

VELOCITY PARAMETER GENERALIZATION OF IDEALLY DISSOCIATED GAS RECOMBINATION

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1. Introductory discussions and starting equation system

In this work it is discussed about a laminar boundary layer on the body of an arbitrary form, which moves at supersonic speed through the earth atmosphere, e.g. supersonic planes, rockets, space ships, Such high speeds lead to the so-called supersonic warming of the aircraft caused by an intensive transition of the mechanical energy into the thermal one, in the impact wave itself, which is formed in front of the aircraft, as well as in the thin layer next to its surface and it happens on account of the force of friction. Because of that high temperature are developed in the space of flow close to the aircraft surface which lead to thermochemical reactions of dissociation and recombination, that is they change the "homogeneous" air into a multicomponent mixture of gases: O_2 , O , N_2 , N , NO and Ar . It is to be remarked that the former thermochemical reactions are followed by the process of diffusion, whereat it is possible that the material of the overflowed aircraft surface enters the chemical reactions with the mentioned component of the mixture.

Taking into consideration the growth of the i^{th} component mass on account of chemical reactions among other components of the mixture, the heat caused by the process of diffusion, as well as the heat resulted from chemical reactions (So-called latent heat), a complete equation system of laminar plane boundary layer for the case of established flow of gases mixture followed by chemical reactions, as it is known, says:

$$\begin{aligned} \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} &= \rho_e u_e \frac{d u_e}{d x} + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right); \\ (1) \quad \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) &= 0; \\ \rho u \frac{\partial C_i}{\partial x} + \rho v \frac{\partial C_i}{\partial y} &= \frac{\partial}{\partial y} \left(\rho D_i \frac{\partial C_i}{\partial y} \right) + \dot{W}_i; \quad (i = q - 1), \end{aligned}$$

$$\begin{aligned} \rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = & -u \rho_e u_e \frac{du_e}{dx} + \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + \mu \left(\frac{\partial u}{\partial y} \right)^2 - \\ & - \sum_i h_i \dot{W}_i + \sum_i c_{pi} \rho D_i \frac{\partial C_i}{\partial y} \frac{\partial T}{\partial y}; \\ p = \rho \bar{R} T, \bar{R} = \sum_i C_i R_i \quad h_i = & \int_0^T c_{pi}(T) dT + h_i^0, \end{aligned}$$

with the corresponding boundary conditions:

$$(2) \quad \begin{aligned} u = v = 0, \quad C_i = C_{iw}, \quad T = T_w \quad & \text{at } y = 0, \\ u \rightarrow u_e(x), \quad C_i \rightarrow C_{ie}(x), \quad T \rightarrow T_e(x) \quad & \text{at } y \rightarrow \infty, \\ u = u_0(y), \quad C_i = C_{i0}(y), \quad T = T_0(y) \quad & \text{at } x = x_0. \end{aligned}$$

The following established symbols in the theory of boundary layer, are used here:

- x — for longitudinal coordinate,
- y — for transversal coordinate,
- u — for longitudinal speed projection in the boundary layer,
- v — for transversal speed projection in the boundary layer,
- ρ — for density,
- μ — for coefficient of dynamic viscosity,
- C — for mass concentration,
- \dot{W} — for the speed of mass forming during chemical reactions,
- D — for diffusion coefficient,
- c_p — for specific heat at constant pressure,
- T — for the temperature
- λ — for coefficient of thermal conductivity,
- h — for enthalpy,
- R — for gas constant, and
- \bar{R} — for gas constant of the mixture.

At the same time the following indexes mark:

- i — the component of mixture,
- q — the number of mixture components,
- w — conditions on the surface of the overflowed body,
- e — conditions on the external border of the boundary layer, and
- o — distribution of physical values in a given cross-section of the boundary layer determined by the abscissa $x = x_0$.

Besides, in the same equations h_i^0 represents the specific enthalpy of the i^{th} component formation.

2. Analogous (automodel) solutions and the parameter of recombination velocity

Equations of the system (1), as it is shown, represent nonlinear partial and simultaneous equations, whose result in a general case can not be determined in a closed form.

