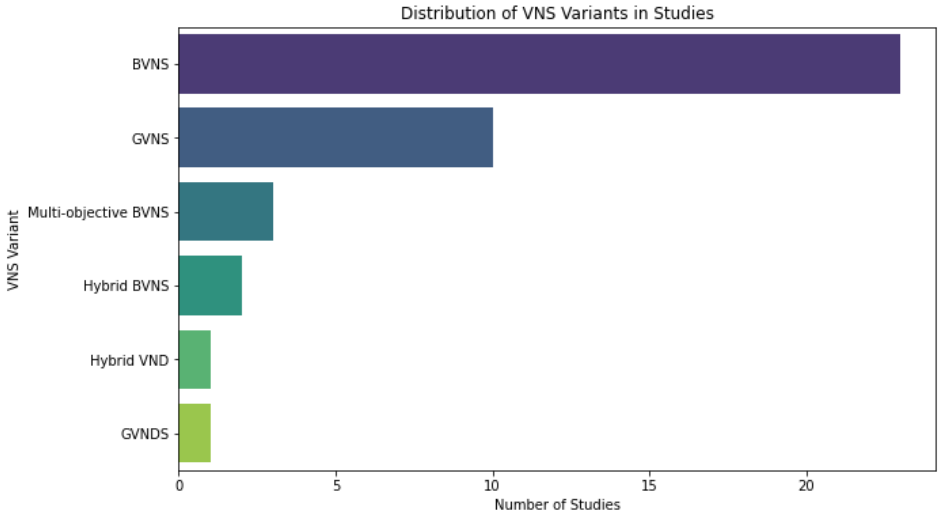


FIGURE 1. Bar plot of different VNS variants

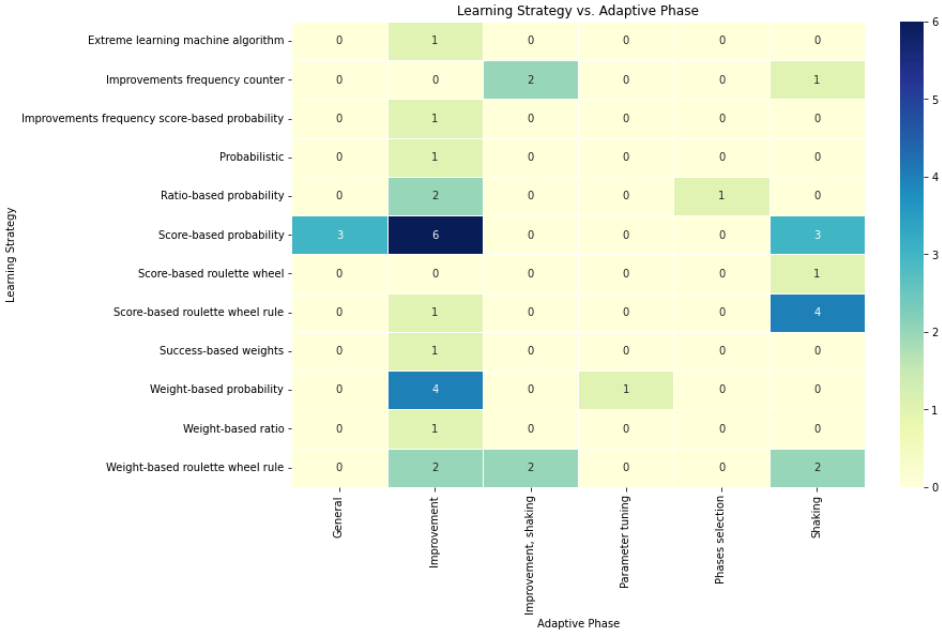


FIGURE 2. Heatmap on the relationship between learning strategies and adaptive VNS phases

