

Saša Popović

**DE INSTITUTIONE MUSICA:
BOETHIUS' ANCIENT SOURCES
AND RECEPTION HISTORY**

Abstract. The present paper contains an exposition of Boethius' treatise *De institutione musica* which is unanimously considered one of the cornerstones of Western musical theory. In the introductory Section 1, we first look into Boethius' ambitious philologico-philosophical project of translating and commentating on Plato's and Aristotle's complete works, and providing a conciliatory synthesis thereof. In this context, we pay special attention to Boethius' authentic method of translating and compiling Greek sources. The introduction closes with an outline of the basic tenets of Pythagorean music theory. Our primary concern in the larger part of the remainder of the paper is the historically important and perplexing question about Boethius' ancient sources for *De institutione musica*. Section 2 opens with preliminary remarks about the sources for Boethius' treatise *De institutione arithmetica* which contains the mathematical basis of the theory from *De institutione musica*. After detecting a specific pattern of citing and referring to other authors in Boethius' texts, we discuss the most probable source for the fifth book of Boethius' musical treatise. Following an analysis of Boethius' original contributions in Section 3 (i.e. the *quadrivium*, the division of different types of music, and Boethius' conception of the "true musician"), in Sections 4 and 5 we continue discussing the possible sources for Books 1–3

and Book 4 of *De institutione musica*, respectively. In the concluding Section 6, we summarise the main insights from the preceding sections, and provide a basic outline of the extensive and multifaceted reception and transmission history of Boethius' treatise on the mathematical foundations of music.

Mathematics Subject Classification (2020): Primary: 00A65, 01A20; Secondary: 01A75

Keywords: Boethius, mathematical foundations of music, Pythagorean quadrivium, Quellenforschung, Textgeschichte, Rezeptionsgeschichte

University of Rijeka Faculty of Humanities and Social Sciences, Centre for
Logic and Decision Theory, Rijeka, Croatia

sasa.popovic@f.bg.ac.rs

ORCID: 0000-0002-8931-8730

DOI: https://doi.org/10.18485/mi_sanu_zr.2024.29.21.ch9

CONTENTS

1. Introduction: Boethius' project	127
2. Boethius' Ancient Sources 1: DIA, DIM, and the Quellenfrage concerning the fifth book of DIM	133
3. Boethius' original contributions: quadrivium – divisio mathematicae (DIA, Proemium), musicae genera (DIM 1.2), quid sit musicus (DIM 1.34)	136
4. Boethius' ancient sources 2: the Quellenfrage concerning the books 1–3 of DIM	140
5. Boethius' ancient sources 3: the “much-maligned” fourth book of DIM	142
6. Conclusion: a note on the reception history of DIM	145
Acknowledgement	147
A general note on abbreviations and some of the references	147
Notes	147
References	152

1. Introduction: Boethius' project

Historians of philosophy agree that Boethius is undoubtedly one the most important intermediaries between the philosophy of late antiquity (i.e. the Hellenistic period) and early medieval philosophy. However, this very fact makes Boethius difficult to precisely situate within the history of philosophy [38, p.11][39]. Is Boethius primarily an ancient or a medieval thinker, is he maybe both, or is he in fact neither of the two? To an extent, this question is connected with the purely chronological issue of dating the “end” of the ancient period, and the “beginning” of the medieval one. One of the most influential historians of medieval philosophy, Étienne Gilson, in *La philosophie au Moyen Age* uses the adjective “medieval” to designate the philosophical doctrines which were developed between the 9th and the 14th century AD [29, p. 3]. This criterion automatically excludes Boethius from the corpus of medieval philosophy¹, and Gilson himself places Boethius alongside other pre-ninth century thinkers in the pre-history of medieval philosophy, i.e. the patristic era which begins with the early Church Fathers and the first contacts between Hellenism and Christianity in 2nd century AD. As we shall see in what follows, this pre-history is in a sense essential because it provided the framework of problems and topics to the medieval Christian philosophers in the West, determining thus the direction in which medieval philosophy developed during the centuries to come,

at least up to the crucial point in the 12th century when classical Greek philosophical and scientific treatises were translated from Arab into Latin, after which they began to rapidly circulate and to drastically change the landscape of the Middle Ages. Especially important in this regard was the rediscovery of the greater part of the Aristotelian corpus devoted to natural philosophy. Up until this moment, medieval thinkers based their understanding of Aristotle exclusively on the certain parts of the *Organon* (ὄργανον), a collection of works devoted mainly to logic, methodology and dialectics (*Categories*, *On interpretation*, *Prior* and *Posterior Analytics*, *Topics*, and *On Sophistical Refutations*). All of these were translated into Latin by Boethius, who may rightfully be considered the founder of medieval logic.² Generally speaking, of all the early Christian thinkers, Boethius was the one who immensely and decisively contributed to the rehabilitation and transmission of the classical tradition, belonging, as Marenbon aptly notes [38, p. 11], to a developmental line going through the Neo-Platonists (Plotinus, Porphyry, and Proclus) directly all the way to Aristotle and Plato.

This is best exemplified by Boethius' ambitiously conceived yet unfinished project announced in the preface to the second book of his second commentary on Aristotle's *On interpretation*, written c. 516 [PL 64, 433]:

“Ego omne Aristotelis opus quodcumque in manus venerit, in Romanum stylum vertens, eorum omnium commenta Latina oratione perscribam, ut si quid ex logicae artis subtilitate, et ex moralis gravitate peritiae, et ex naturalis acumine veritatis ab Aristotele conscriptum est, id omne ordinatum transferam, atque id quodam lumine commentationis illustrem, omnesque Platonis dialogos vertendo, vel etiam commentando in Latinam redigam formam. His peractis non equidem contempserim Aristotelis Platonisque sententias, in unam quodammodo revocare concordiam, et in his eos non ut plerique dissentire in omnibus, sed in plerisque quae sunt in philosophia maxime consentire demonstrarem, haec si vita otiumque supererit, cum multa operis huius utilitate, nec non etiam laude contenderim, qua in re faveant oportet, quos nulla coquit invidia.”

As we may see, Boethius intended to translate all of Aristotle's works in Latin so as to render them in Roman style (*in Romanum stylum vertens*), and to write down in Latin and thus preserve every available ancient commentary.³ Furthermore, he also intended to translate and comment upon all of the dialogues of Plato with the aim of producing — in quite a Neo-Platonist manner [30, p. 132] — a unique *Platonic–Aristotelian* synthesis and an accompanying *extensive conciliatory commentary*. The purpose of the latter would be “to harmonise Plato's and Aristotle's sentences to a certain degree, and to show that, contrary to what most other [interpreters] think, it is not the case that [Plato and Aristotle] do not agree about anything but, rather, that they maximally agree (*maxime consentire*) about most philosophical issues”. It is plausible to assume that Boethius could have been led to such an idea by his study of Porphyry whose *Eisagoge* he translated and twice commented, and who, unlike his teacher Plotinus, did not believe that Aristotle's

logic contradicts Plato's metaphysics. Apart from the Neo-Platonists, principal sources for Boethius' commentaries were primarily of Alexandrian provenance which is yet another confirmation that he was an accomplished Hellenist [32, p. 171]. This should not come as a surprise when we take into account the fact of Boethius' patrician background and privileged social status which allowed him to master the Greek language already in his youth.⁴

As Bower noticed, reading, translating, and commentating constituted an integral tripartite process by means of which Boethius managed to appropriate Greek scientific and philosophical achievements not only for Roman but also, more generally, for medieval culture, science, and philosophy. His early mathematical works alongside the logical treatises "represent one of the most notable projects in intellectual history of preserving and transmitting a corpus of knowledge from one culture to another", from classical Antiquity to the Middle Ages [5, pp. xix–xx]. Speaking of Boethius' mathematical works which are the main focus of our enquiry in this paper, it should be noted that they are similar to his later logical treatises (see n. 2) inasmuch as they too may be seen primarily as *handbooks*, with no special claims to originality, which provide us with an insight in Latin to what Boethius was reading in Greek. This holds for both of the remaining mathematical handbooks of Boethius' *opera mathematica*⁵, namely *De institutione arithmetica* (*DIA*) and *De institutione musica* (*DIM*)⁶. These are Boethius' earliest publications.⁷ One important thing to note when approaching a text such as *DIM* (and this is connected with Boethius' broader critical project of translating-cum-interpreting) is Boethius' specific attitude to translation. In the preface to *DIA*, Boethius writes the following [PL 63, 1080]:

“At non alterius obnoxius institutis artissima memet ipse translationis lege constringo, sed paululum liberius evagatus alieno itineri, non vestigiis, insisto. Nam et ea, quae de numeris a Nicomacho diffusius disputata sunt, moderata brevitae collegi et quae transcurra velocius angustiore intellegentiae praestabant aditum mediocri adiectione reseravi, ut aliquando ad evidentiam rerum nostris etiam formulis ac descriptionibus uteremur”.⁸

Boethius admits he does not slavishly stick to the principle *sola scriptura* when he translates, but, rather, that the degree of literalness in translating is context-dependent, i.e. for the sake of facilitating comprehension of the original text, the translator is permitted to depart from it by making certain emendations, cuts or add-ons, including the insertion of diagrams, explanatory notes or examples. In other words, instead of translating the sourcebooks for *DIM* and *DIA ad litteram*, Boethius produces a redaction *in stylum Romanum*, much like in the case of Aristotle's works, so as to obtain the most palatable and user-friendly version of the text. It is also important to note that Boethius much more frequently expands upon and supplements his sources than he condenses them, or makes cuts [17, pp. 132–133]. By a careful study of Boethius' translations of the Greek logical works, we have discovered another peculiar feature of his translating technique: he oftentimes *directly incorporated* marginal notes, comments and glosses contained in the manuscripts

which he used as sourcebooks for his translations into the translated text itself, for the most part without explicitly acknowledging that he did it, nor where and how he did it [5, p. xxv, cf. especially n. 21]. There is no reason to assume that he did not proceed in much the same way whilst compiling the earlier mathematical treatises, *DIA* and *DIM*. It is clear that this places greater demands on the reader since they are required to be mindful of *traduttore* — *traditore*; in other words, we must always keep in mind the fact that we are not dealing with a mechanical translation faithful to the original but rather with something akin to a not-so-critical edition which may deviate from the original to a lesser or greater extent. Before turning to our main question concerning Boethius' sources in the next section, we should first make a general excursus concerning available evidence and the types of sources for ancient musical theory. How is it that we know anything about ancient musical theorising in the first place?

If we put aside the archaeological findings (remains of instruments, mainly wind-pipes, vase paintings and other forms of pictorial art from Athens and Magna Graecia), the most important resources at our disposal are the textual sources which may be divided into two groups, namely, (i) numerous references to music and music-making in classical literature (especially in early lyric and comic poetry, from the 8th century BC onwards), and — what is more important to our present purposes — (ii) specialised, scientific and mathematico-philosophical musical treatises of a more technical nature, from the late 5th century BC onwards.⁹ The latter ones are particularly significant because they reveal two dominant approaches or two diametrically opposed traditions or “schools of thought” within classical Greek musical theory: the older Pythagorean or, better yet, *Pythagorean-Platonic tradition*, and the slightly younger *Aristoxenian tradition* (which may also be called *Aristotelian*). For the purposes of this introduction, we shall limit ourselves to the Pythagorean theory¹⁰.

