Giuseppe Rega

NONLINEAR DYNAMICS IN MECHANICS: A JOURRNEY TROUGH PERSONAL RESEARCH RESULTS

Abstract. An overview of research results obtained in more than 40 years of scientific activity in collaboration with several colleagues is given. First, the various addressed topics are summarized, by properly framing them within some main stages of development of nonlinear dynamics in mechanics, identifiable over the time period considered. Then, the focus is on two main research fields that have attracted the writer's greatest interest and in which he has probably provided the most significant research results: the nonlinear dynamics of sagged cables, and the fundamental role played by global dynamics in the analysis, control and safe design of mechanical systems and structures.

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1. Preface

A personal preface to this paper is in order. When Professor Katica Hedrih kindly invited me to contribute a paper of "review of my research results" to the Special Issue of the Proceedings, Mathematical Institute SANU, Belgrade - Non-Periodical Issues, illustrating the results of the project 'Dynamics of hybrid systems of complex structures' she coordinated for the MI SANU, I was certainly very much honored. Yet, I wondered whether this would make sense since I was not directly involved in that project and, even more so, because writing about yourself objectively is extremely difficult and quite opinionated. Thus, I kindly declined the invitation. However, Professor Hedrih insisted on the matter, motivating her invitation with my long lasting and fruitful relationships with Serbian mechanicians, and mentioning that, when in 1971 she spent some time in Kiev with the Academician Yu. Mitropolski, she learned that scientists should be free to talk about their results "in the first person singular."

Indeed, I visited Serbia several times, not only for lecturing at the XXII (Vrnjacka Banja, 1997) and XXIII (Belgrade, 2001) *Yugoslav Congresses of Theoretical and Applied Mechanics*, at the 6th *International Symposium on Nonlinear Mechanics: Nonlinear Sciences and Applications*, Nis, 2003, and at the 3rd *Serbian-Greek Symposium Recent Advances in Mechanics*, Novi Sad, 2008, but also for giving a seminar just at the Mathematical Institute SANU, Belgrade, 2008. Moreover, I was a member of the Committee for the defense of a thesis for a Magister's science degree, and I had the pleasure to chair the Scientific Committee of the *IUTAM Symposium Exploiting Nonlinear Dynamics for Engineering Systems*, Novi Sad, 2018. Over this time period, I had nice scientific and personal relationships with Serbian scientists, starting of course with Professor Katica Hedrih, whom I first met at the 2nd *International Conference Asymptotic in Mechanics*, St. Petersburg, 1997. But also including Academician Nikola Hajdin, Prof. Dobroslav Ruzic, Prof. Livia Cveticanin, Prof. Teodor Atanackovic, Prof. Ivana Kovacic, Dr. Andielka Hedrih, and Dr. Julijana Simonovic, a few of them also visiting me in Italy several times.

My fine memories of Serbian science, people, and society (Fig. 1), along with the certainly unopposable lesson learned by the great Academician Yu. Mitropolski, via

Professor Hedrih, convinced me that I could have presented to Serbian researchers a summary of my research results obtained in more than 40 years of scientific and academic involvement first at the University of L'Aquila and then at the Sapienza University of Rome.

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FIGURE 1. Italian and Serbian professors at international conferences in Serbia: (a) Katica Hedrih, Giuseppe Rega and Teodor Atanackovic; (b) Stefano Lenci, Ivana Kovacic and Giuseppe Rega.

2. Embedding Research Topics Within Stages of Development of Nonlinear Dynamics

I have been working on nonlinear dynamics of mechanical systems and structures for about 40 years. Interest in nonlinear dynamics originated on the background of my academic education in structural mechanics and, in particular, in connection with the investigation of large amplitude oscillations in sagged cables. Nonlinear dynamics of suspended cables was a topic of major interest to me for more than twenty years, as well as a scientific field in which I am still interested, even if being involved only occasionally. It was also the specific, yet comprehensive, topic through which I followed the evolution of the nonlinear dynamics of systems and structures occurred in the last two decades of the past century within the applied mechanics community, in terms of mathematical models, methods of solution, and nonlinear phenomena. Indeed, this was about the period in which the awareness of the importance of regular *nonlinear oscillations* of systems and structures to be addressed with *analytical techniques*, attained in that community since the end of the 70s via a transfer of knowledge from the more mathematically oriented environment of theoretical dynamicists, progressively evolved to the explosion of new interests. First, towards the *bifurcation* events and *complex dynamics*, including chaos, to be addressed via *computational* techniques and *geometrical* understanding; and then, towards the sophisticated *experimental* verification on *small-scale* physical *models* of the nonlinear phenomena observed theoretically and/or numerically [1].

Figures 2-4 present in chronological terms summary maps of the topics which I have been dealt with over my research activity, by referring to the considered systems/models, the employed methods of analysis/solution, the addressed nonlinear phenomena, and the main tools specifically utilized. The topics are framed within three general stages of development of nonlinear dynamics in mechanics. These are distinguished from each other just by virtue of the dominant role played in each of them by the phenomena and related methods of greater scientific interest by the nonlinear dynamicists in mechanics: nonlinear oscillations–analytical techniques; bifurcation/complex dynamics–computational/geometrical techniques; nonlinear experimental dynamics–small-scale models. The prevailing time periods are evidenced in the figures with bold year numbers in the top chronological scale and more intense color gradations in the below maps of topics.