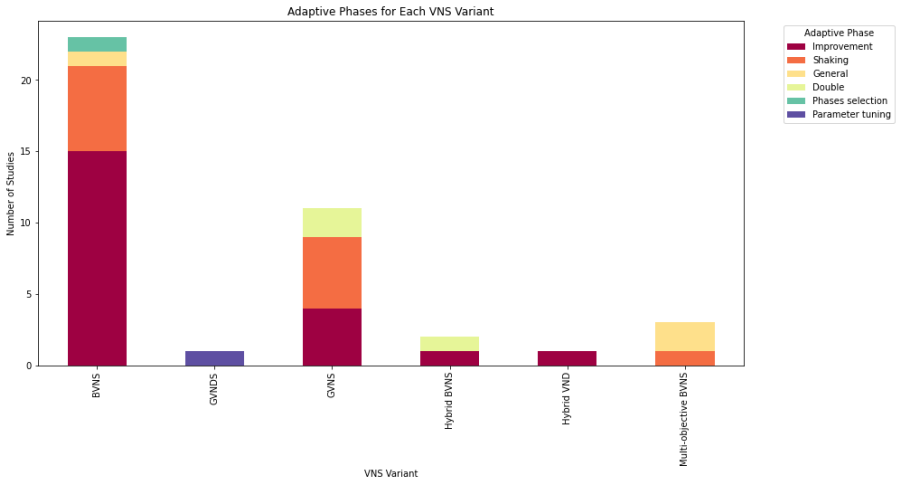


FIGURE 3. Stacked bar chart of adaptive phases distribution per VNS variant

BVNS was subsequently integrated with a heuristic approach to ensure the feasibility of loading decisions concerning the imposed constraints. Extensive comparative analyses were conducted to validate the efficiency of the proposed solution method. However, the authors did not provide any analysis on the effects of the adaptive mechanism on the performance of the VNS method.

[36] examined a location-routing problem incorporating multiple charging stations, capacitated electric vehicles, and time windows. To address this problem effectively, the authors developed a hybrid heuristic approach that integrates an adaptive BVNS with a Tabu List. The adaptive mechanism was implemented in the shaking phase of the VNS method, utilizing a weight-based probabilistic formula in conjunction with a roulette wheel rule. The authors conducted a comparative analysis between the hybrid adaptive solution method and an exact solver on a limited number of small-sized problem instances. Nevertheless, the study did not include any analyses to assess the impact of the adaptive shaking mechanism on the performance of the BVNS. [1] investigated the optimization of the multi-objective node placement problem. To address this challenge, the authors developed an adaptive multi-objective VNS algorithm. The proposed method was augmented with a single-adaptive shaking mechanism that utilized a score-based probabilistic formula for the informed selection of shaking operators. To evaluate the efficiency of the proposed solution method, the authors conducted a comparative analysis with a multi-objective Genetic Algorithm. The computational results demonstrated the efficiency of the proposed method. However, the results section did not include any analyses evaluating the impact of the adaptive mechanism on the performance of the multi-objective VNS.

[59] examined the vehicle routing problem with intermediate stops for specific operations such as replenishment or refueling. The authors introduced an adaptive

BVNS incorporating various selection methods into an adaptive shaking mechanism. This adaptive mechanism utilizes score-based probabilities within a roulette wheel selection procedure to efficiently guide the exploration process of the algorithm. Comprehensive comparative analyses were conducted between the proposed adaptive BVNS solution method, an exact commercial solver, and other efficient heuristic methods from the literature, using several well-established datasets. Additionally, the authors evaluated the impact of incorporating problem-specific features related to intermediate stops within the adaptive BVNS. However, they did not provide similar analyses to assess the effects of the entire adaptive mechanism on the performance of the BVNS. [20] investigated the closed-loop global supply chain network design problem, accounting for uncertainties in product demands and component purchasing costs, within the context of a medical device manufacturer. To address this problem, the authors proposed a hybrid memetic algorithm that integrates an adaptive BVNS method as its core improvement step. The adaptive mechanism is designed to facilitate the informed selection of shaking operators by employing a roulette wheel rule on score-based probabilities. The authors conducted comparative analyses to evaluate the performance of the proposed method against that of a genetic algorithm and a hybrid memetic algorithm incorporating a BVNS. The results demonstrated the efficiency of both the overall proposed solution method and the specifically designed adaptive mechanism.