We have no early biographical or doxographical sources regarding Pythagoras and original Pythagoreanism of the 6th century BC. Even though Pythagoras' life and achievements became subject of a series of texts in later Antiquity and the Hellenistic period, it is clear that most of these documents abound with spurious and unreliable subsequent add-ons, reinterpretations, unsubstantiated misattributions, and even plain forgeries [7, 2, Ch. I, p. 28]. In the absence of reliable historical facts, even Plato and Aristotle barely mention Pythagoras by name¹¹; as a sort of *caveat ad lectorem*, Aristotle (whose treatise *On the Pythagoreans* survives in only a handful of fragments) typically uses expressions such as “the so-called Pythagoreans” (οἱ καλούμενοι Πυθαγόρειοι) or simply “the Italians” (οἱ Ἰταλικοί), and “Italian philosophy” (φιλοσοφία ἰταλική) [6, *Meta.* 985b23, 987a30, 989b29 *et passim*]. It is certain that Pythagoras never wrote anything [cf. Plut. *De Alex.* I, 4, 328a: οὐδὲ Πυθαγόρας ἔγραψεν οὐδὲν]. But, independently of this fact, we may say that the central thesis of the Pythagorean teaching¹² per which *number and ratio* (λόγος καὶ ἀριθμός) underlie everything — i.e. all physical phenomena may be explained mathematically, by means of numerical ratios — was certainly already articulated in the earliest phase of development of that teaching. Moreover, formulating such a thesis might have been motivated by a remarkable discovery attributed

to Pythagoras himself [DL VIII.11–12], namely the *discovery of the essence of musical harmony*: consonant musical intervals (συμφωνοῦντες φθόγγοι), i.e. the octave (διαπασῶν), the perfect fourth (διατεσσάρων), and the perfect fifth (διαπέντε), depend on precise numerical ratios, respectively, 2:1, 3:2, and 4:3. As Xenocrates put it, “Pythagoras discovered also that the intervals in music do not come into being apart from numbers” which is why “he set out to investigate under what conditions concordant intervals come about, and discordant ones, and everything well attuned and ill-attuned”. [Porph. *Comm.* 30.1–31.21, quoted from [7, 2, 1.1 and 9.8]. The apocryphal story about how Pythagoras came to his discovery through “divine providence” when he heard the beating of hammers against anvils in a smithy has been reproduced *ad nauseam* in the classical literature; Boethius also reports it in *DIM* 1.10. In what did Pythagoras’ supposed investigation mentioned by Xenocrates consist?

As the story goes, Pythagoras was walking by the blacksmith’s workshop and he overheard the hammers beating in perfect consonance; realising that such a remarkable coincidence may not be the outcome of mere chance, he was led to conduct a series of tests¹³ in order to find a rational explanation. He first ascertained that the pitches of the hammers depended solely on their weight and no other factor (i.e. weight is directly related to the difference in the sound made by the hammer) and, more importantly, that the weights of the hammers producing consonant intervals — the octave, the fourth, and the fifth — stood in ratios 2:1, 3:2, and 4:3; he also found that the weights of the two hammers which produced a tone between the fifth and the fourth were in 9:8 ratio. In order to confirm these findings, and to establish whether a complete theory of consonances may be formulated on the basis of his insight, Pythagoras returned home and conducted the following *experimentum crucis* [*DIM* 1.11]. Having precisely weighed the hammers, he took from the smithy metal pieces equal in weight to the hammers; he then attached these weights to four strings which were identical in every aspect (length, thickness, etc.), and suspended them from a fixed rod. By plucking the strings, two at a time, he discovered that they produced the same consonant intervals as the hammers.

From this and from the “fact” that Pythagoras managed to successfully reproduce the same experimental results with other instruments, Boethius concluded that Pythagoras has thus “invented the rule” (*itaque invenit regulam*). In his commentary of *DIM* 1.11, Bower remarks that even though the story was uncritically transmitted throughout the ancient period (from Nicomachus, through Gaudentius, Censorinus, and Iamblichus, all the way to Macrobius and Boethius), it is physically impossible that Pythagoras arrived at his conclusions by listening to the ringing of hammers [5, pp. 19–20, n. 75]. That the story was indeed wrong was proved by Galileo’s father, the composer and musical theorist Vincenzo Galilei in the 1589 polemic treatise “Discorso intorno all’opere di messer G. Zarlino”. First, there is the obvious problem: when multiple hammers of differing weights are used in blacksmithing, there can be virtually no variation in the pitch of the different ringing sounds since the anvil and *not the hammers* is what actually rings. More importantly, there is the mathematical problem: namely, as Galilei had shown, in order to produce the fundamental concords, the hammer weights would have to

stand in the following ratio: $4^2:3^2:2^2:1^2$ (and not $4:3:2:1$), and the same holds for weights from Pythagoras' crucial experiment with strings [28, 104ff].¹⁴ This makes a statement by Diogenes Laertius [DL VIII.12] that Pythagoras “discovered musical intervals on the monochord” (τόν τε κανόνα τὸν ἐκ μιᾶς χορδῆς εὐρεῖν) more probable than the fanciful and incorrect story about the hammers. It suggests that Pythagoras could have performed his acoustical tests on some string instrument, by means of manipulating string length.

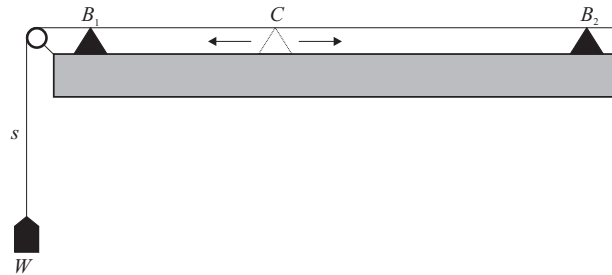


FIGURE 1. The monochord.

In Fig. 1 we may see a schematic representation of the simplest string instrument, the aforementioned *regula* of Boethius' *DIM* 1.1 which is better known as the *monochord* or the *harmonic canon* (κανὼν ἁρμονικός). Even the name of this instrument is interesting because the word *κανὼν* originally denoted a straight-edge or a measuring rod; it received the specialised musicological connotation of a single-stringed tool akin to a ruler used in experimental acoustics much later [27, p. 17].¹⁵ As its name suggests, the monochord is made up of a uniform string s stretched over fixed bridges B_1 and B_2 by means of a weight W over a wooden soundboard whose surface was fitted with a ruler which allowed one to measure the lengths of segments of s obtained by means of a movable bridge (something like a capodastro) C with which we could divide s at any point between B_1 and B_2 . However, the vast majority of historians and musicologists agree that such an instrument could not have been available to Pythagoras, nor that it is possible that he himself invented it since it was invented probably late in the 3rd century BC [14, pp. 6–7, n. 29; cf. 27, p. 11]. For this reason, Creese [27] thinks that we may speak about mathematical harmonics *before* and *after* the invention of the monochord.¹⁶ Independently of the question whether Pythagoras invented the monochord or not, it is indisputable that as early as 6th century BC it was possible to perform at least some sort of rudimentary empirical tests with, e.g. strings or wind-pipes.¹⁷ Then, there is no doubt that some one of the early Pythagoreans, if not Pythagoras himself, could have made the connection between consonant musical intervals and ratios between numbers. Even Hippasus' well-known experiment with bronze discs corroborates this (see [DK 18.12], *scholium* to [*Phaed.* 108D4], in connection with the expression Γλαύκου τέχνη, “Glaucus' skill”). Namely, Hippasus made four bronze discs of equal diameter whose thicknesses stood in the ratio $4:3:2:1$ ¹⁸, and when they were struck they produced a concord (χρουομένους δὲ τούτους ἐπιτελεῖν συμφωνίαν τινά).

Passing from discs to strings, if the string of the monochord is divided by a bridge in two parts, the tone produced by striking the shorter segment will be an octave higher than the tone produced by the entire string; in other words, the length of the entire string stands to the length of the remainder which produces the tone in the ratio 2:1. If that ratio is 3:2, then we obtain a fifth, and if it is 4:3, we obtain a fourth (Fig. 2).

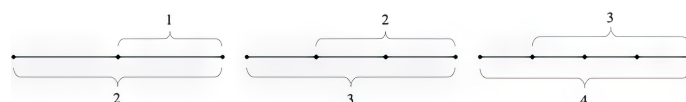


FIGURE 2. An octave, a fifth, and a fourth.

This simple yet fundamental insight paved the way for all subsequent developments which produced the entire classical musical theory whose well-rounded version is found in Boethius' treatise *De institutione musica*.

2. Boethius' Ancient Sources 1: DIA, DIM, and the Quellenfrage concerning the fifth book of DIM

When it comes to the *Fundamentals of arithmetic*, a treatise which not only predates but also provides the key theoretical underpinnings for the *Fundamentals of music*, determining what was Boethius' source is far less complicated. Namely, the very text of *DIA* and Boethius' remarks therein make it clear that his source-book was the *Introduction to Arithmetic* (Ἀριθμητικὴ εἰσαγωγή¹⁹), the *Eisagoge*, of the Neo-Pythagorean Nicomachus of Gerasa (c. 60 – c. 120 AD), which he partly translated and partly paraphrased. Nicomachus' 2nd century treatise was very influential, being used as a handbook in the Neo-Platonist schools of Athens and Alexandria. It was also widely commented by such authorities as, e.g. John Philoponus and Iamblichus, who had also written supplemental scholia concerning it. Even though it is a work of a later date and of a specific manner of composition, Nicomachus' *Eisagoge* is also valuable when it comes to attempts at reconstructing earlier Pythagorean mathematical theories, as well as for providing insights into the mathematical philosophy and the "Pythagorean revival" in the Academy immediately after Plato's death, i.e. during the period in which Speusippus and Xenocrates were at its head. It is of course necessary to emphasise that the teachings of early 5th century BC Pythagoreans (οἱ καλούμενοι Πυθαγόρειοι mentioned by Aristotle in *Meta. A5 983b23 et passim*) which we associate with Pythagoras and his immediate successors and followers should not, under any circumstance, be conflated with Neo-Pythagorean teachings. The latter evolved from the former through a series of modifications, transforming into an eclectic position due to a host of formative influences, from Plato and the Academicians, through Aristotle and the Peripatetics, all the way to Stoics and other Hellenistic schools of philosophy [17, pp. 88–90]. The *Eisagoge* was probably translated into Latin already during Nicomachus' lifetime, thanks to Apuleius of Madauros. However, even though both Cassiodorus and Isidore of Seville mention it,²⁰ Apuleius' translation was lost which means that

Boethius' *De institutione arithmetica* was the only source from which the Latin West could familiarise itself with the contents of Nicomachus' treatise and, more importantly, with Greek arithmetic in general.²¹

In his study of the sources for *DIM*, Bower detected a certain pattern in the way in which Boethius cites and refers to other authors in his mathematical treatises [23, pp. 4–5]. For instance, when it comes to *DIA*, even though the preface makes it clear that Boethius is using Nicomachus' text as a basis, in the text itself he gives almost no references to Nicomachus (he mentions him by name merely three times in the second book of *DIA*). Also, Boethius' references to other authors are typically never first-hand but rather taken over from Nicomachus' *Eisagoge*. Bower thus thinks that it is important to notice the following: Boethius cites Nicomachus only when either some aspect of the mathematical theory he is expounding, or some feature of the language he is translating is quite *specific* and *unique* to Nicomachus [23, p. 4].

