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STRUCTURAL ANALYSIS AND OPTIMIZATION OF LAYERED COMPOSITE STRUCTURES: NUMERICAL AND EXPERIMENTAL INVESTIGATIONS

Abstract. The investigation in this work is focused on developing reliable computation procedure to analyze initial failure load for pin-loaded holes at the layered composite structures. Finite element method (FEM) is used to determine stress distribution around the fastener hole. The model takes into account contact at the pin-hole interface. Combining Chang-Scott-Springer characteristic curve model and Tsai-Wu initial failure criterion are used to determine joint failure. Special attention in this work is on pin-load distributions and its effect on load level of failure and its location. Here is contact finite element pin/lug model analyzed. The influence of stacking sequences of layered CFC composites type NCHR 914/132/300, containing pin-loaded holes is investigated, too. The computation results are compared with own and available experimental results. An efficient optimization method is presented for minimum weight design of the large-scale structural system such as aircraft structural systems. The efficiency of method is based on application of the two-level approach in structural optimization structural systems. Optimization method presented here is based on combining optimality criterion (OC) and nonlinear mathematical programming (NMP) algorithms. Finite element analysis (FEA) are used to compute internal forces at the system level. The two-level optimization approach is applied to minimum-weight design of complex aircraft structures such as aircraft parashute composite beam subject to multiple constraints.

Mathematics Subject Classification (2010): Primary: 70-99, 74-99, 70F35, 92C10, 74L10; Secondary: 70Exx, 70KXX, 74M05, 37H20.

Keywords: composites, failure analysis optimization methods, composite structures, optimality criterions, initial failure constraints, buckling constraints, FEA

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1. Introduction

The application of composite materials in many structural designs has led to extensive research aimed at developing an understanding of the sensitivity of these structures to the presence of holes and inclusions. Because of high strength to weight ratio, composite materials find wide use in aircraft structures. Mechanical fastened joints are used to join the section of aircraft components such as fuselage, wings and empennages. Fiber reinforced composites have been used extensively in aircraft and spacecraft construction because of their high strength/weight ratios [1-8]. Design considerations of aircraft structures based on damage tolerance approach often require prediction of mixed-mode fatigue crack growth. In this approach the propagation path of a crack in a part is an essential aspect for the fatigue life simulation using fracture mechanics methodology [9-15]. Composite structural components are generally connected with other components by means of bolted joints because of low cost, simplicity and ease to assemble or disassemble. However drilled holes significantly reduce the load carrying capacity of composites due to the stress concentration in the vicinity of the boundary of the hole. Mechanical fastening remains a critical aspect of designing composite aircraft joints. The holes drilled for composite joints are the primary source of structural failure in aircraft structures. Developing efficient design procedure is still a challenge for aircraft designers due to anisotropic nature of composite, complex failure response, geometric parameters and stacking sequence. This reduction could be a cause of catastrophic failure. Therefore special attention must be given to design of the bolted joints. Mechanically fastened joints in composite structures are commonly used in aerospace vehicles. Due to the anisotropic and heterogeneous nature, joint problems in composite structures are more difficult to analyze than those in isotropic materials. Due to the significance of the problem many investigators have studied the strength of mechanically fastened joints in composite structures. Factors such as joint geometry and fiber orientation are important parameters for the mechanically fastened joints in composite plates.

This paper considers a computation method in failure analysis of layered composites containing pin-loaded holes.

To evaluate the strength of the mechanically fastened joints, several prediction methods have been proposed. One of the main prediction methods is Chang`s model [16-21]. In model [20] the joint is regarded to have failed when certain combined stresses have exceeded a prescribed value in any of the plies along the characteristic curve. The combined stress limit is evaluated using Yamada–Sun failure criterion [16-19,21]. Another main prediction method is progressive damage model. In this model, the logical methodology for modeling the joint problem is composed of three important steps: stress analysis, failure criteria and property degradation rules. Stress distributions in the plate are calculated and then a failure criterion is tested. If there is no failure, the load is increased. In the case of failure, material properties of failure nodes are reduced to an appropriate property degradation rules. Stresses are then redistributed at the same load and re-examined for any additional failures. The procedure continues until a point where excessive damage is reached [22-32]. In earlier works, Icten et al. [25, 29] established the behavior of mechanically fastened joints in woven glass-epoxy composites with $[(0/90)_3]$s and $[(\pm 45)_3]$s material configurations. The failure analysis based on Hashin and Hoffman criteria was performed and compared with experimental results. Okutan and Karakuzu [30] studied on the response of pin loaded laminated E/glass-epoxy composites for two different ply orientations such as [0/± 45]s and [90/± 45]s. The objective of this work is to study the behavior of graphite-epoxy pin loaded joints both numerically and experimentally, with particular attention given to the sensitivity of the model to different geometric dimensions. The two-dimensional finite element method was used to obtain stress distribution of the material. To determine the failure load and failure mode progressive damage prediction model was selected with Tsai-Wu Criteria. The mechanical properties of the composite material are obtained from standard tests like in [24,25, 33, 34].

In this analysis, based on the Chang et al. strength prediction model, the point stress failure criterion will be used to evaluate the characteristic lengths in tension and compression and a two-dimensional finite element analysis shown in [35-38].

Numerical (mainly finite element based approaches) methods have been used to perform stress analyses on mechanically fastened joints. Following this research line the main task is the deduction of the stress distribution around the fastener hole. To this aim different hypotheses are assumed on the pin-hole interaction concerning, for example, the modelling of the pin, the load distribution at the pin-hole boundary, the stacking sequence of the layered composite plates and so all the through-thickness effects like, for example, friction or clearances. In particular, the analytical methods are mainly based on orthotropic elasticity problems formulated in terms of complex variable theory, the numerical methods are grounded on two-dimensional finite element analyses in conjunction with classical lamination theory.

A pin-loaded plate, under plane stress conditions, has been analyzed and the obtained results have been compared with few experimental ones get from the literature and own experiments. The numerical findings, at least for the examined problem, are quite promising showing the potentialities of the proposed methodology and its competitiveness with respect to a burdensome step-by-step nonlinear analysis.

2. Characteristic length method CH

When a laminate is loaded through a fastener, such as pin or bolt, both sides of the fastener hole are subjected to high tensile stress due to stress concentration. On the other side, the front-area of the fastener hole experiences high compressive stress. Furthermore, as applied load increases and laminate deforms the contact surface between the fastener and the laminate changes.

One of the most common and efficient methods of predicting the strength is the characteristic length method. A practical method considered to predict the failure load of composite joints with the least amount of testing is the characteristic length method. This method was proposed by Whitney and Nuismer [37]. Application of the average stress failure criterion: Part II: Compression, and has been developed by Chang et al. and in [39]. Failure analysis of composite pin loaded joints. In the characteristic length method, two parameters, i.e. compressive and tensile characteristic length should be determined by the stress analysis associated with the results of bearing and tensile tests on the laminates with and without hole. Once the characteristic lengths are determined, an artificial curve connecting the compressive and tensile characteristic lengths named characteristic curve is assumed shown in Chang et al [17]. Failure of a joint is evaluated on the characteristic curve, not on the edge of the fastener hole. In this method the joint is taken to have failed when certain combined stresses have exceeded a prescribed in any of the plies along the characteristic curve.

In order to evaluate the strength of composite pinned joints, Fig. 1, the stress distribution along a characteristic dimension around the hole must first be evaluated. The conditions for failure can then be predicted with the aid of an appropriate failure criterion. The Tsai-Wu failure criterion [40] was used for this analysis. This criterion can be written as:

$$(F.I) = F_1\,\sigma_1 + F_2\,\sigma_2 + F_6\,\sigma_6 + F_{11}\,\sigma_{12} + F_{22}\,\sigma_{22} + \\ + F_{66}\,\sigma_{62} + 2\,F_{12}\,\sigma_1\,\sigma_2 \tag{1}$$

$$F_1 = \frac{1}{X_t} + \frac{1}{X_c} \tag{1.1}$$

$$F_2 = \frac{1}{Y_t} + \frac{1}{Y_c} \tag{1.2}$$

$$F_{11} = -\frac{1}{X_t X_c} \tag{1.3}$$

$$F_{22} = -\frac{1}{Y_t Y_c} \tag{1.4}$$

$$F_6 = 0 \tag{1.5}$$

$$F_{66} = \frac{1}{S^2} \tag{1.6}$$

$$F_{12} = 0 \tag{1.7}$$

where: F.I is failure index, σ_i (i=1, 2, 6) are stress components with respect to material principal axes and $X_{t,c}$, $Y_{t,c}$ are longitudinal and transverse tensile/compressive strengths of a unidirectional lamina and S is the ply shear strength. In this model, failure is expected to occur when the value of F.I is greater than or equal to unity.

