Marija Stamenković Atanasov [1]

BRIEF REVIEW IN DYNAMICS HYBRID SYSTEMS AND DIFFERENT COUPLED NANO-STRUCTURES WITH INFLUENCE OF SEVERAL PARAMETERS

Abstract. The paper contains a brief insight into the results achieved by the author during her participation in the project "Dynamics of hybrid systems with complex structures. Mechanics of materials" - OI 174001. It is also noted where the results were published and at which conference were presented. The greatest attention is paid to works on nonlocal mechanics. Also, papers presented at various conferences are illustrated in a shorter form. The most important results and observations during the research are presented.

Mathematics Subject Classification (2010): Primary: 70-99, 74-99, 70F35, 92C10, 74L10; Secondary: 70Exx, 70KXX, 74M05, 37H20.

Keywords: dynamics of hybrid system, dynamics of rotating of multi body system, nonlocal theory, modified couple stress theory, forced vibrations.

[1] Department of Theoretical and Applied Mechanics, Faculty of Mechanical Engineering, University of Niš, Serbia.
marija.stamenkovic.atanasov@masfak.ni.ac.rs, s_marija86@yahoo.com.

Contents

1. Introduction

Wonderful and successful collaboration binds the author of this review paper with the project leader and with the project participants. From this reason, the review paper is separated in two sections. In the first section, beginnings of author research are briefly presented. Beginnings of author research successfully connect author with Leader of Project professor K. (Stevanović) Hedrih and with researchers with same project ON 174001. The first conference the author attended was 8th European Solid Mechanics Conference-ESMC in Graz, Austria with the paper under the name "Deformable Body Oscillations on a Layer with Visco-Elastic and Inertia Properties", see Ref. [1]. Dynamics of hybrid system with nonlinear elastic and inertia properties is presented in Ref. [2]. This result is presented at the Symposium Nonlinear Dynamics – Milutin Milanković, Multidisciplinary and Interdisciplinary Applications (SNDMIA 2012) in Belgrade. In the papers which are presented in Refs. [3, 4] the vector method based on mass moment vectors and vector rotators for application to systems of multi rigid-bodies rotating about axes of no intersection are developed. The first result is presented at 84th Annual Meeting of the International Association of Applied Mathematics and Mechanics, in Novi Sad, Serbia - GAMM 2013, while two-page paper was printed in PAMM, Proc. Appl. Math. Mech., see Ref. [3]. The series of papers at national and international conferences with the colleague from the project OI 174001, are presented in Refs. [5, 6, 7, 8]. Phase portraits of dynamic system of moving of heavy material particle on smooth circle line are presented in Refs. [5, 6]. Phase portraits of dynamic system of moving of heavy material particle on rough circle line are presented in Refs. [7, 8]. One review paper of the most important results with mini-symposium Non-linear dynamics organizer by K. (Stevanović) Hedrih is presented in Ref. [9].

The second section describes the results that include various higher-order continuum theories involving material length scale parameters. These results were achieved by the successful authorial cooperation of professors from the Faculty of Mechanical Engineering in Nis. Also, one successful authorial cooperation with college D. Karličić, from project OI 174001. In this second chapter, different composed nano-systems used nonlocal theory (see Refs. [10, 11]) and micro-systems used modified couple stress theory (see Refs. [12, 13]), are presented. Including nonlocal theory and modified couple stress theory, the author has published a series of papers.

The dynamic responses of the double single-walled carbon nanotube system for four different cases of external transversal load are considered in Ref. [14]. Also, the paper [14] discusses the effects of the axial magnetic field of coupled nano-system by Winkler elastic medium. Nonlocal forced transversal vibration of an orthotropic double nano-plate system is considered in Ref. [17]. Both nano-plates are rectangular, simply supported and coupled by Winkler elastic medium. The dynamic responses of the orthotropic double nano-plate system for three different cases of external transversal load, by analytical methods are considered in paper [17]. A dynamic analysis of a single rotating nonlocal cantilever nano-beam under external excitations is presented in Ref. [18]. The cases of un-damped and damped forced vibrations are analyzed. The novelty of the study lies in the transient responses of the rotating cantilever nano-beams with the nonlocality magnitude effect taken into consideration. In the parametric study the influences of the varying angular velocity and varying hub radius effects are presented in Ref. [18]. Based on the modified couple stress theory, in Ref. [19] the oscillatory system of two parallel Euler–Bernoulli micro-beams which are continuously joined by a Pasternak elastic layer under the influence of axial loading including with the temperature change effect, is discussed.

Important observations during the research are presented for all mentioned papers in the next paragraphs.

2. The beginnings of the author's research paper

A beginning research paper connects the author with professor K. (Stevanović) Hedrih. With great honor and satisfaction of the authors, with Project Leader ON 174001, several papers were published and presented at the conferences.

2.1. Deformable Body Oscillations on a Layer with Visco-Elastic and Inertia Properties. The paper under the name "Deformable Body Oscillations on a Layer with Visco-Elastic and Inertia Properties", is the first paper exposed from author to 8th European Solid Mechanics Conference- ESMC (see Ref. [1]), by the help of professor K. (Stevanović) Hedrih. The paper in Ref. [1] contains analytical descriptions of coupled deformable body nonlinear oscillations on a layer with different visco-elastic and inert properties see Fig. 1. The partial differential equations of a hybrid multi deformable body system nonlinear transversal oscillation on a discrete continuum layer with nonlinear elastic and translator and rotator inertia properties is also presented in detail in Ref. [1]. Body can be beam, plate and membrane and for all cases, the system of partial differential equations is derived.

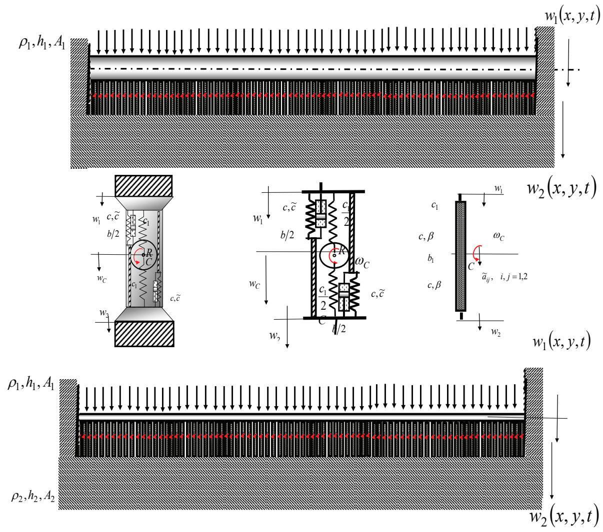

FIGURE 1. Membrane and plate on discrete continuum layer with translator and rotator inertia properties.

Constitutive relations of standard element with rolling sub-element introducing translator and rotor inertia properties are expressed by forces and displacements as element in the final forms:

$$F_{w1} = \frac{1}{4}m\left[\left(\frac{\partial^2 w_2}{\partial t^2}+\frac{\partial^2 w_1}{\partial t^2}\right)-\frac{i_c^2}{R^2}\left(\frac{\partial^2 w_2}{\partial t^2}+\frac{\partial^2 w_1}{\partial t^2}\right)\right]+c\left(w_2-w_1\right)+b\left(\frac{\partial w_2}{\partial t}-\frac{\partial w_1}{\partial t}\right), \quad (1.1)$$

$$F_{w2} = \frac{1}{4}m\left[\left(\frac{\partial^2 w_2}{\partial t^2}+\frac{\partial^2 w_1}{\partial t^2}\right)-\frac{i_c^2}{R^2}\left(\frac{\partial^2 w_2}{\partial t^2}+\frac{\partial^2 w_1}{\partial t^2}\right)\right]-c\left(w_2-w_1\right)-b\left(\frac{\partial w_2}{\partial t}-\frac{\partial w_1}{\partial t}\right). \quad (1.2)$$

Constitutive relations for standard light fraction order elements are

$$F_{w1}(t) = -F_{w2}(t) = -\left\{c_0\left(w_2-w_1\right)+c_\alpha D_t^\alpha\left(w_2-w_1\right)\right\}, \quad (2)$$

$$D_t^\alpha\left[x(t)\right]=\frac{d^\alpha x(t)}{dt^\alpha}=x^{(\alpha)}(t)=\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t\frac{x(\tau)}{(t-\tau)^\alpha}d\tau, \tag{3}$$

where $D_t^\alpha[\bullet]$ is operator of the α^{th} derivative with respect to time t, c, c_α are rigidity coefficients – momentary and prolonged one, and α a rational number between 0 and 1, $0<\alpha<1$.