Unlike the *Fundamentals of arithmetic*, when it comes to the *Fundamentals of music*, the situation gets much more complicated because it is not possible to uniquely and straightforwardly determine the Greek background of Boethius' text.²² Furthermore, the fact that scholars today have 137 MSS (in codex form) or fragments of *DIM* at their disposal, ranging from the 9th to the 15th century, actually makes work on the text itself much more difficult. Also unlike *DIA*, the text of *DIM* is not preserved *in extenso* – it abruptly cuts around the middle of Book 5 (precisely, at 5.19 with the words *generibus nusquam una*), but the titles for eleven more sections (5.20–5.30) are preserved in the contents section of *DIM*, at the beginning of the treatise. Somewhat unexpectedly, this *index capitulorum* is what actually allows us to infer beyond doubt what sourcebook was Boethius using whilst writing Book 5. Furthermore, the aforementioned pattern of citation detected by Bower is evident in the fifth book. If we look at whom Boethius is citing, we shall see that it is most commonly Ptolemy (even nine times), then Aristoxenus (five times), and the Pythagoreans (four times, with two mentions of Archytas). Of course, it goes without saying that a mere mention of an author's name does not automatically mean that Boethius was actually relying on that author's treatise as a source for what he tells us about their views. This is evidenced by the pattern of citation in *DIA* where we have seen that even when Boethius mentions other authors apart from Nicomachus, he does so on the basis of Nicomachus' text as a *secondary source*, and not by consulting primary sources.

Already the first sentence of the *proemium* of Book 5 is indicative [5.1]: “Post monochordi regularis divisionem adicienda esse arbitror ea, in quibus veteres musicae doctores sententiae diversitate discordant, habendumque de omnibus subtile iudicium”. It is clear that Boethius is here signalling a change of direction of his further enquiries, and a transition from a predominantly Pythagorean discussion concerning the division of the monochord to other issues concerning which “ancient musical teachers” (*veteres musicae doctores*, notice the use of the plural) disagree, i.e. make mutually opposed claims. Pizzani finds this sentence “pretentious” because the way in which it is formulated makes the reader believe that the treatment of topics with which Book 5 of *DIM* is concerned is going to be based upon a comprehensive and exhaustive review of the complete ancient musicological literature,

which, as it turns out, is not actually the case; if upon reading this sentence, the reader thinks that Boethius intends to investigate all the issues concerning which ancient musicologists disagree by having previously *read their original writings*, then they will be gravely mistaken because in *DIM* 5 Boethius just recounts what he has found on these topics in the first book of Ptolemy's *Harmonics* (Ἀρμονικόν)²³ [45, pp. 76, 140].

If we look at the contents of the four previous books of *DIM*, we quickly realise that apart from Pythagoras, Boethius also mentions the following “ancient musical teachers”: Hippiasus of Metapontum, Archytas of Tarentum, Plato, Nicomachus of Gerasa, Eubulides of Miletus, Aristoxenus, Philolaus, and Ptolemy; even though he does not mention him by name, Euclid implicitly makes an appearance in Sections 4.1–2, provided we accept the interpretative consensus which makes Euclid the author of the treatise *Division of the Monochord* (Κατατομή κανόνας, lat. *Sec-tio canonis*) which Boethius translates and paraphrases in Book 4 [7, p. 190]. We shall return to the complicated and controversial problem concerning the sources for the fourth book of *DIM* later in the text. Of the authors mentioned, only Aristoxenus and Ptolemy stand out in contrast to the Pythagorean-Platonic tradition in musical theorising but, unlike Aristoxenus whose views he criticises and does not take all that seriously,²⁴ Boethius takes Ptolemy's criticism of Pythagoreanism contained in the *Harmonics* (written at some point in third quarter of the 2nd century AD) quite seriously, and deems it deserving of an in-depth examination [5, p. 162, n. 1]. Moreover, by comparing the text of Ptolemy's *Harmonics* with Boethius' *Fundamentals of Music*, it quickly becomes clear that, beginning from Section 5.2, Boethius translates or paraphrases Ptolemy, cutting, condensing and reorganising the relevant sections of the first book of *Harmonics* as necessary, in the following way²⁵:

<i>DIM</i> 5.2	←	<i>Harm.</i> 1.1	<i>DIM</i> 5.13	←	<i>Harm.</i> 1.9-10
<i>DIM</i> 5.3	←	<i>Harm.</i> 1.2	<i>DIM</i> 5.14	←	<i>Harm.</i> 1.11
<i>DIM</i> 5.4	←	<i>Harm.</i> 1.3	<i>DIM</i> 5.15-16	←	<i>Harm.</i> 1.12
<i>DIM</i> 5.5-6	←	<i>Harm.</i> 1.4	<i>DIM</i> 5.17	←	<i>Harm.</i> 1.13
<i>DIM</i> 5.7	←	<i>Harm.</i> 1.5	<i>DIM</i> 5.18	←	<i>Harm.</i> 1.14
<i>DIM</i> 5.8-10	←	<i>Harm.</i> 1.6	<i>DIM</i> 5.19	←	<i>Harm.</i> 1.15
<i>DIM</i> 5.11-12	←	<i>Harm.</i> 1.7			

Furthermore, a side-by-side comparison of the *Harmonics and Fundamentals of music* additionally corroborates the citation pattern from *DIA* insofar as we see that Boethius takes over all the references to the Pythagoreans, to Archytas, and to Aristoxenus in Book 5 of *DIM* from Ptolemy. There are two striking features of the above correspondence scheme between Boethius' and Ptolemy' text which ought to be noted. First, we may see that the Section 1.8 of *Harmonics* is absent from Boethius' translation in *DIM* 5. According to Pizzani [45, p. 137], this is because Section 1.8 (and especially its second part) already makes an appearance in the last section of Book 4 of *DIM*, i.e. 4. 18. That is why in going from 5.12 to 5.13 Boethius merely recapitulates the contents of *Harm.* 1.8 so as to avoid unnecessary repetitions, and he explicitly refers to what was “previously described in the end of Book 4” (*quem quarto volumine in fine descripsimus*). Even though

he does not entirely agree with Pizzani, Bower [5, p. 161, n. 93 accompanying *DIM* 4.18] also admits that there is a strong similarity between *DIM* 4.18 and *Harm.* 1.8.

Secondly, we may ask ourselves why Boethius who, as a matter of rule, sums up sections of Ptolemy's text in a 1:1 ratio, all of a sudden decides to dedicate eleven missing sections of Book 5 to the exposition of merely two remaining sections of the first book of *Harmonics* (i.e. 1.15–16)? *Prima facie*, the most plausible explanation would be that, starting from *DIM* 5.19, Boethius is expounding Ptolemy's original and quite complicated division of the tetrachord and, if we are to judge by the preserved titles of the sections of Book 5, starting from 5.23 and following his already established *methodus compilandi*, Boethius probably also goes on to construct and describe various diagrams (see, e.g. *DIM* 4 and [5, Appendix 3]) which correspond to Ptolemy's divisions of the tetrachord as a means of facilitating comprehension of Ptolemy's main points [45, p. 153].²⁶ In connection with the Ptolemaic contents of Book 5, it should be noted that Boethius initially probably intended to treat the remaining two books of *Harmonics* as well, which means that *DIM* should have consisted of seven books, last three of which (5–7) containing the translation and a paraphrase of the remainder of Ptolemy's treatise. Bower even made a speculative assumption that Books 5–7 of *DIM* were actually written but that they have been lost in the period between Boethius' death the 9th century, much like his handbooks devoted to geometry and astronomy [5, p. xxxviii].

3. Boethius' original contributions: quadrivium – divisio mathematicae (DIA, Proemium), musicae genera (DIM 1.2), quid sit musicus (DIM 1.34)

Having already mentioned the remaining two *quadrivial* disciplines alongside arithmetic and music, i.e. geometry and astronomy, before proceeding with our discussion concerning the sources for the remaining four books of *DIM*, we should look into what exactly Boethius understood by the term *quadrivium*²⁷ which he himself coined and introduced in the *Fundamentals of arithmetic*, and how that connects with his understanding of music and musicianship. This notion became exceptionally important during the development of medieval science and philosophy, and it occupied a central place within medieval educational theory and practice (especially following the establishment of universities) which revolved around the curriculum founded upon the so-called seven Liberal Arts (*septem artes liberales*).²⁸ No analogous Greek term may be found in Nicomachus' *Introduction to Arithmetic*, apart from one place in 1.4 where there is talk of the “four ways” or “methods” (τῶν τεσσάρων τούτων μεθόδων).

Departing from Nicomachus' original text, in the *proemium* of *DIA* (concerning the division of the mathematical sciences) Boethius speaks of a certain fourfold study — the quadrivium — consisting of arithmetic, music, geometry, and astronomy, in exactly that order. Insistence upon this very sequence is important because, before Boethius, the sequence in which the quadrivial disciplines were studied, following Martianus Capella's *Liber de nuptiis Mercurii et Philologiae* (known colloquially as the *De septem disciplinis*), was the following: geometry,

arithmetic, astronomy, and music.²⁹ Contrary to Capella, in the *Eisagoge* Nicomachus insists upon the absolute precedence of arithmetic with regard to all other mathematical disciplines (calling it *κυριωτέραν αρχής τε και ρίζης και οιονεί προς τας άλλας μητρός*) in which Boethius follows him, though, unlike Boethius, Nicomachus does not strictly specify in 1.4–5 in what kind of sequence should the remaining three disciplines be ordered, not what is their mutual relationship [4, pp. 13–22]. The difference in the ordering of the disciplines may be easily explained: whereas Boethius follows Nicomachus, in *De nuptiis* Capella follows Varro's encyclopaedia *Disciplinarum libri IX* in which the sequence of the disciplines is quite simply different than in the Nicomachean treatise [47, pp. 177–78]. However, whereas Capella had *no special reason whatsoever* for prioritising geometry over arithmetic (which is why he simply copied Varro's order of exposition), Nicomachus and Boethius actually did have *specific philosophical reasons* for prioritising arithmetic.

On the one hand, we have the venerable Pythagorean tradition of *arithmetising geometry*³⁰ within which arithmetic, and especially theory of proportions (*λογιστική*³¹), was considered logically prior to geometry because geometrical problems were treated logistically, by means of the method of “reciprocal diminution”³², i.e. “the application of areas” (*παραβολή τῶν χωρίων*)³³. Furthermore, for the Pythagoreans, arithmetic also turns out to be *ontologically prior* to geometry within the process of cosmogony because its objects — numbers — are “first by nature” (*φύσει πρώτοι*; alternatively: “first things in nature”) and, furthermore, they are first among mathematical entities because geometrical objects are secondary to numbers in the order of nature [cf. 6, *Meta.* 992b13, 1080b25, 1085a7 *et passim*]. As Boethius puts it in *DIA* 1.1, if we are to remove numbers from whatever geometry is about, what, if anything, remains? Nothing, because numbers are implicit (*implicitum*) in geometry.³⁴ On the other hand, “the logical force of numbers” (*numerorum vis*) is, as we have already seen, “also prior to music”. This is confirmed by Pythagoras' (or Hippasus') insight that musical harmony essentially depends on numerical ratios (as explained in Section 1). Finally, since astronomy depends on geometry, and since the latter was shown to essentially depend on arithmetic, by transitivity of the dependence relation it follows that astronomy is also dependent on arithmetic [*DIA* 1.1]. Thus, as Nicomachus puts it (in Aristotelian terms) in the *Eisagoge* [1.4.2.], arithmetic “abolishes (*συναναφεῖται*) other [mathematical] sciences with itself, but is not abolished along with them”.