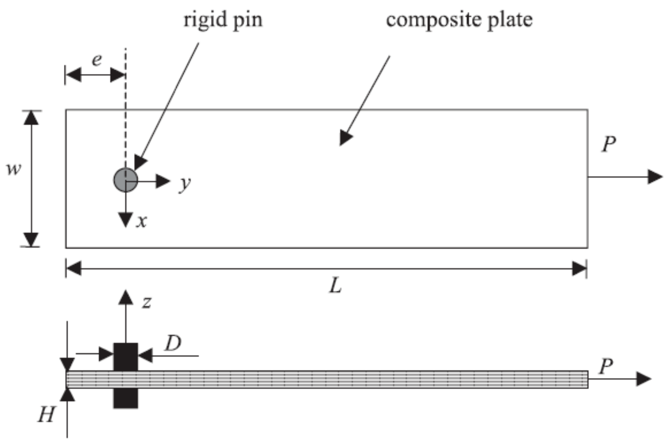


FIGURE 1. Geometry of the composite plate with a circular hole, subjected to pin

The characteristic curve is an artificial curve made of compressive and tensile characteristic lengths. Since the characteristic lengths are determined just for pure compression and tension, other combined failure modes are evaluated on the characteristic curve.

A popular method to construct the characteristic curve is proposed by Chang and Scott. The characteristic curve is expressed as follows:

$$r_c(\theta) = R + R_{0t} + (R_{0c} - R_{0t}) \cos\theta \tag{2}$$

where R_{oc} and R_{ot} are compressive and tensile characteristic lengths, respectively. The angle θ is measured counterclockwise or clockwise from the loaded direction toward the sides of the fastener hole as shown in Fig. 2.

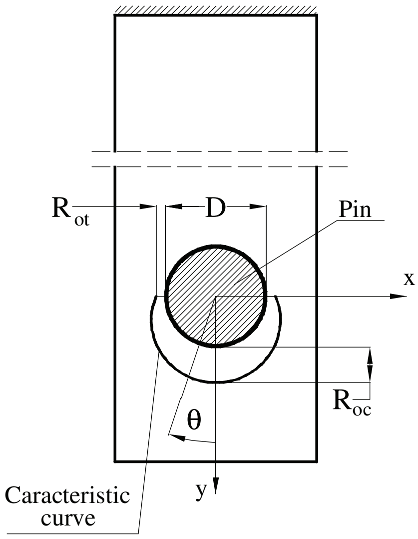


FIGURE 2. Characteristic curve schematic diagram.

The ultimate failure of a joint is generally divided into three modes depending on the failure location, θ_f - Chang et al [21].

$0 \leq \theta_f \leq 15^0$	Bearing mode	
$30^0 \leq \theta_f \leq 60^0$	Shear-out mode	(3)
$75^0 \leq \theta_f \leq 90^0$	Net-tension mode	

Mechanically-fastened bolted-joints under tensile loads frequently damage in four basic modes that is named as cleavage mode, net tension mode, shear out mode and bearing mode. These failure modes are shown in Fig. 3, from P.K.Mallick [41], Fiber-reinforced composites materials, manufacturing, and design.

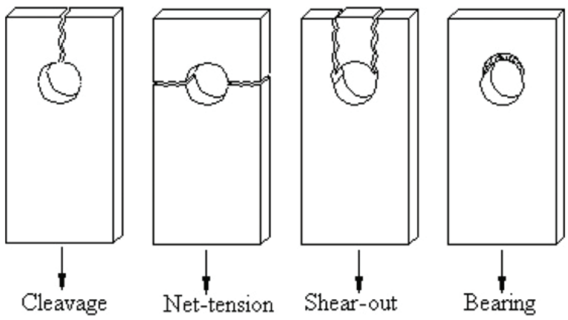


FIGURE 3. Common failure modes in bolted composite plates

There are, in general, four main failure modes: net tension, shear out and bearing as shown in Fig. 3. Net tension and shear out modes are catastrophic and result from excessive tensile and shear stresses. Bearing mode is local failure and progressive, and related to compressive failure. Cleavage failures are associated with both an inadequate end distance and too few transverse plies. Net tension and shear out modes can be avoided by increasing the end distance (e) and width (W) of the structural part for a given thickness but bearing failure cannot be avoided by any modification of the geometry.

Characteristic lengths (CLs) are key factors in the characteristic curve method (CCM), which is widely used in engineering to predict the failure of composite multi-bolt joints. They directly affect the accuracy of predicting the joint failure.

Both the characteristic lengths in tension, Rot, and compression, Roc, must be determined by stress analysis associated with the results of bearing and tensile tests on notched and unnotched plates before employing an appropriate failure theory along the characteristic curve, r_c as shown in Fig. 2. By definition, the characteristic length (in tension or compression) is the radial distance from the hole boundary over which the plate must be critically stressed to initiate a sufficient flaw that can cause failure.

2.1. Tensile characteristic length (R_t). Konish and Whitney [42] in Approximate Stress as in an Orthotropic Plate Containing a Circular Hole, proposed the following approximate solution for the normal stress distribution σ_y (x, 0) in an infinite orthotropic plate with an open hole loaded in tension

$$\frac{\sigma_y(x,0)}{P} \cong 1+\frac{1}{2}\xi^{-2}+1+\frac{3}{2}\xi^{-4}-\frac{(K_T^\infty-3)}{2}(5\xi^{-6}-7\xi^{-8}) \tag{4}$$

where

$$\xi=\frac{X}{R} \tag{5}$$

and K_T^∞ is the orthotropic stress concentration factor defined as

$$K_T^\infty = 1+\sqrt{\frac{2}{\bar{S}_{22}}(\sqrt{(\bar{S}_{11}\bar{S}_{22})-\bar{S}_{12}+\frac{(\bar{S}_{11}\bar{S}_{22}-\bar{S}_{12}^2)}{2\bar{S}_{66}}})} \tag{6}$$

The point stress failure criterion can be applied to Eq. 3 to determine the characteristic dimension at which failure is expected to occur. This criterion assumes that failure occurs when the transverse stress at some distance R_{0t}, away from the opening reaches the unnotched tensile strength, σ_F, of the material. This criterion is expressed as

$$\sigma_y(R+R_t,0)=\sigma_F \tag{7}$$

Substituting Eq. (6) into Eq. (3), the ratio of the notched to the unnotched strength is obtained as

$$\frac{\sigma_N^{\infty}}{\sigma_F} = \frac{2}{2 + \xi^{-2} + 3\xi^{-4} - (K_T^{\infty} - 3)(5\xi^{-6} - 7\xi^{-8})} \tag{8}$$

where σ_N^{∞} , the tensile strength of the notched laminate is equal to P_f, the applied stress at failure, and at $x = R + R_t$

$$\xi = \frac{R + R_t}{R} \tag{9}$$

Values of tensile characteristic length R_t can be determined from Eq. (7) and Eq. (8), if data for both the notched and unnotched strengths are available.

2.2. Compressive characteristic length, R_c. As stated previously, in order to evaluate failure of mechanically fastened joints based on the characteristic curve model, the characteristic length in compression must also be evaluated. This value is obtained from evaluation of the stress distribution in a plate with an inclusion subjected to compressive loading. For determination compressive characteristic length R_{0c} here analytical method proposed by Kweon et al. [43]. A new method to determine the characteristic lengths of composite joints without testing, without bearing tests to evaluate characteristic length in compression. This method utilizes any arbitrary load to compute the mean bearing stress.

As previously emphasized, in order to estimate the failure of mechanical joints, based on the characteristic curve model, the characteristic pressure length must also be determined. This value was obtained based on the stress distribution in a multilayer composite panel with pressure-loaded reinforcement.The stress distribution on an infinite orthotropic plate, which contains reinforcement and is exposed to the action of normal and transverse forces at infinity, can be analyzed using various complex approaches according to Lechitski [68]. The stresses in the multilayer composite can be expressed as follows:

$$\begin{aligned}
\sigma_x &= \sigma_x^0 + 2R_e\left[\mu_1^2\phi_1'(z_1) + \mu_2^2\phi_2'(z_2)\right] \\
\sigma_y &= \sigma_y^0 + 2R_e\left[\phi_1'(z_1) + \phi_2'(z_2)\right] \\
\tau_{xy} &= \tau_{xy}^0 - 2R_e\left[\mu_1\phi_1'(z_1) + \mu_2\phi_2'(z_2)\right]
\end{aligned} \tag{10}$$

where σ_x^0, σ_y^0 i τ_{xy}^0 in Eq. (10) they represent the stresses in the plate without reinforcement which is exposed to a given external force.

