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RETRIAL G-QUEUE WITH PRECIPITOUS BREAKDOWN AND NON-TENACIOUS CUSTOMERS

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Abstract: We perform an analysis on an $M/G/1$ system that has bulk arrival Poisson process as well as instantaneous service, Precipitous breakdown and random repair, G-queue, and operation under an MBV (Modified Bernoulli vacation) policy. The positive outcome of a customer retrial is lost when an Anti-positive customer arrives during a positive (good) customer's assistance period. If a new customer enters and notices that the server is currently undergoing repairs or on vacation, they may renege or balk from the system. To obtain a wide variety of additional outcomes, we use the details provided by the supplementary variable method regarding the rates at which different events occur and it is possible to determine the probability-generating functions of queue length distributions as well as the explicit formulations of important performance metrics. The numerical data confirm the analytical findings about the major performance metrics.

Keywords: Retrial queue, bulk arrival, non-tenacious customers, precipitous breakdown, modified Bernoulli vacation.

MSC: 60K25, 90B22, 60K30.

1. INTRODUCTION

Finding prompt assistance in any system is not always feasible in our typical day-to-day lives. It is predicted that clients will have to wait in line to obtain a service when there is a great demand. Booths of petrol pumping stations and toll plazas, primary health clinics, banks, railway ticket booking counters, etc. are some places where queues may occur. When clients arrive and join the orbit retry waiting, if the server is found busy. By checking the orbit, customers can check if a server is available and retry their service. Clients may not join the queue and exit the service area if they come during a busy server's proceed. We refer to this occurrence as balking. When a client arrives at a wholesale dealership, for instance, they can notice a long line because of an unforeseen high demand for servicing and decide to not visit.

Our proposed model deals with an enhancement of BARQS (Batch Arrival Retrial Queueing System) that provides service for MBV, orbit search and a maintenance server for that provides service. Performance measurements of systems are conducted using both the SVM (Supplementary Variable Method) and SSPGF (Steady-State Probability Generating Function) for the system. Applications for this model include stochastic production techniques, ICS (Inventory Control Systems) with a multi-production facility and issues with machinery replacement, and medical service systems for mobile consultation.

This paper is organized as follows. The background for the research is described in Section 2. Section 3 and Section 4 delves into the mathematical model and notations of the queueing system. The steady-state distributions of the orbit and system size are computed recursively and examined in Section 5. The performance metrics of the queueing model are explained in Section 6. Some special cases and numerical examples demonstrating how the settings affect various performance aspects are given in Section 7 and Section 8. In concluding remarks are represented in Section 9.

2. LITERAURE REVIEW

The study of systems for queuing with G-networks, non-positive clients, and advanced applications has been performed by [1]. Queueing models that include breakdowns and repairs are explored by [2]. Such models are further inspired by current sophisticated applications in WCS (Wireless Communication Systems), CNS (Communication Network Systems), DNS (Data Networking System), Communication Networks,and MRP (Machine Replacement Problems). Non-Markovian retry queueing was explored by [3] in which the consumer is allowed to balk when the server is overloaded or on vacation, and the server provides two phases of services, with the second phase being optional.

They discovered that modifying the Bernoulli vacation schedule leads to an improvement in the system's performance in terms of both its stability and its use of the server resources. In [4], investigated several queueing issues, including balking (which means refusing to join the queue) and reneging (leaving the queue after entering). They were able to determine the Steady-State Probabilities (SSP), the average number of clients in the system and line, the probabilities that there will be R or additional clients in the system, the probabilities that clients will accept service, balk, or renege the client a loss rate and the average value for the time spent in line for clients who accept service, and the corresponding outcomes for clients who initially renege. Reduced the reneging of the parameters to 0, every one of these conclusions is obtained for an absolute balking system.