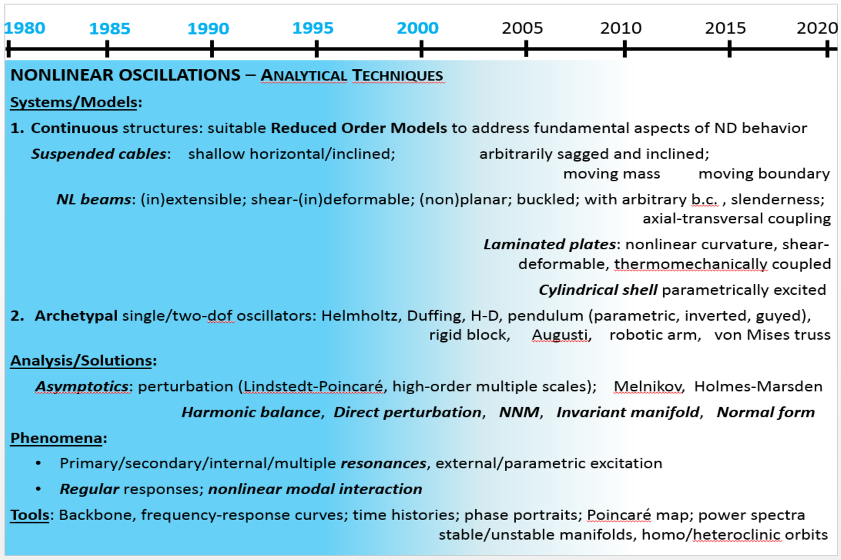

Figure 2. Addressed topics framed in the Nonlinear Oscillations stage of nonlinear dynamics in mechanics.

Topics relevant to nonlinear dynamics of cables, as jointly and fruitfully addressed with the three complementary techniques (analytical, computational/ geometrical, experimental) needed for achieving a comprehensive understanding of different involved nonlinear phenomena, are easily recognizable in the three summary maps (see also Sect. 3 forward). However, several other addressed topics are mentioned, too. They range from archetypal nonlinear oscillators to different one/two-dimensional continua and reduced order models (ROMs) of interest in structural mechanics, up to actually discrete systems playing a role in a variety of mechanical applications. Moreover, they include asymptotic and other analytical/ numerical methods for the investigation of mostly local dynamics, as well as a variety of computational/geometrical/experimental techniques and tools properly suited to global and complex dynamics, which were variably used over the years for the analysis of the mentioned systems and models.

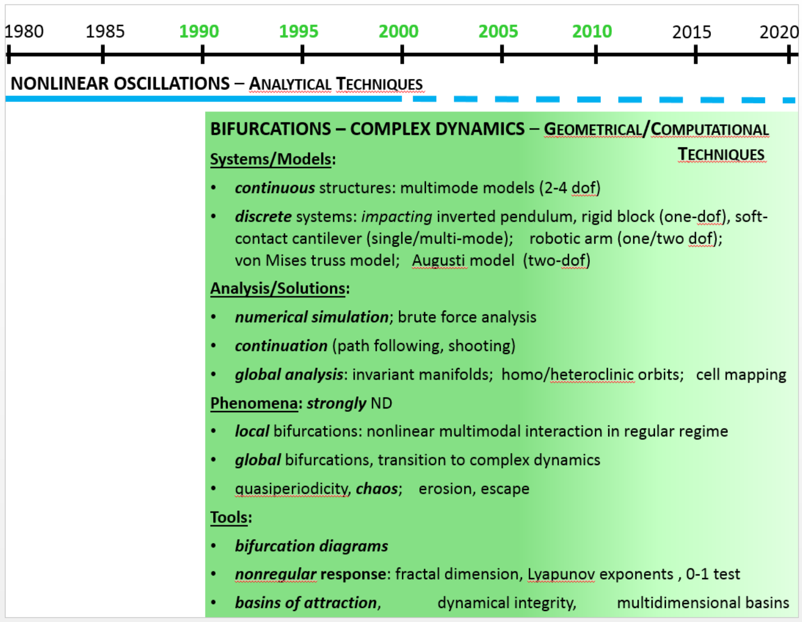

FIGURE 3. Addressed topics framed in the Bifurcations and Complex Dynamics stage of nonlinear dynamics in mechanics.

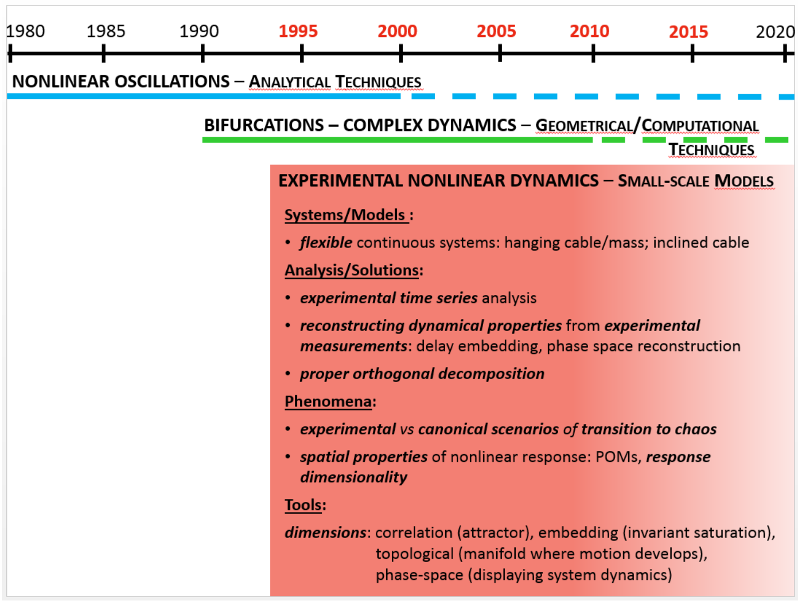

FIGURE 4. Addressed topics framed in the Experimental Nonlinear Dynamics stage of nonlinear dynamics in mechanics.

