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MATHEMATICAL MODELING IN MUSIC COMPOSITION: STRATEGIES, INSIGHTS & PERSPECTIVES

Abstract. Mathematical modeling in music composition is a powerful tool for mathematical reflection as well as for enriching artistic inspiration for composers. In this paper, we analyze the concept of a structural space for music composition, transformed to a multidimensional space of musical parameters. This approach facilitates the construction of appropriate mappings (mainly homeomorphic) from numerical sets or topological spaces into spaces of musical parameters, which may serve as processes of elaborating musical material and texture. Four case studies of music compositions are featured, in order to shed some light into the practicalities of this approach.

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Contents

1. Introduction

The problem of introducing a structural space in music composition has significantly changed during the 20$^{\text{th}}$ century, when standard forms – already violated previously – were gradually put aside, thus leaving to the composer the structural decisions.

After 1950, the problem of form has been in the epicenter of composers' research, resulting to several attitudes, examples of which are: the fusion of form and content (found in many techniques, from total serialism to spectral forms), mathematical models, open forms with aleatoric elements, etc. In all cases, the vivid concern of composers about structural problems in music composition has been emphasized.

In this paper, we present indicative strategies implemented in the recent history of mathematical modeling in music composition, proposing mathematical tools, concepts and relays to specific situations in building musical forms. The liturgical potential of this approach is presented through case studies, from Iannis Xenakis's - the pioneer of mathematical modeling in music composition - works, as well as from the author's own music creations.

2. Methodological background

2.1. Mathematical reflection and creative approaches. The power and significance of mathematical reflection is of great importance, like a fueling material for mathematical research to grow and flourish. The very essence of mathematical research has always been a creative process, with a great number of characteristics being in common with artistic creation. As Carl Friedrich Gauss (1777–1855) famously stated: “The lack of mathematical reflection is never more blatant than in the excess of precision introduced in numerical calculations”.

But what is mathematical reflection indeed? It can be described as an effort aiming to a deeper understanding of a mathematical object or concept. Moreover, it could be enhanced by a creative approach to the potentiality of the object or concept, by understanding properties, relations, implications and perspectives.

To this end, applications of mathematics in art can play a significant role and contribute to a deeper understanding of mathematical concepts, by offering artistic “materializations” that can objectify abstract ideas and shed light to their potentialities. At the same time, mathematics can benefit the arts by introducing advanced structural ideas into art works, in other words enrich artistic inspiration.

2.2. The structural space of music composition. As mentioned in the introduction, the structural space of music composition has significantly changed in the 20$^{\text{th}}$ century. Before the 20$^{\text{th}}$ century, Form, Motifs, Harmony, Counterpoint, Narrative were the basic key words about music structure. In the course of the 20$^{\text{th}}$ century and still to this day, major changes and new approaches have been proposed, generally giving way to new keywords, mainly Texture and Processes.

But what did form consist of? It mainly consisted of sections, arranged in musical time in a way to emphasize periodicity, create symmetries and evoke musical (mostly melodic) memory. These sections were articulated in terms of melodic themes and harmonic phenomena, exploiting motifs and their derivations (which by the way can be interpreted as geometric relations). How is this structural space modified from the 20$^{\text{th}}$ century onward? Texture handling often replaces the old techniques, while Processes emerge to introduce a generalization of motivic elaboration and expand it to the rest of the musical material.

However, mathematical modeling in music composition has produced possibilities that can be broader and more universal, enabling the composers to step further than the intuitive handling of texture: by introducing the concept of multidimensional spaces of musical parameters, it is possible to create a deeper insight concerning the functions of texture and efficiently organize its transformations. These musical variables may be extended to all lengths and creative visions, starting from the typical variables like pitch, durations and intensity and expand to custom music variables, which can be even complex, hybrid and all sorts of invented parameters suitable to the creator's imagination (see Fig. 1).

As for processes - the dynamical structuring tool - there have been proposed several kinds of solutions, from graphs, functions and algebraic formulae to algorithms - many of these have also been broadly experimented with in the field of electroacoustic music (see Fig. 2).