It is clear that the quality of Capella's text which, like Boethius', is primarily a certain compilation or a paraphrase, necessarily depends on the quality of the sourcebook in its base. The fact that both of Boethius' handbooks, *DIA* as well as *DIM*, have, so to say, quasi-instantly surpassed and overshadowed Capella's by then very popular treatise on the Liberal Arts may be illustrated by one quite striking and telling example. When it comes to music, Capella deviated to a large extent from the Pythagorean–Platonic tradition, and even ran counter to it insofar as he decided to completely leave out the key element of that tradition from his discussion in *De nuptiis*, namely, the investigation of the numerical ratios which form the foundation of musical harmony. In his book on Roman science, Stahl [47, p. 190] notes in passing that the last book of Capella's encyclopaedia devoted to

music “may be appropriately omitted from the present study” since it “avoids the usual mathematical approach to harmonic theory” (*sic!*).³⁵

In an entirely Pythagorean–Platonic fashion, Boethius takes the study of mathematics and of the mathematical foundations of music (i.e. harmonics) as the necessary prerequisite or a propaedeutic for the study of philosophy.³⁶ Based upon Boethius’ discussion of the sequence of mathematical disciplines in *DIA* 1.1 which closely follows Nicomachus and Section 2.3 of *DIM*, we may form the following schematic representation of the division of different kinds of magnitudes and of the disciplines corresponding to them (Fig. 3):

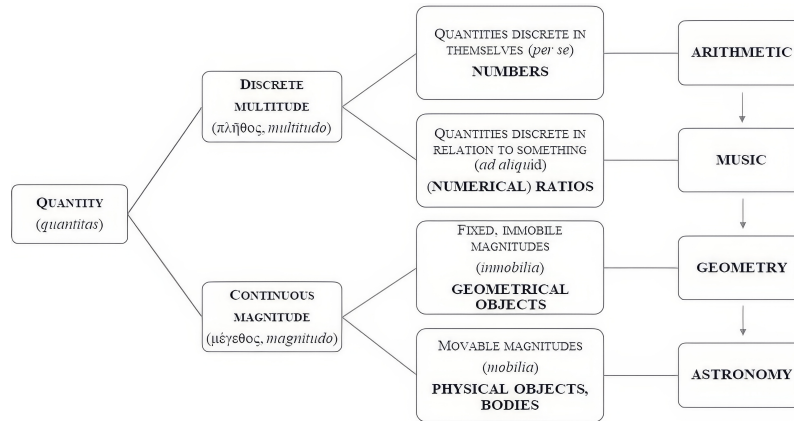


FIGURE 3. Boethius’ division of quantities and corresponding quadrivial disciplines.

It should not come as a surprise that another important original contribution of Boethius — in the not-so-original mathematical treatises — also has to do with division, this time of the kinds of music [*DIM* 1.2]. It is unsurprising because it fits perfectly with the prevailing scholastic predilection for the Peripatetic “science of division” (*scientia dividendi*), as evidenced by Boethius’ own logical treatise *De divisione*, written in approximately the same period as *DIA* and *DIM*. In addition, it could be claimed that Boethius’ threefold division of the kinds of music inspired by the Pythagorean–Platonic tradition is one of the historically most significant and influential passages of the *DIM*. We may thus distinguish the following three kinds of music:

- (i) *musica mundana* – cosmic music: harmony of celestial spheres, of seasons, and of the constitutive elements of nature cf. [18, *Tim.* 32C, 35–36; *Symp.* 188A, *Laws* 889B–C];
- (ii) *musica humana* – music which is uncovered via introspection (*quisquis in sese ipsum descendit intellegit*): “fine tuning” or tempering (*coaptatio et temperatio*) which unites reason with body, that which holds together (*coniugant*) the rational and irrational parts of the soul, as well as the different parts of the body intermingled (*quod corporis elementa permiscet*);

- (iii) *musica instrumentalis*³⁷ – music which rests in certain instruments (*in instrumentis*): depending on the type of instrument, music is generated either by tension (*intentione*) as in strings, or by breath (*spiritu*) as in winds, or by a certain percussion.

Interestingly, even though in *DIM* 1.2 Boethius promises to return to the topic of cosmic and human music in greater detail, he in fact does not seem to keep this promise anywhere in the extant text; on the contrary, as the text progresses from the first to the fourth book, Boethius occupies himself exclusively with the “music of the instruments” (*instrumentorum musica*), investigating the mathematical principles which underlie it [5, p. 10, n. 44]. The reader’s expectation that Boethius will eventually get back to an in-depth study of cosmic and human music remains unfulfilled insofar as, like we said before, text of *DIM* is not preserved in *extenso*. However, if Bower’s speculative hypothesis concerning the existence of Books 6 and 7 of *DIM* were true [cf. Section 2 above], and if we had the entire text of *DIM*, i.e. all seven books at our disposal, then we would potentially be able to find a more detailed discussion devoted to cosmic and human music, probably somewhere near the end of Book 7 which — in this hypothetical scenario — should contain the translation or a paraphrase of the concluding sections of the third book of Ptolemy’s *Harmonics*, notably Sect. 4–9 and 3.10–14 (or even 16) which thematically correspond to what Boethius would call, respectively, human and cosmic music [5, p. xxxviii].³⁸ We may also find a development or a variation of some of the elements of *DIM* 1.2 corresponding to cosmic and human music in Boethius’ literary and philosophical master-piece *De consolazione philosophiae* [*Cons.* i m. 5, 1–24; ii m. 8; iv m. 6; iii m.] [9, pp. 1–17] [26, p. 101].

Connected with Boethius’ threefold division of music is his classification of musicians which we encounter within the enquiry titled *Quid sit musicus?* [*DIM* 1.34]³⁹:

- (i) *Instrumentalists (quod instrumentis agitur)*: they are mere executants who are entirely dependent upon instruments and, as such, they are “excluded from comprehension of musical science since [...] they act as slaves. None of them makes use of reason; rather, they are totally lacking in thought”.
- (ii) *Composers of songs and poets (quod fingit carmina, poetae)*: they are led to song and music-making “by a certain natural instinct rather than by thought and reason” for which reason this class of musicians, much like the instrumentalists, is “also separated from music (*musica segregandum est*)”.
- (iii) *Musical theorists/critics (quod instrumentorum opus carmenque diiudicat)*: they have acquired the ability for judging (*iudicandi*) and are “totally grounded in reason and thought (*totum in ratione ac speculatione positum est*)” which allows them to systematically analyse and evaluate (*perpendere*) a given musical piece as a whole, from every relevant aspect (structure or composition, rhythm, style, harmonic qualities, etc.).

As is evident, for Boethius the only “true musician” (*vero musicus*) is described under the heading (iii). According to his famous definition from *DIM* 1.34, a true musician is one who has arrived at an abstract and pure science of music-making by means of rational evaluation, i.e. through *musical theory* rather than through

the actual *musical practice*.⁴⁰ In this regard we once more find in Boethius a clear representative of the Pythagorean-Platonic tendency of devaluing musical practice in favour of musical theory [26, pp. 85–87]. As Chadwick ad loc. rightly notes, part of the explanation for this tendency lies in *social* factors, i.e. a generally negative attitude among the Roman patrician elite towards professional music performers. The other more important reason is, as in the case of the quadrivium, *philosophical*, and is to be found in Plato, the Neo-Pythagoreans such as Nicomachus, or Augustine. For all of them, not the art but the abstract mathematical science of music is the proper object of intellectual enquiry.

From a historical point of view, it is interesting to note that this tradition had its critics as early as the 4th century BC, namely, Aristotle's pupil Aristoxenus of Tarentum whom we have already mentioned in Section 1, and who is most notable for the idea of equal temperament. Unlike the Pythagorean mathematical science of harmonics, Aristoxenus' musicological approach as evidenced by his *Elements of Harmonics* (Ἀρμονικὰ στοιχεῖα) represents a *par excellence* Aristotelian *natural*, i.e. *empirical science of acoustics*. Aristoxenus starts from the phenomena or data of real musical experience with the intention of describing and analysing it, as it is perceived, i.e. heard [18, p. 56]. More importantly, this raw auditory data which is grasped as musically relevant is treated in *autonomously musical terms*, mostly derived from the everyday vocabulary of the practising musicians; in other words, in his explanations and descriptions, Aristoxenus tends to avoid concepts borrowed from other sciences, notably mathematics [7, pp. 4–5]. Furthermore, even the guiding principles or rules of Aristoxenian science must be abstracted or extracted from the perceived musical sense-data and the underlying patterns into which it is organised, rather than being sought outside it, say in mathematics. Putting it somewhat metaphorically, for Aristoxenus, unlike for Boethius, the (trained musical) ear rather than mathematising reason ought to be considered as the supreme judge in music. We may now return to the question concerning sources for the first four books of *DIM*.⁴¹

4. Boethius' ancient sources 2: the Quellenfrage concerning the books 1–3 of DIM

Considering the list of ancient musicologists from Section 2 to whom Boethius refers, we can ask whether any one of the aforementioned authors may be regarded as the exclusive source of Boethius' text in the first three books of *DIM*. In *Variarum* 1.45, Cassiodorus reproduces Theodoric's letter to Boethius, probably from 507, in which we read the following: "Translationibus enim tuis Pythagoras musicus, Ptolemaeus astronomus leguntur Itali, Nicomachus arithmeticus, geometricus Euclides audiuntur Ausonii". In other words, it turns out that in Boethius' translations, the Italian peoples read, among others, "Pythagoras the musician" (*Pythagoras musicus*). However, that is hardly helpful in our attempts to uncover the sourcebook for *DIM* because Pythagoras—contrary to the tendency of later doxography and a series of *Pseudopythagorica* to ascribe most Pythagorean insights and discoveries to him—never actually wrote anything. Thus, Cassiodorus' reference, unlike the other three (Ptolemy, Nicomachus, and Euclid), is in no way indicative and should

be understood as a rhetorical means whose main purpose was to point out the style and character of DIM [45, pp. 89–90, n. 1][23, p. 6]. Namely, the content, the range of topics, the manner of discussion, and the general pattern of argumentation in the first three books points to the fact that the source (or sources) had to be of a Pythagorean provenance. If we look at Boethius' characteristic pattern of citation [cf. Section 2] in the first four books of DIM, it is clear that the most commonly mentioned author, especially in Books 1 and 2, is Nicomachus whose *Eisagoge*, as we have seen, served as the basis for *De institutione arithmetica* (this is substantiated by the quote from Cassiodorus).⁴²

It was already said that we owe two of the most authoritative and detailed studies devoted to Boethius' ancient sources for DIM to Ubaldo Pizzani and Calvin Bower [see n. 20]. Even though their respective approaches to Boethius' text, as well as their understandings of Boethius' project stand in stark contrast, especially in connection with the difficult and controversial question concerning the composition of the fourth book of DIM, their general conclusions about the sources for the first three books of DIM tend to mostly agree. First of all, they both subscribe (as do I) to the general scholarly consensus based upon Miekley's 1898 hypothesis per which the two primary sources for DIM are Nicomachus and Ptolemy [41]. Having previously shown that Book 5 of DIM is indeed based upon Ptolemy's Ἀρμονικόν, as well as that the sole source for DIA is Nicomachus' Ἀριθμητικὴ εἰσαγωγή [Section 2], we may now turn to the much more complex problem of ascertaining in exactly what kind of relation do Boethius' and Nicomachus' musicological treatises stand [45, p. 10]. The only extant writing on music attributed to Nicomachus is the so-called *Manual of Harmonics* (Ἀρμονικὸν ἐγχειρίδιον), a brief introductory text or a sort of a general summary, consisting of merely twelve sections, written in an informal style, omitting most technicalities and mathematical details, and presenting what may be considered the core tenets of Pythagorean musical theory.⁴³ Most of Boethius' definitions of basic musical concepts (e.g. “sound”, “interval”, “consonance”, etc.) are reproduced verbatim from Nicomachus' manual.⁴⁴ However, as Bower rightly notices, most of the theories which Boethius attributes explicitly to Nicomachus in Books 1 and 2 of DIM [1.20, 1.31–32, 2.20, 2.27] are nowhere to be found, neither in the *Enchiridion*, nor in any other of Nicomachus' extant works [23, p. 7]. This automatically suggests that Boethius had in his possession another treatise of Nicomachus which got lost in the meantime. The hypothesis that some such treatise did indeed exist is corroborated by the following passage from the *Enchiridion* [MSG, p. 238, lines 6ff, *italics mine*]:

“When the gods allow, as soon as I have some leisure time and take a break from my journey, *I will compose for you a longer and more detailed introduction* (ἄκριβεστέραν εἰσαγωγὴν) *into the same topics, completed with elaborate argumentation and comprising several books* (ἐν πλείοσι βιβλίους), and I will send it to you at the earliest opportunity, wherever I hear that you and your family are residing. In order for it to be easier to follow, I will begin at the same point at which I started the instruction when I was explaining the same things to you”.