One more map of topics is shown in Fig. 5. It focuses on the most recent, and indeed current, stage of development of the area, in which, to the writer's opinion, nonlinear dynamics has been more or less markedly 'hybridized' with a variety of companion, and generally independent, research areas. In this last respect, only companion areas actually visited by the writer over his activity are reported, which of course are far from being exhaustive, although representing a meaningful subset. Topics and problems summarily listed within each hybridizing area are those that I addressed with different younger collaborators in the new millennium, i.e. in about the 'second half' of my scientific activity. With respect to those listed in the previous three figures, these are generally, if not even definitely, novel topics and problems, however addressed in the background of the nourishing concepts, methods, operational tools and nonlinear/complex phenomena developed and variably explored in the 'first half' of my activity; this being the non-trivial meaning of the 'hybridization'. In general terms, the link with previous stages of development has consisted of exploiting the available knowledge on nonlinear and complex dynamics to the best possible way, in order to effectively deal with problems and systems in different physical contexts and at variable technological scales. More specifically, relating different areas of knowledge via also feedback interactions (as per the green highlights in Fig. 5), the research has mostly, although not exclusively, aimed at formulating proper, yet reliable, ROMs capable to unveil the strong role that a thorough understanding of the global dynamics of engineering systems and structures may play for effective analysis, control, and a novel, yet safe, design. Indeed, roughly speaking, exploiting nonlinear and complex dynamics for a possibly advanced design of actual engineering systems, characterized by high multidimensionality and non-negligible uncertainties, is the general new frontier of the research in the area, which of course particularizes itself in a great number of challenging sub-problems.

Within this general framework, a few main topics, out of the many addressed ones, are selected and more extensively dealt with in the sequel, although only qualitatively. Features of cable nonlinear dynamics and relevant outcomes are overviewed in Sect. 3. Details on models, methods, and nonlinear/complex phenomena of shallow cables, as formulated and obtained until about the beginning of the new millennium, are reported in the reviews [2,3], whereas modelling and dynamic phenomena of arbitrarily sagged and inclined cables are more extensively addressed in [4], along with updates and new results. Section 4 generally dwells with the fundamental role played by global dynamics in the analysis, control and safe design of mechanical systems and structures. Specifically, the focus is on research achievements obtained in about the last two decades on issues like global bifurcation control, dynamical integrity and its control, possible contributions of global dynamics to systems' analysis, and expectations about the role that a dynamical integrity viewpoint can play for an innovative engineering design. More detailed treatments can be found in [5,6].

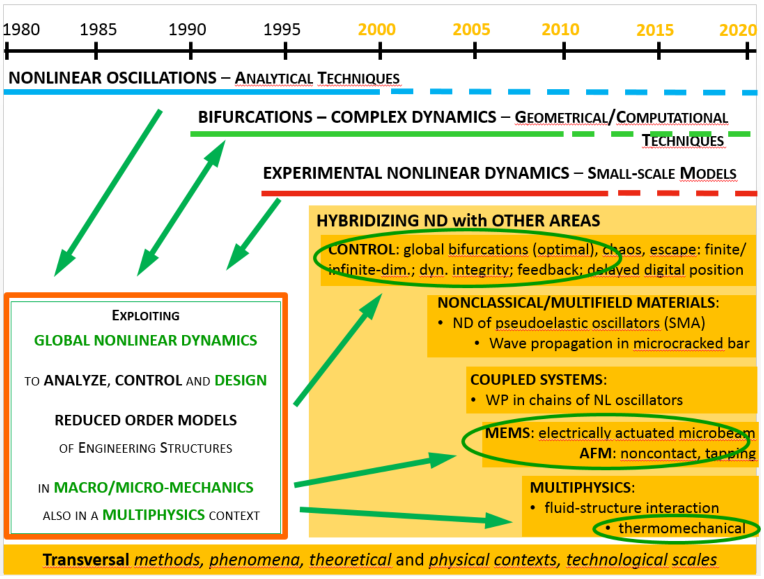

FIGURE 5. Addressed topics framed in the Hybridization stage of nonlinear dynamics in mechanics.

3. Cable Nonlinear Dynamics

The sagged cable is a flexible structural element conveniently used in many applications of mechanical, civil, electrical, ocean and space engineering. It is also an archetypal system for the analysis of a variety of geometrically nonlinear phenomena exhibited by elastic one-dimensional structures with initial curvature—which entail even nonlinearities, besides the odd ones of taut strings and rectilinear beams due to axial stretching—prone to finite amplitude vibrations. The initially curved configuration of the system has meaningful consequences already in linear dynamics, where it gives rise to a modal spectrum of clearly distinct in-plane and out-of-plane frequencies. The former markedly depend on an elastogeometric parameter summarizing the cable properties, and undergo the frequency crossover phenomenon of corresponding symmetric and antisymmetric in-plane modes, where a multiple internal resonance also involving out-of-plane modes can always be activated in the nonlinear regime.