Similar results (see Ref. [2]) are presented at Symposium Nonlinear Dynamics – Milutin Milanković, Multidisciplinary and Interdisciplinary Applications (SNDMIA 2012) in Belgrade. The paper under the title "Hybrid system dynamics on layer with nonlinear elastic and inertia properties" is presented.

2.2. Mass moment vector applications to the multi body dynamics rotating about axes of no intersection. In the paper which are presented in Refs. [3,4] the vector method based on mass moment vectors and vector rotators for application to systems of multi rigid-bodies rotating about axes of no intersection is developed. The mass moment vectors are used the express the linear momentum and angular momentum for the multi body dynamics, by which the derivatives with respect to time are determined. The multi body system contains a finite number of rigid bodies which are coupled with relative rotations to each other in chain of bodies. There are no intersecting axes of relative coupled rotations. These are axes of the component velocities of rigid body system dynamics, see Fig. 2.

For a model in Fig. 2, a rigid body coupled three-rotations around three no intersecting axes are considered, first oriented by unit vector $\vec{n}_1$ with fixed position and second and next-third oriented by unit vectors $\vec{n}_j$, j = 2,3 , which are rotating around fixed axis with angular velocity $\vec{\omega}_1=\omega_1\vec{n}_1$, and with next two, second and third, coupled component angular velocities $\vec{\omega}_j=\omega_j\vec{n}_j$, $j=2,3,$. Three axes of rotations are no intersecting in general cases.

Rigid body is skew, and eccentric positioned on the moving third rotating axis of self-rotation oriented by unit vector $\vec{n}_3$. Also, rigid body is positioned on the moving self-rotating axis eccentrically and skew positioned (in general case body mass center is not positioned on the self-rotation axis and any of the body main central mass inertia moment axes are not parallel to self-rotation axis). Rigid body rotates around rotating self-rotation axis with angular velocity $\vec{\omega}_3=\omega_3\vec{n}_3$, and around mowing axis oriented by unit vector $\vec{n}_2$ with component angular velocity $\vec{\omega}_2=\omega_2\vec{n}_2$, , and around fixed axis oriented by unit vector $\vec{n}_1$ with angular velocity $\vec{\omega}_1=\omega_1\vec{n}_1$ (see Fig. 2).

In Ref. [4], the three theorems are presented in following forms:

Theorem 1. *Let a mechanical system contain K rigid bodies coupled K rotations around K no intersecting axes, when each single body is placed on the corresponding axis, first oriented by unit vector $\vec{n}_1$ with fixed position and second and next oriented by unit vectors $\vec{n}_j$, j = 2,3, ..., K, which are rotating around fixed axis with angular velocity $\vec{\omega}_1=\omega_1\vec{n}_1$, and with next coupled angular velocities $\vec{\omega}_j=\omega_j\vec{n}_j$, j = 2,3, ..., K. Rigid body N_1 rotates only around fixed axis oriented by unit vector $\vec{n}_1$ with angular velocity $\vec{\omega}_1=\omega_1\vec{n}_1$, and next rigid bodies N_2, N_2, ..., and N_K rotates around corresponding rotating self-rotation axis with corresponding self-rotation angular velocity $\vec{\omega}_j=\omega_j\vec{n}_j$,* *j = 2,3, ..., K. and all bodies rotate around fixed axis oriented by unit vector* $\vec{n}_1$ *with angular velocity* $\vec{\omega}_1 = \omega_1 \vec{n}_1$, *and body* N_3 *around axis oriented by unit vector* $\vec{n}_2$ *with component velocity* $\vec{\omega}_2 = \omega_2 \vec{n}_2$ *and all next bodies rotate with the sum of component angular velocities corresponding to their order defined place on the corresponding no intersecting axis from the set of the axes of coupled rotations. For defined system vector expressions for linear momentum, angular momentum and their derivatives is possible to compose by linear vector superposition of corresponding vector expressions of linear momentum, angular momentum and their derivatives of corresponding single rigid body dynamics single, two, three ... and K coupled rotations, previously derived.*

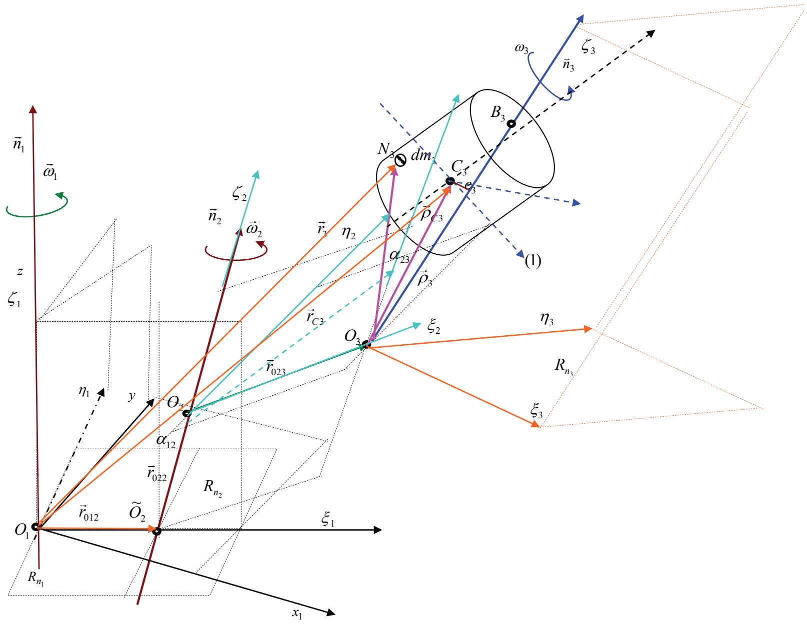

FIGURE 2. A rigid body three coupled rotations around three no-intersecting axes.

Theorem 2. *Vector expression of linear momentum derivative of the rigid N bodies, K multi coupled rotations, around no intersecting axes in all cases, placed bodies on the each axis, between other terms, contain the sum of products by intensity of rigid N bodies mass linear moment vectors* $\left|S_{i\,\vec{n}_j}^{(O_K)}\right| = \left|\iiint_{V_i}\left[\vec{n}_j,\vec{\rho}_i\right]dm_i\right|$, $i = 1, 2, 3 \ldots N$, $j = 1, 2, 3 \ldots K$, *for the axes oriented by unit vectors of component coupled rotation axes through pole on the rigid N bodies self-rotation axis and vector rotators defined by*

$$\vec{R}_{i_{Ojj}} = \dot{\omega}_j \frac{\vec{S}_{i\,\vec{n}_j}^{(O_K)}}{\left|S_{i\,\vec{n}_j}^{(O_K)}\right|} + \omega_j^2 \left[\vec{n}_j, \frac{\vec{S}_{i\,\vec{n}_j}^{(O_K)}}{\left|S_{i\,\vec{n}_j}^{(O_K)}\right|}\right], \quad i = 1, 2, 3 \ldots N,\ j = 1, 2, 3 \ldots K,$$

where are i=1,2,3,…,N number of bodies, j=1,2,3,…,K number of axes.

Theorem 3. *Vector expression of angular momentum derivatives of the rigid N bodies, N multi coupled rotations, around no intersecting axes in all cases, placed bodies on the each axis, between other terms, contain sum of products by intensity of rigid N bodies mass deviation moment vectors* $\left|D_{i\,\vec{n}_j}^{(O_K)}\right| = \left|\vec{n}_j, \left[\left[\iiint_{V_i}\left[\vec{n}_j, \vec{\rho}_i\right] dm_i, \vec{n}_j\right]\right]\right|$ *i* = 1, 2, 3 ...*N*, *j* = 1, 2, 3 ...*K*, *for the axes oriented by unit vectors of component coupled rotation axes through pole on the rigid N bodies self-rotation axes and vector rotators defined by,*

$$\vec{R}_{ij} = \dot{\omega}_j \frac{\vec{D}_{i\,\vec{n}_j}^{(O_K)}}{\left|D_{i\,\vec{n}_j}^{(O_K)}\right|} + \omega_j^2 \left[\vec{n}_j, \frac{\vec{D}_{i\,\vec{n}_j}^{(O_K)}}{\left|D_{i\,\vec{n}_j}^{(O_K)}\right|}\right], \; i = 1, 2, 3 \ldots N, \; j = 1, 2, 3 \ldots K,$$

where are i=1,2,3, ...,N number of bodies, j=1,2,3,…,K number of axis.