[74] addressed the bi-objective flow shop scheduling problem, incorporating sequence-dependent setup times. To solve this complex problem, the authors developed a Pareto-based adaptive bi-objective VNS algorithm. This adaptive mechanism facilitates the dynamic selection of neighborhood structures, which are employed in both the shaking and improvement phases of the algorithm. The adaptive mechanism operates based on score-based probabilities, calculated from the number of successes and failures associated with each neighborhood structure. Extensive computational experiments were conducted to evaluate the overall efficiency of the proposed solution method and specifically the efficacy of the adaptive mechanisms. The results demonstrated the effectiveness and efficiency of both the overall approach and its individual components. [35] investigated the prize-collecting vehicle routing problem, introducing a two-level self-adaptive VNS algorithm. This approach extends the BVNS into a two-level method, with the first level serving as a learning phase and the second one functioning as the main optimization process. The adaptive mechanism calculates a score-based selection probability from the successes and failures of each neighborhood structure in the first phase, which is then applied in the second phase. The authors conducted comprehensive comparative analyses between the proposed method, a conventional two-level VNS, and other heuristic approaches documented in the literature. The computational results demonstrated the effectiveness of the overall solution method and highlighted the advantages of the proposed adaptive mechanisms.

[77] introduced adaptive GVNS heuristics to efficiently solve the unit commitment problem. The proposed GVNS heuristic integrates two pipe-VND methods, which are adaptively selected in each iteration based on their respective weights. These weights are dynamically updated according to the historical success of each

method. The authors performed extensive computational and comparative analyses to assess the efficiency of the proposed adaptive GVNS heuristics relative to other state-of-the-art heuristic and exact approaches. Additionally, they evaluated the impact of the adaptive mechanisms on the performance of the GVNS methods. The reported results demonstrated the overall effectiveness of the solution method, particularly highlighting the significant contribution of the adaptive procedures. [8] investigated a machine reassignment problem and proposed a hybrid hyper-heuristic method that integrates an adaptive VNS within a Simulated Annealing framework. The adaptive VNS method employs a score-based probabilistic formula, which considers the frequency of improvements achieved by each local search operator and the total number of times each operator has been used. The selection of local search operators is conducted using the roulette wheel rule. The authors provided comprehensive computational and comparative analyses across various datasets to evaluate the efficacy of the proposed solution method. Despite these extensive evaluations, the study did not include an analysis of the impact of the adaptive mechanism on the performance of the BVNS.

[69] proposed a hybrid heuristic that integrates an adaptive VNS with a large neighborhood search to solve the vehicle routing problem. The proposed solution method is composed of two distinct stages. The first stage serves as the learning procedure, wherein a score is computed for each local search operator based on its success rate, utilizing the best improvement search strategy. At the conclusion of this stage, a probability selection for each local search operator is also determined. The second stage constitutes the main improvement phase, where the most promising local search operator is selected and applied to potentially enhance the incumbent solution. The authors conducted extensive computational and comparative analyses to assess the efficiency of the overall solution method and to evaluate the contribution of each critical algorithmic component. Furthermore, the impact of the learning mechanism on the performance of the BVNS was systematically examined. The analysis revealed consistent and significant improvements attributable to the incorporation of the adaptive learning mechanisms into the BVNS method. [21] expanded upon the vehicle routing problem with intermediate stops by developing a battery-swap location-routing problem that incorporates capacitated electric vehicles. They effectively adapted their previously introduced adaptive BVNS algorithm, originally designed to address the vehicle routing problem variant as per [59], to solve this new problem. This adaptation involved the integration of additional search operators specific to the battery-swap context. Importantly, the adaptive mechanism utilized remains consistent with the heuristic algorithm outlined in [59]. To evaluate the efficacy of their modifications, the authors conducted numerous computational experiments, assessing both the contribution of individual algorithmic components and the overall performance of the adaptive solution method. Although the impact of the adaptive mechanism itself was not directly analyzed, the corresponding adaptive GVNS was formulated by considering a fixed-order BVND. The experimental results demonstrated the superior efficiency of the adaptive BVNS method, outperforming a commercial

exact solver, an improved heuristic VNS-based method from the literature, and the corresponding adaptive GVNS.