Single-mode model. In the reduced order modelling perspective, the asymmetric Helmholtz–Duffing oscillator (with quadratic and cubic nonlinearities), obtained through condensation of longitudinal dynamics and the Galerkin technique, was reliably used to analyze the planar nonlinear dynamics of a single-mode model of horizontal shallow cables with sag-to-span ratio up to about 1/20 and technical values of the axial rigidity-to-initial tension ratio—whose elastogeometric parameter is lower than the first crossover one—, subject to symmetric transverse harmonic excitation. Weakly forced oscillations at primary, subharmonic or superharmonic external resonances were investigated by the method of multiple time scales. Moving to the strongly nonlinear regime, numerical simulations highlighted the occurrence of involved bifurcation scenarios and varied complex dynamics, with the harmonic balance method furnishing valuable theoretical predictions as to the regions in excitation parameters space where searching for chaotic responses in more detail. Chaos characterization was accomplished via both qualitative (time histories, phase portraits, Poincaré maps) and quantitative (frequency power spectra, Lyapunov exponents, fractal dimensions) measures. Global topological aspects were addressed by constructing basins of attraction of coexisting solutions, and investigating in detail the role played by the invariant manifolds of the governing saddles on the onset/enlargement/destruction of chaotic attractors via boundary and interior crises, when varying a control parameter. Single-mode results were of basic theoretical interest in themselves for exhibiting a rich variety of local and global dynamical phenomena associated with the coexistence of quadratic and cubic nonlinearities. Yet, similar response features, although modified/enriched by the contribution of also bending stiffnesses, were later encountered in the nonlinear dynamics of shallow arches and buckled beams, too. This highlights how the archetypal Helmholtz–Duffing oscillator, formerly studied with rather arbitrary values of the nonlinear coefficients, and first used in a physically meaningful context for the shallow cable, plays a paradigmatic role in the analysis of nonlinear, bifurcation and chaos phenomena of a large class of elastic mono-dimensional systems with initial curvature, of interest in both applied mechanics and structural engineering.

Multi-mode model. Single-mode planar oscillations of sagged cables were mostly of interest for establishing a reference background from which moving forward to investigating the three-dimensional dynamics always occurring in practice, due to the out-of-plane cable flexibility, as the excitation amplitude overcomes quite low threshold values. Moreover, it can only take place when no further modes are involved in the system dynamics through some mechanism of in-plane and/or out-of-plane nonlinear coupling, this being strongly enhanced by the occurrence of internal resonances entailing meaningful contributions of non-directly excited modes to the overall response. In this respect, the spectrum of cable natural frequencies is particularly rich, because of exhibiting a variety of 1:1, 1:2 and 1:3 internal resonances between in-plane, out-of-plane and in/out-of-plane modes, with a special role played by crossover points. Accordingly, a four-mode model with fundamental planar and nonplanar, symmetric and antisymmetric, modes accounting for the multiple 2:2:1:2 resonance occurring at first crossover, and the first symmetric in-plane mode excited at primary resonance, was formulated and addressed with multiple time scales, highlighting the richness and variety of cable dynamic phenomena produced by the nonlinear modal interaction already in regimes of regular vibrations. As regards transition to nonregular responses at higher excitation amplitudes or in specific frequency ranges, complementary indications were provided by the numerical investigation of non-stationary solutions of the reconstituted modulation equations ensuing from the multiple scales analysis, and by computer simulations of the system's original ODEs.

A meaningful improvement towards a reliable analysis of internally resonant responses was associated with directly solving cable PDEs with multiple scales, obtaining a distinct spatial discretization for each time scale due to the expansion of the nonlinear boundary conditions, without preliminarily getting a ROM via an assumed mode technique (direct vs discretized perturbation approach). The advantage ensues from the possibility to capture the spatial dependence of cable motion, and of the associated tension, by including the effects of the infinite number of modes from the spectrum of the eigenvalue problems, via the solution of a number of second order boundary-value problems resulting from the direct procedure. An approach also fruitfully used to obtain resonant nonlinear normal modes of a general class of weakly nonlinear one-dimensional continuous systems with quadratic and cubic nonlinearities, along with a detailed picture of the conditions for the actual activation of various resonant bimodal interactions, in connection with the non-orthogonality of the relevant nonlinear modes.

Experimental cable-mass suspension. A fundamental step forward in the analysis of regular and, mostly, nonregular classes of motion was accomplished through the design and investigation of a refined, small-scale model of experimental cable-mass suspension at about first crossover (Fig. 6), able to account for the flexibility, high modal density and variable modal contributions to the response of actual cables more realistically than the theoretical ROMs with constrained modal shapes. Systematic dynamic analysis under in-phase or out-of-phase support motions in the neighbourhoods of primary, order-1/2 subharmonic, and order-2 superharmonic resonances of first in/out-of-plane antisymmetric modes with nearby frequencies, highlighted rich bifurcation scenarios of regular and nonregular responses, and overall behaviour charts in the excitation parameters plane.

Quantitative characterization of global properties of experimental spatiotemporal dynamics was obtained through different means. (i) The characterization of attractors in terms of dimensionality, strangeness and possible chaoticity, (ii) the identification of number and shape of space configuration variables mostly contributing to the nonregular response, (iii) the local and global characterization and evolution of flow structure in phase space, which is necessary for understanding bifurcation scenarios. All this information was obtained with rather sophisticated techniques requiring considerable experimental and computational efforts. Analysis of the asymptotic motion in a nonregular condition was performed on attractors reconstructed by means of the delay-embedding technique, which provides indications on the actual number of degrees of freedom taking meaningful part in the response. The embedding dimension was evaluated at saturation of an attractor dimension invariant, while the analysis of response spatial properties was performed by means of the proper orthogonal decomposition (POD). Proper orthogonal modes (POMs) were computed starting from simultaneous time series data measured at different positions throughout the system, with the corresponding eigenvalues standing for the amount of energy captured by the eigenfunctions. Furnishing the basis for capturing more power per mode than any other basis, the POD allowed to identify the mechanical configurations most visited, on average, during a temporal evolution of the response, to be also used in a theoretical context for decomposing the spatial flow via a reduction method.