The straightforward investigation of impatient customers in multi-server queues was provided by [5]. They presumptively assumed that a consumer who arrived at the queue system would be informed of its current status. As a consequence of this, we regard reneging and balking to be functions of the system's state. In this article, several of different performance metrics are offered, each with an exact and CFF (closed-form formulation). In [6] have delved into negative customer feedback on the orbital search after service. As soon as there's an issue with the server, it is sent to be repaired, and in the meanwhile, the backup server is in charge of offering service to the customers. This work describes a retry queueing model with a multi-stage service and a notion of the second server that offers service during the original server's repair periods. This concept is provided as part of a retrial queueing model. This investigation is a continuation of the findings that [7] established in the previous research.

Vacation queueing signifies unexpected server absences for several reasons, such as maintenance, handling additional queues, hunting for new work (a common feature in various telecommunications), or just taking a vacation from work. The reason for this could be varied, including a vacation a period during which the server is not accessible to core users. Based on Bernoulli scheduling and generic retry times, researchers [8] and [9] developed single-server retry queues. Various applications involving CSC (client-server communication), and EMS (electronic mail services) on the Internet, all of which use a single server analyzing messages in multiple phases, have demonstrated the usefulness of these queuing systems. Several researchers, such as [10, 11] and [12] have performed research and analysis on such systems. The models used in the [13] study and that being used by [14] in their investigation are quite different from one another. The concept of G-queues was initially introduced by [15, 16] to illustrate neural networks with both positive and negative signals in addition to product-form solutions. An optimum operation strategy for the model in terms of its overall anticipated cost has been established at a reduced cost, and according to the Bernoulli timetable that was established by [17], it is allowed to utilize single vacation at the same time. In [18] expressed an unreliable server, collisions in the $M/M/1$ retrial queue, and transmission errors. In [19, 20] are investigated the retrial model with negative consumers, repair and transmission errors. Reviewed [21, 22, 23] to find more about the researchers $G-$ queueing approach.

3. DESCRIPTION OF THE MATHEMATICAL MODEL

Arrival Procedure:

Clients arrive in bulk according to a CPP (Compound Poisson process) with rate λ. Let G_k be the number of clients that involve the k^{th} arrival (batch), $k = 1, 2, 3\dots$ with a common distribution $P[G_k = n] = G_n, n = 1, 2, 3...$ and G(z) denotes the probability generating function (PGF) of G.

Retrial Procedure:

While a batch of clients arrives and the server is available (there is no waiting space), the primary client in the batch starts receiving service, and the remaining clients join a group of blocked clients known as an orbit. Otherwise batch of clients chooses the option of leaving the service area with a probability (prob) of f or joining the orbit with a probability of $1-f$. The distribution of retrial times is arbitrary and i^{th} corresponds to LST (Laplace-Stieltjes Transform) $\omega^*(\nu)$.

Service Procedure:

Service times are followed by generic distribution. It is presumable that servicing time adheres to the generic random variable (r.v) ι with distribution function (d.f) $\iota(t)$, LST $\iota^*(\nu)$.

The Removable Process and Precipitous Breakdown:

Based on a Poisson arrival rate δ, the negative clients enter the system from outside. These negative clients only show up during the positive clients' usual service hours. The system will eliminate the positive clients who are receiving service if negative clients cannot build up in a line and do not obtain service. These kinds of negative clients lead to server malfunctions and short-interval channel failures. Assume that the server undergoes a precipitous breakdown (failure) only when it is inactive. When a negative customer comes up, the system no longer has the positive customer in service, which forces the server to breakdown. When a server breakdown, it stops service and waits for repair to begin. The server's waiting period is known as the delay time. If there is a service failure with accompanied probability $\bar{\gamma} = 1-\gamma$, then the probability γ is the main and retrial customer, is active and energetic at the beginning of service. The length of κ the assistance's repair time involves a distribution function $\kappa(t)$ and LST $\kappa^*(\nu)$.

Modified Bernoulli Vacation:

After the completion of service of each customer, the server waits for the next customer and then, the server may take a vacation with probability m, and with prob, $1-m$ it waits to serve the next customer. The vacation time of the server is of random length τ with distribution function $\tau(t)$ and LST $\tau^*(\nu)$.

