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ALLUSIONS AND ILLUSIONS. GYÖRGY LIGETI, DRAWING, MATHEMATICS AND MUSIC

Abstract. The purpose of this article is to explore the Hungarian composer György Ligeti's (1923–2006) links with mathematics. He himself wanted to pursue a career in science, and it was only because of the numerus clausus imposed on Jews that he was forced to change his path and devote himself to music. One of his great sources of inspiration was graphic art, in particular the works of Piranesi and Escher, two engravers particularly appreciated by mathematicians.

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1. Introduction

Hungarian György Ligeti (1923–2006), one of the most important composers of the second half of the 20th century, admits to having always been fascinated by questions of form. In particular, the problems of construction and composition in the plastic arts. "For example, he says, I have a great love for Piranesi, his views of squares and buildings [...], the fantastic architecture of the *Carceri*. Precision and labyrinth. It has a huge impact on me". Ligeti also cited the engraver Escher as one of his great passions, this "master of false perspective, of false three-dimensionality" [11, p. 13].

The link he drew between Baroque Mannerism, Piranesi, the deceptive figures of Maurits Cornelis Escher, and even the vertiginous infinity of Constantin Brancusi's *Endless Column*, Ligeti extended to the geometric speculations of Benoît Mandelbrot. It was here that he drew inspiration for his work, endeavoring to translate into music the visual impressions he gleaned from contemplating the engravers' plates or the computer screens on which Sierpiński's triangle or Lévy's dragon are printed.

2. Piranesi

"There were corridors that led nowhere, unreachably high windows, grandly dramatic doors that opened onto monklike cells or empty shafts, incredible upside-down staircases with upside-down treads and balustrades. Other staircases, clinging airily to the side of a monumental wall, petered out after two or three landings, in the high gloom of the cupolas, arriving nowhere" [18]. These lines by Jorge Luis Borges, describing the palace in the city of the Troglodytes, where the lost narrator wanders in the short story "The Immortal" that opens the collection *L'Aleph*, could be applied to Piranesi's *Carceri*. What is striking about the engraver's work is the apparent accuracy and meticulousness of detail, masking the impossibility of the architectural whole. Already, in his *Vedute* and *Varie vedute di Roma*, the infinite

breadth of spaces [11, p. 13][1] and the enormity of architectures are illusory, rendered so by drawing devices ("ellipse or irregular polygon of the bare squares", "sudden ruptures of balance", "very deliberate readjustment of perspectives") on the one hand, and by the swarming of tiny figures ("reduced to minute proportions") that distort scale, on the other. In the *Carceri*, however, this search for false perspectives is amplified, culminating in the purely imaginary. While some commentators have insisted on the phantasmagorical side of the work, claiming that it is the product of hallucinatory delirium due to Piranesi's bout with malaria in 1742, others, such as Bruno Ernst[2], have drawn attention to the deliberate desire of the engraver — trained as an architect — to create impossible figures that he consciously inserted into labyrinthine masonry to conceal them.

In two books [4], Bruno Ernst provides a scientific commentary on engraving no. 14 from the 1761 album entitled *Carceri d'invenzione* (sixteen plates), which is a reworked and reworked reprint of the original fourteen-plate *Invenzioni capriciose di carceri* (1745). In the first book, precisely entitled *L'Aventure des figures impossibles* (The Adventure of Impossible Figures), he brings to light the impossible four-herringbone object in the category "Surfaces both near and far", which is hidden there. In his second book, *Le Monde des illusions d'optique* (The World of Optical Illusions), he returns to the impossibility of the object drawn by Piranesi. An impossibility that distinguishes it from a simple illusion due to a lure of our perceptive apparatus — the object is real, but our eye-brain system interprets it falsely — and also distinguishes it from ambiguous (or two-reading) figures — whether their ambiguity is pictographic or stereographic.

In fact, Piranesi was already dealing with a type of illusion that Escher later found and systematized — and whose troubling peculiarity Ligeti expresses with mathematical precision when he speaks of "false three-dimensionality": while a possible object can always be interpreted as a projection, i.e. as the representation of a three-dimensional object, for an impossible object there is no such thing as a three-dimensional object. In the case of *Carceri* no. 14, the masking effect is accentuated by the presence of a staircase — impossible, of course.

