

PREFACE

Operator theory is an essential part of mathematics, with numerous applications in applied mathematical analysis, physics and engineering. Results which originate in this research area combine methods and techniques from real and complex analysis, functional analysis, measure theory, general algebra, linear algebra, matrix analysis and topology.

Throughout its history, operator theory has further developed in several different directions, each guided by the most suitable scientific inquiry. This monograph, *Topics in Operator Theory*, represents a collection of results obtained by its authors, which cover significant topics in contemporary operator theory: invertibility and generalized invertibility of linear operators on Banach spaces, Hilbert spaces and Hilbert C^* -modules, Fredholm theory, Riesz operators, sequence spaces, stochastic operators, majorization theory, partial orderings, spaces with indefinite inner products, as well as applications to some singular matrix and operator equations.

The monograph contains 10 papers, giving over 500 pages of material. Each paper is written separately, so an interested reader can read any article independently from the others. The papers are listed as follows.

- Dragan S. Djordjević, Milica Z. Kolundžija, Sonja Radosavljević, Dijana Mosić: *A Survey on 2×2 Operator Matrices and Their Applications*. This paper presents some of the author's results which concern Fredholm properties and generalized invertibility operator matrices on Banach spaces and in Banach algebras. New results on generalized and hypergeneralized projections are presented.
- Dijana Mosić: *Various Types of the Reverse Order Laws for the Group Inverses in Rings*. This original paper contains various characterizations of the reverse order laws for the group inverses in rings, which are related to the generalized invertibility of products of operators.
- Snežana Č. Živković-Zlatanović: *An Introduction Into Fredholm Theory and Generalized Drazin-Riesz Invertible Operators*. The paper begins with a brief introduction to classical Fredholm theory and Riesz operators. Afterwards, this survey paper presents the author's results which concern polynomially Riesz operators, generalized Drazin-Riesz invertible operators and generalized Kato-Riesz decomposition for bounded linear operators on Banach spaces.
- Ivana Djolović: *A Note on Matrix Transformations and Some Classes of Operators*. The paper is a survey of the author's results on the so-called classical sequence spaces: the sets ℓ_∞ , c and c_0 , equipped with their basic properties, β -duals and the appropriate characterizations of matrix transformations between them. Afterwards, some results related to general linear operators from the space c into each of the classical spaces are established.
- Nebojša Č. Dinčić and Dragan S. Djordjević: *Survey on Reverse Order Laws for the Moore-Penrose Inverse of Hilbert Space Operators*. The survey article contains results from seven original research papers published by the authors in past ten years on the topic of the reverse order laws for the Moore-Penrose inverse of the Hilbert space operators.

- Dragan S. Rakić and Dragan S. Djordjević: *Review of Partial Orders in Rings Defined by Generalized Inverses*. This article reviews the recent known results concerning generalizations of matrix partial orders to the setting of arbitrary rings with or without involution. The minus, star, sharp, core and dual core partial orders are included.
- Ivana Stanišev: *Survey on Star Partial Order in Indefinite Inner Product Spaces*. This is a short survey on the star partial order for matrices, which are considered in the spaces with indefinite metric. The specific geometry of these spaces affects even the definition of a star partial order, demanding the existence of the Moore-Penrose inverse of a certain matrix.
- Martin Z. Ljubenović, Dragan S. Djordjević: *A Survey of Weak Majorization Relations on $\ell^1(I)^+$ and Their Linear Preservers*. This paper is a survey on the most important properties of the extended three weak majorization relations: \prec_w , \prec_s and \prec^{ws} , determined by stochastic operators on the discrete Lebesgue space $\ell^1(I)$. Linear preservers of these majorizations are characterized.
- Bogdan D. Djordjević: *The Equation $AX - XB = C$ Without a Unique Solution: the Ambiguity Which Benefits Applications*. This paper is a survey of the author's results regarding the equation $AX - XB = C$, in the case when it is without a unique solution. The paper presents sufficient conditions for the existence of infinitely many solutions, as well as the methods for obtaining these solutions.
- Stefan G. Ivković: *On Generalized Spectra of Operators on Hilbert C^* -Modules*. This original paper considers generalized spectra in C^* -algebras of operators on Hilbert C^* -modules. Descriptions of such spectra for shift operators, unitary, self-adjoint and normal operators on the standard Hilbert C^* -module over a unital C^* -algebra, are presented. Generalized spectra induced by various subclasses of semi- C^* -Fredholm operators are also investigated.

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Finally, as the guest editor, I am thankful to the authors, who are also my close collaborators, for their effort and contribution devoted to this monograph.

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