According to analogy with other problems of the boundary layer theory, Fay and Riddell in [2] found the solution of the equations (2) bringing them to automodel form. In order to reduce the system of equations (1) to the system of common differential equations, they introduced the following transformations of variables:

$$(3) \quad \xi(x) = \int_0^x \rho_w \mu_w u_e dx, \quad \eta(x, y) = \frac{u_e}{\sqrt{2\xi}} \int_0^y \rho dy,$$

as well as the transformation of dependent variables:

$$(4) \quad \frac{u}{u_e} = \frac{\partial \varphi}{\partial \eta}, \quad \frac{C_i}{C_{ie}} = S_i, \quad \frac{T}{T_e} = \theta.$$

In that way the system of equations (1) is brought to the form:

$$\frac{\partial}{\partial \eta} \left(\frac{\rho \mu}{\rho_w \mu_w} \frac{\partial^2 \varphi}{\partial \eta^2} \right) + \varphi \frac{\partial^2 \varphi}{\partial \eta^2} + 2 \frac{d(\ln u_e)}{d(\ln \xi)} \left[\frac{\rho_e}{\rho} - \left(\frac{\partial \varphi}{\partial \eta} \right)^2 \right] = 2 \xi \left(\frac{\partial \varphi}{\partial \eta} \frac{\partial^2 \varphi}{\partial \xi \partial \eta} - \frac{\partial \varphi}{\partial \xi} \frac{\partial^2 \varphi}{\partial \eta^2} \right);$$

$$(5) \quad \begin{aligned} & \frac{\partial}{\partial \eta} \left(\frac{\rho \mu}{\rho_w \mu_w} \frac{L_{ei}}{P_r} \frac{\partial S_i}{\partial \eta} \right) + \varphi \frac{\partial S_i}{\partial \eta} + \frac{2 \xi \dot{W}_i}{\rho u_e C_{ie} (d\xi/dx)} = \\ & = 2 \xi \left(\frac{\partial \varphi}{\partial \eta} \frac{\partial S_i}{\partial \xi} - \frac{\partial \varphi}{\partial \xi} \frac{\partial S_i}{\partial \eta} \right) + 2 \frac{\partial \varphi}{\partial \eta} S_i \frac{d(\ln C_{ie})}{d(\ln \xi)}; \end{aligned}$$

$$\begin{aligned} & \frac{\partial}{\partial \eta} \left(\frac{1}{P_r} \frac{c_p}{c_{pw}} \frac{\rho \mu}{\rho_w \mu_w} \frac{\partial \theta}{\partial \eta} \right) + \frac{c_p}{c_{pw}} \varphi \frac{\partial \theta}{\partial \eta} - \sum_i \frac{2 \xi \dot{W}_i}{\rho u_e (d\xi/dx)} \frac{h_i}{T_e c_{pw}} + \\ & + \frac{u_e^2}{T_e c_{pw}} \frac{\rho \mu}{\rho_w \mu_w} \left(\frac{\partial^2 \varphi}{\partial \eta^2} \right)^2 + \sum_i \frac{c_{pi}}{c_{pw}} C_{ie} \frac{L_{ei}}{P_r} \frac{\rho \mu}{\rho_w \mu_w} \frac{\partial S_i}{\partial \eta} \frac{\partial \theta}{\partial \eta} = \\ & = \frac{\partial \varphi}{\partial \eta} \left[2 \frac{c_p}{c_{pw}} \theta \frac{d(\ln T_e)}{d(\ln \xi)} + 2 \frac{u_e^2}{T_e c_{pw}} \frac{\rho_e}{\rho} \frac{d(\ln u_e)}{d(\ln \xi)} \right] + 2 \xi \frac{c_p}{c_{pw}} \left(\frac{\partial \varphi}{\partial \eta} \frac{\partial \theta}{\partial \xi} - \frac{\partial \varphi}{\partial \xi} \frac{\partial \theta}{\partial \eta} \right), \end{aligned}$$

where:

$$P_r = \frac{\mu c_p}{\lambda} \quad \text{Prandtl's number,}$$

$$L_{ei} = \frac{\rho D_i c_p}{\lambda} \quad \text{Lewis's number,}$$

- φ — dimensionless function of flow,
 S — ratio of concentrations, and
 θ — dimensionless temperature.