Similar promises are repeated by Nicomachus throughout the *Enchiridion* in connection with various specific points of the theory [*MSG*, pp. 242, 260–61, 264–65]. They are exceptionally important because a side-by-side comparison of these passages with the aforementioned sections of *DIM* where Boethius refers to Nicomachus leaves little doubt that Boethius indeed had the entire *Introduction to Music* (Μουσική εισαγωγή) at his disposal whilst compiling the first two books of *DIM* [45, p. 29][23, pp. 8–9].⁴⁵ We may take this as decisive evidence in support of the claim that the first two books of *DIM* are based on a single author, Nicomachus, and on his two *Introductions*.

So far so good. The situation starts to get more complicated once we get to Book 3, for several reasons. First of all, unlike *DIM* 1–2 where Boethius’ references to Nicomachus follow a certain regular pattern, nothing of the sort is available to researchers when it comes to the text of *DIM* 3; what is more, there is not even a single reference to Nicomachus in the entire Book 3, nor any precise reference to the work of any other author [45, p. 84]. However, both Pizzani (ad loc.) and Bower [23, p. 10] agree that the key to resolving the *Quellenfrage* for the third book is to look into “the very close link” (*lo strettissimo legame*) which connects it to Books 1 and 2. Especially indicative are the phrases such as “*superiore volumine demonstratum est...*” [3.1] or “*ex secundi voluminis...*” [3.12] which explicitly connect the discussions in the third book with results arrived at in the second one. Not only that, demonstrations of certain mathematical propositions in Book 3 logically depend on propositions previously established in Book 2 (e.g. 3.2 ← 2.28, 3.3 ← 2.21, etc.). In other words, a coherent reading and understanding of Book 3 depends necessarily on Books 1 and 2, and, furthermore, Book 3 rounds up and completes the major points of the theory which were merely glanced at or anticipated in Books 1 and 2 [45, p. 85][23, pp. 10–11]. So, to recapitulate, *DIM* 3 is in no way exceptional in comparison with *DIM* 1–2 in that we find in it a natural and organic continuation and elaboration of the topics introduced in *DIM* 1–2, the manner of treating these topics is much the same, and even stylistically all three texts are entirely Pythagorean. Given these facts, we may conclude that Boethius’ sourcebook for Book 3 was the same one he used for Books 1 and 2 – Nicomachus’ *Introduction to Music*.

5. Boethius’ ancient sources 3: the “much-maligned” fourth book of *DIM*

We now arrive at what is probably the most perplexing issue in all of Boethian scholarship, namely the question of sources for Book 4 of the *Fundamentals of music*, aptly called “the jungle” (*selva selveggia*) by Pizzani [45, p. 9]. Content-wise, it is the most technically demanding and difficult part of *DIM*. Part of the difficulty arises from the fact that Boethius’ possible source or sources are not evidently discernible; quite the contrary, at face value the text suggests the so-called *disparate sources hypothesis* advanced by Pizzani,⁴⁶ per which Book 4 is the only part of *DIM* in which Boethius relies upon several sources of differing provenances but, unfortunately, ends up conflating them in such a way so as to arrive at a contaminated and confused text full of “grave misunderstandings”, i.e. a truly “much-maligned section” (*sezione bistrattatissima*) [45, pp. 87–88]. If we

look back at Boethius' rather consistent application of his *methodus componendi* in Books 1, 2, 3 and 5 analysed in previous sections, what could explain such sudden deviation from it in Book 4?

Pizzani believed that the apparent confusions in Book 4 had to arise from some sort of cross-contamination of various source-books Boethius must have used in compiling it. According to him, even though Miekley's hypothesis about Greek sources for *DIM* is correct, it ought to be accepted in all and only those cases in which it is possible to ascertain unequivocally, by means of a textual analysis and a side-by-side comparison of Boethius' with extant Greek musical treatises, that some given Boethian text is based upon some given Greek original, as its translation or paraphrase [45, pp. 89–90]. However, when it comes to those parts of the Boethian corpus where it is impossible to conduct such comparative analyses, the possibility that Boethius might have used a source other than Greek — specifically a Latin one — cannot be excluded *a priori*. Having stated this caveat, Pizzani goes on to examine the actual content of Book 4. But what about Bower's interpretation? Interestingly and contrary to Pizzani, he claims that upon careful scrutiny, Book 4 may actually be perceived as a unified whole, its theory being logically related both to the preceding Nicomachean three books, as well as to the Ptolemaic Book 5 [23, p. 11]. *Prima facie*, whichever interpretation manages to render the entirety of *DIM* as coherent as possible ought to be given precedence. Let us now in turn explore and compare Pizzani's and Bower's competing interpretative hypotheses.

When it comes to the opening sections of Book 4 [4.1 and 4.2], both Pizzani and Bower agree that Boethius' text is based upon some version of the Euclidean *Sectio canonis*⁴⁷ [45, p. 88][23, p. 12]. Where they disagree is the question concerning the quality of Boethius' rendering of the Euclidean text. For Pizzani, it constitutes “a faithful, almost literal translation” (*una fedele traduzione, quasi ad verbum*), whereas Bower points out several important deviations from the original Greek text in Boethius' Latin which indicate that either Boethius was not a competent translator, or — more probably — that the deviations are actually due to some alternate source which contained of itself an already unfaithful rendering of the Euclidean text [45, pp. 105, 108][23, pp. 12–14]. Since we know for a fact that none of Boethius' extant translations of Greek logical or mathematical works contain blatant mistranslations, this lends credence to Bower's claim that the textual deviations between *DIM* 4.1–2 and the *Sectio canonis* could be the result of an attempt to render the opening of the fourth book consistent and in tune with Nicomachean Books 1–3.⁴⁸

Next point of contention between Pizzani and Bower concerns *DIM* 4.3–4. The appearance of both Greek and Latin terms for musical notation in these sections constituted for Pizzani an “evident proof” that Boethius must have based them upon a Latin source-book, specifically Mutianus' lost Latin translation⁴⁹ of Gaudentius' *Introduction to Harmonics* [45, pp. 89–105]. Similarly to the previous case of *DIM* 4.1–2 and the *Sectio canonis*, Pizzani claims that there is “a perfect textual correspondence” between Boethius' and Gaudentius' text (*una perfetta corrispondenza testuale fra i due autori*) [45, p. 99]. This, however, is simply not true; as

Bower claims [23, p.15], such a description by Pizzani is “a misleading exaggeration” since the passages in question [*DIM* 4.3 and *MSG*, 347, 11ff]⁵⁰, although similar, are only quite loosely connected. Ironically, Pizzani himself was fully aware of this fact yet he still believed that he could rightfully claim that the *surplus* of words and expressions in Boethius in comparison with Gaudentius does not in the slightest call into question the hypothesis per which the Boethian text is directly derived from the Gaudentian one! This shows that Pizzani’s statement that “the influence of a Latin source on the entire section stands out as an incontrovertible fact” [45, p.94] actually lacks the necessary factual, i.e. textual support.

However, even if Pizzani is wrong, the very appearance of notation in Boethius’ treatise is still perplexing because most pre-Boethian theoretical works devoted to music shunned away from notation, most notably Boethius’ confirmed source-books, i.e. Nicomachus’ *Manual* (the same was quite probably true of his *Introduction to Music*) and Ptolemy’s *Harmonica* [23, pp.16–17]. On the one hand, this in itself is an indication that some other source probably might have been available to Boethius but, on the other, would not Boethius then at least refer to the author in question, in line with his pattern of citation, especially since it would have to be some musicologist not belonging to Boethius’ Pythagorean–Platonic tradition? When we look at *DIM* 4.3, we find no such reference,⁵¹ This means that the only possible conclusion regarding the question of sources for *DIM* 4.3–4 is that Boethius must have used a text which is no longer extant, more probably of Greek than of Latin origin.

We may now turn to the important sections of Book 4 devoted to the division of the monochord [*DIM* 4.5–12]. Interestingly, even though one would naturally be inclined to expect these sections to be based upon the *Sectio canonis* of Euclid (bearing in mind the very name of the treatise and the titles of the sections in question as given by the *index capitulorum*), this in fact is not the case. But why would Boethius open Book 4 with Euclid and then abruptly abandon Euclid’s text just to skip to another source which is clearly not Euclidean [45, p.108]? By comparing *DIM* 1.11 with a passage from Nicomachus’ *Enchiridion* [*MSG*, 260, 12ff], we may see that Boethius and Nicomachus alike promise to return to the division of the monochord, which they both consider as the culmination of musicological enquiries [cf. *DIM* 4.1: *ad regulae divisionem quo tota tendit intentio*]. This gives rise to the possibility that Boethius returns to Nicomachus’ *Introduction to Music* in Book 4 [23, p.19]. Such a possibility was also recognised by Pizzani [45, pp.115–121], however, unlike Bower, he came to the realisation that all relevant evidence⁵² “forces us to exclude the [possibility of a] hypothetical derivation of the Boethian *sectio* from Nicomachus”, for which reason he concluded that some unknown source must have served as the basis for *DIM* 4.5–12 [45, p.122]. Without going into the very long and complicated Bower’s attempt at a demonstration of a high degree of correspondence between Nicomachus’ and Boethius’ division of the monochord [23, pp.19–26], I would just like to add one circumstantial piece of evidence in favour of the thesis that Book 4 is also *predominantly* Nicomachean, and this is something about which Pizzani and Bower both agree, namely, the fact that section 4.13 of *DIM* is clearly taken directly from Nicomachus [45, pp.121–122][23,

pp. 26–27]. And not only that, *DIM* 4.13 also “serves to unify Book 4 with the [entirely Nicomachean] Book 1” [23, ad loc.]. Finally, as to the *Quellenfrage* concerning the closing sections of the fourth book [*DIM* 4.14–18], even though their respective arguments differ, Pizzani and Bower agree that the source in question must be a Ptolemaic one, either Ptolemy himself (sec. [45, p. 136], *Harmonics* are “the essential, if not the unique source”), or “a source which was either acquainted with Ptolemy’s theory or with the source upon which Ptolemy based his thought” [23, p. 32]. Not unexpectedly, this “Ptolemaic source” on Bower’s reading turns out to be Nicomachus himself, i.e. Nicomachus’ rendition of Ptolemy’s *Harmonics* [23, p. 38].

