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PREFACE

This collection of articles explore connections between various areas of mathematics. They are written for the general mathematical audience but they try to bring the interested reader to the very frontier of current research on these topics. A typical feature of connections presented here is that certain areas of combinatorics such as, for example, Ramsey theory are related to particular area of topological dynamics, geometric group theory and functional analysis. However some articles in this collection explore also other equally important connections such as, for example, connections between combinatorial and descriptive set theory and graph theory and Tukey theory.

We give a very brief general summary of each of the contribution leaving the interested reader go and directly consult the abstracts of the corresponding articles for the more just summaries. The contribution of Avilés explores the classification problem for finite analytic monotone families $\mathcal{A}_1, \dots, \mathcal{A}_k$ of infinite sets of integers motivated by a particular problem about norms of averaging operators. This classification problem has seen much of the progress in recent years and it is a program that could find many other applications. The article of DiPrisco shows that the classical theory of graph-homomorphisms gets some new and quite interesting phenomena when moved into the context of descriptive set theory. These new graph-theoretic phenomena have in turn played an important role in recent developments of the descriptive set theory itself. Dobrinen's contribution relates the study of ultrafilters on $\mathbb{N}$ relative to their classical Rudin–Keisler ordering and the most recent theory of ultrafilters on $\mathbb{N}$ relative to the coarser Tukey ordering. In particular, Dobrinen shows how this program is closely related to a particular kind of Ramsey-theoretic classification problems that are of independent interest. The article of Dodos presents recent advances in the density Ramsey theory, a very active field of current research. It reveals the new phenomenon that even the infinitary Ramsey-theoretic results can have their deep density versions. Koszmider's article explores combinatorial set-theoretic methods in some areas of functional analysis and in particular in the study of the function space $C(K)$. The contribution of Kurilić surveys the recent study of the classification problem of relational structures $\mathbb{X}$ using the associated posets $\mathbb{P}(\mathbb{X})$ of isomorphic substructures of $\mathbb{X}$ as forcing notions. Lopez-Abad's contribution explores the connection between the classical Nash–Williams' theory of fronts and barriers of finite subsets of $\mathbb{N}$ with some problems in the infinite-dimensional geometry of Banach spaces. The article of Moore takes a particular problem in geometric group theory and relate it to a Ramsey-theoretic problem that is still open and of independent interest. Moore shows that the amenability problem for Thompson group has a natural translation to a Ramsey-theoretic problem related to Hindman's theorem or, more precisely, to the Galvin–Glaser proof of Hindman's theorem using the semigroup $(\beta\mathbb{N}, +)$ of ultrafilters on $\mathbb{N}$. The article of Nguyen Van Thé presents advances on the newly discovered connection between the structural Ramsey theory and some areas of topological dynamics. This is also one of the very active area of current research that brings together several areas of mathematics. Solecki's article is devoted to

Tukey theory of topological orders in the context of descriptive set theory. It shows that some classical orders appearing in topology or functional analysis such as, for example the ideal NWD of nowhere-dense subsets of $\mathbb{R}$ or the Banach lattice ℓ_1 occupy particularly important places in this coarse classification scheme.

Finally we mention that the most of the articles give lists of open problems pointing thus the readers towards interesting directions for further research. In particular, each of these articles end with extensive bibliographies that would support such readers.

Contents

Antonio Avilés:
AN INTRODUCTION TO MULTIPLE GAPS 7

Carlos A. Di Prisco and Stevo Todorčević:
BASIS PROBLEMS FOR BOREL GRAPHS 33

Natasha Dobrinen:
SURVEY ON THE TUKEY THEORY OF ULTRAFILTERS 53

Pandelis Dodos:
SOME RECENT RESULTS IN RAMSEY THEORY 81

Piotr Koszmider:
UNIVERSAL OBJECTS AND ASSOCIATIONS BETWEEN CLASSES OF BANACH SPACES AND CLASSES OF COMPACT SPACES 93

Miloš S. Kurilić:
POSETS OF ISOMORPHIC SUBSTRUCTURES OF RELATIONAL STRUCTURES 117

Jordi Lopez-Abad:
FAMILIES OF FINITE SUBSETS OF N 145

Justin Tatch Moore:
HINDMANN'S THEOREM, ELLIS'S LEMMA AND THOMPSON'S GROUP F 171

Lionel Nguyen Van Thé:
A SURVEY ON STRUCTURAL RAMSEY THEORY AND TOPOLOGICAL DYNAMICS WITH THE KECHRIS–PESTOV–TODORČEVIĆ CORRESPONDENCE IN MIND 189

Sławomir Solecki:
TUKEY REDUCTIONS AMONG ANALYTIC DIRECTED ORDERS 209