It is to be remarked that the automodel condition demands all dependant variables (φ , S_i , and θ) as well as the members characterizing the forming of mass components on account of chemical reactions to be exclusively the functions of the newly introduced transversal coordinate η . This, however, happens only next to the front critical point of the overflowed body [3]. In that case, namely, the system (5) is brought to:

$$\begin{aligned}
 & \frac{d}{d\eta} \left(\frac{\rho\mu}{\rho_w\mu_w} \frac{d^2\varphi}{d\eta^2} \right) + \varphi \frac{d^2\varphi}{d\eta^2} + \frac{1}{2} \left[\frac{\rho_1}{\rho} - \left(\frac{d\varphi}{d\eta} \right)^2 \right] = 0; \\
 & \frac{d}{d\eta} \left(\frac{\rho\mu}{\rho_w\mu_w} \frac{L_{ei}}{P_r} \frac{dS_i}{d\eta} \right) + \varphi \frac{dS_i}{d\eta} + \left[2 \left(\frac{du_e}{dx} \right)_1 \right]^{-1} \frac{\dot{W}_i}{\rho C_{i1}} = 0; \\
 & \frac{d}{d\eta} \left(\frac{1}{P_r} \frac{c_p}{c_{pw}} \frac{\rho\mu}{\rho_w\mu_w} \frac{d\theta}{d\eta} \right) + \frac{c_p}{c_{pw}} \varphi \frac{d\theta}{d\eta} - \left[2 \left(\frac{du_e}{dx} \right)_1 \right]^{-1} \sum_i \frac{\dot{W}_i}{\rho} \frac{h_i}{c_{pw}T_1} + \\
 & + \sum_i \frac{c_{pi}}{c_{pw}} C_{i1} \frac{L_{ei}}{P_r} \frac{\rho\mu}{\rho_w\mu_w} \frac{d\theta}{d\eta} \frac{dS_i}{d\eta} = 0,
 \end{aligned}
 \tag{6}$$

where all values relating to the former critical point, for which $u_e^2 \ll h_1$, have index 1.

The system of simple differential equations (6) was solved by Fay and Riddell [2] for the case of ideally dissociated gas flow (that is the air), supposing the dissociated air consists only of two components: the atomic A and the molecular A_2 between which the reactions determined by the scheme happen exclusively:



In that case the speed of the atom mass formation \dot{W}_A on account of molecule dissociation is determined by the expression [2].

$$\dot{W}_A = - \frac{2 K_1 \rho^3 T^{-1,5}}{M^2} \left[\frac{(1+\alpha)(\alpha^2 - \tilde{\alpha}^2)}{(1 - \tilde{\alpha}^2)} \right],
 \tag{7}$$

where

- K_1 — the constant of recombination velocity,
 M — the molecule mass,
 $\tilde{\alpha}$ — the concentration of atom mass at the equilibrium dissociation.

By means of dependance (7) Fay and Riddell transformed the members of equation system (6), which characterize the chemical reactions, to the form:

$$\left[2 \left(\frac{du_e}{dx} \right)_1 \right]^{-1} \frac{\dot{W}_A}{\rho} = - C_1 \theta^{-3,5} \frac{\alpha - \tilde{\alpha}^2}{1 - \alpha}.
 \tag{8}$$

where

$$(9) \quad C_1 = \frac{2 K_1 p_1 T_1^{-3.5}}{R^2} \left[\left(\frac{du_e}{dx} \right)_1 \right]^{-1},$$

is so-called parameter of recombination velocity, which represents the relation of the so-called "diffusion time" to the so-called "living time" of an atom in the surrounding of the front critical point of the aircraft [3].

Introducing into equations the parameter (9) Fay and Riddell at [2] solved the system (6), for the boundary values of this parameter $C_1=0$ (the case of the so-called "frozen" boundary layer) and $C_1=\infty$ (the case of the so-called equilibrium dissociation), as well as for the case of final recombination velocity $0 \leq C_1 < \infty$, (what represents the general case of ideally dissociated gas flow).

It is to be emphasized that the parameter of recombination velocity C_1 as well as the determined solutions relate exclusively to the flow next to the front critical point of the overflowed body. In this work, however, the generalization of parameter (9) is done in the whole boundary layer region of dissociated gas. To do that we start from the basic general equation system (1) and the corresponding boundary conditions (2).

3. Transformation of initial equations

Instead of physical coordinates x, y the variables s and z are introduced by means of relations:

$$(10) \quad s(x) = \frac{1}{\rho_n \mu_n} \int_0^x \rho_w \mu_w dx; \quad z(x, y) = \frac{1}{\rho_n} \int_0^y \rho dy,$$

where index n marks the arbitrary known value of the considered physical value.