A closer look at these "Prisons" will also reveal a multitude of Lilliputian figures, adding to the monumentality of the vaults, columns, domes and arches. Whether captives or gaolers, they are linked to fantastical machines that the observer's eye is quick to mistake for instruments of torture, but which are perhaps no more than the ordinary tools — pulleys, wheels, winches, scaffolding — of the builders. Finally, before leaving Piranesi's engravings, let us recall the false reminiscences they aroused in Coleridge and which, collected by De Quincey, appear in his *Confessions of an Opium Eater* [3, p. 679–685]. Here is an extract, translated by Marguerite Yourcenar: "Creeping along the sides of the walls you perceived a staircase; and upon it, groping his way upwards, was Piranesi himself: follow the stairs a little further and you perceive it come to a sudden and abrupt termination without any

[1]In his work, the Piazza Navona differs from reality, with an infinite scale. "Chez lui, la piazza Navona est autre que dans la réalité, d'une ampleur infinie"

[2]Bruno Ernst is the pseudonym used by the mathematician J. A. F. Rijk to write his works on optical illusions in the visual arts.

balustrade, and allowing no step onwards to him who had reached the extremity except into the depths below. Whatever is to become of poor Piranesi, you suppose at least that his labours must in some way terminate here. But raise your eyes, and behold a second flight of stairs still higher, on which again Piranesi is perceived, but this time standing on the very brink of the abyss. Again elevate your eye, and a still more aërial flight of stairs is beheld, and again is poor Piranesi busy on his aspiring labours; and so on, until the unfinished stairs and Piranesi both are lost in the upper gloom of the hall" [17, p. 106–107]. It seems that the power of fascination and the incitement to dream and create that Ligeti recognized in Piranesi was already exerting its influence, for, as Marguerite Yourcenar comments: "What strikes one immediately in this admirable page is firstly its complete fidelity to the spirit of Piranesi's work, and secondly its extraordinary infidelity to the letter. [...] In the eighteen plates that make up the entire *Carceri* series, one would search in vain for this delirious staircase continuing its ascent interrupted here and there by absent steps, and where the same character who would be Piranesi reappears a little higher each time, on new steps separated from the previous ones by the abyss".

3. Escher

At first glance, the works of Dutch artist Mauritz Cornelis Escher (1898–1972) appear completely realistic, to such an extent that Ligeti lamented that "aesthetics and poetry are not his forte" [11, p. 13] and that he did not recognize him as a great artist. But if there is one art in which the engraver is a master, it is that of surprises. As Doris Schattschneider and Wallace Walker point out, there are two kinds of surprises: those "that result from serendipity and those that are meticulously prepared, even cunningly concealed, so as to appear natural"; and they cite Escher as the brilliant author of surprises of the second kind. For, at second glance, what seemed self-evident turns out to be impossible: "The key to his disconcerting effects lies in mathematics [...], geometry, both classical and modern" [14, p. 6].

In fact, from his very first engravings, Escher, inspired by the designs of the majolica mosaics seen at the Alhambra, became fascinated by the techniques of filling a flat surface — a major mathematical theme: the periodic tiling of the plane — what Ligeti calls his "marquetry and seemingly infinite interlacing of patterns and shapes" [11, p. 13]. These were to give rise to the first optical illusions represented by the engraver, figures with pictographic ambiguity. Then came the progressive deformations in which contours curved and dilated, until they were transformed into new motifs — a metamorphosis that captivated Ligeti, and which echoed some of his compositional preoccupations dating back to his early Western period[3], perhaps even the beginnings of which can be found in his first string quartet (1953–1954), subtitled, coincidentally, if not presciently, "Métamorphoses nocturnes"[4]. As Ligeti

[3]Ligeti's work is usually divided into three periods: the Hungarian period (until 1956), the Western period (until the mid-1970s) and the final period (until his death in 2006).

[4]Escher's best-known engravings on this theme are *Métamorphose* (1939–1940), a xylogravure that was later "transformed" into *Métamorphose II* (1967–1968), and which appears, enlarged six times, as a wall decoration in the office of the central post office in The Hague.

acknowledged, "in *Continuum* [for harpsichord, 1968], and in my music in general, there are transformations that are very close to his procedures [i.e. the engraver's 'metamorphoses'] [...] and Escher's work provided me with some of my inspiration when I wrote *San Francisco Polyphony* [1973–1974] and the *Trois pièces pour deux pianos* [1976]" [13, p. 173].