2.3. Mappings: the core of mathematical modeling in music composition. The concept of multidimensional spaces of musical parameters offers a rich potential for introducing mathematical models into the designing of the structural space in a music composition. With this strategy, the applications of, e.g., algebraic formulae

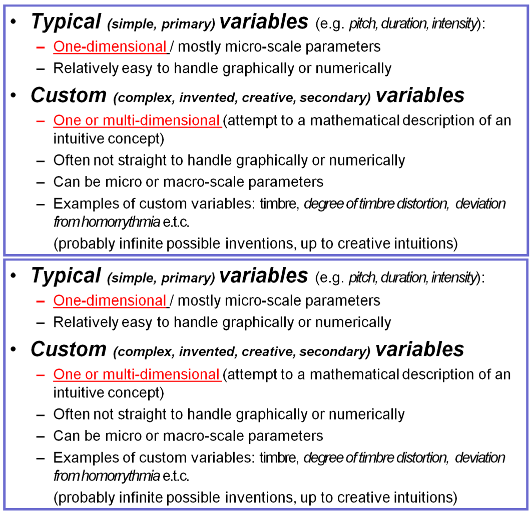

FIGURE 1. Typical and Custom Music Variables.

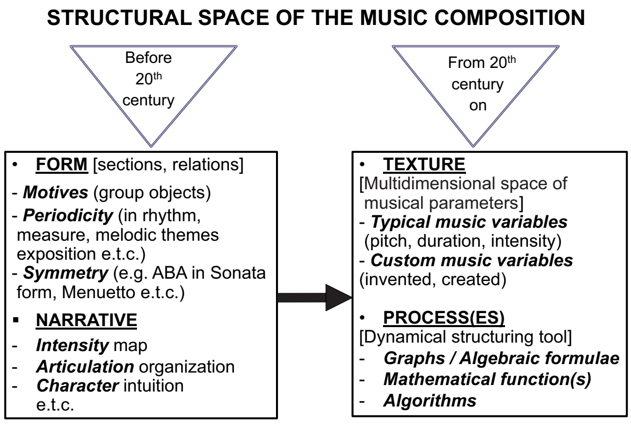

FIGURE 2. The Structural Space of Music Composition.

as generators of elaboration processes can become straightforward to practice. To that end, one more step is required: the creation of mappings from numerical sets or even topological spaces to the musical parameters multidimensional space.

In order to create appropriate mappings, one has to consider which category(ies) of properties have to be prioritized. Homeomorphisms (i.e. one to one, continuous and reversible to continuous) mappings are of key importance and handy in most cases, because they preserve sequences, convergences and topological properties. Moreover, isomorphisms can be of interest, as they preserve order; isometries as well, due to the fact that they preserve metric. The specific choices in defining the appropriate mapping greatly influence the resulting composition. A vast deposit of creative possibilities is available and each composer can act with a great freedom in this terrain, letting his/her imagination expand to all directions.

3. Case studies

3.1. I. Xenakis: Metastaseis. Iannis Xenakis's "Metastaseis" [1] for symphony orchestra makes use of a pitch - time graphic representation (see Fig. 3), resulting to a large scale glissando texture (see Fig. 4). He introduces a complex pitch manipulation in a continuous way, considering also independently varying velocities in these combined moves.

3.2. F. Kosona: A. T. "A. T." for Brass Quintet is based on an application of "Affine Transformations" for triple vectors (see Fig. 5 for a number of aspects of vector transformations).

The vector coordinates handled with the mathematical model stand for:

- *density slope*: a complex custom variable, controlling the density envelope of motivic cells (static density, augmenting density / accelerating cells and vice versa).
- *Bouncing interval*: a variable influencing the magnitude of intervals used in every note - cell.
- *Timbre*: in terms of distortion (by the use of extended techniques).

A few examples from the resulting 3-dimensional cells can be observed in Fig. 6.

