José Manoel Balthazar[1,2,6], Angelo Marcelo Tusset[2],
Rodrigo Tumolin Rocha[2], Jorge Luis Palacios Felix[3],
Marcus Varanis[4], Clivaldo de Oliveira[3], Mauricio Aparecido Ribeiro[2],
Atila Madureira Bueno[5] and Rafael Henrique Avanço[6]

A SHORT REVIEW ON HYBRID VIBRATING SYSTEMS WITH LIMITED POWER SUPPLY (RNIS)

Abstract. We are interested in vibrating systems, with applications of nonlinear behavior. The subject of this work deals with a special class of hybrid systems, that is, a vibrating system that combines electrical and mechanical drivelines. This paper is devoted to present a review concerning some properties of recent research progresses of non-ideal rotary support structures (RNIS) in the recent years restricted, however, to English publications only. In summary, we presented mathematical modelling of problems related to (RNIS), which render descriptions that are close to real situations found in practice. We were interested to what happens to the motor (or electro-mechanical shaker), input, output, as the response of the rotary system (RNIS) support structure changes, that is, we considered nonlinear resonances, including periodic, quasiperiodic, and chaotic characteristics in the steady state motions, and energy transfer between the energy sources and the support structures. Their possible control approaches are also considered to pass to resonance. This work is in honor for MI SANU.

Mathematics Subject Classification (2010): Primary: 70-99, 74-99, 70F35, 92C10, 74L10; Secondary: 70Exx, 70KXX, 74M05, 37H20

Keywords: hybrid systems, non-ideal systems, rotary machines, nonlinear dynamics, chaos, control

[1]UNESP-Universidade Estadual Paulista, Bauru, SP, Brasil. jmbaltha@gmail.com
[2]Universidade Tecnológica Federal do Paraná, Ponta Grossa, PR, Brasil,
a.m.tusset@gmail.com, digao.rocha@gmail.com
[3]Universidade Federal de Fronteira do Sul (UFFS), Cerro Largo, RS, Brasil,
jorge.felix@uffs.edu.br
[4]Universidade Federal de Grande Dourados, Dourados, MS, Brasil,
marcus.varanis@gmail.com, ClivaldoOliveira@ufgd.edu.br
[5]UNESP-Universidade Estadual Paulista, Sorocaba, SP, Brasil,
atila.bueno@unesp.br
[6]Federal University of Maranhão, Campus Balsas, State of Maranhão, Brazil,
avancorafael@gmail.com

Contents

1. Introduction

The subject of this work is deal with a special class of hybrid system, that is, a vibrating system that combines electrical and mechanical drivelines in honor for MI SANU, though references [1-74].

It is well known that the word vibrations come from Latin *vibrationem* ("shaking, brandishing"). The transmission of vibrations to the human body can cause several damages, such as pain, discomfort, loss of efficiency and concentration at work, dizziness, nausea, blurred vision, speech disorder, fatigue, neurological or muscular disorders, bones and articulations injuries, pathologies in the lumbar region and even lesions of the spine. The parts of the human body are sensitive to vibrations in many different frequency ranges (values), for instance, in the head around 25Hz; eyeballs 30-60 Hz; in the rib cage 60 Hz; in the spine 10-12Hz; on the arm 16-30Hz; in the hand 50-150Hz; in the pelvic mass and buttocks 4-8Hz; and in the legs 2-20Hz. The risk of exposure to vibration depends on the amplitude, frequency, direction, time of exposure and behavior of the vibration over time (continuous, intermittent, or transient). This issue of human exposure to vibration is so important that there are several international standards that addressing this problem. As an example, the ISO 2631-1 (1997) sets out the general requirements regarding human exposure to whole-body vibration, covering measurement methods, vibration severity and human comfort and possible health effects. Additionally, the ISO 2631-2 (2003) standard presents a guide for the application of ISO 2631-1 on the human response under the action of building vibration [3].

Differential equations of motion for vibrating systems are commonly differential equations for partial derivatives in their generalized spatial and temporal coordinates. In the case where the variables of the considered problem are not separable, the use of classic methods makes it possible to eliminate variables, simplifying the generalized spatial equations, obtaining in this way, a set of ordinary equations in the time variable, either time-dependent or time-independent coefficients.

It is also especially important to analyze the stability of the solutions to the obtained governing equations of motion. Usually, the Lyapunov stability definition is adopted. It is said that an equilibrium point of the dynamic system is stable if, whatever the disturbance imposed to the equilibrium state, the system remains sufficiently close to that state; otherwise, there is instability. If the disturbed system tends to the state of initial equilibrium over time, it is said that the system equilibrium is asymptotically stable [10,13].

The conditions of stability, or dynamic instability, are commonly expressed in mechanical vibrating theory. Certain relations between the frequencies of external excitations and the natural frequencies of the system, called frequency relations, are an associated mechanical phenomenon called resonance [12-14].

The steps to be used in solving a resonant vibrating problem are shown in the sequence. Establishment of differential equations of motion, obtaining time differential equations of motion; use of asymptotic representations of the solutions of the equations obtained, this procedure determines the relations of resonances; solution of differential equations of motion (analytical or numerical) and, determination of the regions related to system stability and instability and possible control techniques. It is noteworthy that the resonant vibrations existence (resonance relations) or its characteristics (transient or steady state) depends on several factors related to systems and external excitation characteristics [12-14].

The features commonly found in vibrating engineering are nonlinearities (geometric or physical characteristics); Dissipation of energy (internal or external); Gyroscopic Systems; Imperfections of material; Stationary and non-stationary modes; Unlimited power sources (or Ideal sources (IS)) or limited (non-ideal sources (RNIS)) and; Non-conservative dynamic (follower) or forced.

On the other hand, monitoring the integrity of nonlinear structures and machines is an ever-growing concern in modern engineering applications. Better knowledge of structural conditions allows optimized maintenance cycles, increasing the availability and return of investment, and preventing failure of various systems, ranging from manufacturing equipment to air and land vehicles. A way of evaluating the integrity of mechanical systems, used in this study and in modern engineering applications, is capturing and analyzing vibration signals during operations in structures coupled to non-ideal motors (RNIS) (such that the phenomenon of resonance capture can occur) [16-29].

We also mention that according to [30], we may summarize the researches on the nonstationary vibrations problems into three groups: the first one is the study of a nonstationary vibration of a linear system when the rotor passes through resonance with constant acceleration, the second one it the study of the nonstationary vibrations of nonlinear systems, and the third one is the study of the phenomena in systems which have mutual interaction between a driving source and a rotor motion non-Ideal Systems (RNIS).

In summary, the vibrations of nonlinear systems have been studied exhaustively over the last years grounded in H. Poincare, W, Ritz, V.G. Galerkin, A. Lyapunov, A. Stodola, etc. At the end of 19th and beginning of the 20th century. Around 1920 appeared the first works of the authors G. Duffing, Van der Pol, M.N. Krylov, N.N.

Bogolyuubov, SA, Mitrosposky, J. J. Stoker, N. Minorky , C, Hayashi, H. Kauderer , etc. and other significant contributions have been also made to the theory of vibrations of nonlinear dynamical systems. However, (RNIS) cannot be completely explained by means of the current theory (we discuss them over this paper).

It is also well known that, over the years, the construction of mathematical models has played an important part in the discovery and dissemination of knowledge. The degree of realism desired from a mathematical model often depends upon many considerations. This process involves keeping certain terms, neglecting others, and approximating others. There are no best models, only better models. Ideally, modeling leads to a complete understanding of the phenomenon being studied**.** The study of problems involving of the coupling of several systems, was widely explored, in the last years, essentially in function of the change of constructive characteristics of the machines and structures.