Another of Escher's "incredible ideas", in Ligeti's words, has to do with what Bruno Ernst calls his "supersuggestion of space", the distant origins of which are perhaps to be found in Baroque mannerism: individual objects are depicted with the most meticulous accuracy and total fidelity to nature, yet true relationships are annihilated, resulting in visually unsettling arrangements. "The faithful resemblance between image and subject *is no longer important*; it's all about optical illusion. It's pure imposture, just for the pleasure of cheating" [4, p. 5]. Figures with stereographic ambiguity draw on this vein. And when impossible constructions are integrated into them, dynamizing any notion of perspective, we arrive at these fantastic worlds where the same is the opposite, and where the primary distinctions between top and bottom, empty and full, inside and outside are neutralized. The affinity of some of the piano *Etudes* with these imaginary constructions can be seen in a remark made by musicologist Paul Griffiths about *Etude* no. 1: "In the particular case of *Désordre*, the initial tonal ambiguity could be phrased thus: is the right-hand melody in the Aeolian mode on A or in the Locrian on B? Unable to make up its mind, the melody takes a step upwards" [9, p. 120], and is also evident in the former title of En suspens: Etude no. 11 was originally called Convexe-concave, a title almost identical to that of Escher's lithograph no. 56: Concave et convexe. "There are fantastic worlds in Escher, but it's worth seeing them very slowly... at times you think it's convex, at other times you think it's concave, you don't really know", says Ligeti, "It's a question of optical illusions, or more simply of the ambiguity of representing three-dimensional, perspectival space on a two-dimensional surface" [4, p. 173][6] — ambiguity indeed, but to which is added, in the case of etching no. 56, the impossibility of including the represented construction in a three-dimensional space.

These impossible figures, the ultimate gradation of illusion, also inspired the composer. Ligeti mentions the direct influence these "impossible perspectives" had on certain layering procedures he developed with the intention of eliciting imaginary impossible acoustic effects: "In my piano *Etudes*[5] there are pieces in which different speeds coexist, in which the pianist has to play simultaneously at four or five speeds. In fact, he can't: it's an illusion; in the same way as in Piranesi or Escher, perspective is illusory" [11, p. 14].

What goals did Escher pursue in his research into illusion? Although his talent and craft as an engraver were technically irreproachable, engraving seems to have been for him merely the means by which he could express "ideas which, he said, fascinated me so much that I felt a strong desire to communicate them". These ideas, translated into plastic form, gave rise to a series of prints collected in a book

[5]Cycle begun in 1985 and left unfinished. As it stands, the cycle comprises 18 Etudes divided into three Books.

L'Œuvre graphique, composed "with the intention of expressing a development of thought": "By exposing my senses to the enigmas of the universe, by reflecting on these sensations and analyzing them, I approach the realm of mathematics. Although I totally lack knowledge and training in the exact sciences, I feel closer to mathematicians than to my fellow artists" [7, p. 5–6]. It seems to us that these words could have been uttered by Ligeti — once the restriction on scientific knowledge and training had been removed [14, p. 6][6, p. 24].

4. Mathematics

Escher's print *La Goutte de rosée* (mezzo-tinto, 1948) is part of the ensemble "Reflets dans une sphere", and shows three worlds simultaneously: the leaf of a fat plant, an enlarged portion of the leaf behind the drop of water, and the reflection of its surroundings on the front of the drop. Escher comments: "A dewdrop reflects a window and at the same time acts as a lens that magnifies the leaf's vein structure" [7, p. 13 and plate no. 53]. In fact, what jumps out at today's observer is the fractal structure he or she recognizes.

Ligeti often cited fractal theory as one of his sources of inspiration. More generally, he was keen to affirm his love of the exact sciences — a love that dated back to his youth and never faded, even though he was unable, despite his desire, to pursue university studies in these subjects: "Hungarian Jews couldn't enter university unless they passed a strict numerus clausus. I took the entrance exam for mathematics and physics at the University of Cluj. I was admitted, but not accepted, as only one place was reserved for Jews"[6]. This in no way implies that, in his work as a composer, he engaged in learned calculations involving sophisticated fields of mathematics or physics. In 1987, in an interview with Clytus Gottwald, he declared: "In recent years, I've been working a lot with polymetric and complex formal developments and, very consciously, I've introduced more calculus, but this is very simple calculus, rudimentary arithmetic and geometry". And he denies any resemblance in his relationship to mathematics with the approach of a Xenakis, for example, "where very often it's an algorithm or a calculation that plays the decisive role [1, p. 88]. Even Stockhausen has this tendency to abstractly project a certain process [...]. My approach is completely different. There's nothing abstract about me. I don't start from some reflective process, some method of thinking, these reflections are immediately linked to my musical conception. The constructive element is not suppressed, it's always present, never becomes abstract, but is always thought through together with the phenomenon of sound". He insists: "If, for example, I greatly appreciate Xenakis, who always works on the basis of algorithms, I am totally opposed to his method: I accept him as a composer, but my thinking is different" [8, p. 226].