Introducing into consideration the function ψ of flow by means of relations:

$$(11) \quad u = \frac{\partial \psi}{\partial z}; \quad \bar{v} = \frac{\rho_n \mu_n}{\rho_w \mu_w} \left(u \frac{\partial z}{\partial x} + \frac{\rho}{\rho_n} v \right) = - \frac{\partial \psi}{\partial s},$$

the initial equation system (1), written for the case of ideally dissociated gas flow [1], states:

$$(12) \quad \begin{aligned} \frac{\partial \psi}{\partial z} \frac{\partial^2 \psi}{\partial s \partial z} - \frac{\partial \psi}{\partial s} \frac{\partial^2 \psi}{\partial z^2} &= \frac{\rho_e}{\rho} u_e \frac{du_e}{ds} + v_n \frac{\partial}{\partial z} \left(N \frac{\partial^2 \psi}{\partial z^2} \right); \\ \frac{\partial \psi}{\partial z} \frac{\partial \alpha}{\partial s} - \frac{\partial \psi}{\partial s} \frac{\partial \alpha}{\partial z} &= v_n \frac{\partial}{\partial z} \left(\frac{N}{S_m} \frac{\partial \alpha}{\partial z} \right) + \frac{\rho_n \mu_n}{\rho_w \mu_w} \frac{\dot{W}_A}{\rho}; \\ c_p \left(\frac{\partial \psi}{\partial z} \frac{\partial T}{\partial s} - \frac{\partial \psi}{\partial s} \frac{\partial T}{\partial z} \right) &= - \frac{\rho_e}{\rho} u_e \frac{du_e}{ds} \frac{\partial \psi}{\partial z} + v_n \frac{\partial}{\partial z} \left(\frac{N}{P_r} c_p \frac{\partial T}{\partial z} \right) + \\ + v_n N \left(\frac{\partial^2 \psi}{\partial z^2} \right)^2 &+ v_n \frac{N}{S_m} (c_{pA} - c_{pM}) \frac{\partial \alpha}{\partial z} \frac{\partial T}{\partial z} - \frac{1}{\rho} \frac{\rho_n \mu_n}{\rho_w \mu_w} \sum_{i=A, M} h_i \dot{W}_i, \end{aligned}$$

with boundary conditions:

$$\begin{aligned}
 & \psi = \frac{\partial \psi}{\partial z} = 0, \quad \alpha = \alpha_w, \quad T = T_w \quad \text{at } z = 0; \\
 (13) \quad & \frac{\partial \psi}{\partial z} \rightarrow u_e(s), \quad \alpha \rightarrow \alpha_e(s), \quad T \rightarrow T_e(s) \quad \text{at } z \rightarrow \infty; \\
 & \frac{\partial \psi}{\partial z} = u_0(z), \quad \alpha = \alpha_0(z), \quad T = T_0(z) \quad \text{at } s = s_0.
 \end{aligned}$$

In the previous equations S_m marks Schmidt's number ($S_m = \mu/\rho D$), while the function N is determined by

$$\begin{aligned}
 (14) \quad & N = \frac{\rho \mu}{\rho_w \mu_w}, \quad \text{at the same time} \\
 & N = 1 \quad \text{for } z = 0 \quad \text{and } N \rightarrow \frac{\rho_e \mu_e}{\rho_w \mu_w} = N(s) \quad \text{for } z \rightarrow \infty.
 \end{aligned}$$

According to the ideas from the work [4], the following transformations are introduced into the system (12):

$$(15) \quad s = s, \quad \eta = \frac{B_L}{\Delta^{**}} z, \quad \psi(s, z) = \frac{u_e \Delta^{**}}{B_L} \Phi(s, \eta), \quad \bar{T} = \frac{T}{T_1},$$

where

- B_L — so-called standardizing Lojczanskij's constant,
- Φ — dimensionless flow function,
- \bar{T} — dimensionless temperature, and
- Δ^{**} — the thickness of impulse loss determined by the expression

$$(16) \quad \Delta^{**} = \int_0^{\infty} \frac{u}{u_e} \left(1 - \frac{u}{u_e} \right) dz = \Delta^{**}(s).$$

It is remarked, that according to [4], that is [1], we simply come to impulse equation of this case of flow, which can be written in one of three following forms:

$$(17) \quad \frac{dZ^{**}}{ds} = \frac{F}{u_e}, \quad \frac{df}{ds} = \frac{u'_e}{u_e} F + \frac{u''_e}{u'_e} f, \quad \frac{\Delta^{***}}{\Delta^{**}} = \frac{u'_e F}{2 u_e f}.$$