We also remark that in the Design of structures it is necessary to investigate the relevant dynamics to predict the structural response due to the excitations. It is well known that the integration of mechanical, electromagnetic, and computer elements (electro-mechanical) to produce devices and systems that monitor, and control machines and structural systems has led to the need for integration of mechanical and electrical design, and industrial applications to nonlinear mechatronic systems *(Industry 4.0).*

While the materials used in electro-mechanical designs are often new, the basic dynamic principles of Newton and Maxwell still apply. The governing equations of motion are then integral forms of the basic PDEs and result in coupled ordinary differential equations (ODEs). This methodology will be explored in this paper. The analysis of the motion of real electro-mechanical systems (Mechatronics) were carried out by means of mathematical models, which have been always simplified and therefore describe the “exact” behavior with the same degree of approximations.

The study of problems, involving the coupling of several vibrating systems was widely explored in the recent years, essentially in function of the change of constructive characteristics of the machines and the support structures.

Accordingly, vibrating processes can be divided into the following types: free, forced, parametric and self-excited oscillations and we also remark that two or more oscillations can interact in the same oscillatory system. This fact is of important scientific and practical interest, nowadays.

We also note that many oscillatory (vibrating) phenomena of real systems cannot be explained, nor solved, based on linear theory. That is why it is important to introduce nonlinear characteristics on the mathematical models of the considered vibrating systems (and to electro-mechanical systems). The main difficulty, in comparison to linear systems, is due to the absence of the superposition principle. This means that every nonlinear vibrating system must be solved individually, and a special methodology must be developed for each class of problems.

Fractional stiffness and damping are appearing in different contexts for systems with memory and hysteresis. Such damping is defined by a fractional derivative in contrast to classical viscous damping term with the first order derivative. As the memory of a dynamical system induces extra degree of freedom, for the phase space, the standard methods of dynamical response analysis and system identification, which relies on the knowledge of system dimensionality, cannot be used. A review on different aspects of Fracional derivatives may be seen in [47-50]

In the industrial environment there are numerous sources of vibration: rotating machines, alternative machines, impact processes, conveyors, vehicles, machine tools, and others. It is worth highlighting some results of vibrations in the industrial environment: excessive noise, premature wear, cracks, loosening of screws, leaks, malfunction of machinery, equipment, and structural failures. In the context of industrial environment, the imbalance stands out, which is one of the most common causes of vibration in rotating equipment, especially in: electric motor rotors, fan rotors, blower rotors, turbines, centrifugal pumps, and elastic couplings (mechanical couplings). Rotary unbalance is present in numerous Engineering applications, for example it is stated that most drive motors have a problem related to mechanical vibration, due to the inherent imbalance that exists in the motors.

With the development of modern technology, machineries and equipment's are becoming more and more complex every day. Among all machineries, we will restrict ourselves to a particular rotary one and the full interaction of their support structures. The rotary machineries and the full interaction with their support structures are a major, and critical component, of many mechatronic systems, in industrial plants, aerial and ground transportation vehicles, and in many other applications on modern engineering and applied sciences. It is well known that rotor unbalance is the most common reason in rotary machineries to present undesirable vibrations. Most of the rotary machinery problem, may be solved by using the rotor balancing and misalignment.

The mass unbalance in a rotating system often produces excessive synchronous forces, which reduces the life span of various mechanical elements. A very small amount of unbalance may cause severe problem in high-speed rotary machinery. The vibrations caused by unbalance may destroy critical parts of the machine, such as bearings, seals, gears, and couplings. Rotor unbalance is a condition, in which the center of mass of a rotary assembly, typically the shaft and its fixed components, like disks and blades etc., is not coincident with the center of rotation. In practice, rotors can never be perfectly balanced because of manufacturing errors such as porosity in casting, non-uniform density of material, manufacturing tolerances and gain or loss of material during operation. As a direct result of mass unbalance, a centrifugal force is generated and must be reacted against by bearing and support structures. In this way, some phenomena were observed in a composed vibrating system supporting structures and rotating machines, where the disbalance of the rotating parts is the major cause of the vibrations.

This review paper aims to provide a short forum for the discussion and dissemination of the latest approaches, methodologies results and current challenges in nonlinear vibrations, and in the field of electro-mechanical systems (EMS).

The EMS is not just a marriage of electrical and mechanical systems and is more than just a control system; it is a complete integration of all of them. Topics of interest on (EMS) include Interdisciplinary approaches and complex nonlinear phenomena in problems encountered in emergent engineering and science practice.

Therefore, this paper attempts to provide a short review of some of the latest efforts in the development and applications of recent exciting research progresses of rotary non-ideal systems (RNIS) that is considering the full interaction between (RNIS) with their support structures. The main purpose is to provide the researchers and engineers, who are working in this emergent vibrating field, with a comprehensive knowledge to help them with a better applying of the theory of rotary non-ideal systems (RNIS), to solve problems, related to rotary machines and the interaction with the support structure. and nowadays trends show a marked tendency for solutions of vibrating systems analysis of drive systems as early as the design stage.

In this point, it is important to clarify what rotary non-ideal systems (RNIS) means, in order, to avoid future confusions.

Non-ideal systems (RNIS) have appeared in the literature with several meanings; as an example: some researchers use the concept of (RNIS) solutions for concentrated solutions, that is, the solutions can occur in two ways: when intermolecular forces between solute and solvent molecules are less strong than between molecules of similar (of the same type) molecules, and when intermolecular forces between dissimilar molecules are greater than those between similar molecules. Here, we deal with an energy transfer between the energy sources and the support structures and their possible control approaches, that is, we are interested to what happens to the motor (or electro-mechanical shaker), input, output, as the response to the rotary system support structure changes.

We will organize this work into five parts, including this introduction section. In section 2, we will give some aspects of the recent state-of-the art of (RNIS). Section 3 presents the background on (NIS). Also, in this section, we will provide a brief introduction on the so-called Sommerfeld effect and the Saturation phenomenon. Then, Section 4 reviews applications of (NIS) rotary machinery and the full interaction with their support structures, over the past five years. In Section 5, we discuss some models and governing equations of motion After that, Section 6 discusses some new trends of (NIS) rotary machineries. Finally, concluding remarks are drawn in Section 6, and the main bibliographic references are listed. The present work doesn't claim literature completeness since the available literature is dispersed over many distinct sources. It is restricted to the main references on the non-ideal vibrating dynamical systems and some related papers on this subject

2. A Brief Review towards the State-of-the art on (RNIS)

Carl G. P. de Laval was the first Engineer to perform an experiment with a steam turbine [2,3], he observed that quick passage though critical speed would significantly reduce the levels of vibration when compared to steady state excitation. This procedure would require a DC motor, with enough power to be accelerated quickly in the range of resonance frequency. However, in some cases, DC motors have limited power to perform such operations, and the angular velocity increases so slowly that the passage through resonance becomes a problem. Probably [41], was the first approach to a non-ideal rotor described in the literature. We also mention that considered variations of the acceleration rate in order to minimize the motion during passage through resonance, and three earlier works in [67-68] in the (RNIS) field. The problem of passing through resonance has been investigated independently by Japanese, German and American groups leading to important results.

A large class of problems, related to unbalanced DC motors with limited power supply was discussed in a classical book [35], entirely devoted to this subject. In [35] presented the first detailed study on the non-ideal problem (RNIS) related to the passage through resonance. He noted that they proposed an experiment of a DC motor mounted on a flexible wooden table and observed that the energy supplied to the DC motor, was partially converted in the form of a table vibration, instead of increasing the angular speed of the DC motor.