Different bifurcation paths from regular to nonregular dynamics were exhibited by the cable-mass system, depending on the kind of support motion and external resonance, and on cable dynamic properties. They were traced back to two canonical scenarios of dynamical systems theory, also possibly competing with each other, namely (i) the quasiperiodic (three-tori breakdown) scenario, and (ii) a scenario involving global bifurcation of a homoclinic invariant set of the symmetric flow. The quasiperiodic scenario showed various types of bifurcations, including Hopf from 2-torus to 3-torus, transition to chaos through 3-tori breakdown and phase-locking (Fig. 7). Classes of motion were characterized based on topological dimension of manifolds where the motion develops, and correlation dimension of attractors. The spatial coherence analysis showed successive involvement in the system dynamics, at subsequent Hopf bifurcations, of different cable-mass configuration variables, with a meaningful amount (more than 90%) of power of the chaotic response captured by the first three POMs, resembling the first in/out-of-plane symmetric modes and the first out-of-plane antisymmetric mode.

The quasiperiodic scenario was not found for the cable at first crossover, whose nearly perfect multiple (2:2:1:2) internal resonance prevents quasiperiodic couplings and transition to chaos from occurring, while replacing them in parameter space with wider regular resonant couplings. The homoclinic bifurcation scenario was of more general interest for being concerned with each frequency zone where ballooning type classes of motion, involving couples of in/out antisymmetric (symmetric) modes in the case of out-of-phase (in-phase) support motion, are present. In-depth characterization of classes of motion and transition scenarios required working with a proper, thermally conditioned, experimental setup, such to guarantee a steady temperature and stabilize the response of the cable-mass system, making it mechanically accessible without the cable loosening possibly entailed by too high values of excitation amplitude.

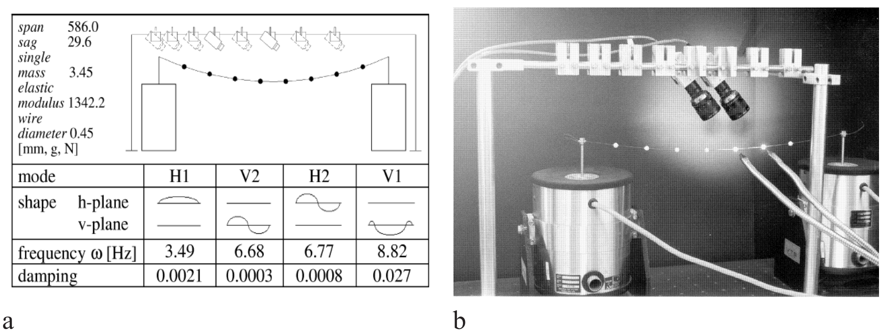

FIGURE 6. Experimental cable-mass system. (a) Mechanical model with parameters and dynamical characteristics; (b) Hanged nylon wire carrying eight small concentrated masses, with shakers and two movable optical cameras.

Bifurcation to homoclinic chaos occurred from a couple of coexisting (e.g. antisymmetric) ballooning periodic solutions, differing from each other for the orbit clockwise or anticlockwise rotation in the configuration plane. The ensuing chaotic attractor showed the lowest observed dimensionality, since transition from regular to nonregular behaviour happened without increasing the number of involved modes over the two of the periodic ballooning already present in adjacent regular zones. Overall, the availability of temperature as a third control parameter allowed various achievements: (i) to qualitatively refer the experimental unfolding of the dynamics to the theoretical one of the divergence-Hopf (d-H) bifurcation normal form; (ii) to unfold the dynamics not only in the strict neighbourhood of the organising d-H bifurcation but also in the ensuing post-critical regions, where the dependence of material damping on temperature affects secondary bifurcations to homoclinic chaos; (iii) to show the variable involvement of a further POM, with respect to the reference two-mode normal form scenario ending up to homoclinic chaos. Construction of an experimentally driven low-dimensional phenomenological model allowed to interpret the physical response scenario in the framework of the symmetry breaking of a highly degenerated bifurcation set describing an O(2) symmetric Takens–Bogdanov bifurcation. This paved the way towards the independent formulation of a refined theoretical ROM characterized by all necessary prerequisites for reliably reproducing the experimentally observed phenomena.

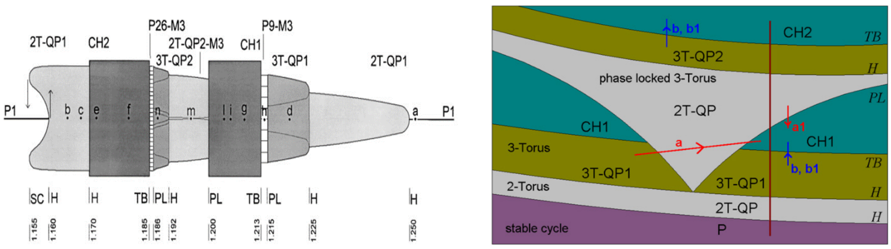

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FIGURE 7. Quasiperiodic scenario to chaos. (a) Experimental bifurcation diagram, (b) canonical two-parameter bifurcation chart for 3-Torus breakdown, with partially similar qualitative scenario along the vertical brown line (H: Hopf, PL: phase-locking, TB: torus breakdown, SC: saddle cycle; periodic (Pm-M3), quasiperiodic (nT-QPm-Mk) and chaotic (CH1, CH2) attractors) [4].

Arbitrarily sagged and inclined cables. Later, attention was focused on arbitrarily sagged and possibly inclined cables, with the formulation of more general models based on a refined kinematical description of cable deformation. As for the shallow cable, they exhibit quadratic and cubic nonlinearities, however, due to the interaction between longitudinal and transverse dynamics—ensuing from accounting for the overall inertia effects—the former occur also in the absence of initial sag. Longitudinal and transverse (in/out-of-plane) dynamics are nonlinearly coupled, so the cable model was referred to as a kinematically non-condensed in order to distinguish it from the condensed model considered in the shallow cable literature.