That is why, if there's one interpretation of his works that Ligeti has always opposed, it's that of seeing them as applications of scientific theories. For him, the

[6]Personal communication. One of Ligeti's major references was Douglas R. Hofstadter's book [10], whose lower banner reads "A metaphorical fugue on minds and machines in the spirit of Lewis Carroll" — apt to arouse the enthusiasm of Ligeti, a great admirer of the British novelist and mathematician.

formal beauty of certain theories is an aesthetic emotion, and therefore a creative stimulus: thus, Ligeti's reverie on fractals is more akin to the contemplation of a painting à la Piranesi or a sculpture à la Brancusi than to the practical use of stochastic calculations in composition, as Xenakis was wont to do. If, when composing, Ligeti refuses any deliberate exploitation of mathematical theories (or even simple techniques, such as computer assistance), he likes to let his thoughts wander in geometric spaces. Reminiscences of these meditations then cross his mind during the process of musical creation, and thus permeate many of his works. Commenting on his first fifteen *Etudes*, he explains that they contain "ideas that come from geometry, from topology: drawings, structures, deformations" [5]. Later, in response to pianist Pierre-Laurent Aimard's[7] use of the words "explosion" and "catastrophe", he corrects this by speaking of "contraction". Yet continuous deformation — dilation or contraction — is the basis of topology. By Ligeti's own admission, this is the source of inspiration for many of his compositions.

It is no surprise that the theory of fractals, among mathematical theories, particularly caught his attention. However, to avoid any doubt, let us repeat that while Ligeti may have drawn inspiration from Mandelbrot, this does not imply he sought to extend Mandelbrot's theory into the musical realm. What he discovered in fractals was a material that was "good to think about" — perhaps more so in its artistic than in its mathematical aspects — a material capable of arousing musical correspondences in his creative imagination. Ligeti's sense of illusion is reflected in the paradox underlying this branch of geometry: the dissolution of the notion of dimension, as the distinction between line and surface is abolished. Ligeti's principle of repeating the same form in an infinitely complex form[8] bears a striking resemblance to some of Escher's engravings, from which he borrowed thought patterns for apprehending labyrinthine fillings.[9]

The mathematical influences Ligeti assumes are simply for their suggestive power: "In the fourth movement of my Piano Concerto [1985–1988], I used forms that have certain analogies with fractal structures, but I did so without calculations. I prefer to work by hand: art doesn't have to be exact. I draw inspiration from scientific data [...], but what I do is art, not science" [1, p. 91]. Admittedly, Ligeti may have said that "time [was] in music the equivalent of space" in geometry [11, p. 14], but despite the parallelism he drew between the two, he never wanted to copy mathematical recipes and transcribe them term by term in his musical compositions: "I maintain my attitude of rejection towards pseudo-scientific composition, which I consider to be pure ideology" [12, p. 21]. This mistrust of the

[7]Pierre-Laurent Aimard is one of the pianists with whom Ligeti worked to finalize his *Etudes* for piano. He is the dedicatee and creator of several of these works.

[8]One of the properties of fractal objects is self-similarity: an enlarged part is identical to the whole object. However, we distinguish between exact self-similarity, such as that of Sierpiński's triangle, and statistical self-similarity, when a detail is not rigorously identical to the whole, but the statistical properties are preserved. Most natural fractal patterns are only statistically self-similar" [16]. Natural fractal objects include snowflakes, clouds, mountains, trees and coastlines.

[9]The labyrinth is one of the favourite themes of Lewis Carroll, Franz Kafka and Jorge Luis Borges — Ligeti's favorite authors, many of whose preoccupations he shared.

jargonized discourse of false science led him to say: "I am in complete agreement with Alan Sokal and Jean Bricmont. I very much appreciated their book *Impostures intellectuelles* [15],[10] because I deplore the fact that, over the last few decades, philosophy in France has tended to move away from Cartesian thought, its coherence and elegance" [2].

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