Orbital search :

Once the service is done, the server either seeks the consumer in the orbit with prob e $(0 \leq e \leq 1)$ or remains idle with prob $1-e$.

Let $C(t_{a*})$ be the sever state, where

$$C(t_{a*}) = \begin{cases} 0 & \text{Server is Idle} \\ 1 & \text{Server is busy on Service} \\ 2 & \text{Server is on Repair} \\ 3 & \text{Server is on Vacation} \end{cases}$$

4. NOTATIONS

1. $\Upsilon_1(x_{a*}) \equiv$ the HR(hazard rate) for retrial of $\omega(x_{a*})$
   i.e., $\Upsilon_1(x_{a*})dx_{a*} = \frac{d\omega(x_{a*})}{1-\omega(x_{a*})}$.

2. $\Upsilon_2(x_{a*}) \equiv$ the HR for service of $\iota(x_{a*})$
   i.e., $\Upsilon_2(x_{a*})dx_{a*} = \frac{d\iota(x_{a*})}{1-\iota(x_{a*})}$.

3. $\Upsilon_3(x_{a*}) \equiv$ the HR for repair of $\kappa(x_{a*})$
   i.e., $\Upsilon_3(x_{a*})dx_{a*} = \frac{d\kappa(x_{a*})}{1-\kappa(x_{a*})}$.

4. $\Upsilon_4(x_{a*}) \equiv$ the HR for vacatin of $\tau(x_{a*})$
   i.e., $\Upsilon_4(x_{a*})dx_{a*} = \frac{d\tau(x_{a*})}{1-\tau(x_{a*})}$.

5. $\Upsilon^0(t_{a*}) \longrightarrow$ ERT (Elapsed Retry Time)

6. $\iota^0(t_{a*}) \longrightarrow$ ETS (Elapsed Time of Service)

7. $\kappa^0(t_{a*}) \longrightarrow$ ERT (Elapsed Repair Time)

8. $\tau^0(t_{a*}) \longrightarrow$ EVT (Elapsed Vacation Time)

9. $D_0(t_{a*}) \longrightarrow$ the probability that the system is idle at time t_{a*}.

10. $D_n(x_{a*}, t_{a*}) \longrightarrow$ the probability that there are precisely n clients in the orbit at time t_{a*}, where x_{a*} is the elapsed retrial time of the test customer undergoing retrial.

11. $\varphi_n(x_{a*}, t_{a*}) \longrightarrow$ the probability that there are precisely n clients in the orbit at time t_{a*}, where x_{a*} is the elapsed retrial time of the test customer undergoing service.

12. $\Theta_n(x_{a*}, t_{a*}) \longrightarrow$ the probability that there are precisely n clients in the orbit at time t_{a*}, where x_{a*} is the elapsed retrial time of the test customer undergoing repair.

13. $Y_n(x_{a*}, t_{a*}) \longrightarrow$ the probability that there are precisely n clients in the orbit at time t_{a*}, where x_{a*} is the elapsed retrial time of the test customer undergoing modifed vacation.

5. STEADY-STATE PROCESS AND ANALYSIS

The probability for this process to occur may be defined as follows:

$D_0(t_{a*}) = P\{C(t_{a*}) = 0, N(t_{a*}) = 0\}$ and the probability densities are

$$D_n(x_{a*}, t_{a*})dx_{a*} = P\{C(t_{a*}) = 0, N(t_{a*}) = n, x_{a*} \leq \Upsilon^0(t_{a*}) < x_{a*} + dx_{a*}\}, \quad \text{for} \ \ t_{a*} \geq 0, x_{a*} \geq 0, n \geq 1$$

$$\varphi_n(x_{a*}, t_{a*})dx_{a*} = P\{C(t_{a*}) = 1, N(t_{a*}) = n, x_{a*} \leq \iota^0(t_{a*}) < x_{a*} + dx_{a*}\}, \quad \text{for} \ \ t_{a*} \geq 0, x_{a*} \geq 0, n \geq 0$$