For the case of an infinite plate with reinforcement, which is loaded in the y direction by an external load P at infinity (Fig. 2), the stress function can be expressed as follows [8]:

$$\phi_1(z_1)=\frac{1}{2(\mu_1-\mu_2)}\left[\mathrm{iRA}-\mu_2\mathrm{R}(\mathrm{B}-\mathrm{P})\right]\frac{1}{\varsigma_1}$$
$$\phi_2(z_2)=\frac{1}{2(\mu_1-\mu_2)}\left[\mathrm{iRA}-\mu_1\mathrm{R}(\mathrm{B}-\mathrm{P})\right]\frac{1}{\varsigma_2} \quad (11)$$

where is

$$\varsigma_k=\frac{z_k+\sqrt{z_k^2-R^2\left(1+\mu_k^2\right)}}{R\left(1-i\mu_k\right)} \quad (12)$$

and

$$z_k=x+\mu_k y \qquad k=1,2 \quad (13)$$

In Eq. (11), the magnitudes A and B, which denote the normal stresses in the reinforcement along x and y axes, are determined on the basis of the following relations:

$$A=\frac{P}{\Delta}\left[\overline{S}_{11}\overline{S}_{22}(k+n)+\overline{S}_{11}\overline{S}'_{22}k(1+n)+\overline{S}_{22}\left(\overline{S}_{12}+\overline{S}_{66}+\overline{S}'_{12}\right)\right]$$
$$B=\frac{P}{\Delta}\left[\overline{S}_{22}\left(\overline{S}_{11}-\overline{S}'_{11}\right)+\overline{S}_{11}\left(\overline{S}_{12}-\overline{S}'_{12}\right)k(1+n)\right] \quad (14)$$

where the quantities marked ' represent the coefficients of elasticity for reinforcement

$$\Delta=\left(\overline{S}_{11}\overline{S}_{22}+\overline{S}'_{11}\overline{S}'_{22}\right)k+\overline{S}_{22}\left(\overline{S}_{66}+2\overline{S}_{12}\right)+\left(\overline{S}_{11}\overline{S}'_{22}k+\overline{S}_{22}\overline{S}'_{11}\right)n-\left(\overline{S}_{12}-\overline{S}'_{12}\right)^2 k \quad (15)$$

and

$$k=-\mu_1\mu_2 \quad \mathrm{i} \quad n=-i\left(\mu_1+\mu_2\right) \quad (16)$$

From Eq. (10) and Eq. (11), the normal stress, for an infinite orthotropic plate without aperture exposed to a uniform stress P at infinity, can be written as follows:

$$\frac{\sigma_y(x,0)}{P}=1+R_e\left\{\frac{1}{\mu_1-\mu_2}\left[\frac{-\left(1-\overline{B}\right)\left(\mu_1-i\mu_1\mu_2\right)-\left(i+\mu_1\right)\overline{A}}{\psi_1}+\frac{\left(1-\overline{B}\right)\left(\mu_1-i\mu_1\mu_2\right)+\left(i+\mu_2\right)\overline{A}}{\psi_2}\right]\right\} \quad (17)$$

where is

$$\overline{A}=\frac{A}{P},\quad \overline{B}=\frac{B}{P} \quad (18)$$

and

$$\psi_k=\frac{1}{\sqrt{\left(\xi^2-1-\mu_k^2\right)}\left(\xi+\sqrt{\xi^2-1-\mu_k^2}\right)},\quad k=1,2 \quad (19)$$

Eq. (17) represents the stress distribution near the reinforcement on an infinite orthotropic plate without hole. Instead of applying the refractive criterion to determine R_c, a program was written to determine the real solutions of this equation. Based on the analysis, the real solutions of Eq. (17) are:

$$\frac{\sigma_y(x,0)}{P} = 1 + \left(\frac{\psi_{11} + \psi_{12} + \psi_{21} + \psi_{22}}{\Delta_T} \right) \tag{20}$$

where is

$$\psi_{11} = -\bar{A}\beta^2 + (\bar{B}-1)\beta^3 + \bar{A}\alpha^2(1+\beta) - (\bar{B}-1)(1+\beta)\alpha^3 + \bar{A}\xi\left(\sqrt{\xi^2-1+\alpha^2} - \sqrt{\xi^2-1+\beta^2}\right)$$

$$\psi_{12} = \beta\left(1 - \bar{A} - \bar{B} - \xi^2 + \bar{A}\xi^2 + \bar{B}\xi^2 + \bar{A}\xi\sqrt{\xi^2-1+\alpha^2} - (1-\bar{B})\xi\sqrt{\xi^2-1+\beta^2}\right)$$

$$\psi_{12} = -\alpha\bar{A}\left(\xi^2 - 1 + \beta^2 + \xi\sqrt{\xi^2-1+\beta^2}\right)$$

$$\psi_{22} = \alpha(\bar{B}-1)\left(1 - \xi^2 + \beta^3 - (\beta+1)\xi\sqrt{\xi^2-1+\alpha^2} + \beta\xi\sqrt{\xi^2-1+\beta^2}\right)$$

$$\Delta_T = (\alpha-\beta)\left[\sqrt{\xi^2-1+\alpha^2}\sqrt{\xi^2-1+\beta^2}\left(\xi+\sqrt{\xi^2-1+\alpha^2}\right)\left(\xi^2\sqrt{\xi^2-1+\beta^2}\right)\right] \tag{21}$$

and

$$\alpha = \frac{1}{2}\left\{ \sqrt{\frac{E_x}{G_{xy}} - 2\nu_{xy} + 2\sqrt{\frac{E_x}{E_y}}} + \sqrt{\frac{E_x}{G_{xy}} - 2\nu_{xy} - \sqrt{\frac{E_x}{E_y}}} \right\}$$

$$\beta = \frac{1}{2}\left\{ \sqrt{\frac{E_x}{G_{xy}} - 2\nu_{xy} + 2\sqrt{\frac{E_x}{E_y}}} - \sqrt{\frac{E_x}{G_{xy}} - 2\nu_{xy} - \sqrt{\frac{E_x}{E_y}}} \right\} \tag{22}$$

The failure criterion can then be applied to obtain the compressive characteristic length, R_c. The criterion can be written in the following form:

$$\sigma_y(R + R_c, 0) = \sigma_{Fc} \tag{23}$$

and by substituting in Eq. (20) gives

$$\frac{\sigma_{Nc}^{\infty}}{\sigma_{Fc}} = \frac{1}{\left[1 + \left(\frac{\psi_{11} + \psi_{12} + \psi_{21} + \psi_{22}}{\Delta_T}\right)\right]} \tag{24}$$

where σ_{Nc} is the compressive strength of a multilayer composite without a stress concentration, σ_{Nc} is the compressive strength of multilayer composite with stress concentration, is equal to the introduced failure load, P_f, at distance $x = R + R_c$

$$\xi = \frac{R + R_c}{R} \tag{25}$$

In order to determine R_c from Eq. (24) and Eq. (25) it is necessary to know the value of the compressive strength of the plate without stress concentration and the strength of the plate with the source stress concentration.

3. Failure loading joints

3.1. Experimental verification. Experimental tests of mechanically fastened joints carried out to servo-hydraulic SCHRENCK RM 100 system, Fig. 4.

Here is tested five specimens made from carbon type composite material NCHR 914/132/300, with stacking sequence $[\pm 45/0_2]_{3S}$. Load-displacement diagrams for tested specimens, Fig. 5, are registrated by means of analogy-digital HEWLETT PACKARD 7090 A system.

FIGURE 4. Servo-hydraulic SCHRENCK RM 100 system

In the experimental study, every composite joint was loaded until tear occurred. The general behavior of the composite was obtained from the load/displacement curves as shown in Fig. 5. The load-displacement curves are linear until the sudden lost of load.

In the experimental study, every composite joint was loaded until tear occurred. The general behavior of the composite was obtained from the load/displacement curves as shown in Fig. 5. The load-displacement curves are linear until the sudden lost of load.

Results of tested specimens type of mechanically fastened joints with stacking sequence $[\pm 45/0_2]_{3S}$ are given in Table 1. Initial failure loads of these specimens are given in Table 1.

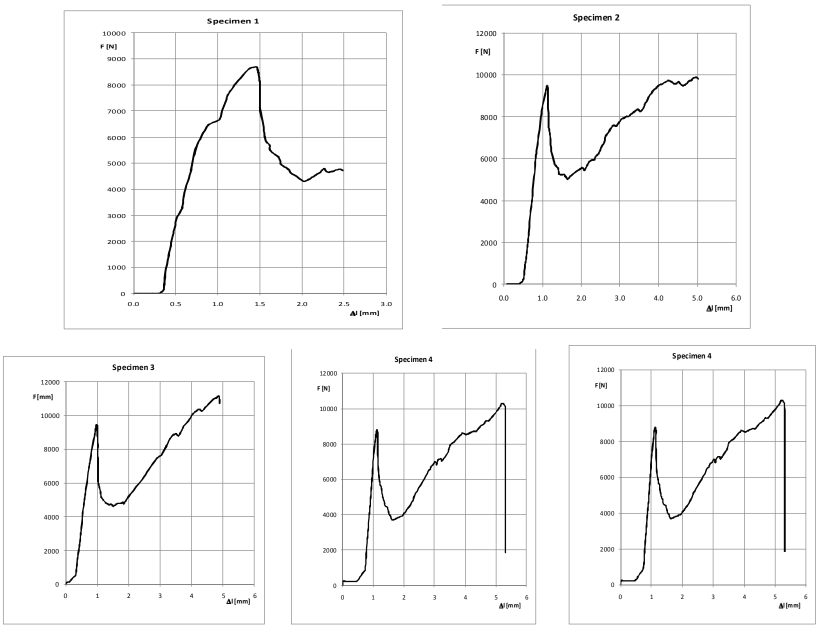


FIGURE 5. 2 Force-displacement curves of different specimens

TABLE 1: Experimentally determined initial failure loads of specimens with sequence $[\pm 45/0_2]_{3S}$

	Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5
F_{exp}[daN]	900	950	950	885	820

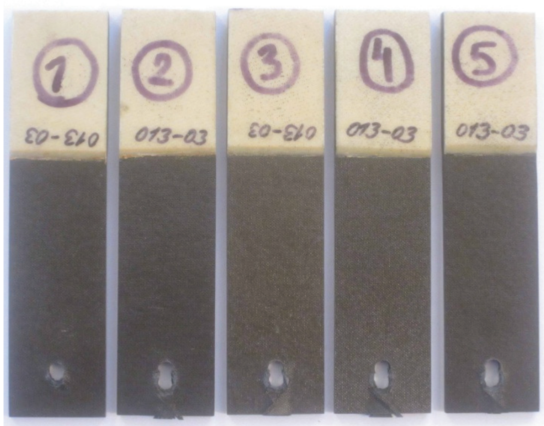


FIGURE 6. Image of damaged specimens for stacking sequence $[\pm 45/0_2]_{3S}$

By using conventional statistical procedure Maksimovic [35], Instability finite element analysis of fiber reinforced composite structures based on the third order theory mean value of experimentally determined initial failure load is defined in the next form: Descriptive statistics (mean force=901 daN, std=53.9). It means that mean value of experimental determined initial failure load of composite specimens, Table 1, is 901 daN with standard deviation of 53.9 daN.

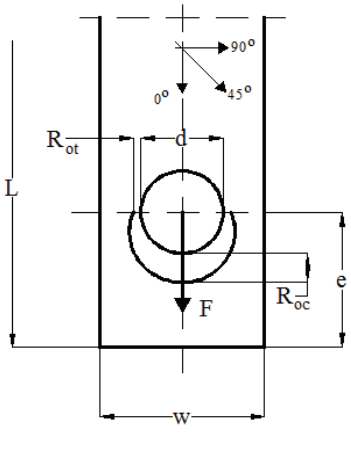
Experimental results are shown that bearing failure mode occurs to all tested specimens, Fig. 6.

3.2. Numerical validation. To validate computation procedure of mechanical fastened joints numerical examples are included. Geometry properties of mechanical fastened joint at composite structure are shown in Fig. 7. To simulate the contact between the pin and the composite, the pin circumference is modeled as a rigid surface and the hole edge as a deformable surface. Contact pairs are then defined between the two surfaces.

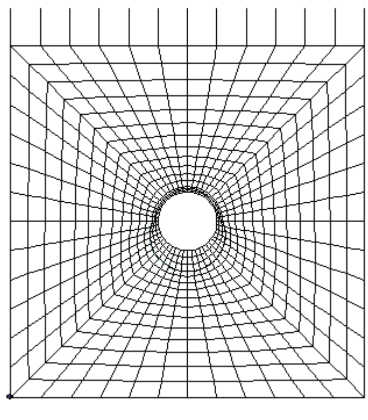
A two-dimensional finite element model is developed using MSC/NASTRAN software [44]. Finite element model of contact problem of pin-loaded joint is shown in Fig. 8. Lug and pin are made from CFC composite and steel materials, respectively.

Mechanical properties of these materials are given in Tables 2 and 3. Mechanical tests were carried out to measure the engineering constants by using standard test methods due to the determination of graphite–epoxy composite material properties.

For purpose of comparison failure analysis of mechanical fastened joints is carried and using cosine load distribution too.



a)



b)

FIGURE 7. Geometry properties of mechanical fastened joint at composites a) Geometrical model of contact problem of pin-loaded joint and b) Finite Element mesh with characteristic curve

To determine initial failure load of mechanical fastened joint the procedure is composed of stress analysis and failure analysis using adequate initial failure criteria along characteristic curve. Tsai-Wu failure criteria association with material property degradation is used in the analysis to predict to failure load and to differentiate failure modes.

The strategy for the finite element modeling of the joints is the same as in the finite element model of the laminate for bearing tests shown in Fig. 7. Nonlinear finite element analysis for the joints is conducted by MSC/NASTRAN. Interface between fasteners and laminates is modeled by the slide line contact element provided by the software. The slide line element in MSC/NASTRAN was adopted to simulate the contact between the pins and the laminates. The pin and the laminate were modeled using CQUAD4 shell elements.

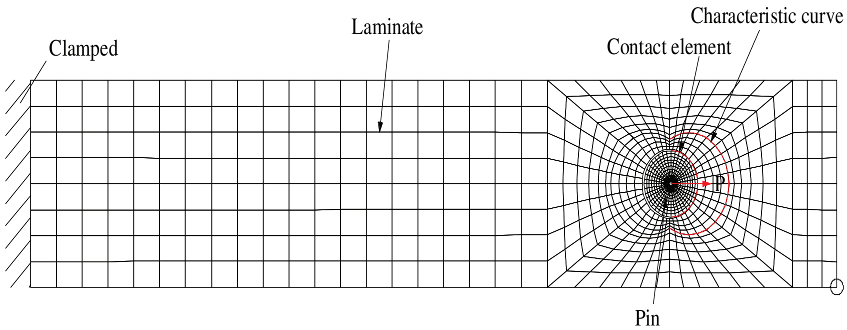


FIGURE 8. Finite element model of contact problem of pin-loaded joint

To study mechanically fastened joints of a layered composite plate here frictional contact conditions are considered. For this purpose Coulomb friction law is used. The contact constraints are handled by extended interior penalty methods. To include the non-differential term due to the Coulomb friction the perturbed variation principle is adopted. The presented computed results are compared with own experimental results. It`s well known that the geometric factors, clearances and static friction coefficients play important roles in determining contact stress. As expected with variation of these parameters, it can be found that the each point takes a different magnitude of pin loading, and extended parametric studies on these factors may be needed for design consideration [45]. In this paper variation of static friction coefficients is considered.

The first is made calculation of composite joint for experimentally obtained values of the tensile characteristic length, R_{ot}, and the compressive characteristic length, R_{oc}. The values of characteristic lengths are given in Table 4. In the same table are given a comparison of numerical and own experimentally obtained failure loads.

TABLE 2: Mechanical properties of CFC material

Longitudinal Young's Modulus	$E_{11} = 13300 \dfrac{daN}{mm^2}$
Transverse Young's Modulus	$E_{22} = 850 \dfrac{daN}{mm^2}$
Shear Modulus	$G_{12} = 580 \dfrac{daN}{mm^2}$
Poission's Ratio	$\nu = 0.33$
Longitudinal Tensile Strength	$F_{11}^{T} = 175.7 \dfrac{daN}{mm^2}$
Longitudinal Compressive Strength	$F_{11}^{C} = 135 \dfrac{daN}{mm^2}$
Transverse Tensile Strength	$F_{22}^{T} = 7.2 \dfrac{daN}{mm^2}$
Transverse Compressive Strength	$F_{22}^{C} = 23 \dfrac{daN}{mm^2}$
Rail Shear Strength	$F_{12} = 12.5 \dfrac{daN}{mm^2}$
One Layer Thickness	$t = 0.133\,mm$

TABLE 3: Mechanical properties of pin

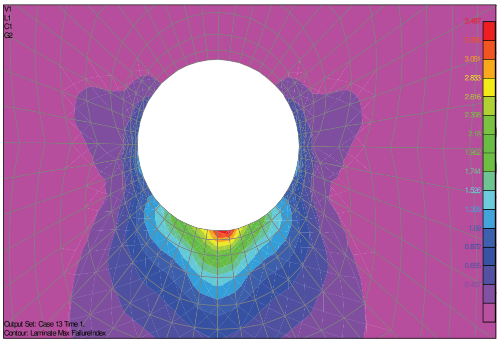
Young's Modulus	$E_{12} = 21000 \dfrac{daN}{mm^2}$
Shear Modulus	$G_{12} = 8140 \dfrac{daN}{mm^2}$
Pooission's Ratio	$\nu = 0.29$
Ultimate Tensile Strength	$\sigma_{doz} = 125 \dfrac{daN}{mm^2}$
Ultimate Shear Strength	$\tau_{doz} = 80 \dfrac{daN}{mm^2}$
Static Friction Coefficient	$\mu = 0.25$

TABLE 4: Comparisons computation with experimental results

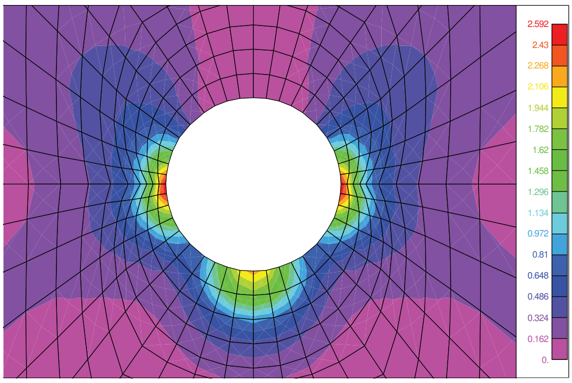
Stacking sequence $[\pm 45/0_2]_{3S}$							
d [mm]	e [mm]	w [mm]	L [mm]	Rot [mm]	Roc [mm]	F^{exp} [daN]	Failure mode
5	15	30	100	0.41	2.201	901	B

Case	Failure load (F.I=1)				Difference F^{exp} and F^{num}	
	Contact FEM		Cosine		Contact	Cosine
	F^{cont} [daN]	FI cont	F^{cos} [daN]	FI cos		
1	975	0.99	1100	1.03	7.6 %	18.1 %

In Figures 9 and 10 are shown the distributions of coefficient of the initial failure (FI) in the composite lug and along the characteristic curve for the numerical determined failure load.