Based on the results from Ref. [4] the linear and angular momentums and their derivatives of three rigid bodies rotating about three non-intersecting axes are devised through combination of the results which presented in Ref. [4] and using properties of linear vector superposition. Also, two theorems for derivatives of linear momentum and angular momentum and vector rotators of *N* rigid bodies rotating about *K* no intersecting axes are inductively defined. From previous vector expressions and corresponding analysis, is conclude that vector rotators appear into expressions of the kinetic reactions of the shaft bearings of the structures of the rigid body multi-coupled rotations and that are very important to analyze their intensity as well as their relative angular velocity and angular acceleration around directions to the directions of axes of coupled multi-rotations. A series of kinematical vector rotators coupled to the axis and pole of self-rotation, last body in chain multi body system are defined and identified, which are expressed by the component angular accelerations and velocities. The results obtained for the mass moment vectors applications to the multi body systems dynamics open a new possibility in devising a new software tool for broad applications of the method to engineering problems. A lot more of this model can be seen in Ref. [4]. Summary results, see Ref. [3] are presented at the 84th Annual Meeting of the International Association of Applied Mathematics and Mechanics, GAMM 2013, in Novi Sad, Serbia, and printed in a shorter version in PAMM, Proc. Appl. Math.

2.3. Parametric testing of singularity and position of non-linear dynamics relative balance of heavy material particle. The series of papers at national and international conferences with the colleague from the project OI 174001, Marija Mikić are presented. The titles of papers are "Three-parametric testing of singularity and position of non-linear dynamics relative balance of heavy material particle on eccentrically rotating smooth circle line", presented at SNDMIA, Belgrade 2012, (see Refs. [5,6]), "Testing of singularity and position of non-linear dynamics relative equilibrium of heavy material particle on eccentrically rotating rough circle line, with constant angular velocity", presented at Fourth Serbian Congress on Theoretical and Applied Mechanics, Vrnjačka Banja 2013, (see Ref. [7]), "Three parametric testing of singularity and position of non-linear dynamics relative equilibrium of heavy material particle on eccentrically rotating rough circle line, with constant angular velocity", presented at ENOC 2014, in Vienna, Austria, (see Ref. [8]). The all papers contain analytical descriptions of heavy material particle which moves on a rotating circular smooth line, radius R, which rotate around vertical axis, eccentrically positioned in relation to center of circle line on distance e, angular velocity Ω, see Fig. 3.

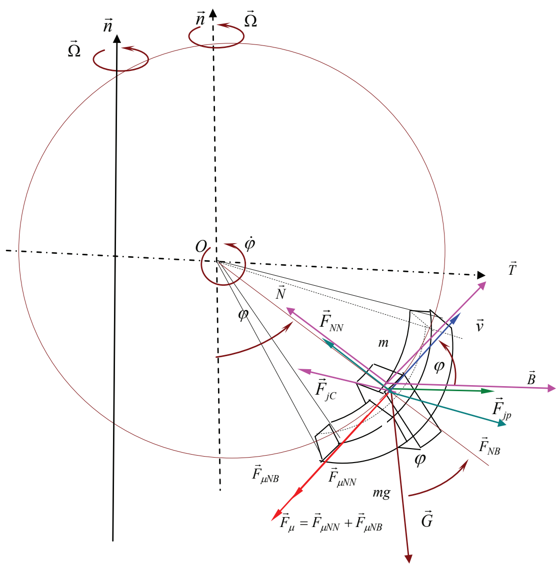

FIGURE 3. Moving of heavy material particle on rough circle line, radius R, which rotates around vertical axis, eccentrically positioned in relation to center of circle line on distance e, by angular velocity Ω.

Differential equation of heavy material particle moving illustrated in Fig. 3 is:

$$\ddot{\phi} \pm \mu\dot{\phi}^2 + \Omega^2\left[\left\langle \lambda - \cos\phi \right\rangle \sin\phi - \varepsilon\cos\phi \pm \mu\lambda\cos\phi \pm \mu\left\langle \varepsilon + \sin\phi \right\rangle \sin\phi\right] \pm \mu_1 2\dot{\phi}\Omega\cos\phi = 0, \quad (4)$$

where the following symbols $\lambda = g/(R\Omega^2)$, $\varepsilon = e/R$ and coefficient $\mu = tg\alpha$ of the Coulomb's type friction for the rough surfaces with normal in the radial directions and $\mu_1 = tg\alpha_1$ of the Coulomb's type friction for the rough surfaces with normal in the binormal directions, are introduced.

Phase portraits of dynamic systems are presented in detail in Refs. [5, 6, 7, 8].

2.4. The most important results from Congress of Serbian Society for Mechanics IConSSM 2011, in one review paper. In the paper "Nonlinear differential equations in current research of system nonlinear dynamics" presented in Ref. [9], basic nonlinear differential equations which describe non-linear phenomena in system dynamics with one degree of freedom with same or more degree of mobility are illustrated. The most important results are shown in the papers presented at mini-symposium Non-linear dynamics organizer by K. (Stevanović)

Hedrih, and which is held on Congress of Serbian Society for IConSSM 2011. While, the review paper was published in journal of Scientific Technical Review.

The following works were specially selected: J. T. Katsikadelis; L. Cvetićanin; Z. Rakaric and I. Kovačić; K. (Stevanović) Hedrih and Lj. Veljović; A. Hedrih and K. (Stevanović) Hedrih; S. Jović and V. Raičević; R. M. Bulatović1 and M. Kažić; C. Frigioiu; A. Obradović, S. Šalinić, O. Jeremić, Z. Mitrović. All the above-mentioned papers can be found in the reference list of the paper presented in Ref. [9].

3. Forced vibration of different composed nano-systems

Modern science and technology became interested in micro and nano structures after they had been invented. They possess important mechanical, electrical and thermal performances that are higher than conventional structural materials. In recent years, nonlocal elastic theory has attracted many attentions because of the necessity of modeling and analysis of very small sized mechanical structures in the developments of nanotechnologies. It is known that the effect of nanostructures size is important for their mechanical behavior because their dimensions are small and comparable to molecular distances. There are various higher-order continuum theories involving material length scale parameters. In this review paper, different composed nano-systems used nonlocal theory (see Refs. [10,11]) and micro-systems used modified couple stress theory (see Refs. [12,13]), author is presented.

3.1. Nonlocal forced vibration of a double single-walled carbon nanotube system under the influence of an axial magnetic field. In the paper M. Stamenković et al. presented in Ref. [14], compressive nonlocal double single-walled carbon nanotube (SWCNT) system which is under the influence of an axial magnetic field is considered, and shown in Fig. 1a. Presented paper was published in Journal of Mechanics of Materials and Structures. The nonlocal double SWCNT system is assumed to be modeled as a system composed of two parallel nanobeams, which have the same length and are continuously joined by a Winkler elastic layer. The stiffness modulus of the Winkler elastic layer is denoted with k. The transversal displacement over the two nanobeams is denoted by $w_1(x,t)$ and $w_2(x,t)$, respectively, Fig. 4b. For the sake of simplicity, both nanobeams are identical, where geometric and physical properties are the same and defined as: A is the cross-sectional area, E is the Young's modulus, ρ is the mass density, I is the moment of inertia and L is the length of the nanobeam. Also, we assume that nanobeam-1is subjected to axial compression F_1 and nanobeam-2 is subjected to axial compression F_2, that are positive in compression, and arbitrarily distributed transverse continuous loads $f_1(x,t)$ and $f_2(x,t)$, that are positive when they act downward. The influence of the Lorentz magnetic force on the double nanobeam system is caused by the axial magnetic field $\eta A H^2_x$ which acts in the x direction, as shown in Fig. 4.

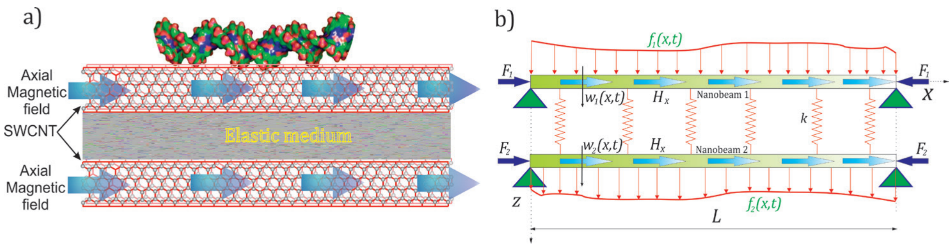

FIGURE 4. The system of double SWCNT affected by an axial magnetic field; (a) The physical model of external excited DSWCNTS coupled by an elastic medium and influenced by an axial magnetic field, and (b) The equivalent mechanical model.