$$\Theta_n(x_{a*}, t_{a*})dx_{a*} = P\{C(t_{a*}) = 2, N(t_{a*}) = n, x_{a*} \leq \kappa^0(t_{a*}) < x_{a*} + dx_{a*}\}, \quad \text{for} \ \ t_{a*} \geq 0, x_{a*} \geq 0, n \geq 0$$

$$Y_n(x_{a*}, t_{a*})dx_{a*} = P\{C(t_{a*}) = 3, N(t_{a*}) = n, x_{a*} \leq \tau^0(t_{a*}) < x_{a*} + dx_{a*}\}, \quad \text{for} \ \ t_{a*} \geq 0, x_{a*} \geq 0, n \geq 0$$

The differential difference equations for the model were created using the following SVM based on the aforementioned assumptions:

$$f\lambda D_0 = \int_0^\infty Y_0(x_{a*})\Upsilon_4(x_{a*})dx_{a*} + (1-m)\int_0^\infty \varphi_0(x_{a*})\Upsilon_2(x_{a*})dx_{a*} + \int_0^\infty \Theta_0(x_{a*})\Upsilon_3(x_{a*})dx_{a*} \tag{1}$$

$$\frac{dD_n(x_{a*})}{dx_{a*}} + (\lambda + \Upsilon_1(x_{a*}))D_n(x_{a*}) = 0, \ \ n \geq 1 \tag{2}$$

$$\frac{d\varphi_0(x_{a*})}{dx_{a*}} + (\lambda f + \delta + \Upsilon_2(x_{a*}))\varphi_0(x_{a*}) = 0 \tag{3}$$

$$\frac{d\varphi_n(x_{a*})}{dx_{a*}} + (\lambda f + \delta + \Upsilon_2(x_{a*}))\varphi_n(x_{a*}) = \lambda f \sum_{k=0}^{\infty} G_k \varphi_{n-k}(x_{a*}) \tag{4}$$

$$\frac{d\Theta_0(x_{a*})}{dx_{a*}} + (\lambda f + \Upsilon_3(x_{a*}))\Theta_0(x_{a*}) = 0 \tag{5}$$

$$\frac{d\Theta_n(x_{a*})}{dx_{a*}} + (\lambda f + \Upsilon_3(x_{a*}))\Theta_n(x_{a*}) = \lambda f \sum_{k=0}^{\infty} G_k \Theta_{n-k}(x_{a*}) \tag{6}$$

$$\frac{dY_0(x_{a*})}{dx_{a*}} + (\lambda f + \delta + \Upsilon_4(x_{a*}))Y_0(x_{a*}) = 0 \tag{7}$$

$$\frac{dY_n(x_{a*})}{dx_{a*}} + (\lambda f + \delta + \Upsilon_4(x_{a*}))Y_n(x_{a*}) = \lambda f \sum_{k=0}^{\infty} G_k Y_{n-k}(x_{a*}) \tag{8}$$

The boundary conditions are

$$\begin{aligned} D_n(0) = \int_0^{\infty} Y_n(x_{a*})\Upsilon_4(x_{a*})dx_{a*} + (1-m)\int_0^{\infty} \varphi_n(x_{a*})\Upsilon_2(x_{a*})dx_{a*} \\ + \int_0^{\infty} \Theta_n(x_{a*})\Upsilon_3(x_{a*})dx_{a*} \end{aligned} \tag{9}$$

$$\begin{aligned} \varphi_n(0) = \int_0^{\infty} D_{n+1}(x_{a*})\Upsilon_1(x_{a*})dx_{a*} + \lambda \sum_{k=1}^{\infty} G_k \int_0^{\infty} D_{n-(k-1)}(x_{a*})dx_{a*} \\ + \gamma \lambda f D_0 \end{aligned} \tag{10}$$

$$\Theta_n(0) = \delta(1-e)\int_0^{\infty} \varphi_n(0)(x_{a*})dx_{a*} + \bar{\gamma}\lambda f D_0 \tag{11}$$

$$Y_n(0) = m\int_0^{\infty} \varphi_n(x_{a*})\Upsilon_2(x_{a*})dx_{a*} \tag{12}$$