The inherent asymmetry of inclined sagged cables entails an important qualitative modification in the natural frequency spectrum, with replacement of the frequency crossover of symmetric horizontal cables by the frequency veering of asymmetric inclined ones. The latter entails occurrence of hybrid, i.e. asymmetric, modes resulting from a mixture of symmetric and antisymmetric shapes, which also affects the system nonlinear behavior. Multimode discretization of the non-condensed continuous model provided low-dimensional reduced ODEs suitable for analytical solution via multiple scales. A major concern was related with the need to include in the ROM also some selected non-resonant modes, whose effects are generally overlooked in the analysis of modal interactions at crossovers, where only the resonant ones are considered. Indeed, depending on the role played by second-order effects of quadratic nonlinearities entering the perturbation analysis, non-resonant (higher-order) modes may furnish non-negligible contributions to the overall response of cables with significant sag/inclination. This highlighted how the lowest dimensional discretization may yield quantitatively inaccurate or even qualitatively crude results with respect to the infinite-dimensional discretization, or the direct application of the asymptotic method to the original PDEs with no a priori assumptions of the displacement solution form, whose outcomes are equivalent provided enough modes are retained in the discretization. Again, a very rich pattern of nominally activable internal resonances involving different in/out-of-plane modes occurs at both crossover (avoidance) frequencies of horizontal (inclined) cables and away from them. Although not all internal resonances are actually activated due to the nonlinear orthogonality of the correspondingly involved modes, the activated ones may entail strong modal interaction and energy exchange between involved modes.

The analysis of planar vibrations under vertical harmonic excitation at primary resonance with a 1:1 or 1:2 internally resonant mode allowed a general description of uncoupled and/or coupled solutions, the former only involving the directly excited resonant mode, the latter also including a non-excited mode driven into the response by an internal resonance enhanced mechanism of energy transfer. In the 1:1 internal resonance of horizontal (inclined) cables at crossovers (avoidances) of different order, modification from symmetric/antisymmetric to hybrid modes entails meaningfully different scenarios of nonlinear response.

One more aspect of interest in the second-order multiple scales analysis of non-condensed models was associated with the availability of coupled dynamic configurations accounting for the spatial corrections, with respect to the reference linearly resonant modes, due to the quadratic nonlinearity effects of all non-resonant modes considered in a finite discretization. This was also of major importance as regards the evaluation of cable nonlinear dynamic tension. Indeed, allowing for space-varying distribution of the tension along the cable, against the spatially constant one inherently associated with the condensed model, the non-condensed model revealed nontrivial effects also on the induced space/time-varying tension, which may increase up to unwanted tensile/compressive values to be carefully considered in the dynamic design.

Overall, research results on cable nonlinear dynamics, obtained with colleagues and younger collaborators, included modeling, analysis and description of a variety of phenomena, accomplished according to various classification criteria. These consisted of considering and distinguishing between: (i) horizontal/inclined and (ii) small-sag (parabolic)/large-sag (catenary) configurations, (iii) condensed/non-condensed kinematics and continuum modeling, (iv) analytical/numerical/geometrical/experimental techniques, (v) single-mode/multi-mode ROMs, (vi) free/forced and planar/nonplanar dynamics, (vi) non-resonant/resonant (single or multiple) dynamics, under external/parametric/combination excitations, (vii) weak/strong nonlinearities, (viii) 'exact'/approximate solutions, (ix) uncoupled/coupled and regular/non-regular response regimes, (x) local/global bifurcation scenarios and the ensuing dynamics.

4. Global Dynamics for Analysis, Control and Safe Design of Engineering Systems

Since about the end of the past century and over the last twenty years, my research efforts have been mainly devoted to a wider set of issues in the nonlinear analysis, control and design of a variety of structures and mechanical systems in a dynamic environment. Starting from the analysis of the response scenarios through the local dynamics approach necessary to grasp their basic features in predominantly analytical terms, interest has progressively shifted to the equally fundamental role that the corresponding global analysis plays for the full description, the reliable understanding, and a possible fruitful addressing of the system dynamics. This generally claims for the essential contribution of the latter.

The interest towards global aspects of the cable response, needed to fully catch its complex behavior, already entailed a substantial widening of perspective on systems' nonlinear dynamics. In the last few years of the past millennium, this was meaningfully complemented with a novel and more general aim: investigating whether an aware exploitation just of the global features of a generic dynamical system may help in effectively controlling its unwanted complex responses. In this respect, the collaboration with Stefano Lenci, a former young researcher nourished with nonlinear issues precisely in the dynamical systems-oriented environment of the last part of the millennium, showed to be fundamental.

Global bifurcation control. Transition to complex dynamics (i.e., chaos and/or escape from a potential well) is triggered by the occurrence of the homoclinic (heteroclinic) bifurcation of the stable and unstable manifolds of a (two) main saddle(s) organizing the system phase space. Properly controlling such global event may thus entail avoiding, or at least delaying, the onset of unwanted dynamic phenomena when varying a governing system parameter. Aiming at an overall control of the dynamics in phase space, instead of the local one more commonly pursued in the OGY and feedback techniques of chaos control [7], an optimal and 'nearly universal' procedure to control global bifurcations was developed. It consists of modifying the shape of the reference harmonic excitation without affecting its frequency, which would not make much sense from the application viewpoint. Adding suitable, and generally not too 'expensive', controlling superharmonics, properly tailored according to whether control of the whole phase space or only a part of it is pursued [8], allowed to move the overall threshold of homoclinic (or heteroclinic) bifurcation in the excitation parameter space towards higher amplitude values. In this way, the region where topological mechanisms leading to complex dynamics are prevented turns out to be meaningfully expanded (see Fig. 8 for the exemplary Helmholtz oscillator). The idea was not completely new. However, its aware theoretical implementation, and the following, nearly systematic, application to a huge variety of dynamical systems, allowed to highlight the great advantages that global bifurcation control (or even possibly its anti-control) may provide in terms of delaying (or facilitating) the occurrence of chaos and/or escape, provided the saddle manifolds actually responsible for those topological mechanisms are correctly detected and handled. This is a relatively easy issue if the involved saddle is (are) the hilltop one(s) governing a system potential well, which enables to apply the procedure in substantially analytical terms by means of the Melnikov method [7,8]. Instead, if the involved saddle(s) is a (are) secondary one(s) associated with a formerly stable periodic orbit, the Melnikov method cannot be applied and a fully numerical, and generally more onerous, procedure has to be implemented.