The discussion in this and the previous sections then leaves us with the following two alternatives concerning the source(s) of *DIM*:

	Pizzani 1965	Bower 1978
Boethius' <i>Fundamentals of Music</i> Books 1–3	Nicomachus' <i>Introduction to Music</i>	Nicomachus' <i>Introduction to Music</i>
	Euclid's <i>Sectio canonis</i> (<i>DIM</i> 4.1–2)	A version of Euclid's <i>Sectio canonis</i> as transmitted by Nicomachus in the <i>Introduction to Music</i> (<i>DIM</i> 4.1–2)
Books 4	Mutianus' Latin translation of Gaudentius' <i>Introduction to Harmonics</i> (4.3–4)	An unknown Greek source (4.3–4)
	An unknown source (4.5–12)	Nicomachus' <i>Introduction to Music</i> (4.5–13)
	Nicomachus' <i>Introduction to Music</i> (4.5–12)	A version of Ptolemy's <i>Haimonics</i> as transmitted by Nicomachus in the <i>Introduction to Music</i> (4.14–18)
Books 5	Ptolemy's <i>Harmonics</i> (4.14–18)	
	Ptolemy's <i>Harmonics</i> , Book 1	Ptolemy's <i>Harmonics</i> , Book 1

Bower’s analysis of Book 4, in contrast with Pizzani’s disparate sources hypothesis, is more in line with the general interpretative consensus based upon Miekley’s thesis. In other words, on his reading, the fourth book can be seen not only as being internally coherent but also as structurally and logically connected with the previous three books. Cross-references in Books 1–3 to the contents of Book 4, as well as the introductory statement of *DIM* 5.1 clearly point to the fact that Books 1–4 represent a unified and well-rounded theoretical discourse on instrumental music [cf. Section 3], of which Book 5 is, as we have seen in Section 2, a natural continuation and elaboration [23, pp. 39–40][5, p. xxxv]. We may now turn to the question of *Rezeptionsgeschichte*, i.e. the enduring legacy of Boethius’ musical treatise.

6. Conclusion: a note on the reception history of *DIM*

As we already mentioned in the Introduction, Boethius’ historical importance as a transmitter of classical learning can hardly be overestimated; the same holds for his role of arbiter or moderator between conflicting views expressed by classical authors, of which his ambitiously conceived yet unrealised project of conciliating Plato and Aristotle is a telling example [cf. Section 1].⁵³ It is true that he might

not have been the most original of authors [cf. however Section 3 and *Consolatio*], but we may say that his originality consists precisely in the selection of subjects to pursue, problems to explore, sources to follow and emulate, as well as in his authentic method of translating, compiling and editing the works of other authors [cf. Sections 2, 4 and 5]. Were it not for Boethius' classical background and his understanding of the importance of preserving classical tradition, we would not be able to read Nicomachus' lost *Introduction to Music* in the first four books of *DIM*. It should be underlined that Boethius' translations typically are not *ad litteram*, but rather somewhat loose, and more akin to paraphrases. These paraphrases are, as Pizzani noted [45, pp. 155-156], at the same time both faithful and unfaithful: faithful insofar as Boethius exactly follows and reproduces the order of exposition of the original text, replicating characteristic mannerisms and expressions, as well as other relevant elements of some author's style; unfaithful because of the many and frequent cuts of the original text due to condensing, which are alternated with Boethius' expansions or clarifications of certain more complicated or less palatable points, motivated by his didactical concerns.

Boethius' two extant quadrivial treatises, *De institutione arithmetica* and *De institutione musica*, alongside his logical works and commentaries became go-to schoolbooks within the Roman educational system, as well as cornerstones of the university curricula. Boethian texts were indispensable not only throughout the Middle Ages, but they were also widely used during the Renaissance, and even well into the modern period; for instance, *DIM* served as a textbook at Oxford as late as the 18th century [22, p. ii].⁵⁴ This made Boethius one of the leading, if not the most influential, schoolmaster of the liberal arts, surpassing both Capella and Cassiodorus [47, p. 199]. And not only that, contemporary authors continue to study, translate⁵⁵, and re-evaluate Boethius' works to this day for a plethora of different reasons: historical, philological, mathematical, musicological, and philosophical (see Claude Palisca's editorial preface to the English translation of *DIM* in [5]). This clearly shows that the reception and transmission history of Boethius' musical treatise is not only very long — over fifteen centuries — but, more importantly, that it is multifaceted and complex.⁵⁶

It is difficult to say with precision what happened in the initial phases of the reception history of *DIM*, in the period between Boethius' death and the Carolingian era. Digested versions of Boethius' handbooks, especially *DIA*, appeared already in the sections of the *Institutiones* of Cassiodorus⁵⁷ devoted to the quadrivium; Cassiodorus' digest was then picked up by Isidore of Seville in the *Etymologies* in the first third of the 7th century [31, pp. 153, 155–156]. First musical manuscripts in which it is possible to detect a Boethian influence and which contain references to *DIM* are the 9th century *Musica* and *Scolica enchiridis* [46, pp. 453–454]. Gerbert of Aurillac, later Pope Sylvester II, further popularised Boethius' works by means of his numerous *Epistolae* (written before ascending to papacy) during the 10th century, by basing his educational programme in Paris upon Boethian quadrivium, and by distributing copies of *DIA* and *DIM* throughout Europe. Most importantly, during this first period of transmission, Latin medieval musicology took all of its

key notions and principles, as well as the entirety of its technical terminology directly from Boethius' *DIM* [43, p. 451]. Following the establishment of universities, especially in Oxford and Paris, Boethius' treatises continued to exert direct influence on generations of musical theorists. Even though there was a slight decline of interest in the study of quadrivial disciplines following the rediscovery of Aristotle's writings on natural philosophy (primarily via Arab translations) in the 12th and 13th centuries, 15th and 16th century humanists brought Boethius back to the forefront of extensive research, *DIM* becoming one of the first musical treatises to be printed (Venice, 1491–1492) and subjected to critical editorial work (cf. Palisca's preface in [5, pp. xiii–xiv]). It should be noted that *DIM* never really attracted the attention of musical practitioners (composers or performers) which, if we look at what Boethius himself had said in the context of classifying the types of music and musicians [cf. Section 3], should not come as a surprise. His treatise was never intended to be used as a manual for the training of musicians, nor did he write it with the ambition of it ever actually being applied in practical music-making; rather, it was conceived as a theoretical study into the mathematical foundations of music.⁵⁸

Acknowledgement. The paper is based on a talk which I delivered at the Seminar for Mathematics and Music of the Mathematical Institute of the Serbian Academy of Sciences and Arts in Belgrade on 23rd January 2023. I would like to thank Professor Vesna Todorčević, the Seminar Chair, for the kind invitation. My thanks also go to the participants for a stimulating discussion, and for their helpful and insightful remarks.

A general note on abbreviations and some of the references. When referring to Migne's edition of Boethius' works, i.e. [1], I will use the standard abbreviation PL, followed by tome and page number. Next, *GMW* stands for [7], *DK* for [11], *Harm.* for [12], *DL* for [13], *MSG* for [15], and *Intr.* for [16]. Standardly, I will refer to Aristotle's and Plato's works, i.e. [6] and [18], by means of Bekker and Stephanus numberings. Finally, I will use [4] to refer to Masi's accompanying notes and comments in his translation of Boethius' *De institutione arithmetica*, [5] to refer to Bower's notes and comments in his translation of *De institutione musica*, [7] to refer to Barker's commentary in his collection and translation of Greek musical writings, [14] to refer to Solomon's remarks in his translation and commentary of Ptolemy's *Harmonics*, and [17] to refer to the comments and accompanying studies of D'Ooge et al. in their English translation of Nicomachus' *Introduction to Arithmetic*.

Notes

1. Boethius was born around 476, and executed for treason most probably in 524, which means that he was active at the turn of the 6th century. Even though accusations against him were considered false for a long time, Stahl [46, p. 198] points out that Boethius was “deeply implicated in religious and political controversies before his imprisonment”.
2. In order to illustrate the dimensions of Boethius' significance for the establishment of medieval logic, it should be said that alongside the translations of the abovementioned logical works by Aristotle, Boethius also wrote an extensive commentary of the *Categories*, as well as an improved

Latin translation (superior to that of Victorinus) followed by two commentaries of Porphyry's *Introduction* (Εἰσαγωγή) to the *Categories*. *Eisagoge* is historically important because it became the focal point of the dispute concerning the ontological status of universals. Furthermore, Boethius wrote two commentaries on Aristotle's *De interpretatione*, a treatise of great historical importance not only for temporal and modal logic, but also because of Aristotle's "Sea Battle Problem" about future contingents, and the discussion concerning the status of logical principles of bivalence and excluded middle. Finally, one should also mention Boethius' original contributions contained in his logic handbooks, *Introductio ad categoricos syllogismos*, *De syllogismo categorico*, *De syllogismo hypothetico*, *De divisione*, *De deffinitione* i *De differentiis topicis*. Taken together with Aristotle's *Organon*, Porphyry's *Eisagoge*, and Cicero's *Topics* (also commented on by Boethius), these writings constitute the so-called "old logic" (*logica vetus*). As Gilson put it, "we may say that, thanks to all of his treatises, Boethius became the chief teacher of logic of the Middle Ages, up to the 13th century" [29, p. 139, *my emphasis*]. All of the aforementioned works by Boethius may be found in [PL 63–64].

3. It is sufficient to merely glance at Boethius' opus in order to realise that he was deeply familiar with Aristotle's commentators.
4. Even though he was primarily devoted to the study of classical texts and his own philosophical and theological work, Boethius was also a member of the imperial administration during the reign of the Ostrogoth emperor Theodoric. He became a senator aged just 25, consul in 510, and already in 522 he was the *magister palatii* (*officiorum*), one of the highest ranking bureaucrats, somewhat of a *chargé d'affaires* of the imperial court in Ravenna. For a more detailed biography of Boethius, see [26, pp. 1–68].
5. It seems that Boethius had also written handbooks devoted to the remaining two quadrivial disciplines, i.e. geometry and astronomy, however, these are unfortunately not extant (apart from several spurious fragments under the title *Ars Geometriae*). In the section of *Institutiones Divinarum et Saecularium Litterarum* devoted to geometry [II, 6], Cassiodorus points out that Boethius "translated Euclid into Latin", whereas in the *Variae epistolae* 1.45 we even find a hint that he might have also translated Archimedes. As recorded by Stahl, in 1867 Friedlein edited a pseudo-Boethian treatise in Latin devoted to geometry, but, as well as most historians of mathematics, he also agreed that the text is spurious and not authentic [47, p. 201]. When it comes to the astronomical treatise, the evidence is vague at best (e.g. two letters by Gerbert of Aurillac from 983 and 988 mentioning, respectively, "Boethius' *De astrologia*" and "M. Manlius *De astrologia*" in the Abbey of Bobbio in Italy), and it does not allow us to infer with any degree of probability that a Boethian treatise on astronomy even existed. [40] is an attempt at reconstructing Boethian astronomy and cosmology. For more details on the Boethian quadrivium, see Section 3.
6. *DIA* and *DIM* are typically rendered in English as, respectively, *Fundamentals* (also: *Foundations or Elements*) of *Arithmetic* and *Fundamentals of Music*. Notice that another possible translation may be *On the teaching of arithmetic (music)*.
7. From the middle of the first decade of the 6th century, probably around 505. For the chronology of Boethius' writings, consult [24]. A somewhat different chronology is given by [44]. Brandt [24, p. 154] is right in claiming that *DIA* and *DIM* were probably written "in one sitting"; the close connection between the two treatises is evidenced by a series of references to *DIA* in *DIM*. More on this in Section 2.
8. A similar remark concerning translation may be found in Boethius' commentary of Porphyry's *Eisagoge* (*Comm. in Porphyrium a se translatum*, PL 64, 71):

“Secundus hic arreptae expositionis labor nostrae seriem translationis expediet, in qua quidem uereor ne subierim fidi interpretis culpam, cum uerbum uerbo expressum comparatumque reddiderim. cuius incepti ratio est quod in his scriptis in quibus rerum cognitio quaeritur, non luculentae orationis lepos, sed incorrupta ueritas exprimenda est.”