This observation was used to explain a class of motors called non-ideal energy sources (RNIS). The non-ideal energy source (RNIS) has a reciprocal influence on the system near the resonance regime. When considering a DC motor, usually the angular velocity increases according to the power supplied by the source. However, due to the Sommerfeld effect, near the resonance and with additional energy, the average angular velocity of the DC motor remains unchanged, until it suddenly jumps to a much higher value, upon exceeding a critical input power. Simultaneously, the amplitude of oscillations of the excited system jumps to a much lower value. Before the Jump, the non-ideal oscillating system (RNIS) cannot pass through the resonance frequency of the system or requires an intensive interaction between the vibrating system and the energy source to be able to do so.

In summary, Sommerfeld suggested that the structural response or vibrations provide an "energy sink" and thus, we pay to vibrate our structure rather than operate the machinery [35].

One of the problems often faced by designers is how to drive a system through resonance and avoid the "energy sink", described by Sommerfeld. Note that, however, when the interaction above mentioned is present, the final rotation speed of the motor does not depend only on the characteristics of the DC motor and of the power supplied, but also on the parameters of the beam, as well as on the initial conditions, and on the range of the physical parameters, such as, the mass of the motor, the mass of the rotating unbalanced disks, the eccentricity of the disks, the moment of inertia of the rotor, the electrical and mechanical characteristics of the motor, the mass, the inertia and length of the beam.

The angular velocity can increase up beyond the resonance condition (passage through resonance) or remain close to the natural frequency of the dynamical system (capture by resonance).

Obviously, the time instant of the passage through resonance depends also on the initial conditions imposed to the imposed to the system. Due to the nonlinear stiffness, it presents a complex behavior, and the system's response is not always periodic. Additionally, depending on the angular speed constant of the motor, it is possible to find a chaotic (irregular) response. The nonlinear stiffness adopted in the mathematical model of mechanical systems that present elastic potential energy, with two potential wells (Duffing system), is the characteristic that increases the complexity of the observed dynamic response.

The presence of chaos in the response of (RNIS), can be verified through the phase picture and the FFT graphs, which presents a very disturbed spectrum, indicating a characteristic of the chaotic response, and verified by the evaluation of the Lyapunov exponents, or the 0-1 test [14]. The appearance of chaos in (RNIS) was treated, in first time in by [38]. Later in [45] pointed out the scenario of the hyperchaotic attractor formation (an attractor with at least two positive Lyapunov exponents) in a class of models governed by the Sommerfeld effect.

The Sommerfeld effect is the result of the energy conservation law of physics, and when a dynamical system is coupled to a power source, it acts like an "energy sink", and a portion of the source's energy is driven to deform the system, rather than to increase the speed. The Sommerfeld effect involves the riddling bifurcation which explains the creation of the hyperchaotic attractor.

Recently, a tribute to the scientific heritage of Kononenko was done by [19] and by a scientific paper by [11].

An important point is that the interaction in (RNIS) shows a quasi-periodic regime movement, since its vibrations are predominant in the capture by resonance, i.e., the Sommerfeld Effect, mentioned above. This property of (RNIS) enabled the advance of research in engineering applications, as in the case of using, as an excitation of the system, an electro-mechanical vibrator instead of a direct current motor, in agreement as it does, classically. This property made it possible to increase the range of research emergent possibilities on this topic to Macro and MEMS scales [11].

An additional property of (RNIS) is the observed fact that when the adopted model is calibrated, a 2:1 internal resonance occurs between the frequencies of the second mode (the first symmetrical mode) and the first mode (the "sway" mode). As an example, in a portal frame, the external resonance is imposed between the angular speed of the supported rotating machine and the second natural mode of the structure. It is intended to demonstrate that the energy pumped into the system via the second mode, leads to the saturation phenomenon [12,46], passing the energy balance to the first mode, not directly excited, which starts to develop wide amplitudes, potentially dangerous and not predicted in theory. It is also important to consider synchronization, in this case, with two unbalanced DC motors.

The (RNIS) subject were studied and presented by a few numbers of authors in the current literature. In the following, some are mentioned.

An overview of various aspects on vibrating problems excited by limited power supply: as the physical phenomena involved, the adequate methodology to deal with them and a report of selected papers published recently, and in the past, on non-ideal dynamical systems are shown in the following papers [8-12]; in the book chapters of by [8, 11], and in the books of [2, 19, 24].

The above-mentioned authors present comprehensive reviews of the (RNIS) vibrating problems considering the dynamical coupling between the energy sources and structural response, that must not be ignored in real engineering problems since real motors have limited output power. The present models, for certain problems, render descriptions that are closer to real situations, where the excitation of the vibrating system is always limited in two senses: by the characteristic curves of the energy source, and by the dependence of the system's motion on the energy source, *i.e.*, the coupling between the governing equations of motion and the energy source. In particular, the saturation phenomenon in [12] used to harvesting energy in the works of [53,54]. In [51] deal with friction induced nonideal vibrations: a source of fatigue [29] included a discussion on (RNIS) health monitoring and [50] presented comments on nonlinear response of a nonideal systems with shape memory alloy.

Some authors have explored (RNIS) in various other aspects, for example, [16] and [17] discussed the motion of an unbalanced rotor when passing through a resonance zone, solved by the iteration method, combined with the method of the direct separation of motions. The approach presented by [5] can be used to separate the vibration from rotations in many other mechanical and mechatronic systems. The behavior of the considered non-ideal system near two simultaneously occurring resonances is examined using the Krylov–Bogolyubov averaging method. The stability analysis of the resonant response is also carried out. The method to avoid resonance capture by switching on and off a mechanism, changing the stiffness of an engine mount. A (RNIS) was also using similar approach. It was considered a flexible supported vertical shaft with damping; it was assumed that the bending stiffness of the shaft could be switched from one value to another. The rotational speed of the shaft increased, and the bending stiffness was changed at a certain time to avoid passage through resonance. The transient motion induced by this change of stiffness could be larger and hence the decrease in maximum response may not always be significant.

The dynamic behavior of a rectangular plate excited by two accelerated unbalanced DC machines moving along a rectangular plate in opposite direction was explored by [21]. The dynamic behavior of a rectangular plate excited by two accelerated unbalanced DC machines moving along a rectangular plate in opposite direction is explored. Both motors are considered as non-ideal oscillators and act as external excitation on a specific straight line of the plate. The effects of the moving acceleration of both motors and their initial moving velocities on the plate deflection are investigated. The impact of the way of crossing of the motors on the plate amplitude is also analyzed, and it was shown that the physical characteristics of the motors contribute to the reduction in the plate vibration. The analytical approach used leads to some mathematical expressions, which allows to some predictions on vibration amplitude in the system. It follows that the reduction in amplitude of vibration depends on the characteristics of the DC motors.

In [43] discussed vibrations of the snap-through motions successfully and in [61] deals with the construction attraction basins, used for the analysis of nonlinear dynamical systems presenting multistability. Two versions are considered, one for multi-core and another for many-core architectures, both based on a SPMD approach. The algorithm is tested on three systems, the classic nonlinear Duffing system, a non-ideal system (RNIS) exhibiting the Sommerfeld effect and an immunodynamic system. The results for all examples demonstrate the versatility of the proposed parallel algorithm, showing that the multi-core parallel algorithm using MPI has nearly an ideal speedup and efficiency.