Over about the last two decades, global bifurcation control has been successfully applied to a meaningful variety of models, ranging from archetypal oscillators also describing discrete mechanical systems, up to single-degree-of-freedom models of continuous structures [6,7,9-11]. They include: softening Helmholtz-like oscillators of interest in macromechanics (with reference to, e.g., ship capsizing and rigid block overturning) and micro/nano-mechanics (as regards dynamic pull-in in electrically actuated microcantilevers or unwanted sample contact in non-contact atomic force microscopes); various kinds of Duffing oscillators with their underlying mechanical/ structural significance; the impacted inverted pendulum; the shallow von Mises truss; the Augusti model and the guyed pendulum under specific symmetry conditions.

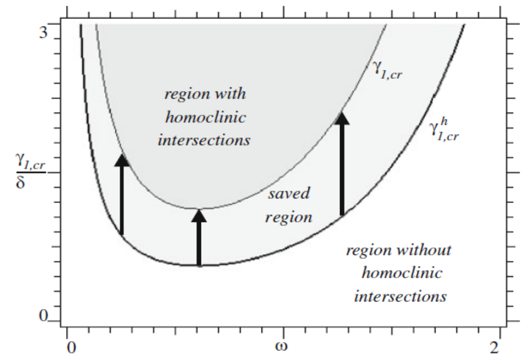

FIGURE 8. The optimal saved region for the Helmholtz oscillator [7].

Dynamical integrity and its control. Since the early conceptual and operational achievements by Thompson and coauthors [12], global effects on a system dynamics in phase space, entailed by the variation of some main governing parameter (e.g., the excitation amplitude), may be evaluated in terms of dynamical integrity. This provides the robustness of a (generally regular) attractor against non-infinitesimal variations of initial conditions, and of the relevant basin of attraction against variations (possibly also small) of system parameters. Since about the beginning of the new millennium, this important, yet somehow overlooked, geometrical and computational topic has been resumed and deepened. The concept of safe basin was generalized. Novel integrity measures (e.g., the integrity factor) capable to effectively assess the actual robustness of a given attractor, which strongly depends on the compactness of its basin, were formulated. Integrity tools (such as 1D profiles and 2D/3D response charts) have been comparatively and systematically used for analyzing the more or less abrupt (i.e., dangerous) evolution of the topological erosion of a safe basin, ending up to escape of the forced dynamics to an unbounded attractor [13]. This holds for either a single bounded attractor or a potential well encompassing competing bounded attractors. Escape corresponds to a variety of physical failures of engineering systems, such as capsizing of a ship, overturning of a rigid block, dynamic pull-in in a micro/nano-electro-mechanical-system (MEMS/NEMS), jump-to-contact in a non-contact atomic force microscope (AFM), loss of stability in discrete systems and flexible structures liable to unstable buckling, and so on. All of this is concerned with the *analysis* of systems' dynamical integrity and its ensuing safety.

Their possible *control* is a meaningfully related aspect. To this aim, the above mentioned technique of global bifurcation control also succeeds in meaningfully shifting towards higher excitation amplitudes the abrupt and dangerous fall-down of dynamical integrity ending up to escape, provided the invariant manifolds intersection of the (hilltop or non-hilltop) saddles actually triggering the basins' erosion is the target to be avoided by the control. This can be seen in Fig. 9 where, by contrast, the highly dangerous deterioration of dynamical integrity entailed by the locally-tailored feedback control, commonly used in AFMs to stabilize a given operational solution, is also highlighted. Global bifurcation control is seen to be effective for all of the mentioned systems/models, although with variable features depending on a number of mechanical, dynamical, bifurcation and control issues, which affect the obtained performances.

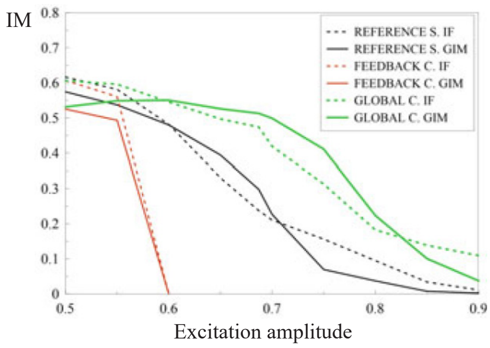

FIGURE 9. AFM erosion profiles for the reference uncontrolled system (black), and for the system with feedback control (orange) or global bifurcation control (green), with two different integrity measures (IM) [11].

A summary schematic flowchart of the system's dynamical integrity scenario and its global bifurcation control is shown in Fig. 10, by considering erosion and escape to either infinity or an adjacent well, and distinguishing between its being unwanted (control), as in most technical applications, or wanted (anti-control), as it sometimes occurs, e.g. for switching purposes in MEMS.