[70] introduced a modified variant of their adaptive two-stage BVNS, building upon their prior work [69]. This modification incorporated specific search operators tailored for the cumulative capacitated vehicle routing problem, addressing either min-sum or min-max objectives. Consistent with their earlier research, the adaptive mechanism employed in the first stage utilizes a score-based probabilistic learning procedure aimed at refining the heuristic algorithm during its improvement phase. The authors meticulously examined the advantages of integrating the second stage along with the proposed diversification strategy within the first stage of the adaptive BVNS. Their findings underscored the substantial performance improvements facilitated by this integration. Additionally, further comparative analyses demonstrated the high efficiency and effectiveness of the proposed adaptive BVNS method relative to existing solutions. [23] examined a robust vehicle routing problem which considers hard time windows, as well as uncertainties in demand and travel time. To address large problem instances, they developed a two-stage adaptive BVNS. The first stage of their solution method focuses on minimizing the number of routes and, consequently, the number of vehicles used. In contrast, the second stage aims to minimize the total travel distance. Both stages employ a single-adaptive shaking mechanism to inform the selection of shaking operators. The adaptive mechanism incorporates various route- and node-based selection methods to determine which routes and nodes will be involved in the shaking process. This shaking adaptive procedure utilizes a score-based probabilistic formula and a roulette wheel selection rule. Through extensive computational experiments, the proposed method demonstrated significant efficiency. However, the study did not explore the specific effects of the adaptive mechanisms on the overall performance of the BVNS.

[38] introduced an adaptive GVNS method aimed at optimizing the reconfiguration of supply networks to enhance their robustness against both random and targeted disruptions. The authors incorporated a low-level adaptive mechanism that intelligently selects either the improvement or shaking phase, leveraging a randomized ratio-based formula as a core component of a simple reinforcement learning process. To evaluate the efficacy of their adaptive GVNS, they conducted comparative studies across three real-world scenarios, juxtaposing their approach against alternative edge addition-based network reconfiguration methods. Despite the comprehensive nature of their analysis, the authors did not investigate the impact of adaptiveness on the performance of the traditional GVNS method. [27] proposed several adaptive GVNS methods for solving complex integrated supply chain network optimization problems, which simultaneously address strategic, tactical, and operational decisions. The authors introduced adaptive mechanisms for reconfiguring the developed shaking operators within a GVNS framework. These adaptive mechanisms utilized improvement frequency counters to record the number of improvements achieved by a pipe-VND following the application of a specific shaking operator. This data was then used to reconfigure the order of the shaking operators in subsequent iterations. Several comparative analyses were conducted to

verify the efficiency of these novel adaptive schemes. Furthermore, comparisons between the adaptive and conventional GVNS methods, considering the same critical parameters (i.e., execution time limit), revealed marginal but consistent benefits of incorporating adaptive mechanisms into the traditional GVNS approach.

[40] investigated a practical variant of the static bike-sharing repositioning problem. They proposed a hybrid heuristic algorithm that integrates a destroy-and-repair method with an adaptive VNS. The destroy-and-repair method focuses on optimal station clustering, while the adaptive VNS is utilized to optimize routing decisions. The proposed VNS method is an improvement-based, adaptive variant of the BVNS, incorporating a weight-based probability learning mechanism to select the most promising local search operator at each iteration of the heuristic algorithm. The efficiency of the proposed hybrid algorithm was verified through extensive computational comparative analyses against exact solvers and state-of-the-art solution approaches from the literature. However, the authors did not assess the impact of incorporating the adaptive mechanism within BVNS. [81] developed two metaheuristic-based solution approaches for optimizing a multi-compartment vehicle routing problem. One of these methods is an adaptive GVNS, which incorporates a self-learning mechanism to enhance its performance. This self-learning mechanism leverages data generated during the improvement phase of the algorithm to reconfigure the order of local search operators in the VND. Specifically, in each iteration, a ratio is calculated for each local search operator by combining the metrics of achieved improvement and the computation time required to achieve this improvement. The authors conducted extensive computational comparisons between the developed adaptive VNS variant, the exact solver CPLEX, and other state-of-the-art heuristic methods, revealing the promising performance of the adaptive VNS. However, the study did not include comparisons between the adaptive and conventional GVNS methods, nor did it assess the effects of the adaptive mechanism.