Normalization Condition is

$$\begin{aligned} D_0 + \sum_{n=1}^{\infty}\int_0^{\infty} D_n(x_{a*})dx_{a*} + \sum_{n=0}^{\infty}\int_0^{\infty} \varphi_n(x_{a*})dx_{a*} + \sum_{n=0}^{\infty}\int_0^{\infty} \Theta_n(x_{a*})dx_{a*} \\ + \sum_{n=0}^{\infty}\int_0^{\infty} Y_n(x_{a*})dx_{a*} = 1 \end{aligned} \tag{13}$$

The following set of PDE (partially differential equations) obtained by multiplying the eqns. (2–12)by the power it needs for z_{a*}^{n}, then adding them all up for n:

$$\frac{dD(x_{a*},z_{a*})}{dx_{a*}}+(\lambda+\Upsilon_1(x_{a*}))D(x_{a*},z_{a*})=0 \tag{14}$$

$$\frac{d\varphi(x_{a*},z_{a*})}{dx_{a*}}+(\lambda f(1-G(z_{a*}))+\delta+\Upsilon_2(x_{a*}))\varphi(x_{a*},z_{a*})=0 \tag{15}$$

$$\frac{d\Theta(x_{a*},z_{a*})}{dx_{a*}}+(\lambda f(1-G(z_{a*}))+\Upsilon_3(x_{a*}))\Theta(x_{a*},z_{a*})=0 \tag{16}$$

$$\frac{dY(x_{a*},z_{a*})}{dx_{a*}}+(\lambda f(1-G(z_{a*}))+\Upsilon_4(x_{a*}))Y(x_{a*},z_{a*})=0 \tag{17}$$

$$\begin{aligned}D(0,z_{a*})=&\int_0^\infty Y(x_{a*},z_{a*})\Upsilon_4(x_{a*})dx_{a*}+(1-m)\int_0^\infty \varphi(x_{a*},z_{a*})\\&\Upsilon_2(x_{a*})dx_{a*}+\int_0^\infty \Theta(x_{a*},z_{a*})\Upsilon_3(x_{a*})dx_{a*}-\lambda fD_0\end{aligned} \tag{18}$$

$$\begin{aligned}\varphi(0,z_{a*})=&\frac{1}{z_{a*}}\int_0^\infty D(x_{a*},z_{a*})\Upsilon_1(x_{a*})dx_{a*}+\frac{\lambda G(z_{a*})}{z_{a*}}\\&\left[\int_0^\infty D(x_{a*},z_{a*})dx_{a*}+\gamma fD_0\right]\end{aligned} \tag{19}$$

$$\Theta(0,z_{a*})=[\delta(1-e)+z_{a*}\delta e]\int_0^\infty \varphi(x_{a*},z_{a*})dx_{a*}+\bar{\gamma}\lambda fD_0 \tag{20}$$

$$Y(0,z_{a*})=m\int_0^\infty \varphi(x_{a*},z_{a*})\Upsilon_2(x_{a*})dx_{a*} \tag{21}$$

Solving the partial differential equations (14) to (17), accordingly for $(1\le i\le k)$

$$D(x_{a*},z_{a*})=D(0,z_{a*})[1-\omega(x_{a*})]e^{-\lambda x_{a*}} \tag{22}$$

$$\varphi(x_{a*},z_{a*})=\varphi(0,z_{a*})[1-\iota(x_{a*})]e^{-d_1(z_{a*})x_{a*}} \tag{23}$$

$$\Theta(x_{a*}, z_{a*}) = \Theta(0, z_{a*})[1 - \kappa(x_{a*})]e^{-d_2(z_{a*})x_{a*}} \tag{24}$$

$$Y(x_{a*}, z_{a*}) = Y(0, z_{a*})[1 - \tau(x_{a*})]e^{-d_2(z_{a*})x_{a*}} \tag{25}$$

where $d_1(z_{a*}) = \lambda f(1 - G(z_{a*})) + \delta$, and $d_2(z_{a*}) = \lambda f(1 - G(z_{a*}))$