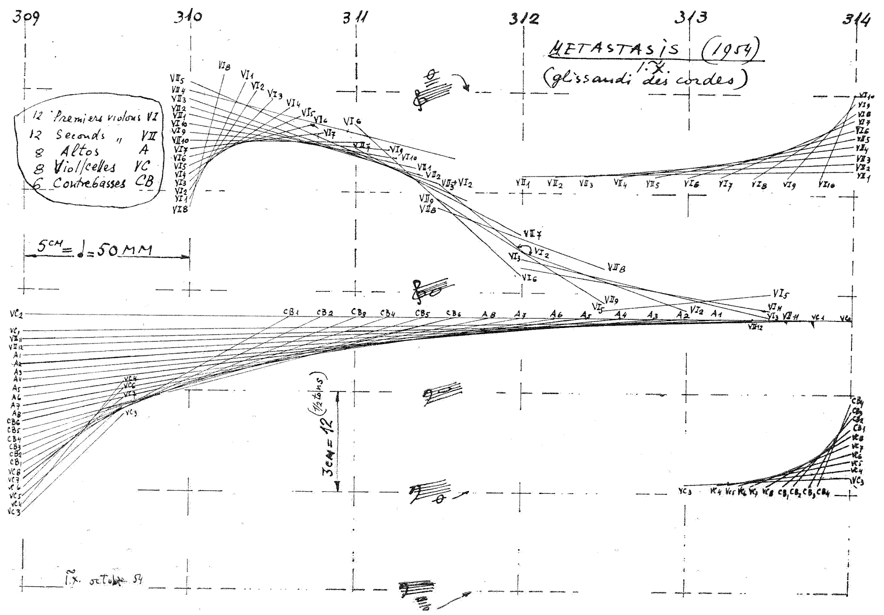

FIGURE 3. Sketch of the beginning of "Metastaseis" [1], also used as part of the design of Philips Pavilion.

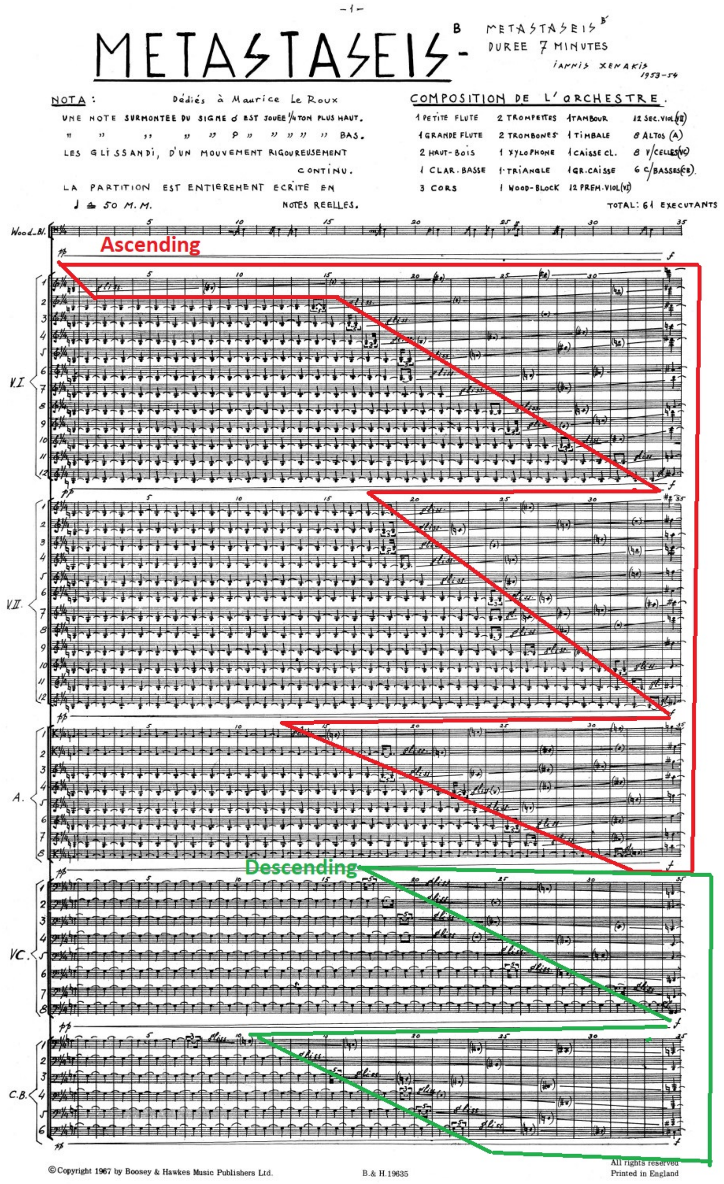

FIGURE 4. First page of the score of "Metastaseis".