Global dynamics for analysis. Since the 90s (see Fig. 3), investigation of systems' global dynamics has been increasingly complementing local bifurcation analyses in either theoretical or, to a wider extent, computational terms, mainly concerned with the construction of basins of attraction and the analysis of their evolution with a varying parameter. Indeed, cross-correlating local and global outcomes has allowed a more comprehensive description and understanding of the system response, even though the obtained enhancement of knowledge has been sometimes mostly incremental. Yet, there are situations in which the investigation of a system global dynamics is necessary not only, e.g., to detect basins of additional (e.g., rare or hidden) attractors with respect to the sole main ones usually uncovered by a local bifurcation analysis, but also to reveal possible underlying dynamic phenomena whose non-observance would meaningfully prevent from a reliable overall understanding. As to the writer's experience, this is the case of, e.g., coupled multiphysics problems, such as a plate in a thermo-mechanically coupled environment. Therein, the coexistence of mechanical and thermal fields evolving on non-trivially different time scales prevents the local bifurcation analysis—which pays attention to the solely steady dynamic regimes—from catching the entire response scenario in the multidimensional phase space. In the thermomechanical plate, the phenomenon unveiled by the global dynamics consists of the important effect entailed by the quite slow thermal dynamics on the steady outcomes of the much faster mechanical vibrations, which strongly depend on the thermal conditions [6,14].

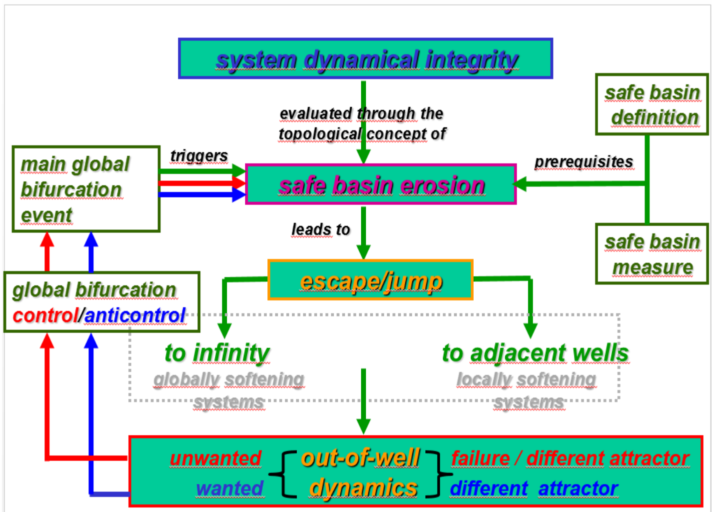

FIGURE 10. Dynamical integrity (erosion) scenario and its control.

Dynamical integrity for safe design. Analysis of dynamical integrity turns out to be a fundamental step not only for a comprehensive description of the system global response in the uncontrolled or controlled context, but also for the interpretation of the often meaningful discrepancies observed between theoretical/numerical results and actual experimental outcomes, due to the unavoidable disturbances and uncertainties always characterizing real systems. As a matter of fact, regions of practical stability of given solutions in parameters space, i.e. those actually attainable in physical systems, are often non-trivially narrower than those obtained for a corresponding theoretical/numerical model via a local (i.e., Lyapunov) stability analysis. The global stability assessment provided by the dynamical integrity approach allows to catch the rationale behind the observed discrepancies. It represents an invaluable criterion for predicting a possibly reduced safe region of the dynamic regime actually realized in the physical system, with respect to the desired operational one provided by the local stability analysis of the companion theoretical model, under expected environmental disturbances and/or system inherent uncertainties [15,16].

Of course, extension and exploitation of global dynamics and integrity concepts/tools to the multidimensional models actually representing real systems and structures is a challenging and still unexplored frontier of nonlinear dynamics [6].

Anyway, in the long-term perspective, a conscious assumption of the residual dynamical integrity to be reliably considered still admissible for safe system operation under expected disturbances/uncertainties may allow to establish a novel paradigm for the design of mechanical and structural systems. Specifically, an aware assessment of the relevant nonlinear dynamics may enable to exploit the actual load carrying capacity of engineering systems according to a safe, knowledge-based, design criterion, generally variable from one technological field to another. Indeed, this allows to meaningfully expand the safely usable (i.e., practically stable) region of system operation in parameters space, with respect to the one allowed by conventional, and highly conservative, design criteria (see the schematic in Fig. 11 [5,13]), possibly paving the way to the conception and realization of innovative systems and structures.

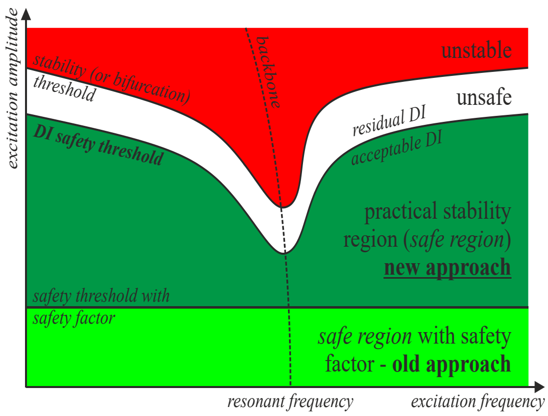

FIGURE 11. Schematic safety chart for a softening system subjected to harmonic excitation [5].

5. Conclusions

A journey through the writer's research in the nonlinear dynamics of mechanical systems and structures, accomplished over about the last forty years, has been illustrated, in the background of the more general evolution of knowledge and applications occurred in the area over the same time interval. Cable nonlinear dynamics and the meaningful role played by global dynamics in the analysis, control and design of engineering systems have been discussed in some detail. The meaningful contribution of a number of excellent colleagues and younger collaborators, who coauthored most of the relevant research papers, is gratefully acknowledged.

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