The probability generating function of orbit size when the server is idle, busy, waiting for repair and on vacation (Modified Bernoulli) are respectively

$$D(z_{a*}) = \left\{ \frac{\begin{array}{r} \{z_{a*}(1-\omega^*(\lambda))fD_0\Big(\gamma(d_1(z_{a*}))\iota^*(d_1(z_{a*})) \\ (1-m+m\tau^*(d_2(z_{a*}))) + \gamma[\delta(1-e)+z_{a*}\delta e] \\ (1-\iota^*(d_1(z_{a*})))\kappa^*(d_2(z_{a*})) + d_1(z_{a*})(\bar{\gamma}\kappa(d_2(z_{a*})-1))\Big) \end{array}}{\begin{array}{r} z_{a*}(d_1(z_{a*})) - \{[\omega^*(\lambda)+G(z_{a*})(1-\omega^*(\lambda))] \\ [(d_1(z_{a*}))\iota^*(d_1(z_{a*}))(1-m+m\tau^*(d_2(z_{a*}))) \\ +\gamma[\delta(1-e)+z_{a*}\delta e](1-\iota^*(d_1(z_{a*})))\kappa^*(d_2(z_{a*}))]\} \end{array}} \right\} \tag{26}$$

$$\varphi(z_{a*}) = \left\{ \frac{\begin{array}{r} z_{a*}(1-\iota^*(d_1(z_{a*})))fD_0 \\ \{\gamma z_{a*} + [\omega^*(\lambda)+G(z)(1-\omega^*(\lambda))](\bar{\gamma}\kappa(d_2(z_{a*})-1))\} \end{array}}{\begin{array}{r} z_{a*}(d_1(z_{a*})) - \{[\omega^*(\lambda)+G(z_{a*})(1-\omega^*(\lambda))] \\ [(d_1(z_{a*}))\iota^*(d_1(z_{a*}))(1-m+m\tau^*(d_2(z_{a*}))) \\ +\gamma[\delta(1-e)+z_{a*}\delta e](1-\iota^*(d_1(z_{a*})))\kappa^*(d_2(z_{a*}))]\} \end{array}} \right\} \tag{27}$$

$$\Theta(z_{a*}) = \frac{(1-\kappa(d_2(z_{a*})))\lambda f D_0 U}{d_1(z_{a*})} \tag{28}$$

where

$$U = \left\{ \frac{\begin{array}{r} \{\bar{\gamma} z_{a*}(d_1(z_{a*})) + \gamma z_{a*}[\delta(1-e) \\ +z_{a*}\delta e](1-\iota^*(d_1(z_{a*})))- \\ [\omega^*(\lambda) + G(z_{a*})(1-\omega^*(\lambda))](\bar{\gamma} d_1(z_{a*})\iota^*(d_1(z_{a*}))) \\ (1-m+m\tau^*(d_2(z_{a*}))) \\ +(1-\iota^*(d_1(z_{a*})))[\delta(1-e)+z_{a*}\delta e]\} \end{array}}{\begin{array}{r} z_{a*}(d_1(z_{a*})) - \{[\omega^*(\lambda) + G(z_{a*})(1-\omega^*(\lambda))] \\ [(d_1(z_{a*}))\iota^*(d_1(z_{a*}))(1-m+m\tau^*(d_2(z_{a*}))) \\ +\gamma[\delta(1-e)+z_{a*}\delta e](1-\iota^*(d_1(z_{a*})))\kappa^*(d_2(z_{a*}))]\} \end{array}} \right\}$$

$$Y(z_{a*}) = \left\{ \frac{\begin{array}{c} (a(1-\tau^*(d_2(z_{a*})))\iota^*(d_1(z_{a*}))fD_0) \\ \{\gamma z_{a*} + [\omega^*(\lambda) + G(z_{a*})(1-\omega^*(\lambda))] \\ (\bar{\gamma}\kappa(d_2(z_{a*})-1))\} \end{array}}{\begin{array}{r} z_{a*}(d_1(z_{a*})) - \{[\omega^*(\lambda) + G(z_{a*})(1-\omega^*(\lambda))] \\ [(d_1(z_{a*}))\iota^*(d_1(z_{a*}))(1-m+m\tau^*(d_2(z_{a*}))) \\ +\gamma[\delta(1-e)+z_{a*}\delta e](1-\iota^*(d_1(z_{a*})))\kappa^*(d_2(z_{a*}))]\} \end{array}} \right\} \tag{29}$$