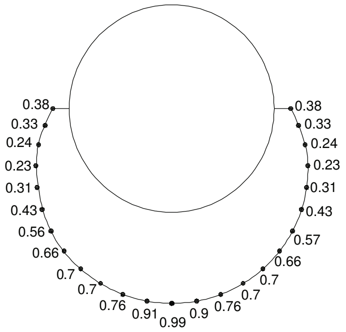
a) Contact (F= 975 daN)



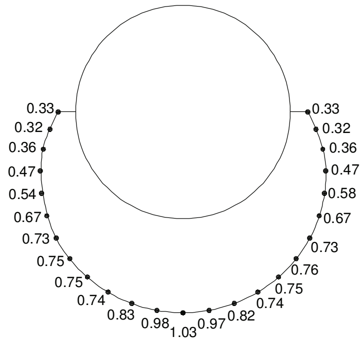
b) Cosine (F= 1100 daN)

FIGURE 9. Distributions of F.I at composite lug for numerical failure load for contact and cosine distributions

With the variation of ply orientations in the laminate, the stiffness properties change and consequently the stress levels pertaining to the same boundary conditions differ. For optimum strength requirements, one needs to compute stress levels in the laminate for given boundary conditions with different ply orientations. Because, in many practical situations, the closed form elasticity solutions are not available, a finite element method has to be implemented. In the present investigation, the finite element method has been used to conduct the stress analysis of the laminate for a number of ply orientations. The present study consists of the computation of stress distributions in composite laminates with a free or a loaded fastener hole.



Contact (F= 975 daN)



Cosine (F= 1100 daN)

FIGURE 10. Distribution of F.I along characteristic curve for numerical failure load

The stress levels at various points in the constituent angle ply laminates are used to approximate the states of stress for multidirectional composite laminates with different ply.

Numerical values of initial failure loads are obtained using two procedures: (1) is based on using contact pin/lug finite element model and (2) is based on using cosine distribution between pin and lug. In Table 4 is shown difference between the numerical and own experimental results for failure loads and better agreement is in the case of using contact pin/lug finite element model, than using the cosine distribution of the load. From location of maximum value of Failure Index along characteristic curve, Fig. 10, is shown that numerical simulation gives bearing failure mode.

Coulomb friction law is used and the contact constraints are handled by extended interior penalty methods. The perturbed variation principle is adopted to treat the non-differential term due to the coulomb friction. The computed results by previous formulation are compared with own experimental results. Good agreement between computation and experimental results is obtained. In previous analyses static friction coefficient, $\mu = 0.25$, is considered.

TABLE 5: The Effects of Static Friction Coefficient on Contact load F^{cont}

Static friction Coefficient [μ]	F^{con} [daN]	Difference F^{exp} and F^{num} [%]
0.15	1100	18.1
0.20	981	8.2
0.25	975	7.6
0.30	1050	14.2

For this value of static friction coefficient obtained the contact load F^{cont} has the best agreement with experimental load (F^{exp} =901 daN), as shown in Table 5.

4. Conclusions about mechanically fastened joints

This paper investigates failure loads and failure modes in CFC layered composite plates, with circular hole which is subjected to rigid pin. In order to obtain failure modes in pin hole, parametric studies are performed, experimentally and numerically. The presented numerical results were compared with the own experimental results concerning failure load and failure modes. The following points are concluded:

- In present numerical study, combining finite element method for stress analysis together with Chang-Scott-Springer characteristic curve model and Tsai-Wu initial failure criterion are used to predict the failure load and failure mode of mechanically fastened joint. Two type pin/lug finite element models for stress distributions at the layered composite lugs are used.
- Summarizing the results, it is shown that the proposed numerical method, based on contact finite element pin/lug model predicts the strength of composite joints under pin loading within the maximum of 7.6% difference from the experimental results. Numerical finite element method, based on cosine pin/lug model predicts the strength of composite joints under pin loading within the maximum of 18.1% difference from the experimental results. In the case of the composite materials, both percent differences are correct and may be acceptable.

It means that proposed computation procedure based on combining the Chang-Scott-Springer characteristic curve model and Tsai-Wu initial failure criterion together with contact finite element pin/lug model for the stress analyses are efficient method to predict the failure load and failure mode of mechanically fastened joints at layered composites

5. An Efficient Optimization Method to Minimum Weight Design of Large-Scale Structural Systems

5.1. Introduction. One of the primary requirements in aircraft design is to ensure the minimum mass of the structure while satisfying the appropriate strength and rigidity where composite materials have found their significant application. Due to their good mechanical properties on the one hand and the possibility of their use for modeling the required strength and rigidity on the other hand, they have found significant application in the design of aircraft structure. Certainly, for that purpose, it is necessary to use modern methods for structural analysis and optimization. These are basically numerical methods based primarily on the application of the finite element method (FEM), which have found application in the field of structural analysis and optimization. When it comes to optimization, this primarily means providing a minimum weight while meeting certain limitations in terms of strength and stiffness.

Composite materials are widely used in the industry because of their superior mechanical, thermal, and chemical properties, e.g. high stiffness-to-weight and strength-to-weight ratios, corrosive resistance, low thermal expansion, vibration damping.

The growing use of high performance fiber composite materials has simulated interest in the development of optimization procedures for the design of laminates. The use of finite element methods in parallel with optimization techniques such as non-linear mathematical programming or optimality criteria make it possible to attack large and complex problems. Discretized optimality criteria methods were developed since non-linear mathematical programming techniques were inefficient for large structural problems. However during previous period considerable efficient gains had been achieved in the mathematical programming approach by using approximation concepts. The two approaches are now comparable not only in their efficiency but also in their basic concepts as pointed out by Fleury and Schmit [53] who deduced a mixed method from these two approaches.

Optimization problems that purely involve the sizing of members are especially crucial when complex structural forms are involved and when composite materials are employed. In these cases it becomes difficult, if not impossible, for the designer to have an intuitive understanding of the structural mechanics that is sufficient to lead to optimal sizing of the various members.

Today, it is a common practice to use numerical optimization methodologies to deal with multidisciplinary industrial design problems. One of the major tasks in the design of aircraft wing structures is the sizing of the structural members to obtain the desired strength, weight, and stiffness characteristics. Optimization algorithms have been coupled with structural analysis programs for use in this sizing process. Most of the difficulties associated with large structural design are solution convergence and computer resources requirements. Structural optimization problems traditionally have been solved by using either the mathematical programming (MP) or the optimality criteria (OC) approach. More recently, the works in Refs [46-52, 59] have illustrated the uniformity of the methods. Nevertheless, each approach offers certain advantages and disadvantages. The MP methods are extremely useful in defining the design problem in proper mathematical terms. When the design variables are few the these methods can be used quite effectively for optimization. However, in the presence of a large number of variables these methods are very slow. The rate of convergence for OC methods is initially very fast, step size determination is critical closer to the local optimum where the number of active constraints' increases and the computations of Lagrange multipliers becomes more complex. Power and weakness of the various MP methods are given in Ref. [53]. Ideally, a methodology that exploits the strength of both approaches could be employed in a practical system. The object of the present research effort is to develop such design method that can efficiently optimize large structures that exploit strengths (power) of the MP and OC methods. The motivation of this study is to come up with a multilevel optimization method using optimality criteria and mathematical programming techniques. Multilevel optimization permits a large problem to be broken down into a number of smaller ones, at different levels according to the type of problem being solved. This approach breaks the primary problem statement into a system level design problem and set of uncoupled component level problems. Results are obtained by iteration between the system and component level problems. The decomposition of a complex optimization problem into a multilevel hierarchy of simpler problems often has computational advantages. It makes the whole problem more tractable, especially for the large engineering structures, because the number of design variables and constraints are so great that the optimization becomes both intractable and costly. The nature of an aircraft structure makes multilevel optimization highly practical, not only in terms of reducing the computing cost but also because the individual tasks in the traditional design process are preserved. The suitability of multilevel optimization in more complex design problem tested on a structure representative of a wing box in composite material, with buckling limitations in each panel, and another problem in which reliability requirements are included. Multilevel approach for optimization of the composite structures subject to stress, displacement, buckling and local failure constraints is developed [60-63].