In [23] investigated the dynamical behavior of a (RNIS) Duffing oscillator, to identify new features on Duffing oscillator parameter space due to the limited power supply. An extensive numerical characterization in the bi-parameter space by using Lyapunov exponents is provided. Following this procedure, a remarkable new organized distribution of periodic windows is identified, the ones known as Arnold tongues and shrimp-shaped structures. In addition, intertwined basins of attraction for coexisting multiple attractors connected with tongues are identified. In [50] analyzed the dynamic integrity of a (RNIS) considering two different integrity measurements, that are used to quantify the magnitude of the safe basin. After obtaining the basins of attraction as functions of a variable parameter, the so-called erosion profiles are given, which is the key tool for the study of the dynamic integrity. The erosion curves for each measure of integrity are constructed numerically and compared to each other. The dynamic integrity of the periodic solution is studied, and the basin erosion is evaluated. The erosion profiles obtained allows to identify the practical thresholds that guarantee a priori a safe project to be developed.

In [73] a (RNIS) with memory due to a fractional damping term is considered, to distinguish between periodic and non-periodic behaviors, three different mathematical tools are used, namely, the 0-1 test, scale index and wavelet technique. In [20] studied the influence of the order of nonlinearity on the dynamic properties of the (RNIS). The authors considered the motor with the torque as a cubic function of the angular velocity, and the nonlinear oscillator, with a certain order. The numerical calculation and analytical solutions were also done.

In [72] discussed an application of the continuous wavelet transformation for the characterization of the Sommerfeld effect, and [73] presented the modelling of (NIS) composed of a cantilever beam with two motors positioned on the top.

In [4] analyzed a (NIS) pendulum behavior, using bifurcation diagrams, exhibiting doubling-period and saddle-node bifurcations, with chaos. In [32] the appearance of chaotic behavior due to the coupling of the manipulator with the motors was investigated and the feedback control was designed using the state dependent Riccati equation to control of the positioning of the manipulator and the torque applied on the MR damper.

In [74], it is numerically and experimentally investigated the dynamics of a pendulum vertically excited by a crank-shaft-slider mechanism driven by a DC motor. The power supplied to the DC is small enough to observe the return influence of the pendulum dynamics on the motor angular velocity. In the performed experiments, the motor is supplied with constant time voltages. A series of experimental periodic solutions allowed to estimate the model parameters and, in a further step, predict the bifurcation phenomena observed in the experiment.

In [40] studied the pendulum horizontally excited by a DC motor and a slider-crank mechanism. The mathematical modeling is realistic and based on an experimental rig, considering details such as friction in the joints as well as a realistic mass distribution for the elements of the system. Using basic nonlinear tools as phase portraits, Poincaré maps, and Fourier spectra, reporting various solutions including periodic, quasi-periodic and non-periodic ones. To identify chaotic solutions, the 0-1 test [14] was used, and the simulation results were qualitatively confirmed by experiments.

In [15] it was studied the features of vibrational motion of an orthogonal mechanism with disturbances, such as restricted power in the presence of a fixed load on the horizontal link. Dynamic and mathematical models were developed, and the fields of existence of the operating conditions for the vibration mechanism in terms of the driving power were defined.

In [20] the authors suggested to develop the control method for the motion in the non-ideal mass variable oscillatory system, and in [71] the authors considered the application of the time-delayed feedback control in a (RNIS) with cubic nonlinearity to suppress chaotic behavior, considering the velocity of the rotating angle as a parameter to determine the time delay. In addition, another control was proposed. Therefore, two control signals are considered in which one is the nonlinear feedforward controller to maintain the system in periodic orbit and the other one is the feedback controller obtained by the SDRE, which takes the system trajectory to the desired periodic orbit. Numerical simulations demonstrated the effectiveness of the control strategy.

In [71] a perturbation theory is applied to the analysis of an electromechanical pendulum system. The frequency response behavior of the system is studied, and the existence of unstable poles is detected using the Routh–Hurwitz criterion. Numerical simulations show the existence of nonlinear behaviors such as hysteresis and the Sommerfeld effect in the resonance region. The SDRE control strategy is applied considering two control signals, a feedback control that force the state trajectory of the system to a previously defined periodic orbit, and a nonlinear feedforward control that keeps the system motion synchronized to the periodic orbit. Additionally, the robustness of the control technique is tested for parametric uncertainties.

In [37] used an actuator consisting of an (SMA) wire to attenuate the vibration and Sommerfeld effect of a non-ideal type of oscillator. The temperature control of the actuator was carried out through the application of an electric current in the wire. The results are presented for different electric current values, to investigate the temperature variation for vibration control applications.

In [48] the authors studied the nonlinear dynamics behavior of a shape memory oscillator (SMO) subjected to an ideal (IS) or nonideal excitation (RNIS). The restoring force of the oscillator was s provided by a shape memory device (SMD), described by a thermomechanical model capable of reproducing the hysteretic behavior via the evolution of a suitable internal variable. Due to nonlinearities in the model, the (SMO) exhibited periodic or non-periodic behaviors. The effects of the external sources on the response of (SMO) were studied through the scalogram analysis of continuous wavelet transform by using a new measure, called the Scale index, with success.

Recently, some results on harvesting (RNIS)energy considering different approaches have been reported. In [39] considered an axisymmetric buoy, which oscillates and is subjected to its natural hydrostatic restoring force.

In [52], through numerical and analytical tools analyzed a (RNIS) with a magnetic levitation system, considering an electrodynamical shaker to base-excite the main system (which is a (RNIS). In (Balthazar et al, 2018), the magnetic levitation system was used to investigate the energy harvesting potential, considering an electrodynamical shaker exciting the system. The modes of vibration of the coupled system and the average harvested power were described by expressions related to the coupling between the mechanical and electrical domains.

In [53] the authors shown that when the excitation frequency is near to the second frequency mode of vibration of the main system, saturation and jump phenomena occur and they explore the possibility to harvesting energy from high amplitudes of vibration.

In [27] showed that the vibration transfer and energy harvesting may be achieved simultaneously by electricity-generation from auto parametric vibration absorber system and a (RNIS). The (RNIS) consisted of a simple portal frame excited by a DC motor and located on the top. The results showed the existence of Sommerfeld effect in (NIS) and the saturation phenomenon in the (NIS) and on the absorber. The portal structure is of two-degrees-of-freedom considering with quadratic coupling between the first and second modes of vibration. 2:1 internal resonance between the first and second modes is set, which is a special condition of this type of system due to the appearance of a saturation phenomenon, as studied by [59] considering the contribution of the linear part of the Piezoelectric.

Also, [27] used the method of Jacobi-Anger expansion in a 2DOF model of a flexible portal frame with harmonic force of varying frequency to an energy harvester. A good performance of the harvester generator was reported, and the authors observed periodic, quasi-periodic or chaotic oscillations, depending on the saturation phenomenon.

On the other hand, on [63], the dynamics of a (NIS) driven single-degree-of-freedom vibrating system was explored, describing the Sommerfeld effect in a reciprocating system; in [62-63] two eccentric rotors were mounted rigidly on a common vibrating base structure. Each of these rotors were separately driven by two motors, which were by nature (NIS) kind. And the phenomena of "self-synchronization" was obtained. Additionally, the presence of the Sommerfeld effect was verified.

The Sommerfeld effect and synchronization analysis in a simply supported beam system excited by two nonideal induction motors was investigated by [36]. In [65] Sommerfeld effect was analyzed with the help of Bond Graph models.