As D_0 is the prob that the server will be inactive while there are no consumers in the orbit, and because this probability can be calculated utilizing a normalizing criteria $D_0 + D(1) + \varphi(1) + \Theta(1) + Y(1) = 1$.

$$D_0 = \frac{\begin{bmatrix} \delta - \lambda f E(c.)(1-\omega^*(\lambda)) - 2E(c.)(1-\omega^*(\lambda))\delta s.^*(\delta) \\ -2\delta s.^{*'}(\delta) - \delta e \lambda E(c.)[\Upsilon_1]\Upsilon_4 + s.^*(\delta)E(g.) \end{bmatrix}}{\begin{bmatrix} \delta - \lambda f(1-s.^*(\delta))[E(c.)((1-\omega^*(\lambda))) - \bar{\gamma}E(g.) + \omega^*(\lambda)(\bar{\gamma}-1)] \\ -2\delta\{E(c.)(1-\omega^*(\lambda))s.^*(\delta) - s.^{*'}(\delta)\} - \delta f \lambda E(c.)[\Upsilon_1\Upsilon_4 + E(g.)s.^*(\delta)] \\ +\delta e[f(1-s.^*(\delta))(1-\omega^*(\lambda)) - s.^*(\delta)] \end{bmatrix}} \tag{30}$$

The parts that follow are the PGF that we specify for the number of consumers in the dormant pool and system are

$H(z_{a*}) = D_0 + D(z_{a*}) + \varphi(z_{a*}) + \Theta(z_{a*}) + Y(z_{a*})$

$K(z_{a*}) = D_0 + D(z_{a*}) + z_{a*}\varphi(z_{a*}) + z_{a*}\Theta(z_{a*}) + Y(z_{a*})$

substitute equ (26) to (29)in $H(z_{a*})$ & $K(z_{a*})$ we get

$H(z_{a*}) = D_0 \left(\frac{E_1}{Dr} + \frac{E_2}{Dr} + \frac{(E_3+E_4)}{d_2(z_{a*}).Dr} \right)$ and

$K(z_{a*}) = D_0 \left(\frac{E_1}{Dr} + z_{a*} \left[\frac{d_2(z_{a*}).E_2+E_3}{Dr} \right] + \frac{E_4}{d_2(z_{a*}).Dr} \right)$

where

$$E_1 = \Bigg[z_{a*}(d_1(z_{a*})) - \{[\omega^*(\lambda) + G(z_{a*})(1-\omega^*(\lambda))][(d_1(z_{a*}))\iota^*(d_1(z_{a*})) \\ (1 - m + m\tau^*(d_2(z_{a*}))) \\ +\gamma[\delta(1-e) + z_{a*}\delta e](1 - \iota^*(d_1(z_{a*})))\kappa^*(d_2(z_{a*}))]\} + \\ z_{a*}(1-\omega^*(\lambda))fD_0\{\gamma(d_1(z_{a*}))\iota^*(d_1(z_{a*})) \\ (1 - m + m\tau^*(d_2(z_{a*}))) + \gamma[\delta(1-e) \\ +z_{a*}\delta e](1 - \iota^*(d_1(z_{a*})))\kappa^*(d_2(z_{a*})) \\ +d_1(z_{a*})(\bar{\gamma}\kappa(d_2(z_{a*}) - 1))\} \Bigg]$$

$$E_2 = z_{a*}(1 - \iota^*(d_1(z_{a*})))fD_0\{\gamma z_{a*} + [\omega^*(\lambda) + G(z_{a*})(1-\omega^*(\lambda))] \\ (\bar{\gamma}\kappa(d_2(z_{a*}) - 1))\}$$