[11] investigated a task scheduling problem for tracking and data relay satellites. They developed a hybrid heuristic method combining an adaptive VND and a Tabu List to optimize working efficiency and task completion rates within a data relay satellite network. The proposed VND incorporates a single-adaptive, weight-based probabilistic mechanism for selecting the most promising local search operator. This selection probability formula includes inertia weighting factors and is based on the ratio of successful applications of each operator to the number of times it was selected in previous iterations. The authors conducted several comparative analyses between the proposed hybrid method and other approaches, including the corresponding adaptive VND and the conventional VND. These analyses revealed the enhanced performance of both the adaptive VND and its hybridization with the Tabu List. [57] developed a hybrid approach combining an adaptive VNS with Simulated Annealing to minimize cycle time in a multi-sided assembly line balancing problem. The authors incorporated weight-based adaptive mechanisms in both the shaking and improvement phases of the VNS. These adaptive mechanisms update weights by a specific factor whenever an improved solution is found. Subsequently, the roulette wheel rule is employed for weight-based probabilistic selection

of shaking and improvement search operators in each iteration of the algorithm. Comparative analyses were conducted to evaluate the efficiency of the proposed method against exact and other heuristic approaches. However, the study did not include analyses on the impact of the hybrid mechanisms.

[84] proposed a two-level adaptive VNS method to optimally address sensor allocation and heterogeneous multi-UAV path planning in integrated mission planning. The authors introduced score-based shaking adaptive mechanisms that utilize three selection methods within a roulette wheel rule to efficiently guide the exploration of the search space. The scoring system adheres to specific statistical rules, while the selection methods leverage probabilistic or problem-specific features. A Monte Carlo study was conducted, comparing the proposed method to a corresponding two-level VNS without the shaking adaptive mechanisms and one with a random selection strategy. The results demonstrated a consistent improvement in solution quality, albeit with a slight increase in computation time. [28] explored the potential benefits of incorporating simple adaptive mechanisms in both the intensification and diversification phases of a GVNS method for the strategic reordering of search operators. Using the classic traveling salesman problem as a reference, they conducted extensive comparative analyses. Initially, they evaluated the performance of the new double-adaptive GVNS against the corresponding single-adaptive and conventional GVNS approaches. The results indicated consistent benefits from employing such simple adaptive mechanisms within the two key phases of GVNS. Subsequently, the proposed adaptive method was compared with other recent, improved variants of metaheuristics. Despite its simple structure, these analyses revealed the highly competitive performance of the proposed algorithm, which was fully attributed to the adaptive mechanisms utilized, in comparison to those advanced metaheuristic variants.

[41] developed a fuzzy clustering-based adaptive VNS method for optimizing a large-scale bike-sharing repositioning problem within a two-echelon network. Building on their previous work [40], the authors employed an adaptive mechanism that leverages a weight-based probabilistic formula to enhance the selection of local search operators in each iteration. Extensive comparative analyses were conducted to assess the performance of the proposed adaptive VNS method against exact solutions and other state-of-the-art approaches. [25] proposed an adaptive VNS to address the rural dial-a-ride problem, considering interrelated trips. They developed an adaptive mechanism that assigns weights to each local search operator. These weights are updated in each iteration based on the previous values and the ratio of improvement frequency to the number of times each operator was executed during the previous iteration. The probabilistic selection of an operator for each subsequent iteration is managed through a roulette wheel rule. Extensive comparative analyses demonstrated the efficiency of the proposed solution method. However, the authors did not investigate the impact of the adaptive mechanisms on the performance of the developed VNS approach.