5.2 Formulation of optimization problem. In general, the function optimization procedure is to find a vector of design variables X_i that minimizes or maximizes an objective function *W(X)* which is defined as a function of a design variable vector *X*;

$$W(X) = f(X_1, X_{2, \dots,} X_m) \tag{26}$$

Subject to the constraints that are defined as

$$G_j(X) \leq \mathbf{0}\ , \qquad \mathbf{0} \leq j \leq m \tag{27}$$

The vector *X* contains the m design variables which depend on the type of optimization problem. With composite materials a new dimension is added. The orientation in which the built-up plies are patterned is also vitally important to the weight of the structure. The most frequently encountered optimization problem in designing of aircraft composite structures is as follows. Given a set of loading conditions, each consisting of combined membrane panel loads, and a set of minimum stiffness, what is the optimum pattern of ply orientation?

In practice, it has been found that a reasonably good design can be determined if only 0, ±45 and 90 degree orientations are treated. In this case, it is only necessary to determine *f= (l, m, n)* from a three-dimensional design space, where *l,m,n* denote the number of 0, 90 and ±45 degree plies, respectively.

The general structural optimization problem of layered composite structures modeled by finite elements can be stated as follows:

Find the vector of design variables *x* such that

$$W = \sum_{i=1}^{n} \rho_i \, l_i \, x_i \quad \Rightarrow \min \tag{28}$$

subject to behavior and side constraints

$$G_j = C_j - \overline{C}_j \ge 0 \qquad j = 1,\ldots,m \tag{29}$$

where:

W - is the weight of structure
$\mathbf{x_i}$ - is design variable assigned to element *i*
$\mathbf{l_i}$ - is a geometrical parameter such that the product $l_i x_i$ is the volume of the element *i*
ρ_i - is the mass density
$\mathbf{G_j}$ - is constraint j
$\overline{C}_j$ - is limiting value of the constraint j
n - is total number of elements
m -is total number of constraints

The constraints imposed on the structure, defined by Eq. (29), may have the global and local character. The global constraints will be defined as system constraints. The system constraints imposed on the structure may include the maximum allowable stress in each element, the displacement limits at one or more locations, system stability, reactive forces, dynamic stiffness, divergence, flutter etc. In addition to these there would be limitations on the minimum and maximum sizes of the elements. In addition to system constraints there are local constraints. These include various buckling loads, various failure types in composite structures, etc.

Inclusion all these constraints in optimization process to large-scale structures are inefficient with computational aspect. However, to develop an efficient algorithm that effectively handles all types of constraints would be impractical and generally unnecessary. In the case of most structures it is likely that one can predict the type of constraint that will be the most active at the optimum and use the algorithm based on that constraint. The multilevel optimization approach may be very efficient for optimization large-scale structural systems because it breaks the primary problem statement into a system level design problem and a set of uncoupled component level problems. Results are obtained by iterating between the system and local level problems. The decomposition of a complex optimization problem into a multilevel hierarchy of simpler problems often has computational advantages. It makes the whole problem more tractable, especially for the large aircraft structures. The nature of an aircraft structure makes multilevel optimization highly practical, not only in terms of reducing the computing cost but also because the individual tasks in the traditional design process are then preserved.

5.3 Theory of multilevel optimization. Multilevel optimization is a promising approach to solving large design problems. In this approach, large optimization problems are broken in smaller problems that are in an iteration fashion. This approach is known as the linear decomposition approach. In it, the design problem is decomposed into a hierarchy of subproblems. At the top level, a subproblem optimizes a simplified model that describes the overall behavior of the system. At the lower levels, subproblems optimize increasingly detailed representations of subsystems.

Let D and d represent the sets of system and component design variables, respectively. Then the problem can be stated as:

Find vectors **D** and **d** such that

$$W(D) \Rightarrow \min \tag{30}$$

subject to

$$G_q(D,d) \geq 0 \quad , \quad q \in Q \tag{31}$$

and

$$g_{lj}(d_j, D) \geq 0 \quad , \quad l \in L \;\; ; \quad j \in M \tag{32}$$

The $\mathbf{G_q}$ **(D,d)** represents constraints that are strongly dependent on the **D** vector and they are implicit functions except for the side constraints. The $\mathbf{g_{lj}(d_j, D)}$ represent constraints that are primarily dependent on the j component variables $\mathbf{d_j}$, and they are either explicit or implicit functions of $\mathbf{d_j}$, depending on the type of constraints and the type of local failure analysis. The symbols **Q** and **L** denote the set of system and component level constraints respectively, **M** denotes the number of components and $\mathbf{d^T = [d_1^T, d_2^T, \ldots, d_M^T]}$.

The system design variables can be expressed symbolically as explicit functions of the detailed design variables, that is

$$D_j = \Psi(d_j) \qquad j = 1, \ldots, M \tag{33}$$

For each component the number of detailed design variables are larger than the number of corresponding system design variables.

Therefore, casting the problem entirely at the system level by expressing $\mathbf{D_j}$ as functions of $\mathbf{d_j}$ and solving it using mathematical programming methods are an impractical task for large-scale problems. The multilevel approach presented here is decomposed into two levels of design modification; one with the constraints that are strongly dependent on system design **D** and the other with the constraints that are primarily dependent on local design variables $\mathbf{d_j}$. Then system and local analyses and optimizations are carried out separately and tied together by an iterative scheme going from one level of design modification to the other and visa-versa seeking an overall optimum design.

The structural optimization problem given by Eqs. (30)-(32) is recast as a multilevel optimization problem following form:

i.) System level:

Find vector D (34)

such that $W(D) \Rightarrow \min$ (35)

and $G_q(D, d^*) \geq 0 \quad ; \quad q \in Q$ (36)

where **d*** implies that the parameters strongly dependent on the detailed design variables **d** (i.e., failure loads and local buckling), do not change during a system level design modification stage.

ii.) Component level:

Find vectors $\mathbf{d}_j$ (37)

such that $m_j(d_j) \Rightarrow \min$ (38)

and $g_{lj}(d_j, D^*) \geq 0 \quad ; \quad l \in L$ (39)

where **D*** implies that the parameters strongly dependent on the system level design variables are kept constant during each component design modification stage.

5.4 The system level optimization. An efficient optimality criterion method is used for the system level optimization of large-scale complex structures subjected to constrains which are included at the system level. Optimality criteria approach will be used for the optimization structures with system level constraints. Optimality criteria methods for structural optimization involve:

1. derivation of set of necessary conditions that must be satisfied at the optimum design, and
2. the development of an iterative redesign procedure that drives the initial trial design toward a design which satisfies the previously established set of necessary conditions.

In order to establish the optimality conditions for the problem defined by (34)-(36) we need the associated Lagrangian which is given by the expression

$$L(D,\lambda) = \sum_{i=1}^{N} \frac{w_i}{D_i} + \sum_{j=1}^{Q} \lambda_j G_j \tag{40}$$

where λ_j's are the Lagrange multipliers. The Kuhn-Tucker optimality conditions are now obtained, in part, by differentiating the Lagrangian and the complete set is given by

D* is possible (41)

$$\lambda_q \, G_q(D^*) = 0 \qquad \lambda \geq 0 \;,\; q \in Q \tag{42}$$

$$\nabla W(D^*) + \sum_{q \in Q} \lambda_q \, \nabla G_q(D^*) = 0 \tag{43}$$

If the problem is assumed to be convex then these conditions are necessary and sufficient for the solution of vector D^*, λ^* to represent a global optimizing point otherwise they define a local optimum. The optimum structure must satisfy Eqs. (41)-(43). These are the Kuhn-

Tucker conditions or the optimality conditions. Equation (43) is the ratio of the weighted sum of the gradient of the constraints to the gradient of the objective function, which must be equal for all elements in an optimum design. Eq. (40) and Eq. (41) ensure satisfaction of the constraint equations. The constraints G_q in Eq. (36) may be displacement limits at the different node points in a structure, the relative nodal displacements corresponding to maximum allowable stress in each element, system stability, frequency constraints, flutter requirements, various failure criterions in layered composite structures such as the Tsai-Wu criterion.

The real optimum structure must satisfy conditions (42)-(43). To develop a computational algorithm that handles all these constraints efficiently would be difficult and generally unnecessary. In practical design problem what may be required is a design which is near minimum weight and not a design that exactly satisfies the mathematical optimality criteria. This can generally be achieved by designing the structure based on one or two of the must important constraints, and checking the design for the other constraints.