Using the Euler-Bernoulli beam theory and Eringen nonlocal elasticity (see Refs. [10, 11]), the governing equations of motion of the nonlocal double nanobeam system (NDNBS) can be given as

$$\rho A\frac{\partial^2 w_1}{\partial t^2}-f_1+k\left(w_1-w_2\right)+F_1\frac{\partial^2 w_1}{\partial x^2}-\eta AH_x^2\frac{\partial^2 w_1}{\partial x^2}+EI\frac{\partial^4 w_1}{\partial x^4}$$
$$=\mu\frac{\partial^2}{\partial x^2}\left[\rho A\frac{\partial^2 w_1}{\partial t^2}-f_1+k\left(w_1-w_2\right)+F_1\frac{\partial^2 w_1}{\partial x^2}-\eta AH_x^2\frac{\partial^2 w_1}{\partial x^2}\right], \tag{5.1}$$

$$\rho A\frac{\partial^2 w_2}{\partial t^2}-f_2-k\left(w_1-w_2\right)+F_2\frac{\partial^2 w_2}{\partial x^2}-\eta AH_x^2\frac{\partial^2 w_2}{\partial x^2}+EI\frac{\partial^4 w_2}{\partial x^4}$$
$$=\mu\frac{\partial^2}{\partial x^2}\left[\rho A\frac{\partial^2 w_2}{\partial t^2}-f_2-k\left(w_1-w_2\right)+F_2\frac{\partial^2 w_2}{\partial x^2}-\eta AH_x^2\frac{\partial^2 w_2}{\partial x^2}\right], \tag{5.2}$$

where, $\mu=(e_0\tilde{a})^2$ is the nonlocal parameter.

The dynamic responses of the DSWCNT system for four different cases of external transversal load, Uniformly distributed harmonic load, Concentrated harmonic force, Moving constant force and Moving concentrated harmonic force are considered. Closed form solutions for natural frequencies, amplitude ratio and forced vibration response under the influence of the magnetic field and the nonlocal parameter for four cases of external excitation are obtained by applying the method of separations of variables.

Shown results for the lowest natural frequency of DSWCNT can be used to validate them with the results obtained for the free vibration of a SWCNT via molecular dynamics simulation in Ansari et al. [15].

Table1 present the values of fundamental frequency obtained from MD simulations and also the Euler–Bernoulli beam models based on the nonlocal elastic theory. The results predicted by the present models are found to be in excellent agreement with the ones obtained from MD simulation which indicates the capability of the present approach in accurately predicting frequencies of SWCNT. This table show that the frequency of SWCNT decreases with increasing length-to-diameter ratio.

TABLE 1. Fundamental frequencies (THz) for (8, 8) armchair SWCNT obtained from MD simulations of Euler–Bernoulli beam models, $R/l=3$, $l=\mu^{1/2}$, $H_x=1\cdot10^8\ A/m$, $\eta=4\pi\cdot10^{-7}$.

L/d	MD simulation [15]	Present study ($\eta H^2_x = 0$)	Present study ($\eta H^2_x \neq 0$)
8.3	0.5299	0.5485	0.8284
10.1	0.3618	0.3707	0.6306
13.7	0.1931	0.2016	0.4267
17.3	0.1103	0.1264	0.3236
20.9	0.0724	0.0860	0.2613
24.5	0.0519	0.0630	0.2195
28.1	0.0425	0.0479	0.1895
31.6	0.0358	0.0379	0.1674
35.3	0.0287	0.0303	0.1491
39.1	0.0259	0.0247	0.1341

It can be noticed that the results obtained by using the Bernoulli-Fourier method, when is $\eta H^2_x = 0$, are in agreement with the results presented by Ansari et al. [15].

TABLE 2. Analytical validation of the steady-state vibration amplitude ratios ψ_1 and ψ_2.

	$\psi_1 = A_{nI} / A^0_{nI}$	$\psi_2 = A_{nII} / A^0_{nII}$
Zhang et al. [16], $MP=0,\ \eta=0$	1.1965	1.3019
$MP=50,\ \eta=0.3$	1.1499	1.2277
$MP=50,\ \eta=0.5$	1.0872	1.1309
$MP=100,\ \eta=0.3$	1.0779	1.1175
$MP=100,\ \eta=0.5$	1.0436	1.0658

$\chi=0.5$.

In order to compare the results of the presented study and with those in the existing study by Zhang et al. [16], the values of the steady-state vibration amplitude ratios ψ_1 and ψ_2 for the uniformly distributed harmonic load and the concentrated harmonic force are represented in Table 2. It is found that the ratios ψ_1 and ψ_2 in this case are totally the same. From the presented we can conclude that the influence of the longitudinal magnetic field MP and the nonlocal parameter η on the relationship between ratio ψ_1 and ψ_2 causes a decrease in their values. It is presented in more detail in Ref. [14].

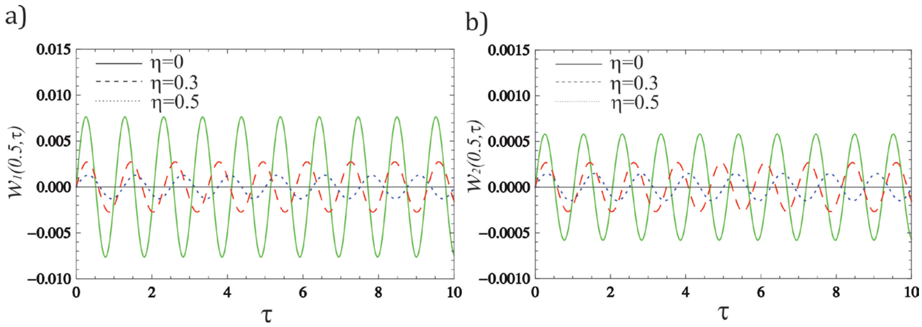

FIGURE 5. The relationship between forced vibrations a) $\bar{w}_1(0.5,\tau)$ and b) $\bar{w}_2(0.5,\tau)$ and dimensionless time for different nonlocal parameters η in the case of the uniformly distributed harmonic load.

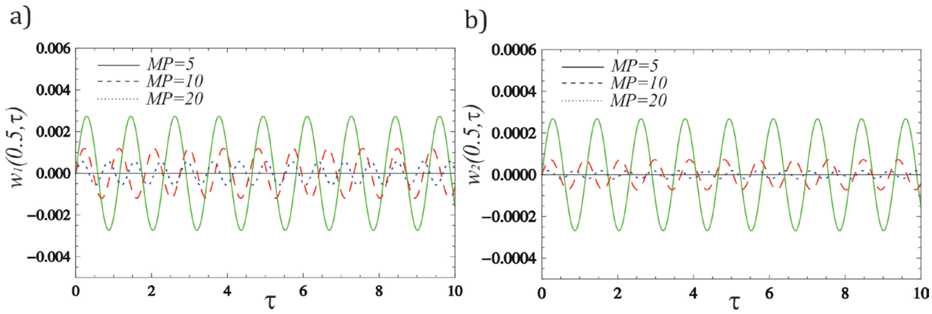

FIGURE 6. The relationship between forced vibrations a) $\bar{w}_1(0.5,\tau)$ and b) $\bar{w}_2(0.5,\tau)$ and dimensionless time for different axial magnetic fields MP in the case of the uniformly distributed harmonic load.

In the paper presented in Ref. [14] the nondimensional parameters is used. From this reason the longitudinal magnetic field and nonlocal parameters are presented by

$$MP = \frac{L^2}{EI}\eta A H_x^2, \; and \; \eta = \sqrt{\mu / L^2}.$$

Analytical expressions for the steady-state vibration amplitudes of the two nanobeams with the influence of the magnetic field and the nonlocal parameter are obtained, and numerical results based on them are presented. From the obtained results, we found that the nonlocal parameter and longitudinal magnetic field have a damping effect on the response vibration amplitude. In order to validate our results, we compared the obtained results for the steady-state amplitude ratios with the results found in the literature and excellent agreement was achieved. It was found that the natural frequencies and response vibration amplitude of the system can change by varying the intensity of the axial magnetic field without the necessity to change any other material and geometric parameter of the DNBS. We analyzed amplitudes of transversal displacements for four cases of external excitation vibration and numerically presented the case of the uniformly distributed harmonic load and the case of the moving harmonic concentrated force with different nonlocal parameters and different axial magnetic fields. The obtained amplitudes of transversal vibration in both cases of external excitation are reduced due to the influence of the axial magnetic field, see Figs. 5 and 6. We noted that the effect of the magnetic field allows a change in the stiffness of carbon nanotubes and therefore a change in the overall stiffness of the DNBS. Changing the stiffness of the system causes changes in the natural frequency of the system, thus avoiding the resonance region for different cases of external excitation. All used parameters are presented in Ref. [14].