[83] investigated the car sequencing problem considering block batches. They developed an adaptive hybrid method combining GVNS and Variable Neighborhood Decomposition Search (VNDS) to solve large-scale problem instances. The

adaptiveness of this method is achieved through a weighted probabilistic-based reinforced parameter tuning process for the dynamic selection of key parameter values. Several computational experiments were conducted to evaluate the performance of the proposed approach. Despite these efforts, the effect of the adaptive mechanism was not examined. [82] investigated an optimization problem aimed at minimizing time and energy consumption in multi-shuttle automated storage and retrieval systems. To address large-scale problems, the authors developed an adaptive VNS algorithm. This algorithm employs a probabilistic formula based on the performance of local search operators in previous iterations to select the most promising operator during the improvement phase of each new iteration. The efficiency of the proposed adaptive VNS was thoroughly evaluated through comprehensive comparative analyses. However, the impact of the adaptive mechanism itself was not examined.

[15] studied a complex variant of the milk collection problem that incorporates multiple features, including time windows, multiple depots, various product types, split deliveries, electric vehicles with multiple compartments, fast chargers, and fleet composition decisions. The author developed a single, shaking-based, adaptive GVNS method to efficiently address this problem. The adaptive mechanism leverages a weight-based probabilistic formula to utilize generated knowledge for effective guidance during the diversification phase. The selection of shaking operators is managed using a roulette wheel rule. Computational analyses demonstrated the efficiency of the algorithm in solving both small and large-scale problem instances. Although extensive computational and sensitivity analyses were conducted, no specific evaluation of the impact of the adaptive mechanism on the GVNS performance was provided. [50] examined a variant of the capacitated cross-docking system problem, considering multiple suppliers, cross-docks, retailers, various product types, and time-dependent demands. They proposed a two-level self-adaptive VNS solution method to address this complex problem efficiently. The adaptiveness is implemented in the second level of the heuristic, utilizing a score-based probabilistic selection of local search operators based on their previous successes and failures. Extensive computational comparative analyses were conducted between the proposed adaptive VNS method and the corresponding conventional VNS heuristic. The results demonstrated the superior performance of the adaptive VNS variant. Notably, the adaptive mechanism positively impacted computational time, with the adaptive VNS reporting lower computational times than the conventional method.

[85] investigated a dual-source inventory routing problem that incorporates air-land transportation, route-dependent lead times, and demand uncertainty within a rolling horizon framework. To address this complex problem efficiently, the authors developed an adaptive GVNS method, embedding a score-based probabilistic learning procedure in the shaking phase. This procedure assigns score-based weights to each shaking operator, utilizing a roulette wheel rule to select the most promising operator in each iteration. Comprehensive computational and comparative analyses were performed. However, the impact of the adaptive mechanism on the performance of the GVNS method was not specifically evaluated. [22] examined a variant of the flow-shop scheduling problem, incorporating budget constraints and

non-identical factories, with the objective of minimizing production makespan. The problem was mathematically formulated as a mixed integer programming model, and three VNS algorithms were proposed for its solution. Among these, a two-level self-adaptive VNS heuristic algorithm was developed, which employs a ratio-based probability formula for the adaptive selection of the most promising local search operator in each iteration. Comparative analyses demonstrated the high efficiency of the adaptive VNS method in comparison to an exact solver and the alternative VNS methods.

[65] studied the dynamic job-shop scheduling problem and developed an improved Gene Expression Programming (GEP) method that incorporates an adaptive BVNS to enhance solution efficiency. The adaptive BVNS employs a ratio-based probabilistic formula to select local search operators during algorithm iterations. The performance of the proposed improved and adaptive method was compared with alternative GEP-based approaches. The comparative analyses demonstrated the increased efficiency of the proposed method. While the authors provided extensive computational analyses to explore the benefits of integrating adaptive BVNS and feature selection into the GEP framework, they did not examine how the inclusion of the adaptive operator selection mechanism specifically affects the performance of the BVNS. [24] examined the identification of vulnerabilities within a public transport network. To address this problem, they developed a multi-objective adaptive VNS method. The proposed VNS approach employed a reduced version of the BVNS by integrating both shaking and improvement phases. Specifically, a score-based probabilistic formula utilizing several dominance rules was used to adaptively select the most promising search operator in each iteration. The selected operator could either be a shaking or an improvement search operator, effectively constituting a general single-adaptive mechanism. The solution method was validated against an analytical brute force method. However, no computational analysis was conducted to examine the effects of the adaptive mechanism on the performance of the multi-objective VNS.