In [42], the effect of nonlinearity on vibration of a rotating (NIS) shaft passing through critical speed was investigated, taking into account that the interaction between a nonlinear gyroscopic (NIS) continuous system (i.e., rotating shaft) and the energy source. In [33] a novel approach was presented to study the attenuation of this Sommerfeld effect on the driven discrete rotor system with the adjustment of fractional order parameter in its external damping term. Additionally, in [34] an eccentric (NIS) shaft-disk system with internal damping exhibiting Sommerfeld effect was analyzed and discussed.

The Sommerfeld effect in a strongly gyroscopic rotor dynamic system is studied by [13], and the coupled bending torsional vibrations of non-ideal energy source rotors under non-stationary operating conditions was studied by [60].

In [66] a forced oscillation of a two-mass system, induced by the unbalanced vibration exciter driven by asynchronous AC motor of limited power are considered. The mathematical model of the system is obtained by considering the static characteristics of the motor. The oscillations of the system in resonance zones are analyzed, and relations between power input and dissipated power are estimated, depending on the system parameters.

3. Modern Rotary non-ideal systems (RNIS) modeling outline

In this section we discuss some possible ways to obtain the governing equations of (RNIS).

3.1. Case 1. Formally, the governing equations of the rotary system (RNIS) with n degrees of freedom, may be written as [27]:

$$\ddot{x}_i + f_i(x,\dot{x}) + f_i(x,t) = f_{0i}\cos[\Omega_{0i}t + a_{0i}\sin(b_{0i}\Omega_{0i}t + c_{0i}) \qquad (1)$$
$$I=1, 2, \ldots n.$$

Where the functions $f_i(x,\dot{x})$ are the damping coefficients; the functions $f_i(x,t)$ are the stiffness coefficients, where ω_{0i}^2 are the natural frequencies (it is possible to have internal resonance if, for instance, $\omega_{0i}^2 \approx m\,\omega_{0j}^2$ and $j, m = integers$. The constants $f_{0,i}$ are the amplitudes of excitations, with excitation frequencies Ω_{0i}. The constants Ω_{0i} are obtained from averaging the angular frequencies at resonance. The control parameters a_{0i}, b_0 and c_{0i} are defined by the active interaction between the vibrating system and the excitation sources. If $a_{0i} = 0$, then $q_{3i} = \Omega_{0i}t$ corresponding to harmonic excitations (IS). In some cases, we can take fractional derivatives for both stiffness and damping, in order to introduce a memory, in the considered system; for example, the damping device acts with a force F_d given by $F_d = c\dfrac{d^v x}{dt^v}$ proportional to the v^{th} derivative of the relative displacement. For $v = 1$ the force is a linear viscous damping force. It is assumed that $0 \le v \le 1$, in general.

Using the Jacobi-Anger expansion, the sine and cosine terms can be written as:

$$\cos(z\sin\theta) = \sum_{k=-\infty}^{\infty} J_k(z)\cos(k\theta) \qquad (2)$$

where $J_k(z)$ is the ki-th classical Bessel function, then we obtain from (1) and (2):

$$\ddot{x}_i + f_i(x,\dot{x}) + f_i(x,t) = f_{0i}\sum_{k=-\infty}^{\infty} J_k(a_0)\cos(\Omega_k t + kc_{0i}) \qquad (3)$$

where $\Omega_k = \Omega_{0i} + kb_{0i}\Omega_{0i}$; i=1,2,n

It is important to note that (Eq. 3) looks like a system with harmonic excitation for each k. It is a generalization important of (RNIS) vibrating governing equations of motion. Note that if in the case of (RNIS) (particular cases of $f_i(x,\dot{x}) + f_i(x,t)$, i=1, k=1) from Eq. (3), we get

$$\ddot{x} + 2\varsigma\dot{x} - \omega_0^2 x + \gamma x^3 = f_0 \cos(\Omega_0 t - a_0 \cos(b_0 \Omega_0 t)) \quad (3a)$$

then if $a_0 = 0$ the equations are the classical Duffing (RNIS) equation.

3.2. Case 2. We reduce the amplitudes of vibrations of the (RNIS) during the occurrence of the Sommerfeld effect in both inside and outside resonance region, respectively, using a Nonlinear-Energy Sink as a Passive Controller [55,57]. Then, adding one more equation to Eq (1),

$$m_2\ddot{y} + c_2\dot{y} + g(y) + \frac{\partial V(x,y)}{\partial y} = 0 \quad (4)$$

where m_2 is the mass, c_2 is the coefficient of the viscous damping and y is the displacement of the (NES), g(y) is the stiffness term of the (NES), V is the potential energy associated with the coupling spring. Both g(y) and V are not necessarily linear.

3.3. Case 3. The attenuation of the *Sommerfeld effect*, may be done by means of a nonlinear electromechanical vibration absorber, called (NEVA) [27], taking into account that the electric part of the controller consisted of a linear inductor L, a nonlinear capacitor C, and nonlinear resistor R. The expression of the voltage over the resistor and the condenser were a nonlinear function of the instantaneous electrical charge q. Now we consider one more equation with Eq (1):

$$L\ddot{q} - R\left(1 - \frac{1}{i_0^2}\dot{q}^2\right)\dot{q} + \frac{1}{C_p}q + i_0^2\alpha_a q^3 + i_0^4\alpha_b q^5 + T\dot{x} = 0 \quad (5)$$

where i_0 is the initial current, in the electrical part, C_0 is the linear value of the capacitive characteristic, and the parameters α_3 and α_5, are nonlinear coefficients depending on the capacitor type. T is the transducer (constant), which relates the current in the coil to the magnetic force acting on the considered coil. The transducer constant is given by $T = 2\pi/nlB$, where n is the number of turns of the coil, l is the radius of the coil, and B is the uniform radial magnetic field strength in the annular gap. The transducer constant T also relates the electrical potential *e*, across the terminals of the coil to the velocity of the coil, with respect to the permanent magnet. In this case Eq (1) must be replaced by:

$$\ddot{x}_l + f_i(x,\dot{x}) + f_i(x,t) + T\dot{q} = f_{0i}\cos[\Omega_{0i}t + a_{0i}\sin(b_{0i}\Omega_{0i}t + c_{0i}) \quad (1b)$$

3.4. Case 4. Now, considering the mathematical model (RNIEH) for energy harvesting, we need to replace Eq (1) for the following governing equations of motion:

$$f_i(x,\dot{x}) + f_i(x,t) + \frac{d(x)}{C}q = f_{0i}\cos[\Omega_{0i}t + a_{0i}\sin(b_{0i}\Omega_{0i}t + c_{0i}) \quad (1c)$$

where the quantity $M = m_1 + m_0$ is the total mass of the (NIEH), x is displacement of the (RNIEH), P1 and P2 are the thin film piezoelectric applied layers and the electrical charge developed in the coupled circuit given by q, the term $\frac{d(x)}{C}q$ represents the piezoelectric coupling to the mechanical component, with a strain-dependent coupling, coefficient $d(x)$. The voltage across the piezoelectric material has the form:

$$V = -\frac{d(x)}{C}x + \frac{q}{C} \tag{6}$$

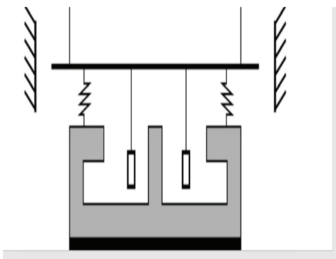
where C represents the piezoelectric capacitance, and with $V = -R\dot{q}$ the (RNIEH) coupled governing equations of motion are:

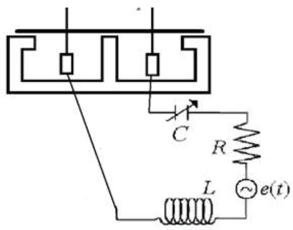
$$\begin{gathered} \ddot{x}_i + f_i(x,\dot{x}) + f_i(x,t) + \frac{d(x)}{C}q = f_{0i}\cos[\,\Omega_{0i}t + a_{0i}\sin(\,b_{0i}\Omega_{0i}t + c_{0i}) \\ R\dot{q} - \frac{d(x)}{C} + \frac{q}{C} = 0 \end{gathered} \tag{7}$$

for I=1,...,n.