Problem optimization defined by Eqs (34)-(36) or (28)-(29) involves; large numbers of design variables, large numbers of inequality constraints and many inequality constraints that are computationally burdensome implicit functions of the design variables. These obstacles have been overcome by replacing the basic problem statement (3417)-(3619) with a sequence of relatively small, explicit, approximate problems that preserve the essential features of the original design optimization problem. This has been accomplished through the coordinated use of approximation concepts. The most important feature of the approximation concepts approach lies in the construction of simple explicit expressions for the set of constraints retained during each stage. This is achieved by linearization of these constraints with respect to linked reciprocal design variables. The linearized behavior constraints Eq. (36) are obtained by using a first order Taylor series expansion as:

$$G_q(D,d^*)=1-\sum_{i=1}^{n} C_{iq}\, D_i \qquad ; \quad i=1,\ldots,Q \tag{44}$$

where $\mathbf{C_{iq}}$ is the partial derivative of **q**-th constraint for **i**-th design variable, a **Q** is the total number of constraints. Eq. (44) represents the current linearized approximations of the retained behavior constraints. Using Eq. (44) the retained behavior constraints system level optimization problem Eqs. (34)-(36) can be expressed as: Find vector **D** such that

$$W(D)=\sum_{i=1}^{N}\frac{w_i}{D_i} \Rightarrow \min \tag{45}$$

subject to constraints

$$G_q(D)=1-\sum_{i=1}^{n} C_{iq}\, D_i \qquad ; \quad q\in Q \tag{46}$$

and

$$D_i^L \le D_i \le D_i^U \tag{47}$$

The $\mathbf{w_j}$ are positive fixed constants corresponding to the weight of the set of elements in the j-th linking group when $\mathbf{D_j=1}$. The set of independent design variables after linking is denoted by N and Eq. (46) represent the linear approximations of the behavior constraints. The $\mathbf{D}_i^{\mathbf{L}}$ and $\mathbf{D}_i^{\mathbf{U}}$ respectively denote lower and upper limits on the independent design variables.

In developing optimality conditions standard approach is to form a Lagrangian:

$$L(D,\lambda)=\sum_{i=1}^{N}\frac{w_i}{D_i}-\sum_{q\in Q}\lambda_q\left(1-\sum_{i=1}^{n}C_{iq}D_i\right) \tag{48}$$

where λ_q are the undetermined Lagrangian multipliers. Approximation problem Eqs. (45) -(47) is convex problem and therefore Kuhn-Tucker conditions are necessary that solutions D^*, λ^* represent global minimum. Conventional optimality criteria methods for structural optimization involve; (i) the derivation of a set of necessary conditions that must be satisfied as the optimum design and (ii) the development of an iterative redesign procedure that drives the initial trial design toward a design which satisfies the previously established set of necessary conditions. Each approximate primal problem of the form given by Eqs (45)-(47) can be transformed to correspond an explicit dual problem. Detail solution methods and optimization algorithms are given in Refs [53-56].

5.4.1 Definition of strength constraints in layered composites. For analysis and optimization fibrous layered composite structures, modeled by laminated shell type finite elements, various failure criterions can be used. The Tsai-Wu criterion [40] is used for failure analysis of orthotropic layers in composite shell. This criterion can be expressed as:

$$T_t=\left[\left(\frac{\sigma_1}{F_1}\right)^2+\left(\frac{\sigma_2}{F_2}\right)^2-\left(\frac{\sigma_1\sigma_2}{RF_1F_2}\right)+\left(\frac{\tau_{12}}{F_{12}}\right)^2\right]^{1/2} \tag{49}$$

where $\mathbf{s_1}$, $\mathbf{s_2}$, $\mathbf{t_{12}}$ are the components of stress tensor $\boldsymbol{\sigma}$; $\mathbf{F_1}$,$\mathbf{F_2}$ and $\mathbf{F_{12}}$ are the stresses of failure in uniaxial tension, compression and shear, respectively and T_t is Tsai's number. By using Eqs (45) and (47) linearized approximations of Tsai-Hill criterion can be written as:

$$G_q=1-\sum_{i=1}^{n}\frac{\partial T_t}{\partial D_i}D_i \tag{50}$$

where:

$$C_{iq}=\frac{\partial T_t}{\partial D_i}=T_1\frac{\partial\sigma_1}{\partial D_i}+T_2\frac{\partial\sigma_2}{\partial D_i}+T_3\frac{\partial\tau_{12}}{\partial D_i} \tag{51}$$

with

$$T_1=\frac{1}{2T_t}\frac{2\sigma_1-\sigma_2}{\left(F_1\right)^2}$$

$$T_2=\frac{1}{2T_t}\left[\frac{2\sigma_2}{\left(F_2\right)^2}-\frac{\sigma_1}{\left(F_1\right)^2}\right]^2 \quad \text{and}$$

$$T_3=\frac{1}{T_t}\frac{\tau_{12}}{\left(F_{12}\right)^2}$$

In similar manner linearized constraints such as displacement, stability, frequency or other system constraints can be defined.

5.4.2 Definition of stability constraints. The linear stability of a structure is defined by eigenvalue problem.

$$[K - \lambda_j K_G]q_j = 0 \tag{52}$$

where K and K_G are respectively system stiffness and geometric matrix of the structure and q_j is the eigenvector associated with the j-th eigenvalue λ_j. For an efficient optimization of buckling problems it is essential to know the sensitivity of the buckling load parameter λ_j. The sensitivity with respect to changes in the design variable t_i (thicknesses of shell layers) is evaluated by

$$\frac{\partial \lambda}{\partial t_i} = q^T \left(\frac{\partial K}{\partial t_i} - \lambda \frac{\partial K_G}{\partial t_i} \right) q \tag{53}$$

The evaluation of sensitivities using equation (53) is not computational efficient. It is better to obtain the sensitivity of the buckling load parameter λ_j at the element load level using (53) in the form

$$\frac{\partial \lambda}{\partial t_i} = \sum_{l \in E} [q_l]^T \left(\frac{\partial K^e}{\partial t_i} - \lambda \frac{\partial K_G^e}{\partial t_i} \right)_l q^e{}_l \tag{54}$$

where E is the number of elements in the structure.

5.4.3. Local Level Optimization. Local level optimization process can include various types of failure modes in laminates or local buckling constrains.

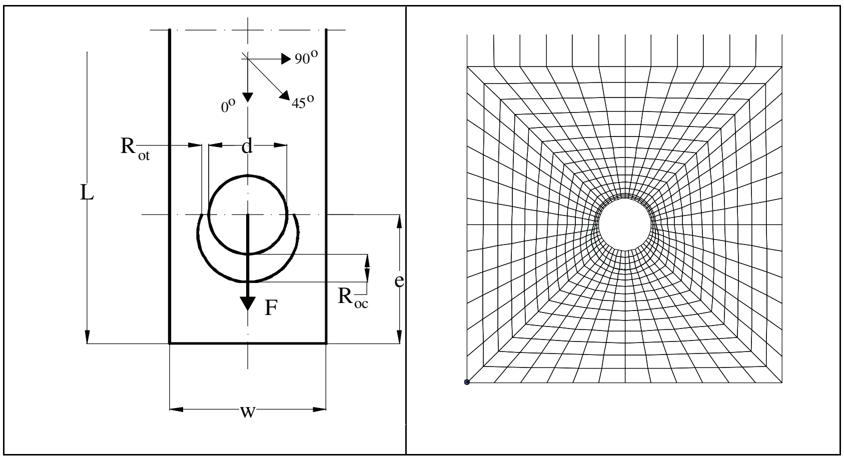


FIGURE 11. Description of the characteristic curve with FE mesh

This optimization problem is solved by algorithms based on nonlinear mathematical programming methods. Classical optimization problem in local level are mechanically fastened joints in composites. Initial failure arises on characteristic curve, as shown in Fig. 11

The characteristic curve with finite element mesh, Fig. 11, is specified by the expression:

$$r_F(\Phi)=\frac{D}{2}+R_t+(R_c-R_t)\cos\Phi \;;-\frac{\pi}{2}\leq\Phi\leq\frac{\pi}{2} \tag{55}$$

where $\mathbf{R_t}$ and $\mathbf{R_c}$ are referred to as the characteristic lengths for tension and compression. In order to determine the load at which a mechanical fastened joint fails and the mode of failure, the conditions for failure must be established. In this paper the joint is taken to have failed when certain combined stresses have exceeded a prescribed limit in any of plies along a chosen the characteristic curve. The combined stress limit is evaluated using the failure criterion proposed by Yamada-Sun in form [21]

$$\left(\frac{\sigma_1}{F_1}\right)^2+\left(\frac{\tau_{12}}{F_{12}}\right)^2\leq 1 \tag{56}$$

where σ_1 and τ_{12} are the longitudinal and shear stresses in a ply, respectively (1 and 2 being the directions parallel and normal to the fibers in the ply). F_{12} is the rail shear strength of a symmetric cross ply laminate $[0^o/90^o]_s$. F_1 is either the longitudinal tensile strength or the longitudinal compressive strength of a single ply.