3.2. Forced transverse vibrations of an elastically connected nonlocal orthotropic double-nanoplate system subjected to an in-plane magnetic field. The forced transverse vibrations of the orthotropic double nanoplate system composed of two orthotropic elastic nanoplates embedded in a Winkler-type elastic medium influenced by the in-plane uniaxial magnetic field is consider in Ref. [17], and shown in Fig. 7. The presented paper was published in Acta Mechanica.

The orthotropic double nanoplate system is modeled as a stack of rectangular simply supported orthotropic nanoplates with the same material and geometric characteristics, with elastic modulus E_1 and E_2, Poison coefficients υ_{12} and υ_{21}, shear modulus G_{12}, mass density ρ, length a, width b and thickness h. The elastic medium located between the two nanoplates of the orthotropic double nanoplate system is modeled via continuously distributed pairs of parallel connected springs with stiffness coefficient k. Both nanoplates in the orthotropic system are subjected to the in-plane uniaxial magnetic field in the x direction and arbitrarily distributed transverse continuous loads are $f_1(x,y,t)$ and $f_2(x,y,t)$. We assume that the transversal displacements of the nanoplates are $w_1(x,y,t)$ and $w_2(x,y,t)$.

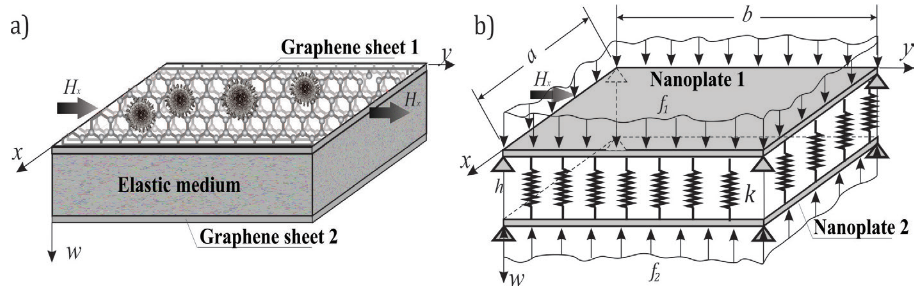

FIGURE 7. The double graphene nano-sheet system coupled by an elastic layer, (a) Physical model, (b) Mechanical model.

Based on the nonlocal constitutive relation and Kirchhoff-Love plate theory, the system of two coupled non-homogeneous partial differential equations of motion is derived in following form

$$\rho h\frac{\partial^2 w_1}{\partial t^2}+D_{11}\frac{\partial^4 w_1}{\partial x^4}+D_{22}\frac{\partial^4 w_1}{\partial y^4}+2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_1}{\partial x^2\partial y^2}+k\left(w_1-w_2\right)-\eta AH_x^2\left(\frac{\partial^2 w_2}{\partial x^2}-\frac{\partial^2 w_2}{\partial y^2}\right)$$
$$=\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left[\rho h\frac{\partial^2 w_1}{\partial t^2}+k\left(w_1-w_2\right)-\eta AH_x^2\left(\frac{\partial^2 w_1}{\partial x^2}-\frac{\partial^2 w_1}{\partial y^2}\right)\right]=f_1-\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)f_1, \quad (6.1)$$

$$\rho h\frac{\partial^2 w_2}{\partial t^2}+D_{11}\frac{\partial^4 w_2}{\partial x^4}+D_{22}\frac{\partial^4 w_2}{\partial y^4}+2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_2}{\partial x^2\partial y^2}-k\left(w_1-w_2\right)-\eta AH_x^2\left(\frac{\partial^2 w_2}{\partial x^2}-\frac{\partial^2 w_2}{\partial y^2}\right)$$
$$=\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left[\rho h\frac{\partial^2 w_2}{\partial t^2}-k\left(w_1-w_2\right)-\eta AH_x^2\left(\frac{\partial^2 w_2}{\partial x^2}-\frac{\partial^2 w_2}{\partial y^2}\right)\right]=f_2-\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)f_2, \quad (6.2)$$

where D_{11}, D_{12}, D_{22} and D_{66} are the bending rigidities of the orthotropic elastic nanoplates, ηAH_x^2 is magnetic field parameter and $\psi=(e_0\tilde{a})/L$ is the nonlocal parameter.

The analytical solutions of the orthotropic double nanoplate system of non-homogeneous differential equations of corresponding dynamic forced processes are obtained by using the Bernoulli–Fourier method.

Authors conducted an analysis of forced vibrations of the orthotropic double nanoplate system for three cases of exciting loads: uniformly distributed harmonic surface load, uniformly distributed harmonic line load and concentrated harmonic force. For all cases, the obtained values of the amplitudes were compared with the values from the literature and presented in tables. It is concluded that the presented results are in very good agreement with the results observed in the literature.

TABLE 3. Validation of the analytical results for the fundamental natural frequency of the single-layer graphene sheet

	Fundamental natural frequency ($\mu=1.34nm^2$)	
$a\times b$	MD Simulations (THz) [20]	Presented model (THz)
10×10	0.0595014	0.0592
15×15	0.0277928	0.0284
20×20	0.0158141	0.0165
25×25	0.0099975	0.0107

Table 3 presents the fundamental natural frequencies, obtained by using the MD simulation and nonlocal continuum mechanics approach, for different sizes of squared single-layer graphene sheets. In this case, a single layered graphene sheet is modeled as a simply supported nonlocal Kirchhoff-Love plate. The value for the nonlocal parameter in the free vibration analyses is $\mu=1.34nm^2$. From the presented results, it can be noticed that the results obtained by using the analytical methods are in excellent agreement with the results presented in Ref. [20].

When the magnetic field and nonlocal parameter are neglected, as is the case in the local theory, the value of amplitudes is lower for the orthotropic plates system than for the isotropic plates system. Also, the values of the amplitudes of the nanoplate system decrease with the introduction of the magnetic field and nonlocal parameters, as expected. Furthermore, the paper presents the effect of the nonlocal and magnetic field parameters on the dynamic response of nanoplate one and two of the system for all cases. From that, it can be seen that without the effects of the nonlocal and magnetic field parameters, the dynamic response for both plates has a much higher value, than when these effects are present. The response of nanoplate two is almost imperceptible with the effects of the nonlocal and magnetic field parameters in comparison with the case without these effects. We have concluded that the nonlocal and magnetic field parameters greatly reduce the dynamic response of nanoplates one and two of the orthotropic system, see Fig. 8. The increasing of the magnetic field parameter leads to the reduction of the forced vibration responses of nanoplates one and two. It was also examined the effects of external excitation for all three cases. Here, it was noticed that with an increase in external excitation, the dynamic response of nanoplates one and two also increases, see Fig. 9. All used parameters are presented in Ref. [17].

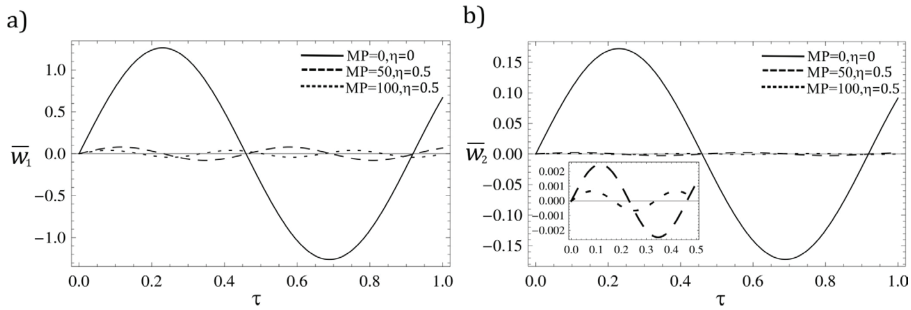

FIGURE 8. Effect of the nonlocal and magnetic field parameters for the uniformly distributed continuous harmonic load case, (a) dynamic response of nanoplate one, (b) dynamic response of nanoplate two.