The power of the harvester is $P = R\dot{q}$ and the average power is $p = \frac{1}{T}\int_0^T P(\alpha)d\alpha$, where T is the period of the vibrations.

3.5. Case 5. Another form of (RNIS) governing equation of motion, can be obtained by taking into account the action of an electro-dynamical shaker (on the support structure of (RNIS)) which consists of a device reproducing harmonic excitations. The interaction of a support structure of (RNIS) and the electromechanical shaker, is due to the existence of quasi-periodic oscillations, the Sommerfeld effect [27]. The schematics of the support of this kind of (RNIS) can be seen in Fig1, where L is the shaker inductance, C is the Shaker capacitance, R is the shaker resistance, e_0 is the input voltage of the shaker, and I is the electrical current of the shaker circuit. The input voltage of the shaker is given by a harmonic force $e(t) = e_0\cos(\omega t)$.





(A) (B)

FIGURE 1. Schematic of the support (RNIS) excited by an electromechanical shaker. (A) scheme of device and (B) electrical scheme.

Therefore, the governing equation of motion are

$$\begin{gathered}\ddot{x}_i + f_i(x,\dot{x}) + f_i(x,t) + T\dot{q} = f_{0i}\cos[\,\Omega_{0i}t + a_{0i}\sin(\,b_{0i}\Omega_{0i}t + c_{0i})\\ L\ddot{q} + R\dot{q} + Tx = e_0\cos(\omega t)\end{gathered} \tag{8}$$

where T is the Transducer gain.

3.6. Case 6. Next, we discuss the contribution of nonlinearities of piezoelectric material to generate harvesting energy.

Because of the constitutive characteristics of piezoelectric materials, the role of nonlinearities must be taken into account in the electro-mechanical coupling of energy harvesting system design [1] and [22].

The behavior of the piezo-electric element was checked experimentally by [18], and the function to the dimensionless piezo-electric coupling coefficient was suggested by [70], where the dimensional coefficient piezo-electric d(x) was approximated by:

$$d(x) = d_{linear}(1 + d_{nonlinear}|x|),\ x = \text{Material's deformation} \tag{9}$$

having defined the dimensionless counterpart as

$$\hat{d}(x) = \theta(1 + \Theta|x|) \tag{10}$$

where the piezoelectric coefficient is constituted by a linear part represented by Θ and a nonlinear part represented by Θ. Recently [59] analyzed the behavior of a (RNS) Portal Frame instability with the parameter Θ. For $\Theta \neq 0$ instability and resonance were observed, with catastrophic displacements for the portal frame system. It was also observed that the system has a reduction in the energy production when increasing Θ.

The contribution of the nonlinear part of the piezoelectric was also discussed in [54, 55]. It was found that the energy harvested is dependent on the piezoelectric linear and nonlinear coefficients, changing the average power generated. However, further research on this subjected is needed.

4. Conclusions

This chapter presented an overview of various aspects on a special class of hybrid vibrating problems excited by a limited power supply (RNIS). We discussed, concerning to (RNIS), and as the physical phenomena was involved, the adequate methodology to deal with them, and reported a selection of papers recently published, for recent and emergent studies.

A new phenomenon was addressed, concerning structures supporting unbalanced machines, capable of a limited output power, that is, Rotary Non-Ideal Systems (RNIS), and the motion of an oscillating structure under the action of such energy source was accompanied by a full interaction between these non-ideal motors and their supports.

We also analyzed possible and practical applications concerning unbalanced non-ideal DC motor type foundation structure (RNIS) in the presence of the Sommerfeld Effect, getting stuck at resonance (energy imparted to the motor being used to excite large amplitude motions on the supporting structure).

Finally, some relevant models in the study of (RNIS) were presented, showing that the nonlinear piezoelectric coupling has relevant contributions to the system, depending on the value of the linear piezoelectric coupling. It is important to note that the coupling makes the system become close to the real movement of the system, so that the simulations results are close to the real problem.

Additionally, this paper did not exhaust the subject, showing the need for further research on piezoelectric (RNIS).

Acknowledgement
The first and second authors acknowledge the financial support by the Brazilian Council for Scientific and Technological Development, CNPq, grants 306525/2015-1 and 307371/2017-4, respectively.

References

[1] A. Abdelkefi, A. Nayfeh, M. Hajj, *Effects of Nonlinear Piezoelectric Coupling on Energy Harvesters Under Direct Excitation*, Nonlinear Dyn. **67** (2012) 1221–1232.

[2] A.A. Alifov, K.V. Frolov, *Interaction of nonlinear oscillatory systems with energy sources*, English Edition, E. Rivin, (Ed.), 1990.

[3] J.D. Anderson, *Modern Compressible Flow: With Historical Perspective,* Aeronautical and Aerospace Engineering Series, McGraw-Hill Education, 2003.

[4] R.H. Avanço, A.M. Tusset, N. Suetake, H.A. Navarro, J.M. Balthazar, A. Nabarrete, *Energy harvesting through pendulum motion and DC generators*, Lat. Am. j. solids struct. **16**(1), (2019).

[5] J. Awrejcewicz, R. Starosta, & G. Sypniewska-Kamińska, *Decomposition of governing equations in the analysis of resonant response of a nonlinear and non-ideal vibrating system,* Nonlinear Dyn. **82** (2015), 299–309.

[6] J.M. Balthazar, R.M.I.R.F Brasil, H.I. Weber, A. Fenili, D. Belato, J.L.P. Felix, *Anoverview on non-ideal vibrations,* Meccanica **38** (2003), 613–621.

[7] J.M. Balthazar, R.M.I.R.F. Brasil, H.I. Weber, A. Fenili, D. Belato, J.L.P Felix, F.J. Garzelli, *A review of new vibration issues due to non-ideal energy sources,* In: F.E.Udwadia, H.I Weber, G. Leitmann (eds.), *Dynamical Systems and Control, Stability and Control: Theory, Methods and Applications*, Chapman & Hall/CRC, Boca Raton, 2004.

[8] J.M. Bathazar, B.Jr. De Pontes, *On Friction induced nonideal vibrations: A source of fadigue,* D. Inman, et al. (eds), John Wiley & Sons, (2005), 91-108.

[9] J.M. Balthazar, R.M.I.R.F. Brasil, J.L.P. Felix, A.M. Tusset, V. Picirillo, *Dynamics behaviour of an elastic non-ideal (NIS) portal frame, including fractional nonlinearities*, Journal of Physics Conference Series **721**(1) (2016), 012004 .

[10] J.M. Balthazar, J.L. P Felix, *Revisited nonlinear and non-ideal vibrations,* Vibration Problems ICOVP 2011, The International Conference on Vibration Problems, Technical University of Liberec, S. Segla et al. (eds), 2011.

[11] J.M. Balthazar, A.M. Tusset, R.M.L.R.F. Brasil et al. , *An overview on the appearance of the Sommerfeld effect and saturation phenomenon in non-ideal vibrating systems (NIS) in macro and MEMS scales*, Nonlinear Dyn. **93** (2018), 19–40.