[37] proposed a double-adaptive GVNS heuristic algorithm to address the routing and scheduling problem of unmanned electric vehicles. They developed weight-based probabilistic mechanisms that utilize the roulette wheel rule for selecting the most promising search operators or sequences of search operators during both the improvement and shaking phases of the GVNS algorithm. The impact of these adaptive mechanisms was evaluated by comparing the double-adaptive GVNS with the conventional GVNS, an adaptive large neighborhood search, and a random GVNS. Both comparative and statistical analyses demonstrated the enhanced efficiency of the double-adaptive GVNS, primarily attributed to the adaptive mechanisms employed. Furthermore, the authors conducted additional computational comparisons to further validate the efficiency of the developed method. [52] developed a three-phase hybrid heuristic method to solve the time-dependent joint location and dispatching problem, which involves multiple types of ambulances and categories of patients. In the third phase of the solution method, an adaptive BVNS is employed. The authors explored two distinct adaptive approaches, one

utilizing a score-based probabilistic formula and the other employing an improvement frequency-based probabilistic formula. A comparative analysis between the adaptive BVNS and its conventional counterpart was conducted to evaluate the impact of the adaptive mechanisms on the heuristic method's performance. Additionally, several computational analyses were carried out to assess the efficiency of the overall hybrid solution approach. The results indicated that the BVNS variants incorporating adaptive mechanisms achieved better outcomes and, notably, required less computational time despite their enhanced performance.

[61] investigated an election logistics problem considering multiple time periods and time-dependent profits. They developed a hyper-heuristic method based on an adaptive BVNS approach. The adaptive BVNS utilized a score-based formula that takes into account the frequency of improvements achieved by each local search operator to calculate their selection probabilities. Using these calculated probabilities, the overall solution method proceeded with a granular selection of search operators. Several comparative analyses were conducted to evaluate the efficiency of the proposed solution approach. However, the effect of the adaptive mechanism on the performance of the BVNS was not examined. [10] examined a home healthcare routing problem involving heterogeneous electric vehicles, considering synchronization constraints related to the working hours of the nurses. They proposed a hybrid heuristic approach that integrates a Greedy Random Adaptive Search Procedure for initialization with an adaptive BVNS for improving the initial solution. The adaptive BVNS employs a weight-based probabilistic formula to select the most promising local search operator during the algorithm's intensification phase. The authors conducted a comprehensive analysis of the effects of the adaptive mechanism on the performance of the BVNS. The reported results indicate that the adaptive mechanism significantly enhances the performance of the VNS method.

[34] proposed a self-adaptive GVNS to efficiently solve the parallel machine scheduling problem, considering unrelated servers. They developed an Extreme Learning Machine algorithm as the core component of the adaptive mechanism, which dynamically adapts both the most suitable VND method and the most promising order of local search operators during the improvement phase. Extensive computational investigations were conducted to examine the effects of the adaptive mechanism on the performance of the GVNS. The results revealed a significant enhancement in the performance of the GVNS due to the adaptive mechanism. [71] proposed a two-stage adaptive BVNS for efficiently solving the dynamic vehicle routing problem. The first stage of the heuristic method functions as the learning phase, focusing on identifying the importance of each local search operator. This stage is further enhanced by incorporating guided diversification through the application of a large neighborhood search method. In the second stage, the generated knowledge is utilized to intelligently select a smaller number of local search operators, which are then applied based on the Kth best improvement search strategy. The authors conducted extensive computational analyses to provide valuable managerial insights. However, the impact of the adaptive mechanism on the performance of the BVNS was not investigated.