In this way we introduce into consideration the parameter form

$$(18) \quad f = \frac{du_e/ds \Delta^{**2}}{\nu_n},$$

as well as the following values:

$$(19) \quad Z^{**} = \frac{\Delta^{**2}}{\nu_n}; \quad \zeta = \left[\frac{\partial(u/u_e)}{\partial(z/\Delta^{**})} \right]_w; \quad H = \frac{\Delta^*}{\Delta^{**}};$$

$$F = 2[\zeta - (2+H)f]; \quad \Delta^* = \int_0^\infty \left(\frac{\rho_e}{\rho} - \frac{u}{u_e} \right) dz,$$

which are known from the oneparametar theory of boundary layer.

By applying transformation (15), impulse equation (17) and corresponding values determined by equations (18) and (19), the equation system (12) is brought to the following form:

$$(20) \quad \begin{aligned} & \frac{\partial}{\partial \eta} \left(N \frac{\partial^2 \Phi}{\partial \eta^2} \right) + \frac{F+2f}{2B_L^2} \Phi \frac{\partial^2 \Phi}{\partial \eta^2} + \frac{f}{B_L^2} \left[\frac{\rho_e}{\rho} - \left(\frac{\partial \Phi}{\partial \eta} \right)^2 \right] = \\ & = \frac{u_e f}{u_e' B_L^2} \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial^2 \Phi}{\partial s \partial \eta} - \frac{\partial \Phi}{\partial s} \frac{\partial^2 \Phi}{\partial \eta^2} \right); \\ & \frac{\partial}{\partial \eta} \left(\frac{N}{S_m} \frac{\partial \alpha}{\partial \eta} \right) + \frac{F+2f}{2B_L^2} \Phi \frac{\partial \alpha}{\partial \eta} + \frac{1}{\nu_n} \left(\frac{\Delta^{**}}{B_L} \right)^2 \frac{\rho_n \mu_n}{\rho_w \mu_w} \frac{\dot{W}_A}{\rho} = \\ & = \frac{u_e f}{u_e' B_L^2} \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \alpha}{\partial s} - \frac{\partial \Phi}{\partial s} \frac{\partial \alpha}{\partial \eta} \right); \\ & \frac{\partial}{\partial \eta} \left(\frac{N C^*}{P_r C_1^*} \frac{\partial \bar{T}}{\partial \eta} \right) + \frac{F+2f}{2B_L^2} \frac{C^*}{C_1^*} \Phi \frac{\partial \bar{T}}{\partial \eta} - \frac{\rho_e}{\rho} \left(\frac{u_e^2}{c_{p1} T_1} \right) \frac{f}{B_L^2} \frac{\partial \Phi}{\partial \eta} + \\ & + N \left(\frac{u_e^2}{c_{p1} T_1} \right) \left(\frac{\partial^2 \Phi}{\partial \eta^2} \right)^2 + \frac{N D^*}{S_m C_1^*} \frac{\partial \alpha}{\partial \eta} \frac{\partial \bar{T}}{\partial \eta} - \\ & - \frac{1}{\nu_n} \left(\frac{\Delta^{**}}{B_L} \right)^2 \frac{\rho_n \mu_n}{\rho_w \mu_w} \left(\frac{h_A^0}{c_{p1} T_1} \right) \frac{\dot{W}_A}{\rho} = \frac{u_e f}{u_e' B_L^2} \frac{C^*}{C_1^*} \left(\frac{\partial \Phi}{\partial \eta} \frac{\partial \bar{T}}{\partial s} - \frac{\partial \Phi}{\partial s} \frac{\partial \bar{T}}{\partial \eta} \right), \end{aligned}$$

with corresponding boundary conditions:

$$\begin{aligned} \Phi = \frac{\partial \Phi}{\partial \eta} = 0, \quad \alpha = \alpha_w, \quad \bar{T} = \bar{T}_w \quad \text{at } \eta = 0; \\ \frac{\partial \Phi}{\partial \eta} \rightarrow 1, \quad \alpha \rightarrow \alpha_e, \quad \bar{T} \rightarrow \bar{T}_e \quad \text{at } \eta \rightarrow \infty; \\ \Phi = \Phi_0(\eta), \quad \alpha = \alpha_0(\eta), \quad \bar{T} = \bar{T}_0(\eta) \quad \text{at } s = s_0. \end{aligned}$$