3.3. I. Xenakis: Mycenae Alpha. In his electroacoustic work "Mycenae Alpha", created with UPIC, Xenakis is applying an algorithmic process on a 4-dimensional space of music parameters: pitch, time, intensity and timbre.

The work is based on a mapping transferring algorithmically a series of sketches with interesting allegories (see Fig. 7) to sound.

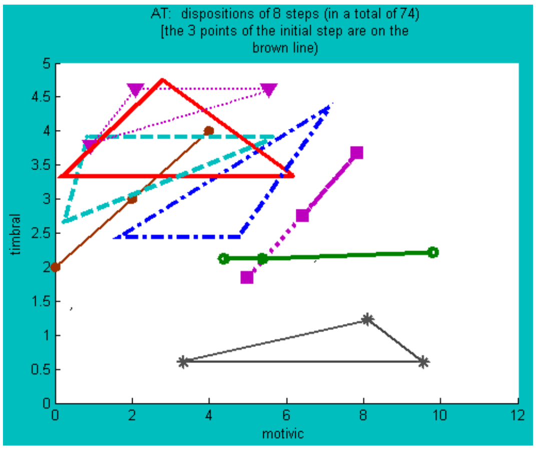

FIGURE 5. Vector Transformations preview.

FIGURE 6. Examples of cell manipulation.

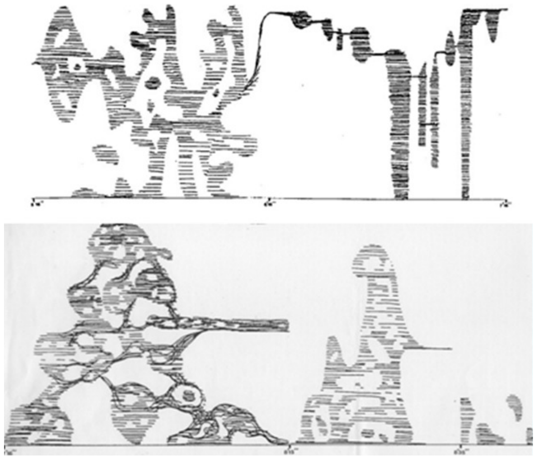

FIGURE 7. Sketches from Mycenae Alpha.

3.4. F. Kosona: Diathlassis. The work "Diathlassis" (the greek word for 'refraction') for flute solo was composed in 2009 for the flutist Katrin Zenz [2], based on the cusp catastrophe model. In this work, the cusp catastrophe model was adopted. The mathematical formula that decribes the catastrophic manifold of the cusp model is:

$$t^3 + Y^* t + X = 0$$

where X and Y are the control variables and T is the state variable. This manifold is a folded surface. The fold projection to the (X, Y) – plane indicates the area where more than one – and up to three – equilibrium states are possible, i.e. the bifurcation set of the cusp catastrophy model (see Fig. 8).

The work is built in a 4-dimensional space (X, Y, t, t^*) [3].

The variable t^* is a triggering variable.

The state variable t of the cusp model is mapped to the musical parameter of pitch. This results in a melodic discontinuousness, correspondingly to the measure of the catastrophic leap calculated in every case. The first control variable X is associated with the rhythmic structure of the piece, representing a factor of rhythmic fragmentation. The second control variable Y is mapped to the timbre, which is considered to evolute continuously from pure air to normal flutistic sound, passing through all the intermendiate stages of aeolian sounds and from normal sound to an amalgam of normal sound with singing notes, using several stages of dynamical balance between the two combined sounds.

Melodic lines are refracted. An example of such manipulation i.e. persisting on the 'obstacle', using repeated notes, bending notes, fluttertongue, until finally managing to get to the other side of the fold is given in Fig. 7.

Another example which refers to the event of forcing on the 'dead end', until the sound dissolves to multiphonics, through which is turned to the passage to the other side of the fold, is depicted in Fig. 10.

