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ON THE QUANTIFIER OF LIMITING REALIZABILITY

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Logical connectives intermediate between $\underset{\cdot}{\exists}$ and $\exists$ as well as between $\underset{\cdot}{\vee}$ and $\vee$ [$\underset{\cdot}{\exists} xF$ denotes $\neg \forall x \neg F$ and $(F_1 \underset{\cdot}{\vee} F_2)$ denotes $\neg(\neg F_1 \& \neg F_2)$] are sometimes useful in searching for interesting "in contents" constructive analogs of theoremes of classical mathematics. We introduce two such logical connectives prompted by the theory of limiting computable (in other terms semicomputable) functions, namely the quantifier of limiting realizability $\underset{\rightarrow}{\exists}$ and the limiting disjunction $\underset{\rightarrow}{\vee}$. They are defined in terms of the basic connectives of constructive logic as follows:

$$\underset{\rightarrow}{\exists} z F \leftrightharpoons \exists y ((y \operatorname{stab}) \& \forall z ((z \lim \cdot \operatorname{val} y) \rightarrow F)),$$

$$(F_1 \underset{\rightarrow}{\vee} F_2) \leftrightharpoons \underset{\rightarrow}{\exists} x ((x = 0 \rightarrow F_1) \& (x \neq 0 \rightarrow F_2)).$$

The expression (y stab) stands for the condition $\ll y$ is a gödelnumber of a stabilizing unary total recursive function $\gg$ (this means: a gödel number of such a total unary recursive function f for which a value x_0 of the argument quasi-exists ($\neg\neg\exists$) such that $f(x_0+x)=f(x_0)$ for any x). ($z \lim \cdot \operatorname{val} y$) stands for the condition $\ll z$ is the limit value of the unary recursive function with gödelnumber $y \gg$. Several properties of $\underset{\rightarrow}{\exists}$, $\underset{\rightarrow}{\vee}$ have been presented in the report. A detailed exposition can be found in [1].

Reference

[1] Н. А. ШАНИН. *О кванторе предельной осуществимости.* — Записки научных семинаров ЛОМИ, т. 60, 1976. стр. 209—220.

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