[12] J.M. Balthazar, *On recent issues on Sommerfelf effect and saturation phenomenon in some elastic structures: a note*, Journal of low frequency Noise*,* Vibration *&* Active Control*,* submitted (2020).

[13] A. Bisoi, A.K. Samantaray & R. Bhattacharyya, *Sommerfeld effect in a gyroscopic overhung rotor-disk system,* Nonlinear Dynamics **88**(3) (2017), 1565–1585.

[14] D. Bernardini, G. Litak, *An overview of 0–1 test for chaos*, Journal of the Brazilian Society of Mechanical Sciences and Engineering **38**(5) (2016), 1433-1450.

[15] K. Bissembayev, Zh. Iskakov, *Oscillations of the orthogonal mechanism with a non-ideal source of energy in the presence of a load on the operating link,* Mechanism and Machine Theory **92** (2015), 153-170.

[16] I.I. Blekhman, *Vibrational mechanics: Nonlinear dynamic effects. General Approach, Applications*, World Scientific, Singapore, 2000.

[17] I.I. Blekhman, V.S. Sorokin, *Extension of the method of direct separation of motions for problems of oscillating action on dynamical systems,* Procedia IUTAM **19** (2016), 75-82.

[18] E.F. Crawley, E.H. Anderson, *Detailed Models of Piezoceramic Actuation of Beams,* J. Intell. Mater. Syst. Struct. **1** (1990), pp. 4–25.

[19] L.Cveticanin, M. Zukovic, J.M. Balthazar, *Dynamics of Mechanical Systems with Non-Ideal Excitation*, DSTA 2018, 978-3-319-54168-6, 2018.

[20] L. Cveticanin, M. Zukovic, D. Cveticanin, *Oscillator with variable mass excited with non-ideal source*, Nonlinear Dyn. **92** (2018)**,** 673–682.

[21] A.A.N. Djanan, B.R.N. Nbendjo, *Effect of two moving non-ideal sources on the dynamic of a rectangular plate,* Nonlinear Dyn. **92** (2018)**,** 645–657.

[22] M.F. Daqaq, R. Masana, A. Erturk, D.D. Quinn, *On the role of Nonlinearities in Vibratory Energy Harvesting: A Critical Review and Discussion*, Journal of Applied Mechanics Reviews, ASME **66** (2014).

[23] De Souza, I.L. Batista, I. Batista, I. Caldas, Balthazar, *A. Physica: Statistical Mechanics and its Applications* **466** (2017) , 224-231,.

[24] R.M. Evan-Iwanowski, *Resonance Oscillators in Mechanical Systems,* Elsevier, 1976.

[25] A. Ertuk, J. Hoffman, D. J. Inman, *A piezomagnetoelastic structure for broadband vibration energy harvesting,* Appl. Phys. Lett. **94** (2009), 254102.

[26] J. L.P. Felix, R.P. Bianchin, A. Almeida, J.M. Balthazar, R.T. Rocha, R.M.L.R.F. Brasil, *On energy transfer between vibration modes under frequency-varying excitations for energy harvesting*, Applied Mechanics and Materials, Trans Tech Publications, Ltd. **849** (2016), 65–75.

[27] J. L. P. Felix, J. M. Balthazar, R. T. Rocha, A. M. Tusset, F. C. Janzen, *On vibration mitigation and energy harvesting of a non-ideal system with auto parametric vibration absorber system,* Meccanica **53** (2018), 3177–3188.

[28] R.F. Ganiev, T.S. Krasnopolskaya, *The Scientific Heritage of V.O. Kononenko: The Sommerfeld–Kononenko Effect,* J. Mach. Manuf. Reliab. **47** (2018), 389–398.

[29] P.J.P. Gonçalves, M. Silveira, *Condition Monitoring of Structures Under Non-ideal Excitation Using Low Cost Equipment*, In Probabilistic Prognostics and Health Management of Energy Systems, Oriseres et al. (eds), (2017), 241-261, Springer.

[30] Y. Ishida, Y.T. Ikeda, T. Yamamoto, S. Murakami, *Nonstationary Vibrations of a Rotating Shaft with Nonlinear Spring Characteristics During Acceleration through Critical Speeds,* Memoirs of the School of Engineering Nagoya University **44**(1) (1992), 1-70.

[31] T. Iwatsubo, H. Kanki, R. Kawai, *Vibration of Asymmetric Rotor Through Critical Speed with Limited Power Supply,* Journal Mechanical Engineering Science **14** (3) (1972), 184-194.

[32] J.J. Lima, J.M. Balthazar, R.T. Rocha, F.C. Janzen, D. Bernardini, G. Litak, D.G. Bassinello, A.M. Tusset, *On Positioning and Vibration Control Application to Robotic Manipulators with a Nonideal Load Carrying* (2019), 5408519.

[33] A.K. Jha, S.S. Dasgupta, *Suppression of Sommerfeld effect in a non-ideal discrete rotor system with fractional order external damping,* European Journal of Mechanics - A/Solids **79** (2019) :10...

[34] A.K. Jha, S.S. Dasgupta, *Attenuation of Sommerfeld effect in an internally damped eccentric shaft-disk system via active magnetic bearings*, Meccanica **54** (2019), 311–320.

[35] V. O. Kononenko, *Vibrating Systems With a Limited Power Supply*, English Translation, life Books, 1969.

[36] X. Kong, J. Jiang, C. Zhou, et al, *Sommerfeld effect and synchronization analysis in a simply supported beam system excited by two non-ideal induction motors,* Nonlinear Dyn**. 100** (2020), 2047–2070.

[37] I. Kossoski et al, *Jump Attenuation in a Non-Ideal System Using Shape Memory Element*, MATEC Web of Conferences **148** (2018), 03003.

[38] T.S. Krasnopolskaya, A.Y. Shevts, *Chaos in Vibrating Systems With a Limited Power-Supply*, Chaos **3**(3) (1993).

[39] T.S. Krasnopolskaya, E. Pechuk, *Energy extraction by an oscillating system from a shaker or a wave field,* International Journal of Nonlinear Dynamics and Control **1**(3) (2019), 304 – 316.

[40] G. Litak, A. Syta, G. Wasilewski, et al., *Dynamical response of a pendulum driven horizontally by a DC motor with a slider–crank mechanism*, Nonlinear Dyn. **99** (2020)**,** 1923–1935.

[41] D. Macchia, *Acceleration of an Unbalanced Rotor through the Critical Speed*, ASME Paper 63-WA-9, (1963), 1-7.

[42] A. Mahmoudi, S.A.A. Hosseini, M. Zamanian, *Nonstationary analysis of nonlinear rotating shafts passing through critical speed excited by a nonideal energy source,* ASME Paper **232**(4) , 572-584.

[43] Y. Mikhlin, A. Onizhuk, *Resonance Behavior of the Non-Ideal System Which Contains a Snap-Through Truss Absorber,* In: Nonlinear Dynamics and Control, W. Lacarbonara, B. Balachandran, J. Ma, J. Tenreiro Machado, G. Stepan, eds. Springer, Cham, 2020.

[44] Y. Mikhlin, A. Onizhuk, J. Awrejcewicz, *Resonance behavior of the system with a limited power supply having the Mises girder as absorber,* Nonlinear Dyn. **99** (2020)**,** 519–536.

[45] L. Munteanu, C. Brişan, V. Chiroiu, *et al.* , *Chaos–hyperchaos transition in a class of models governed by Sommerfeld effect,* Nonlinear Dyn**. 78** (2014)**,** 1877–1889.

[46] H.A. Nayfeh, D.T. Mook, *Nonlinear Oscillations,* Wiley, 1979

[47] I. Petras, *Fractional-order nonlinear systems: modeling, analysis, and simulation*, Higher Education Press, Beijing, 2011.