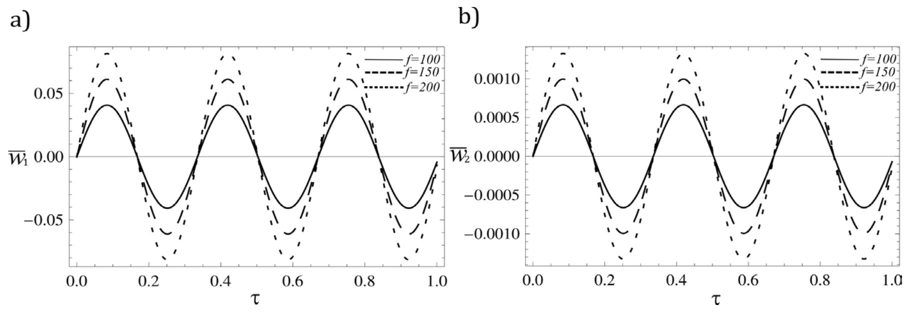

FIGURE 9. Effect of the external excitation for the uniformly distributed continuous harmonic load case, (a) dynamic response of nanoplate one, (b) dynamic response of nanoplate two.

Summary results where are including thermal effect, see Ref. [21] are presented at the conrefence of Nonlinear dynamics–Scientific work of prof. Dr Katica (Stevanovic) Hedrih in Beograd, Mathematical Institute of SASA.

Also, in paper under the name „Thermal and magnetic effects on the forced vibration of an elastically connected nonlocal orthotropic double-nanoplate system" see Ref. [22] which is presented at the 6th International Congress of Serbian Society of Mechanics, in Mountain Tara, the influence of nonlocal parameter on critical scale load ratio of nonlocal and local critical buckling loads is presented. The paper in Ref. [22] presents the effect of the nonlocal and magnetic field parameters on the critical buckling load under constant value of temperature change.

In the field of nonlocal mechanics is presented one more paper at Proceedings of the 8th European Nonlinear Dynamics Conference (ENOC 2014) under the name Nonlinear vibration of nonlocal Kelvin-Voigt viscoelastic nanobeam embedded in elastic medium in Ref. [23].

3.3. Nonlocal forced vibrations of rotating cantilever nano-beams.

The dynamic behavior of a rotating nano-beam is of practical interest, especially when examining an external load. The physical model of a rotating nanotube is important for new miniature devices. The hybridized nano-generator can be presented as a nano-sensor where external load is observed as water or wind. When it comes to nanostructures, the effect of their size is important for their mechanical behavior because their dimensions are small and comparable to molecular distances. Rotating nanotube is represented as cantilever nano-beam. It was adopted that a nano-cantilever has length L which is fixed at point O to a rigid hub and has an external excitation, see Fig. 10. The hub has the radius r with constant rotational speed. Presented paper was published in European Journal of Mechanics A-Solids in Ref. [18].

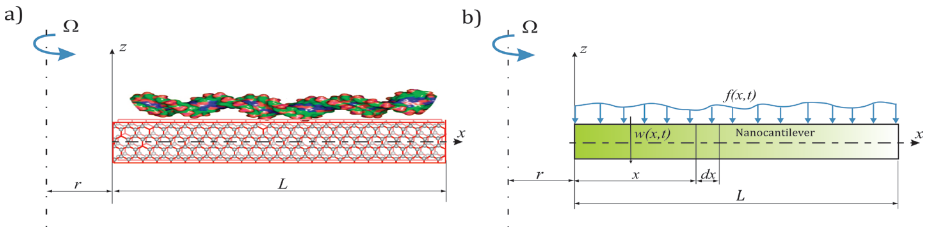

FIGURE 10. Rotating nano-cantilever with external excitations; (a) The physical model of the external excited CNT, (b) The equivalent mechanical model

By employing Eringen's nonlocal elasticity theory Refs. [10,11] and based on Euler–Bernoulli's beam theory, the governing equation of motion of the forced vibration rotating nonlocal cantilever nano-beam is derived in following form

$$EI\frac{\partial^4 w}{\partial x^4}+\rho A\frac{\partial^2 w}{\partial t^2}-(e_0\tilde{a})^2\rho A\frac{\partial^4 w}{\partial x^2\partial t^2}-\rho A\Omega^2\left[(-r-x)\frac{\partial w}{\partial x}+\left(rL+\frac{L^2}{2}-rx-\frac{x^2}{2}\right)\frac{\partial^2 w}{\partial x^2}\right]$$
$$+(e_0\tilde{a})^2\rho A\Omega^2\left[-3\frac{\partial^2 w}{\partial x^2}+3(-r-x)\frac{\partial^3 w}{\partial x^3}+\left(rL+\frac{L^2}{2}-rx-\frac{x^2}{2}\right)\frac{\partial^4 w}{\partial x^4}\right]=f(x,t)-(e_0\tilde{a})^2\frac{\partial^2 f(x,t)}{\partial x^2}. \quad (7)$$

The mentioned equation of motion (7) is discretized by the Galerkin method. The further determination of the dynamic behavior is carried out by the standard modal analysis procedure. Then the forced damped and undamped vibrations of the given model are studied. Detailed procedure is presented in the Ref. [18].

In the paper in Ref. [18], the solutions for the natural frequencies of the rotating nano-model are determined and verified with [24]. The values for the natural frequencies with various effects of nonlocality, angular velocities and hub radius are shown and discussed. Nondimensional frequencies is marked with $\gamma=\sqrt{((\rho A\Omega^2 L^4)/EI)}$. The first four nondimensional frequencies are presented. Slightly higher values of the natural frequencies are obtained when the nondimensional hub radius exists. The obtained values of the first nondimensional frequency for various values of the nonlocal parameter are in very good agreement with the results available in the literature, see Table 5. It is concluded that an increase in the nonlocal parameter leads to an increase only in the first natural frequency (the tendency is contrary for the higher frequencies as shown). Detailed analysis is conducted for variation of nondimensional first, second, third and fourth modes frequency with nondimensional angular velocity γ for different values of nonlocal parameters ψ and nondimension a hub radius δ. The impact of nonlocal and hub radius parameter in the first mode leads to increase of frequency with angular velocity of the rotating nanobeam. For higher modes frequency are the opposite observations. In fact, the increase the nonlocal parameter leads to decrease higher modes frequencies while the impact of hub radius at leads to increase higher modes values of frequencies. The convergence study is also presented. Increased the number of orders in discretization leads to the decreasing values of nondimensional frequencies, see Table 4.

TABLE 4. Convergence study of nondimensional frequencies for $\psi=0$, $\gamma=1$ and $\delta=0$.

	λ_1	λ_2	λ_3	λ_4
n=2	3.68167306	22.18102852		
n=3	3.68167306	22.18102833	61.84178166	
n=4	3.68164789	22.18101484	61.84177366	121.05094094
n=5	3.68164718	22.18101281	61.84176519	121.05092897
n=6	3.68164697	22.18101185	61.84176414	121.05092360

From the detailed analysis of the undamped system and presented time and longer time histories, the impact of contemplated effects leads to the periodically response. The transverse deflections of the undamped system with the impact of nonlocal effects were described by the beat phenomenon, see Fig. 8. The nondimensional nonlocal parameter is marked with $\psi=(e_0\tilde{a})/L$. The nonlocal effects were reduced the deflections of the vibration, see Fig. 8a. With the increase of values of nonlocal effects, more beat periods for the same observed time period can be noticed. The beat period is shorter for higher values of nonlocal effects. An interesting phenomenon was observed in the case of higher values of angular velocity effect. The nondimensional angular velocity is marked with $\gamma=\sqrt{((\rho A\Omega^2 L^4)/EI)}$. Increase in the angular velocity reduced the transverse deflections of the nonlocal undamped cantilever nano-beam, see Fig. 11b. The beat phenomenon described the transverse deflections of the nonlocal undamped cantilever nano-beam for a lower value of angular velocity. The transverse deflections with an impact of hub radius effects were described as periodic variation, see Fig. 11c. The nondimensional hub radius is marked with $\delta=r/L$. Increased hub radius effects caused the decrease of transverse deflections, see Fig. 11c. With the increasing of hub radius effects that increased number of beat and the amplitude in this case was less than in the case of the hub radius absence. Summary results, see Ref. [25] are presented at the 7th International Congress of Serbian Society of Mechanics, in Sremski Karlovci.

TABLE 5. Nondimensional frequencies for the nonlocal parameters $\psi=0;0.1;\ 0.2;\ 0.3;\ 0.4$, for $\gamma=1$ and $\delta=1$.