[29] investigated a practical variant of the traveling salesman problem that incorporates fuel consumption modeled through the comprehensive modal emissions model, as well as refueling decisions. The new combinatorial optimization problem was mathematically formulated as a mixed integer linear programming model, and a two-stage double adaptive GVNS was developed to solve large problem instances. The authors adopted the double-adaptive GVNS [28] and enhanced its performance by considering alternative local search operators. While the prior examination had already established the benefits of the double-adaptive mechanisms within the GVNS, the authors did not replicate this computational analysis. Instead, extensive computational experiments were conducted to examine the contribution of the alternative search operators. [42] examined a novel variant of the truck-drone delivery problem, incorporating considerations for multiple visits and multiple launch and retrieval locations. The authors proposed an adaptive BVNS heuristic algorithm featuring several local search operators. Given the large number of search operators, an initial clustering was applied based on various characteristics, such as the level of change mechanism (i.e., intra or inter-change moves) and the affected vehicles (i.e., truck, drone, or both). Consequently, the heuristic method was divided into two main stages, a learning stage and a primary improvement stage. The primary concept is to leverage the knowledge generated during the learning stage to reduce the number of search operators applied in the main improvement stage. To this end, a score-based probabilistic procedure was designed. This procedure calculates the selection probability of each search operator and the cumulative probability of each cluster of operators, ultimately facilitating the selection of the most promising combination. Extensive comparative computational analyses were conducted to evaluate both the effect of the developed adaptive mechanism on the performance of the proposed BVNS and its efficiency relative to other metaheuristic approaches.

[79] studied the satellite range scheduling problem and proposed a hybridization of an adaptive GVNS with a Tabu List and a Metropolis rule. The adaptiveness in the GVNS was integrated into its shaking phase through a score-based probabilistic formula, which was updated using historical performance data of each shaking operator. Additionally, focusing on non-standard VNS features, the authors developed an adaptive mechanism for the dynamic configuration of the temperature parameter used in the Metropolis rule. Extensive comparative analyses were conducted to assess the performance of the proposed solution method against alternative well-established heuristic approaches, including a corresponding conventional VNS method. The comparisons highlighted the statistically significant contributions of both the adaptive mechanisms and the inclusion of the Tabu List method to the enhanced performance of the proposed hybrid approach.

4. Conclusions

This review provides a comprehensive analysis of recent advances in adaptive VNS methods for solving complex optimization problems. The analysis reveals that most research has adopted probabilistic-based learning strategies, akin to those used in adaptive large neighborhood search. Notably, the enhancement of the intensification phase of VNS methods has been the primary focus of many studies, with

BVNS being the most commonly selected variant for adaptation through adaptive mechanisms.

Despite the significant progress in the development and refinement of adaptive VNS techniques, our review identifies a critical gap in the exploration of the impact of adaptive mechanisms on the overall performance of these methods. There remains a limited understanding of how these mechanisms contribute to the efficiency and effectiveness of VNS algorithms. Addressing this gap requires a systematic investigation into the effects of various adaptive strategies on VNS performance metrics. Also, several other intelligent VNS methods nowadays are based on modern machine learning techniques [26]. Thus, it is also important to study their adaptiveness and compare their performance against other simpler learning strategies.

Moreover, while BVNS has dominated the research focus, it is essential to extend investigations to alternative VNS variants, such as GVNS and SVNS. Exploring these variants can offer a more comprehensive understanding of the adaptability and versatility of VNS methods. Additionally, the current emphasis on improving individual phases, particularly the improvement phase, should be broadened. Future research should aim to develop holistic adaptive mechanisms that concurrently address the improvement and shaking phases and incorporate parameters reconfiguration. This integrated approach could lead to more balanced and effective optimization strategies.

By pursuing these research directions, the scientific community can significantly enhance the understanding and application of adaptive VNS methods, thereby paving the way for more robust and efficient optimization solutions. These efforts will contribute to the evolution of adaptive VNS into a more mature and versatile optimization tool, capable of addressing a wide array of complex optimization problems across various domains.

It is important to note that the methodology employed in this review involved a four-step process, guided by specific research questions and search criteria. In our choice to focus on the specific terminology of adaptive VNS, rather than terms like intelligent or smart VNS, we aimed to align with standard optimization terminology. As a result, certain limitations are acknowledged, such as the exclusion of studies that utilize these alternative terminologies to describe enhancements in VNS algorithms through learning mechanisms for dynamic reconfiguration. These limitations suggest areas for further investigation to ensure a more comprehensive understanding of the field.

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