[48] V. Piccirillo, J.M. Balthazar, Jr.B.P. De Pontes, J.L.P. Felix, *On nonlinear response of a nonideal systems with shape memory alloy,* In: J. Awrejcewicz (ed), *Modeling , Simulation and control of nonlinear engineering dynamics systems*, *DSTA*, (2009), 107-116,.

[49] V. Piccirillo, J. M. Balthazar, A.M. Tusset, D. Bernardini, G. Rega, *Characterizing the nonlinear behavior of a pseudoelastic oscillator via the wavelet transform*, Springer **230** (1) (2015), 120-132

[50] V. Piccirillo, T.G. Do Prado, A. M. Tusset, J. M. Balthazar, *Dynamic Integrity Analysis on a Non-ideal Oscillator. Mathematics in Engineering*, Science & Aerospace (MESA) **11**(3) (2020), 541-547.

[51] B. Pontes, J. M. Balthazar, *On Friction Induced Nonideal Vibrations: A Source of Fatigue,* Damage Prognosis: For Aerospace, Civil and Mechanical Systems, Wiley, 2005.

[52] R.T. Rocha, J.M. Balthazar, A.M. Tusset, S.L.T. Souza, F.C. Janzen, H.C. Arbex, *On a non-ideal magnetic levitation system: nonlinear dynamical behavior and energy harvesting analyses,* Nonlinear Dyn. **95** (2019), 3423–3438.

[53] R,T. Rocha, *On saturation phenomenon in energy harvesting based on nonlinear piezoelectric material coupled to a portal frame foundation with ideal ad nonideal excitations,* PhD dissertation, UNESP-Bauru, Brazil, 2015.

[54] R.T. Rocha, J.M. Balthazar, A.M. Tusset, V. Piccirillo, J.L.P. Felix, *Using Saturation Phenomenon to Improve Energy Harvesting in a Portal Frame Platform with Passive Control by a Pendulum,* In: J. Awrejcewicz et all. eds., *Dynamical Systems: Theoretical and Experimental Analysis*, Springer Proceedings in Mathematics & Statistics **182** (2016), Springer, Cham.

[55] R.T. Rocha, J.M. Balthazar, A.M. Tusset, V. Piccirillo, J.L.P. Felix, *Comments on energy harvesting on a 2:1 internal resonance portal frame support structure, using a nonlinear-energy sink as a passive controller*, Int Rev Mech Eng (IREME) **10**(3) (2016), 147–156.

[56] R.T. Rocha, J.M. Balthazar, A.M. Tusset, D.D. Quinn, *An analytical approximated solution and numerical simulations of a non-ideal system with saturation phenomenon,* Nonlinear Dyn. **94** (2018), 429-442.

[57] R.T. Rocha, J.M. Balthazar, A.M. Tusset, et al., *Comments on Energy Harvesting on a 2:1,* Internal Resonance Portal Frame Support Structure, Using a Nonlinear-Energy Sink as a Passive Controller*,* 2020.

[58] R.T. Rocha, J.M. Balthazar, A.M. Tusset, et al., *Nonlinear piezoelectric vibration energy harvesting from a portal frame with two-to-one internal resonance,* Meccanica **52** (2017), 2583–2602.

[59] R.T. Rocha, A.M. Tusset, et al., *On the positioning of a piezoelectric material in the energy harvesting from a non-ideally excited portal frame,* Journal of computational and nonlinear dynamics (2020), in press.

[60] E. Sghaier, A. Boudon, et al., *Coupled bending torsional vibrations of non-ideal energy source rotors under non-stationary operating conditions,* International Journal of Mechanical Sciences, **163** (2019), 105155.

[61] M. Silveira, J.P. Gonçalves, J.M. Balthazar, *Multi-core and many-core SPMD parallel algorithms for construction of basins of attraction,* Journal of Theoretical and Applied Mechanicsm **57**(4) (2019), 1067–1079.

[62] A. Sinha, S.K. Bharti, A.K. Samantaray, G. Chakraborty, R. Bhattacharyya, *Sommerfeld effect in an oscillator with a reciprocating mass,* Nonlinear Dyn. **93** (2019), 1719–1739.

[63] A. Sinha, S.K. Bharti, A.K. Samantaray, R. Bhattacharyya, *Sommerfeld Effect and Passive Energy Reallocation in a Self-Synchronizing System*, NODYCON (2019), 1-10.

[64] A. Sinha, S.K. Bharti, A.K. Samantaray, R. Bhattacharyya, *Sommerfeld effect in a single-DOF system with base excitation from motor driven mechanism,* Mechanism and Machine Theory **148** (2020).

[65] A. Sinha, S.K. Bharti, A. K. Samantaray, R. Bhattacharyya, *Sommerfeld Effect and Passive Energy Reallocation in a Self-Synchronizing System*, Paper, 10 pages, 2019 NOSYCON (in press).

[66] A. Shokhin, A. Nikiforov, G. Korendyasev, S. Eremeykin, *Analysis of dynamic properties of two-mass system with inertial exciter of limited power. Vibroengineering*, PROCEDIA **6** (2015), 316-320.

[67] S. H. Suzuki, *Dynamic Behaviour of a Beam Subject to a Force of Time -Dependent Frequency,* Journal Sound and Vibration **57** (1978),59-64.

[68] S. H. Suzuki, *Dynamic Behaviour of a Beam Subject to a Force of Time - Dependent Frequency (Continued),* Journal Sound and Vibration **60**(3) (1978), 417-422.

[69] S. Shahlaei-Far, J.M. Balthazar, *Method of Direct Separation of Motions Applied to a Non-ideal Electromechanical Pendulum System,* In: J. Awrejcewicz, et all., eds. *Dynamical Systems in Applications*, DSTA (2017).

[70] A. Triplett, D.D. Quinn, *The effect of non-linear piezoelectric coupling on vibration-based energy harvesting*, Journal of Intelligent Material Systems and Structures **20**(16) (2009), 1959-1967.

[71] A.M. Tusset, J.M. Balthazar, R.T. Rocha, M.A. Ribeiro, W.B. Lenz, F.C. Janzen Time-Delayed, *Feedback Control Applied in a Nonideal System with Chaotic Behavior,* In: W. Lacarbonara, B. Balachandran, J. Ma, J. Tenreiro Machado, G. Stepan, eds. *Nonlinear Dynamics and Control*, Springer, Cham, 2020.

[72] M. Varanis, J.M. Balthazar, A.Silva, A.G. Mereles, R. Pederiva, *Remarks on the Sommerfeld effect characterization in the wavelet domain*. NODYCON **25**(1) (2019), 98-108..

[73] M. Varanis, A. M. Tusset, J.M. Balthazar, G. Litak, C. Oliveira, R.T. Rocha, A. Nabarrete, V. Piccirillo, *Dynamics and control of periodic and non-periodic behavior of Duffing vibrating system with fractional damping and excited by a non-ideal motor*, NODYCON, 2019.

[74] G. Wasilewski, G. Kudra, J. Awrejcewicz, M. Kaźmierczak, M. Tyborowsk, M. Kaźmierczak, *A Pendulum Driven by a Crank-Shaft-Slider Mechanism and a DC Motor—Mathematical Modeling, Parameter Identification, and Experimental Validation of Bifurcational Dynamics,* In: J. Awrejcewicz et all., eds. *Dynamical Systems: Theoretical and Experimental Analysis*, Springer Proceedings in Mathematics & Statistics **182** (2016). Springer.