		λ_1	λ_2	λ_3	λ_4
	Present study	3.889	22.375	62.043	121.263
ψ=0	Pradhan and Murmu [24]	3.890	22.380	62.050	-
	Present study	3.932	21.122	51.603	86.356
ψ=0.1	Pradhan and Murmu [24]	3.932	21.122	51.603	-
	Present study	4.065	18.257	37.744	55.350
ψ=0.2	Pradhan and Murmu [24]	4.065	18.257	37.744	-
	Present study	4.305	15.107	29.519	39.189
ψ=0.3	Pradhan and Murmu [24]	4.305	15.110	29.520	-
	Present study	4.701	12.229	26.018	28.587
ψ=0.4	Pradhan and Murmu [24]	4.701	12.230	26.0201	-

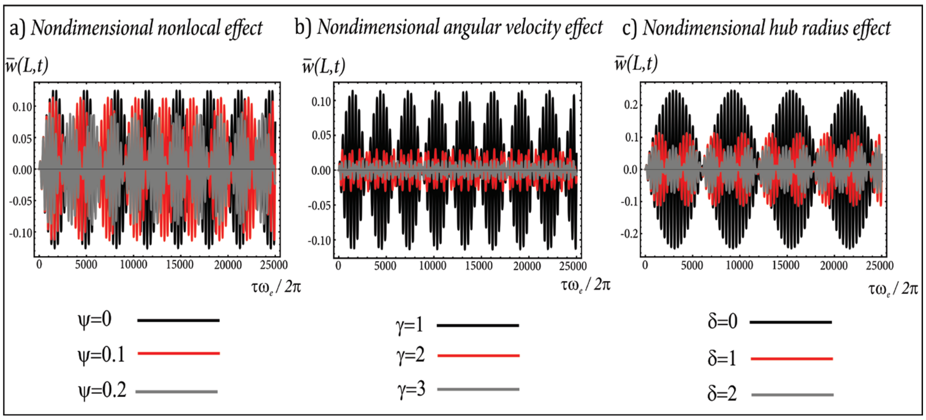

FIGURE 11. Longer time histories of the forced undamped vibration of the rotating nonlocal cantilever nano-beam a) nonlocality effects; b) angular velocities effects; c) hub radius effects.

Nondimensional nonlocal effect

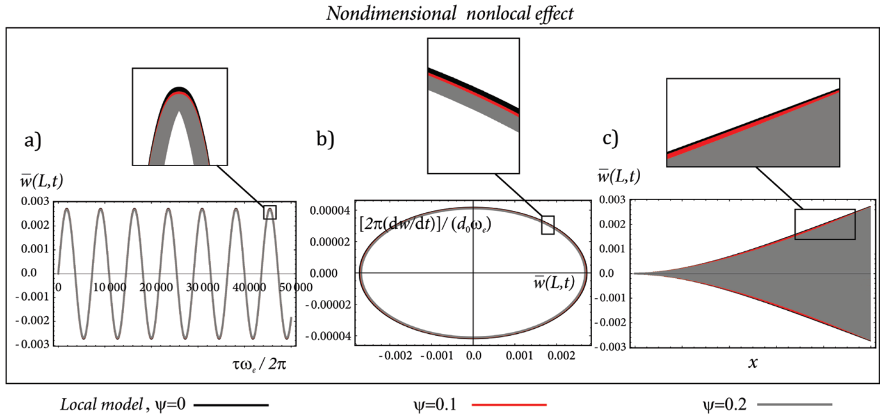

FIGURE 12. Time domain of the damped vibration of the rotating nonlocal cantilever nano-beam with the nonlocality effect; a) time histories; b) phase plots; c) deformed shapes.

Nondimensional angular velocity effect

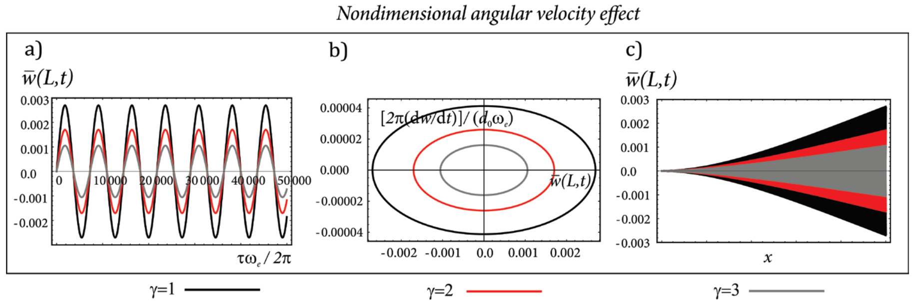

FIGURE 13. Time domain of the damped vibration of the rotating nonlocal cantilever nano-beam with the effect of angular velocity; a) time histories; b) phase plots; c) deformed shapes.

Nondimensional hub radius effect

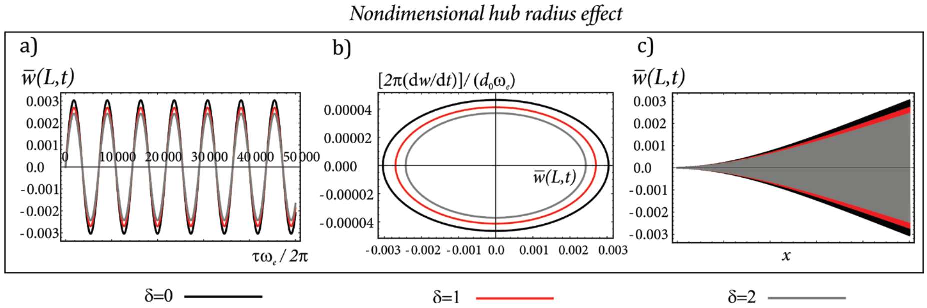

FIGURE 14. Time domain of the damped vibration of the rotating nonlocal cantilever nano-beam with the hub radius effect; a) time histories; b) phase plots; c) deformed shapes.

From the detailed analysis of the damped system, it is important to note that the qualitative character of the increase in the effect of nonlocality, angular velocity and hub radius, leads to a decrease in the transversal deflections of the nano-model. The quantitative characteristic of each parameter is determined and presented. An interesting phenomenon is discovered in the case when the maximal deformed shapes of the nano-model are analyzed under the nonlocality effect. It can be seen that the increase in the nonlocality effect slightly decreases the maximal deflections of the beam's points close to the middle of the beam, see Fig. 12. This means that the nonlocality effect is the highest at the middle of the beam if we observe all beam's points. These changes between the maximal deflections of the beam's points are smooth. The observed phenomenon does not exist in the other examples for the analyzed hub radius and velocity effect. An increase in the hub radius and velocity leads to a decrease in the maximal deflections, see Figs. 13 and 14, and their effect is the highest on the free end of the beam. All used parameters for obtained results are presented in Ref. [18].

The novelty of the study from Ref. [18] lies in the transient responses of the forced excited of the rotating cantilever nano-beams with the nonlocality magnitude effect. The presented dynamic behavior of a rotating nano-beam under external excitation possesses practical interest. Functional properties of rotating structures with various effects of external excitations can be improved to better satisfy the motion of the shafts in hybridized nano-generators.

3.4. Thermal effect on the free vibration and buckling of a double-micro-beam system. Using micro/nano structures in a high temperature environment leads to certain changes in the stiffness. Recently, the vibration and buckling studies of beams with the microstructure effect have been increasingly present in the scientific community. As is well known, the classical continuum mechanic theory does not contain any internal material length scale parameter and they are not able to describe these effects. The structural elements such as beams, plates, and membranes in the micro or nano length scale are frequently used as components in micro/nano electromechanical systems (MEMS/NEMS).

Based on the of the modified couple stress theory - MCST, in Ref. [19] the oscillatory system of two parallel Euler–Bernoulli micro-beams which are continuously joined by a Pasternak elastic layer under the influence of axial loading including with the temperature change effect, have been discussed (see Fig. 15). Both micro-beams are rectangular and have the same length L, thickness h, width b. The beams are simply supported at the ends and under the effect of the axial compressive load with the temperature change effect. For the sake of simplicity, it was adopted that the two parallel beams of the elastically connected double-beam system have the same bending stiffness EI and cross-sectional area A. Both micro-beams have the same material characteristics ρ. Presented paper was published in Facta Universitatis Series: Mechanical Engineering.