9. See [49, pp. 4–8]. Also, see the references in [19, p. 53, n. 1 and p. 54, n. 2].
10. We shall return to Aristoxenus' theory in Section 3.
11. Plato just once [*Resp.* 600A–B], as well as Aristotle [*Rhet.* 1398b14]. The other mention of Πυθαγόρας in Aristotle [*Meta.* 986a30] is a later addition.
12. As reported by Aristotle in [*Meta.* 985b23ff.], cf. also [*Meta.* 1092b17].
13. In recounting his version of the story, Nicomachus' uses the word πείρα, experiment. Cf. [*MSG*, 246, 16].
14. Here is an illustrative passage (I reproduce the text as it appears in [28]): “hora che questo non fusse ne poss'essere in modo alcuno, l'esperienza (com'io ho detto) ce lo dimostra. imperoche colui che da due corde d'ugual lunghezza, grossezza, & bontà, vdir volesse il Diapason, gli sarebbe di mestiere sospenderui pesi che fussino non in dupla (come erano i martelli) ma in quadrupla proportione”. Cf. [27, pp. 82–84] for a detailed explanation of the problematic and dubious points in the Boethian narrative concerning Pythagoras' invention.
15. Creese says [27, ad loc. cit.] that the word κανών began to be used because it was believed that the instrument from Fig. 1 “canonises”, i.e. straightens (κανονίζειν) the flaws and imperfections of auditory perception in terms of the truth (ταῖς αἰσθήσεσιν ἐνδέοντα πρὸς τὴν ἀλήθειαν; cf. Ptol. *Harm.* 5.12–13). See also the entry κανών in *LSJ*.
16. The reader is advised to consult [27] for an exceptionally interesting historical study of the monochord in ancient Greek harmonics.
17. This holds especially of the instrument known as the Greek panpipe or Pan's flute (σύριγξ, pl. σύριγγες) which is mentioned already in the Iliad [X.13, XVIII.526]. The panpipes were mostly made out of hollow reeds of approximately the same length and diameter. Pitch variation in panpipes was achieved by means of a wax plugs which were inserted into the tubes so as to shorten their length and thus effectively change their pitch. For instance, in order to tune a given reed-pipe *A* an octave above some other reed-pipe *B*, one should simply just block the pipe *A* with a wax plug to one half of the speaking length of pipe *B* [27, pp. 94–95]. Martin West remarked that the term σύριγξ initially was not used to denote the entire instrument but rather just these wax plugs which were used for tuning the pitches of the *auloi* [49, pp. 86, 102–103]; on the panpipe, see [49, pp. 109–110].
18. Thickness of the first disk was ἐπίτριτον [sc. 4:3] with regards to the thickness of the second, ἡμόλιον [sc. 3:2] with regards to the thickness of the third, and διπλάσιον [sc. 2:1] with regards to the thickness of the fourth.
19. The standard edition is [16]. For the English translation and an excellent accompanying study, see [17].
20. See *De artibus* etc. in [PL 70, 1208]; also, see *Etymologiae* in [PL 82, 155].
21. For a list of Boethius few departures from Nicomachus' original text, see [17, pp. 134–137]. The passages in question confirm that Boethius was mostly faithful to the original whilst translating or paraphrasing.
22. Two of the probably most detailed and influential studies devoted to Boethius' sources for *DIM* are [45] (an extensive, book-length analysis) and [23] (a critical re-examination of some of the pitfalls of [45]). As we shall see in Section 5, the principal point of disagreement between Pizzani

- [45] and Bower [23] is the question concerning Boethius' source (or sources) for the fourth book of *DIM*.
23. The standard edition is [12]. For the English translation, see [14].
 24. E.g. in Sections 2.1–4 which, as Bower rightly notes, may rightfully be put under the heading *Contra Aristoxenum* [5, p. xxxii].
 25. Probably by following Pizzani [45, pp. 140–151], in the critical apparatus accompanying his English translation of *DIM*, Bower also connected the sections of Boethius' text with the corresponding sections of Ptolemy's *Harmonics* which were used by Boethius as a basis for Book 5 of *DIM*. The major advantage of the above schematic presentation of the correspondences between the two texts which I have opted for lies in its immediateness, i.e. it conveys the same point as Pizzani and Bower, just in a much less cumbersome and diffused way.
 26. The relevant passage concerning the use of diagrams for didactic purposes appears in the already cited Preface to *DIA*: “*ut aliquando ad evidentiam rerum nostris etiam formulis ac descriptionibus uteremur*”.
 27. More precisely, in *DIA* Boethius uses the term *quadrivium* which appears in all older manuscripts, *quadrivium* is a later spelling.
 28. Alongside the quadrivium, we also have the *trivium* which predates Boethius, and which encompasses grammar, rhetoric, and logic (or dialectics).
 29. Plato's sequence of the mathematical disciplines is also different [cf. *Resp.* 522C–617B]: arithmetic, geometry, astronomy, and harmonics. It should be noted however that Plato considers harmonics as the science which studies the motions of celestial bodies.
 30. It was claimed that Pythagoras studied the “arithmetical form of geometry” (τὸ ἀριθμητικὸν εἶδος αὐτῆς [sc. τῆς γεωμετρίας]) [DL VIII, 12].
 31. For Archytas' fragment on logistics, see [DK 47 B 4].
 32. The terms used are ἀντανάφρσις [in Arist. *Topica* VIII, 3] or ἀνθυάφρσις [in Euclid, *Elements*, Book 10, Props. 2–3].
 33. For the Pythagorean background of these methods, cf. [33, pp. 99, 187]; also, cf. [34, pp. xl–xli].
 34. “Rursus cum aliquam geometricam formam dixero, est illi simul numerorum nomen implicatum; cum numeros dixero, nondum ullam formam geometricam nominavi”. (*DIA* 1.1) Nicomachus says that arithmetic is implied by geometry without itself implying geometry (καὶ συνεπιφέρεται μὲν ἐκείνη, οὐ συνεπιφέρει δὲ αὐτήν). [*Intr. arith.* 1.4.5]
 35. For an examination of Capella's quadrivium, see [48, pp. 125–227].
 36. “Quibus quattuor partibus si careat inquisitor, verum invenire non possit, ac sine hac quidem speculatione veritatis nulli recte sapiendum est. Est enim sapientia earum rerum, quae vere sunt, cognitio et integra comprehensio. Quod haec qui spernit id est has semitas sapientiae ei denuntio non recte esse philosophandum, siquidem philosophia est amor sapientiae, quam in his spernendis ante contempserit. [...] Constat igitur, quisquis haec praetermiserit, omnem philosophiae perdidisse doctrinam. Hoc igitur illud quadrivium est, quo his viandum sit, quibus excellentior animus a nobiscum procreatis sensibus ad intellegentiae certiora perducitur”. (op. cit.)
 37. In the text of *DIM* Boethius does not actually use the adjectival form *instrumentalis* but rather the expression *in instrumentis*, as indicated in the brackets above. Cf. [43, p. 63, n. 60].
 38. Bower's thesis is rendered plausible by those very passages in *DIM* 1.2 where Boethius promises that he will later return to those very topics for which we know that they only could have been

based upon Ptolemy's *Harmonics*; the relevant sentences in *DIM* are “de quibus posterius studiosus disputandum est”, and “sed de hac quoque posterius dicam”.

39. I here follow Miekley [41, pp. 29–30] who first formulated the thesis per which *DIM* 1.34 is Boethius' original contribution, independent of any specific Greek or Latin source. See also [45, p. 65].
40. “Is vero est musicus, qui ratione perpensa canendi scientiam non servitio operis sed imperio speculationis adsumpsit”. (op. cit.)
41. According to Pizzani [45, p. 9], the fact that Boethius' musical treatise contains discussions of both major musicological traditions, the Pythagorean and the Aristoxenian (with a clear prejudice in favour of the Pythagorean approach), indicates that he necessarily had to work with different sources (*sfruttare fonti diverse*), with little or no organic or structural similarities. For him, this is a sign of greater maturity of *DIM* in comparison with, say, the single-sourced *DIA*.
42. Pizzani disagrees with this and speaks about *DIM* as of a work which is “rounded in itself and self-sufficient”, as well as about the “autonomy of the section devoted to music (*autonomia della sezione musicale*) within the [Boethian] corpus with regards to the one devoted to arithmetic” [45, p. 5]. In his opinion, Boethius' musical treatise does not presuppose any prior knowledge of the arithmetical one since the main mathematical ideas from *DIA* connected with the musical theory expounded in *DIM* are reproduced *ab ovo* whenever necessary in the actual text of *DIM*. However, on p. 77 he contradicts himself by claiming that *DIA* logically precedes *DIM*, and that it is logically dependent upon it. For a competing view, see [23, pp. 8ff].
43. The Greek text may be found in [*MSG*, pp. 235–265]. For the English translation, see [37].
44. For instance, in the case of “sound”, we have the following:
 Boethius (*DIM* 1.3): “Idcirco definitur Nicomachus (ap. *MSG*, 242, lines 20–21):
 sonus percussion aeris indissoluta usque ad “Καθόλου γάρ φασὲν ψόφον μὲν εἶναι πλῆξιν
 auditum”. ἀέρος ἀθρυπτον μέχρι ἀκοῆς”.

For a detailed comparison of all the parallel passages between the *DIM* and the *Enchiridion*, see [45, pp. 35–62], as well as Bower's notes in the critical apparatus which accompanies his 1989 English translation of *DIM* [5].

45. It is interesting to note that Pizzani [45, pp. 13ff] even attempted a reconstruction of the contents of Nicomachus' *Introduction to music* by cross-examining the *Enchiridion*, the fragments collected in *MSG*, as well as the extant scholia.
46. Pizzani himself does not call it that, Bower does [23, p. 12].
47. For the Greek text, see [*MSG*, 113–166]. For the English translation, see [*GMW* 2, Ch. 8].
48. Pizzani thinks that passages where Boethius is unfaithful to the Euclidean original are few, and that the departures or omissions are minimal and as such insubstantial. Somewhat carelessly, he fails to take into account the abovementioned fact concerning Boethius' well attested competence as translator, and then attempts to explain away the evident inconsistency between Boethius' and Euclid's text nonchalantly in terms of Boethius' “scholastic taste” [45, p. 107].
49. That such a translation actually existed is confirmed by a passage from Cassiodorus' *Institutiones* II, 5 [PL 70, 1208] from which we learn that Mutianus was his friend.
50. They are reproduced in [45, pp. 98–99].
51. In the apparatus accompanying his translation of *DIM*, Bower points out three more possible candidates, apart from Gaudentius, all of them Aristoxenians—M. Capella, Alypius, and an anonymous author found in Bellerman's collection (*Anonymi scriptio de musica*) under “Anon. III”—yet claims that it is unlikely that any one of them actually served as Boethius' source [5, p. 123, n. 20], cf. [23, p. 17].