This criterion is based on the assumption that just prior to failure of the laminate, every ply has failed due to cracks along the fibers. It is very important to say, that local constraints such as expressed by Eq. (56) or similar, can be included in optimization process as direct formulae using Fortran lingue notation in software OPTIS [57]. Direct manner for defining very nonlinear constrains by using direct Fortran description is very efficient in practical optimization of composite or metal aircraft structure. Final dimensions are obtained at local optimization. Optimization algorithms are based on Nonlinear Mathematical programming methods such as: SUMT, CONMIN, method inscribed hyperspheres [51], etc.

5.5 Numerical validation of optimization problems

5.5.1 Optimization of Aircraft Parachute Composite Beam. As very illustrative example for multilevel optimization procedure the fibrous composite parachute beam considered. The structure of parachute beam shown in Fig. 12 idealized with membrane finite elements. The elements consist of four layers in the 0^o,90^o and $\pm45^o$ directions. The 0^o fibers are parallel to the length of the beam. The parachute composite beam was subject to static loading conditions. The aircraft parashute composite beam shown in Fig. 12 used for system level optimization.

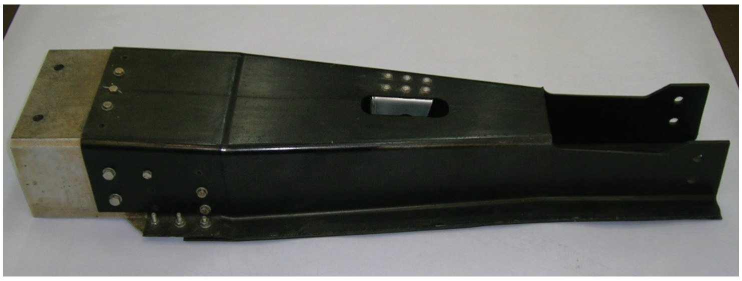


FIGURE 12. Parachute CFC-composite beam

Material of composite beam was graphite/epoxy NCHR 914/34%/132/ T300 with next mechanical properties:

E_{11}=126800 MPa	F_{11}^{t}=1362 MPa
E_{22}=9220 MPa	F_{11}^{c}=1333 MPa
E_{33}=9220 MPa	F_{22}^{t}= 42 MPa
G_{12}=4620 MPa	F_{22}^{c}= 172 MPa
G_{23}=G_{13}=720 MPa	F_{12} = 100 MPa
$\nu_{11}=\nu_{13}=\nu_{23}$	
t_{layer}=0.13 mm	

There are four mechanical fastened joints (holes) on the end of the parachute beam. For the optimum design of bolted joints in composite laminates, a knowledge of stress distribution around the fastener hole due to the applied load is very important. The loads are introduced in these holes. Zone around each hole considered as substructure. This substructure has characteristic curve, as defined in Fig. 11, is modeled by very refined finite element mesh. The substructure (rectangular panel with central hole) is treated as optimization model on the local level. The Yamada-Sun criterion Eq. (56) around characteristic curve Eq. (55) used as constraints in local level optimization. For this purpose, in the local level, SUMT optimization algorithm is used. Optimization results of this substructure are thicknesses of layers:

$t_1(\ 0^o)$ = 2.08 mm
$t_2(+45^o)$ = 0.78 mm
$t_3(-45^o)$ = 0.78 mm
$t_4(\ 90^o)$ = 0.26 mm

Failure load that is in this analysis obtained: **F_f = 2297 daN.** Failure was initiated in layer 0^o, with extension type of mechanism of failure $75^\circ \leq \Theta_f \leq 90^\circ$.

Failure loads that are experimentally obtained: (**F_1 = 2087 daN, F_2 = 2296 daN** and **F_3 = 2390 daN**).

Good agreement between numerical and experimental results is evident. Detail comparisons between numerical and experimental results are given in Ref. [58].

Difference between numerical and experimental results is maximum 5%. In this work optimization results of one substructure are presented only. These results illustrate multilevel optimization process.

6. Conclusions about optimization of layered composite structures

The obtained results demonstrate the practicality of multilevel optimization approach in the design of the complex aircraft structures. In this study two-level optimization algorithm is applied; system- and component level. From the various investigated test problems it becomes clear that the choice of various optimization algorithms at each level play a major role in the efficiency of the whole optimization process. Presented multilevel optimization approach uses optimality criteria's algorithm in conjunction with a Sequential Unconstrained Minimization Technique (SUMT). Optimality criteria's algorithms are used for system level optimization i.e. in case of weight minimization subject to global (system) constraints that can be displacements, system stability, frequencies, flutter etc. Nonlinear Mathematical Programming optimization algorithms are used for local (component) level optimization. Combining FEA, approximation concepts and OC or dual algorithms has led to a very efficient method for minimum weight sizing of large-scale structural systems. The proposed method is suitable for designing practical large-scale structures with a large number of design variables. Finally, minimum weight designs obtained for the aircraft parachute composite beam illustrate the application of the multilevel approach to a relatively large structural system. Most composite optimization research has focused on minimum weight design and strength/buckling/frequency constraints. However, optimization could also be used to increase damage tolerance, aeroelastic envelopes, manufacture and cost [64-67]. Optimal design of joints improves not only structural integrity and performance, but more importantly, it considerably minimizes the weight of the structures and hence, can increase the load-carrying capability.

The introduction of future configurations of unconventional aircraft demand for innovative structural concepts to improve the structural performance, and thus reduce the structural weight. New materials and specific couplings are necessary to cope with such demanding structural design influencing static and dynamic aircraft performances. Moreover, in the design phase, the structural model could be improved by using FEM for stress analysis and numerical optimizations. In particular, the conventional design process can be improved and simplified when a preliminary step in numerical optimization is adopted including two-level optimization approach. The design space for the aeroelastic tailoring is being significantly enlarged with the introduction of innovative solutions such as Variable Angle Tow (VAT) laminates and curvilinear stiffeners [73]. Previously experimental/ numerical comparisons were discussed pointing out specific initial failure analysis of thin-walled composite parashute composite beam. A specific analytical procedure for determining equivalent stiffness of box-beam typical configuration has been developed recently. Innovative configurations accounting for local coupling effects due to the presence of straight and curved stringers have been introduced in [69].

Particularly, the local stiffening effects introduced by innovative configurations may allow unconventional structural coupling and postpone critical aeroelastic phenomena otherwise typical of a wing with a High Aspect Ratio (HAR) [70-72]. An appropriate structural model capable of taking into consideration specific structural behavior of such kind of configurations should be adopted, in order to correctly introduce the terms of geometric nonlinearity in the curvatures of the beam, the effects due to the introduction of the composite material and the effects of local stiffness. [69-72]. Outcomes obtained within the preliminary stages, where low fidelity models are adopted to navigate the entire design space via optimization or parametric analysis, can be further investigated in the successive design phases, by means of detailed FE analysis.

7. General Conclusions

The structural optimization of an aircraft structures is a highly complex problem. This is due to the large number of variables as well as structural and aerodynamics constraints influencing the design of skins and stiffeners. To make it computationally more efficient, a large problem can be decomposed into several smaller subproblems while preserving the couplings among these subproblems. The present research consists of a design tool for the optimization of variable stiffness composite structures (where fibers are not steered), and a method which is developed mainly for the optimization of an aircraft structures. To optimize a variable stiffness composite structure, the proposed method separates the optimization of stacking sequences from the optimization of the thickness distribution. A special method is subsequently introduced for the optimization of interacting skins and ribs of an aircraft wing box. In this work, two types of problems are considered in the domain of strength analysis of structural elements made of multilayer composite materials. The primary attention of the research was focused on the issue of strength analysis of mechanical fastened joints on the one side and optimization on the other side. Both of these problems are basically important problems in the design of aircraft as well as other types of aircraft structural elements. The finite element method (FEM) was used for precise analyzes of stress states in both types of problems. When it comes to the analysis of the strength of structural elements of the type of mechanical joints, special attention was paid to the specifics of the initial fracture in the area around the opening at the mechanical joint (modeling the contact of the metal shaft with the multilayer composite shell). For that purpose, a characteristic curve was used, which is formed around the opening and along which the initial fracture occurs. The results of the computational analysis were compared with the experimental results. In addition to the analysis of the strength of mechanical fastened joints, the paper also considers the optimization of structural elements made of multilayer composite materials. Optimization refers to the minimization of the weight of a structure made of multilayer composite materials while satisfying the requirements of strength and stiffness. In order to provide an efficient numerical optimization method, a two-step optimization approach was used, which refers to the global and local level of optimization. Optimization refers to the minimization of the weight of a structure made of multilayer composite materials while satisfying the requirements of strength and stiffness. In order to provide an efficient numerical optimization method, a two-step optimization approach was used, which refers to the global and local level of optimization. Optimization refers to the minimization of the mass of a structure made of multilayer composite materials while satisfying the requirements of strength and stiffness. In order to provide an efficient numerical optimization method, a two-step optimization approach was applied that refers to the global and local level of optimization.

Acknowledgements
Parts of this research were supported by the Ministry of Sciences of the Republic of Serbia through the Mathematical Institute SANU Belgrade (grant number OI 174001: Dynamics of hybrid systems with complex structures. Mechanics of materials)

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