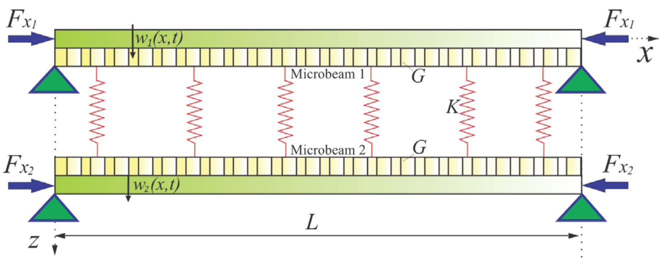

FIGURE 15. Double-micro-beam system coupled by the Pasternak's layer

The equations of motion can be expressed in the terms of the displacements W_{01} and W_{02}. The higher-order main differential equations are derived using the Hamilton principle

$$\left(D_{xx}+A_{xz}l^2\right)\frac{\partial^4 W_{01}}{\partial x^4}+m_0\frac{\partial^2 W_{01}}{\partial t^2}+\left(F_{m1}+F_T\right)\frac{\partial^2 W_{01}}{\partial x^2}-G\frac{\partial^2 W_{01}}{\partial x^2}+K\left(W_{01}-W_{02}\right)=0, \quad (8.1)$$

$$\left(D_{xx}+A_{xz}l^2\right)\frac{\partial^4 W_{02}}{\partial x^4}+m_0\frac{\partial^2 W_{02}}{\partial t^2}+\left(F_{m2}+F_T\right)\frac{\partial^2 W_{02}}{\partial x^2}-G\frac{\partial^2 W_{02}}{\partial x^2}-K\left(W_{01}-W_{02}\right)=0, \quad (8.2)$$

where K and G are the spring constants of the Winkler and Pasternak elastic medium, respectively, D_{xx} and A_{xz} are the stiffness components, $m_0=\rho A$ is the mass of micro-beams, and l is the length scale parameter.

The first variation of the work done by the axial forces is $F_{xi}=F_{mi}+F_T$, $(i=1,2)$, where $F_T=A_{xx}\alpha\Delta T$ is the axial force due to the influence of the temperature change and F_{mi}, $(i=1,2)$ is the axial forces due to the mechanical loading for the first and second micro-beam. The separation of variables method (known as the Fourier method) is used for the main equations to obtain the free vibration frequencies and critical buckling loads of the Euler-Bernoulli double-micro-beam system (EBDMBS). In Ref. [19] in detailed is developed of expression of buckling load in out-of-phase and in-phase modes.

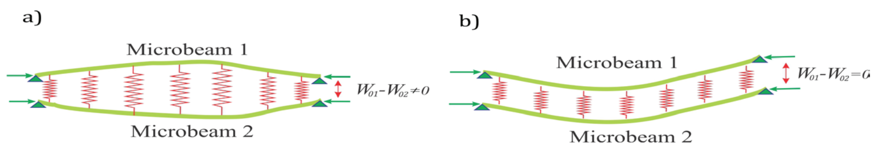

FIGURE 16. Vibration of the double-micro-beam system (a) Out-of-phase modes, (b) in-phase modes

The length scale parameter, temperature change effect, critical buckling load, thickness/material parameter, Pasternak's parameter and Poisson's effect are discussed in detail. Also, the effect of different mentioned parameters on the natural frequency, frequency under compressive axial loading, critical buckling load and critical temperature of EBDMBS with thermal effect are presented. Effect of the material length scale parameter and thermal effect on the two different cases of phase modes of vibration and buckling state are discussed, see Fig.16. Based on the presented analysis we concluded that the in-phase vibration mode and buckling state of the EBDMBS is independent of the stiffness of the connecting springs while is dependent of Pasternak's layer and temperature effect and hence the EBDMBS can be treated as a single micro-beam.

In order to confirm the present study, we have shown in tabular form a comparison of thermal effect on the dimensionless natural frequency of the system for three modes with the results found in the literature, see Ref. [19].

It is concluded that the presented results are in perfect agreement with the results observed in Ke et al. [26]. It is shown that the inclusion of the thermal effect decreases the frequencies of the micro-beam one. Also, the effect of the Pasternak parameter $k_p=G/A_{xx}$ for a greater mode leads to the increase in natural frequencies, but including the temperature change, the frequency is decreased and leads to the decrease stiffness of the system. The numerical results obtained for the natural frequency with Poisson's effect and suggested by the present Euler-Bernoulli beam model are always higher than that without Poisson's effect. The thermal effect on the natural frequency is very low for the micro-beam one of EBDMBS and with a small ratio of h/l, while it is significant for the micro-beam with a large ratio of h/l, see Fig. 17. The impact of compressive axial loading on the natural frequencies of EBDMBS transverse vibration leads to the following observations:

The temperature change effect has an impact on both micro-beams.

The lower and higher natural frequency under compressive axial loading decrease with the increasing axial compressive load and also decrease with a temperature change increase. The reason for that is that the thermal effect leads to the reduction in stiffness and such a behavior leads to the softening of the materials of the EBDMBS.

The effect of compressive axial loading on the lower natural frequency is almost independent from the axial compression ratio, whereas on the higher natural frequency it depends on.

For the higher value of the Pasternak parameter, the critical buckling load has a higher value which decreases with the increasing temperature change.

The critical buckling temperature for the presented systems is always lower than for the classical theories.

The critical scale load ratio of the modified and local critical buckling loads at the low temperature environs increases with the increasing length scale parameter, see Fig. 18.

All these observations can be useful for modern electromechanical systems. Physical views of these paper may be useful for the design and vibration analysis of micro-resonators and micro-sensors applications. We have shown that using the presented system with the temperature change leads to considerable changes in stiffness, i.e. the thermal effect leads to the reduction in stiffness and such a behavior leads to the softening of the materials of the EBDMBS. All used parameters for obtained results are presented in Ref. [19]. Also, the results of this paper are better described in Ref. [19].

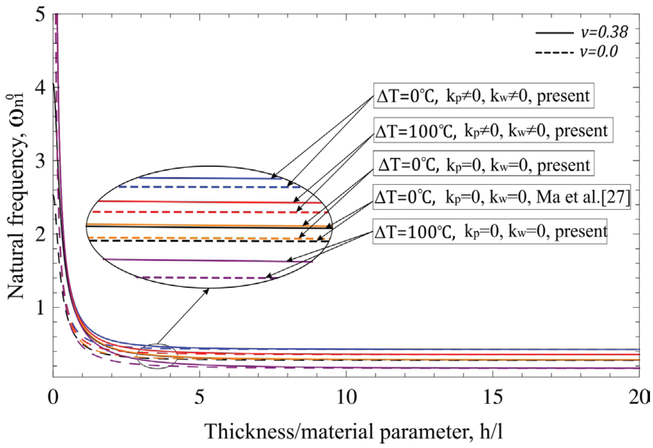

FIGURE 17. The natural frequency of the EBDMBS varying with micro-beam thickness, temperature effect and Pasternak's parameter.

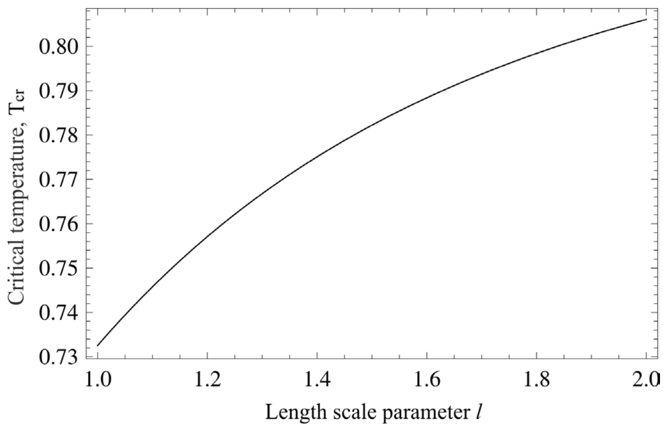

FIGURE 18. Length scale parameter T_{cr} at a low temperature environs

One more paper under the name "The critical load parameter of a Timoshenko beam with one-step change in cross section" is published in Facta Universitatis Series: Mechanical Engineering, (see Ref. [28]).

Based on the modified couple stress theory – MCST, one more result (see Ref. [29]) is presented at 5th International Congress of Serbian Society of Mechanics in Arandjelovac, Serbia, under the name „Dynamic analysis of micro-beam under the action of moving micro-particle". Ref. [29] is shown that the effect of the velocity of the moving micro-particle also plays an important role, with their magnification, value of deflections is increased. Practical applications of this research are in MEMS devices which allow to build sensors and actuators, together with measurement, control, and signal conditioning circuitry, and equipped with power and communications, all in the tiniest space.

4. Conclusion

In the paper, a brief insight into the results achieved by the author during his participation in the project is considered. It is also noted where the results were published and at which conference were presented. Different composed nano-systems and micro-systems which used nonlocal theory and modified couple stress theory are presented. In a shorter form, the papers from various conferences are illustrated. Forced vibration of a double single-walled carbon nanotube system under the influence of an axial magnetic field, forced transverse vibrations of an elastically connected nonlocal orthotropic double-nanoplate system subjected to an in-plane magnetic field, forced vibrations of rotating cantilever nano-beam are presented with significant influences of different parameters on transverse vibrations responses. Important observations during the research for all mentioned papers are presented.

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