By dividing with c_{p1} the energy equation of the system (20) is brought to the dimensionless form during the transformation, while the relations of the specific heats are replaced with corresponding relations of dimensionless functions as follows:

$$(21) \quad \frac{c_p}{c_{p1}} = \frac{C^*}{C_1^*}, \quad \frac{c_{pA} - c_{pM}}{c_{p1}} = \frac{D^*}{C_1^*}.$$

Here are, according to [1], that is [3], those functions determined by the expressions:

$$\begin{aligned} C^* &= \frac{10}{7} \alpha + (1 - \alpha) \left[1 + \frac{2}{7} e^{-(\bar{T}_V/\bar{T})^2} \right]; \\ C_1^* &= \frac{10}{7} \alpha_1 + (1 - \alpha_1) \left[1 + \frac{2}{7} e^{-\bar{T}_V^2} \right]; \\ D^* &= \frac{3}{7} \alpha - \frac{2}{7} e^{-(\bar{T}_V/\bar{T})^2}; \\ &\bar{T}_V T_V / T_1, \end{aligned}$$

in which the characteristic temperature T_V , according to Fay and Riddell, for the ideally dissociated air is $T_V = 800$ K.

4. The introduction of velocity recombination generalized parameter

In the equation system (20), which described the flow in a plane stationary and laminar boundary layer for the common case of ideally dissociated gas, diffusion and energy equations consist of the members which characterize chemical reactions when they differ only in the constant factor $h_A^\circ / c_{p1} T_1$. That is why only the following member is further considered:

$$\bar{W} = \frac{1}{B_L^2} \frac{\Delta^{**2}}{\nu_n} \frac{\rho_n \mu_n}{\rho_w \mu_w} \frac{\dot{W}_A}{\rho},$$

which, taking the pattern (7), situation equation and proportionless temperature in consideration, says:

$$\bar{W} = \frac{1}{B_L^2} \frac{\Delta^{**2}}{\nu_n} \frac{\rho_n \mu_n}{\rho_w \mu_w} \frac{2 K_1 p_e^2}{R^2 T_1^{3,5}} \frac{(\tilde{\alpha}^2 - \alpha^2)}{(1 + \alpha)(1 - \tilde{\alpha}^2)} \bar{T}^{-3,5}$$

that is

$$\bar{W} = g \frac{(\alpha^2 - \tilde{\alpha}^2)}{(1 + \alpha)(1 - \tilde{\alpha})} \bar{T}^{-3,5},$$

where

$$(23) \quad g = \frac{2 K_1}{B_L^2 R^2 T_1^{3,5}} \frac{\rho_n \mu_n}{\rho_w \mu_w} \frac{\Delta^{**2}}{\nu_n} p_e^2 = g(s).$$

Having in mind the transformation form (10), which introduces the longitudinal variable $s(x)$, and taking into account the expressions (18) and (19), the previous dimensionless parameter $g(s)$ can be written in the following form:

$$g(s) = \frac{f}{B_L^2} \frac{2 K_1 p_e^2}{R^2 T_1^{3.5}} \left(\frac{du_e}{dx} \right)^{-1} = \frac{f}{B_L^2} \cdot C \left(p_e, \frac{du_e}{dx} \right) = \frac{f}{B_L^2} \cdot C(x).$$

At the same time the value

$$(24) \quad C(x) = \frac{2 K_1 p_e^2 T_1^{-3.5}}{R^2} \left(\frac{du_e}{dx} \right)^{-1}$$

represents the parameter, which will be marked by the generalized parameter of recombination velocity. It is, namely, for the case of ideally dissociated gas flow in the surrounding of the front critical point brought to:

$$C(x) = \frac{2 K_1 p_1^2 T_1^{-3.5}}{R^2} \left[\left(\frac{du_e}{dx} \right)_1 \right]^{-1} = C_1,$$

where C_1 represents the local parameter of recombination velocity which was introduced into literature by Fay and Riddell in the expression (9). Since the parameter $C(x)$ contains in itself the distribution of pressure and velocity on the exterior border for the whole boundary layer range the proposed term suits it here completely. Now, the parameter of recombination velocity C_1 is a specific case of the introduced generalized parameter of recombination velocity $C(x)$.

It is to be emphasized, at the end, that the way to complete the generalization of the equation system (20), which describes the general case of ideally dissociated gas flow, is opened by the introduction of the parameter $C(x)$.

References

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