52. The evidence in question is partly textual since it concerns the absence from the Boethian treatise of some important features of the Nicomachean division of the monochord which were explicitly mentioned in the *Enchiridion*, and partly mathematical in that it has to do with the inconsistency in one of Boethius' computations. For more details, see [45, ad loc. cit.] and [23, pp. 21–22].
53. See the entry “Boethius” in [30, pp. 131–133].
54. One of the oldest surviving manuscripts kept in Oxford University's Bodleian Library is precisely a copy of Boethius' *Fundamentals of Music*, gifted to Baliol College by Peter Cossington in 1276. [46, p. 458].
55. See for instance the recent translation of *DIM* into Slovenian: *Temelji glasbe — De institutione musica* (translated with notes and an accompanying study by Jurij Snoj), Ljubljana, 2013.
56. For a detailed account, consult the expository articles [46, 43, 35, 42].
57. Cassiodorus succeeded Boethius' as the *magister officiorum* at Theodoric's imperial court.
58. In some contrast to this, it is historically interesting to note that Boethius' *DIM* served as inspiration to the 20th century German composer Paul Hindemith (1895–1963). Namely, during his Charles Eliot Norton Lectures at Harvard in 1949–1950, whilst discussing “everlasting values” in music, Hindemith based his philosophical approach to this question on two medieval musical treatises, Augustine's *De Musica* and Boethius' *De institutione musica* [36, Ch. 2–4]. Furthermore, Hindemith claimed that the strong influence *DIM* exerted on European musical education was decisive in shaping “organised technique of composition and its underlying theories” up to about 1700 [36, pp. 6–7]. Especially important to him was Boethius' threefold division of music, so much so that Hindemith implemented it into his symphony in three movements, *Die Harmonie der Welt* (1951) which was turned into an opera in 1957. The three movements are even entitled *musica instrumentalis*, *musica humana*, and *musica mundana*. For the musicological analysis of the characteristically Pythagorean and Boethian aspects of Hindemith's symphony, see [25, pp. 84ff].

References

- [1] Anicius Manlius Severinus Boethius, *Anicii Manlii Severini Boethii opera omnia*, in: J.–P. Migne (ed.), *Patrologiae cursus completus sive bibliotheca universalis etc*, Series Latina, **63–64**, Paris, 1847, 1891.
- [2] *Anicii Manlii Severini Boethii Philosophiae consolationis libri quinque*, in: G. Weinberger (ed.), *Corpus Scriptorum Ecclesiasticorum Latinorum* **67**, Apud C. Geroldi filium, Leipzig, 1934.
- [3] G. Friedlein, *Anicii Manlii Torquati Severini Boetii De institutione arithmetica libri duo; De institutione musica libri quinque; accedit geometria quae fertur Boetii*, Leipzig, 1867.
- [4] M. Masi, *Boethian Number Theory: A Translation of the de Institutione Arithmetica (with Introduction and Notes)*, Rodopi, Amsterdam, 2006.
- [5] Anicius Manlius Severinus Boethius, *Fundamentals of Music*, C. V. Palisca (ed.), Translated, with Introduction and Notes by C. M. Bower, Yale University Press, New Haven and London, 1989.
- [6] I. Bekkeri, *Aristotelis opera omnia*, De Gruyter, Berlin, 1831.
- [7] A. Barker, *Greek Musical Writings: Volume 2, Harmonic and Acoustic Theory*, Cambridge University Press, Cambridge, 1989.
- [8] F. Bellermann, *Anonymi scriptio de musica: Bacchii senioris Introductio artis musicae*, Albertum Foerstner, Berlin, 1841.
- [9] Magnus Aurelius Cassiodorus, *Institutions of Divine and Secular Learning and On the Soul*, Translated with notes by James W. Halporn and introduction by Mark Vessey, Liverpool University Press, Liverpool, 2004.

- [10] Magnus Aurelius Cassiodorus, *The Variae: The Complete Translation*, Translated by M. Shane Bjornlie, University of California Press, Oakland, California, 2019.
- [11] H. A. Diels, W. Kranz, *Die Fragmente der Vorsokratiker*, griechisch und deutsch **1–2**, Berlin, 1960.
- [12] Claudio Ptolemaios *Die Harmonielehre*, in: I. Düring (ed.), Göteborgs Högskolas Årsskrift **36**, Göteborg, 1930.
- [13] Diogenes Laertius, *Lives of Eminent Philosophers*, **1–2**, Translated by R. D. Hicks, Loeb Classical Library, Harvard University Press, Cambridge, MA, 1972.
- [14] J. Solomon, *Ptolemy Harmonics: Translation and Commentary*, Brill, Leiden–Boston, 2000.
- [15] C. von Jan, *Musici scriptores graeci: Aristoteles, Euclides, Nicomachus, Bacchius, Gaudentius, Alypius et melodiatarum veterum quidquid exstat*, Leipzig, 1895.
- [16] R. G. Hoche, *Nicomachi Geraseni Pythagorei Introductionis arithmeticae libri II*, Leipzig, 1866.
- [17] Nicomachus of Gerasa, *Introduction to Arithmetic*, Translated by M. L. D'Ooge, with studies in Greek arithmetic by F. E. Robbins and L. C. Karpinski, Macmillan, New York, 1926.
- [18] J. Burnet, *Platonis opera*, Scriptorum classicorum Bibliotheca Oxoniensis, Oxford, 1900–1907.
- [19] A. Barker, *Greek Musicologists in the Roman Empire*, *Apeiron* **27**(4) (1994), 53–74.
- [20] A. Barker, *Ptolemy's Pythagoreans, Archytas, and Plato's Conception of Mathematics*, *Phronesis* **39**(2) (1994), 113–135.
- [21] F. Bellermann, *Anonymi Scriptio de Musica: Bacchii Senioris Introductio Artis Musicae*, Albertum Foerstner, Berlin, 1841.
- [22] C. Bower, *Boethius' The Principles of Music: An Introduction, Translation, and Commentary*, PhD dissertation, George Peabody College for Teachers, 1966.
- [23] C. Bower, *Boethius and Nicomachus: An essay concerning the sources of De institutione musica*, *Vivarium* **16**(1) (1978), 1–45.
- [24] S. Brandt, *Entstehungszeit und zeitliche Folge der Werke von Boethius*, *Philologus: Zeitschrift für Antike Literatur Und Ihre Rezeption* **62**(1) (1903), 141–154.
- [25] S. Bruhn, *Explorations of universal order and beauty in Paul Hindemith's Symphony Die Harmonie der Welt*, *The Nordic Journal of Aesthetics* **21**(39) (2010), 80–99.
- [26] H. Chadwick, *Boethius: The Consolations of Music, Logic, Theology, and Philosophy*, Oxford University Press, Oxford, 1981.
- [27] D. Creese, *The Monochord in Ancient Greek Harmonic Science*, Cambridge University Press, Cambridge, 2010.
- [28] V. Galilei, *Discorso di V. Galilei, nobile fiorentino: intorno all'opere di messer G. Zarlino da Chioggia et altri importanti particolari attenenti alla musica*, G. Marsecotti, Florence, 1589.
- [29] É. Gilson, *La Philosophie au Moyen Age*, Payot, Paris, 1944.
- [30] A. Grafton, G. W. Most, S. Settis, *The Classical Tradition*, Harvard University Press, Cambridge, MA and London, 2010.
- [31] J.-Y. Guillaumin, *Boethius's De institutione arithmetica and its Influence on Posterity*, in: N. H. Kaylor, Jr., P. E. Phillips (eds.), *A Companion to Boethius in the Middle Ages*, 135–161, Brill, Leiden–Boston, 2012.
- [32] N. G. L. Hammond, H. H. Scullard, *The Oxford Classical Dictionary*, 2nd ed., Clarendon Press, Oxford, 1970.
- [33] T. L. Heath, *The Thirteen Books of Euclid's Elements: Translated from the text of Heiberg with Introduction and Commentary*, Cambridge University Press, Cambridge, 1908.
- [34] T. L. Heath, *The Works of Archimedes*, Cambridge University Press, New York, 2010.
- [35] A. Hicks, *Music and the Pythagorean Tradition from Late Antiquity to the Early Middle Ages*, in: I. Caiazzo, C. Macris, A. Robert (eds.), *Brill's Companion to the Reception of Pythagoras and Pythagoreanism in the Middle Ages and the Renaissance*, 82–110, Brill, Leiden–Boston, 2022.

- [36] P. Hindemith, *A Composer's World: Horizons and Limitations*, Harvard University Press, Cambridge, 1958.
- [37] F. R. Levin, *The Manual of Harmonics of Nicomachus the Pythagorean*, Phanes Press, Grand Rapids, 1994.
- [38] J. Marenbon, *Boethius: from antiquity to the Middle Ages*, in: J. Marenbon (ed.), *Routledge History of Philosophy: Volume III, Medieval Philosophy*, 11–28, Routledge, London and New York, 1998.
- [39] J. Marenbon, *Anicius Manlius Severinus Boethius*, in: E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, Winter 2021 Edition.
- [40] S. C. McCluskey, *Boethius's Astronomy and Cosmology*, in: N. H. Kaylor, Jr., P. E. Phillips (eds.), *A Companion to Boethius in the Middle Ages*, 47–73, Brill, Leiden–Boston, 2012.
- [41] W. Miekley, *De Boethii libri de musica primi fontibus*, Academic thesis, Jena, 1898.
- [42] A. E. Moyer, *The Quadrivium and the Decline of Boethian Influence*, in: N. H. Kaylor, Jr., P. E. Phillips (eds.), *A Companion to Boethius in the Middle Ages*, 479–517, Brill, Leiden–Boston, 2012.
- [43] C. Panti, *Pythagoras and the Quadrivium from Late Antiquity to the Middle Ages*, in: I. Caiazzo, C. Macris, A. Robert (eds.), *Brill's Companion to the Reception of Pythagoras and Pythagoreanism in the Middle Ages and the Renaissance*, 47–81, Brill, Leiden–Boston, 2022.
- [44] P. E. Phillips, *Anicius Manlius Severinus Boethius: A Chronology and Selected Annotated Bibliography*, in: N. H. Kaylor, Jr., P. E. Phillips (eds.), *A Companion to Boethius in the Middle Ages*, 551–589, Brill, Leiden–Boston, 2012.
- [45] U. Pizzani, *Studi sulle fonti del De Institutione Musica di Boezio*, *Sacris Erudiri* **16** (1965), 5–164.
- [46] M. T. Rimple, *The Enduring Legacy of Boethian Harmony*, in: N. H. Kaylor, Jr., P. E. Phillips (eds.), *A Companion to Boethius in the Middle Ages*, 447–478, Brill, Leiden–Boston, 2012.
- [47] W. H. Stahl, *Roman Science: Origins, Development, and Influence to the Later Middle Ages*, The University of Wisconsin Press, Madison, 1962.
- [48] W. H. Stahl, *Martianus Capella and the Seven Liberal Arts: Volume I, The Quadrivium of Martianus Capella*, Columbia University Press, New York and London, 1971.
- [49] M. L. West, *Ancient Greek Music*, Oxford University Press, Oxford, 1992.