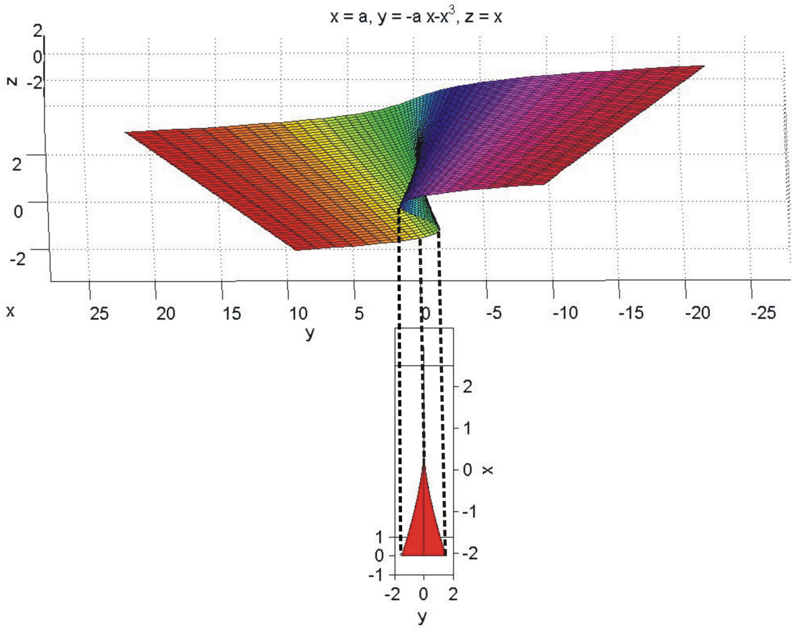

FIGURE 8. The cusp catastrophe manifold and the projection of the cusp to the (X, Y) - plane.

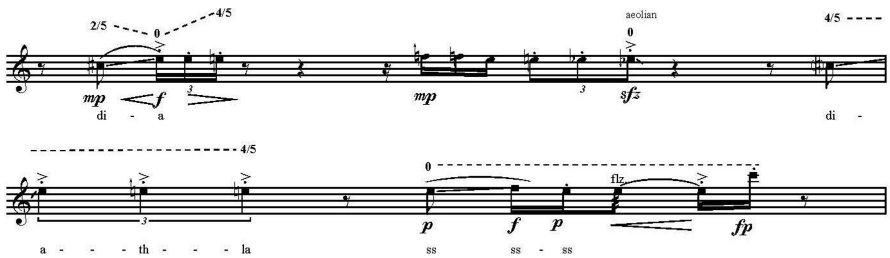

FIGURE 9. Persisting on an obstacle, until the occurrence of the melody forced passage through the discontinuity.

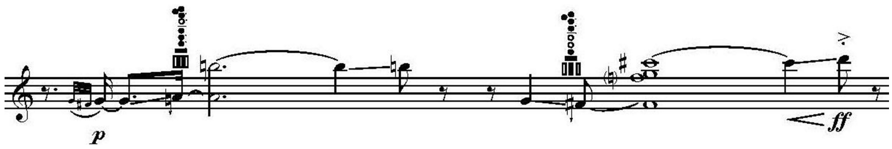

FIGURE 10. Example of forcing the discontinuity through multiphonic sounds.

4. Conclusion

Mathematical modelling in music composition is a powerful method, which can unlock for composers a vast potential for expanding imagination and inspiration. By no means can this analysis be exhaustive of all aspects of practicing this method, but hopefully it can be of assistance, in terms of facilitating access to this practice and create more insight on the underlying potential for composers. It may also serve as a tool for mathematical reflection, benefitting art and science both ways.

References

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[2] K. Zenz, *Flute Recital (Greek Flute Music of the 20th and 21st Centuries)*, Naxos, Greek Classics, Cat. No: 8.572369, 2010, compact disc.

[3] F. Kosona, *"Diathlassis", for flute solo: a composition based on an application of the mathematical model of cusp catastrophe*, in: V. Munteanu, R. Raducanu, G. Dutica, A. Croitoru, V. E. Balas (eds.), AMTA'10: Proceedings of the 11$^{\mathrm{th}}$ WSEAS international conference on Acoustics & music: theory & applications, 182–185, World Scientific and Engineering Academy and Society (WSEAS), Stevens Point, Wisconsin, USA, 2010.