$$E_3 = (1 - \kappa(d_2(z_{a*}))\lambda f d_1(z_{a*}) \Big(\bar{\gamma} z_{a*}(d_1(z_{a*})) + \gamma z_{a*}[\delta(1-e) + z_{a*}\delta e] \\ (1 - \iota^*(d_1(z_{a*}))) - [\omega^*(\lambda) + G(z_{a*})(1-\omega^*(\lambda))](\bar{\gamma} d_1(z_{a*})\iota^*(d_1(z_{a*}))) \\ (1 - m + m\tau^*(d_2(z_{a*}))) + (1 - \iota^*(d_1(z_{a*})))[\delta(1-e) + z_{a*}\delta e] \Big)$$

$$E_4 = a(1 - \tau^*(d_2(z_{a*})))\iota^*(d_1(z_{a*}))d_1(z_{a*})f\{\gamma z_{a*} + [\omega^*(\lambda) + G(z_{a*}) \\ (1-\omega^*(\lambda))](\bar{\gamma}\kappa(d_2(z_{a*}) - 1))\}$$

$$Dr = z_{a*}d_1(z_{a*}) - \Big([\omega^*(\lambda) + G(z_{a*})(1-\omega^*(\lambda))] \\ [(d_1(z_{a*}))\iota^*(d_1(z_{a*}))(1 - m + m\tau^*(d_2(z_{a*}))) + \gamma[\delta(1-e) + z_{a*}\delta e] \\ (1 - \iota^*(d_1(z_{a*})))\kappa^*(d_2(z_{a*}))] \Big)$$

6. PERFORMANCE MEASURES

While the structure is in each of its various states, we can get a few impressive probability values. Importantly, we got to this conclusion on the performance of the system and its features. Nevertheless, the probabilities that the server has been repaired while still being utilized for regular service, or being on vacation is determined from (26) to (29).

$$D(1) = D_0\left[\frac{Nr_1}{Dr'}\right]$$

$$Nr_1 = f(1-\omega^*(\lambda))\Big(- f\lambda\gamma\Omega(\Psi(\delta)+\bar{\gamma}-1) + \delta\Psi(\delta)\lambda f[m\Omega\tau_1 + \Omega\beta_1] + \delta e(1-\Psi(\delta)) + \delta f\lambda\Omega\beta_1\Big)$$

$$Dr^{'} = \delta - f\lambda\Omega(1-\Psi^*(\delta)) + \delta[m\tau_1 + \beta_1\Psi(\delta)] - 2\delta(\Omega(1-\omega^*(\lambda)\Psi^*(\delta))) - \delta e\Psi^*(\delta)$$

$$\varphi(1) = D_0\left[\frac{Nr_2}{Dr'}\right]$$

$$Nr_2 = f(1-\Psi^*(\delta))[\bar{\gamma}f\lambda\Omega\tau_1 - \omega^*(\lambda)(\bar{\gamma}-1)]$$

$$\Theta(1) = D_0\left[\frac{Nr_3}{Dr'}\right]$$

$$Nr_3 = f\lambda\tau_1\Big(\gamma\delta e(1-\Psi^*(\delta)) - \bar{\gamma}\delta\Psi^*(\delta)f\lambda\beta_1 m\tau_1 - \delta e(1-\Psi^*(\delta)) + \omega^*(\lambda)[\bar{\gamma}\delta\Psi^*(\delta) + \delta(1-\Psi^*(\delta))]\Big)$$

$$Y(1) = D_0\left[\frac{Nr_4}{Dr'}\right]$$

$$Nr_4 = fm\lambda\delta\tau_1\Psi^*(\delta)[\bar{\gamma}f\lambda\Omega\tau_1 - \omega^*(\lambda)(\bar{\gamma}-1)]$$

We obtain the model's system performance. After differentiating $K(z_{a*})$ w.r (with respect) to z_{a*} and analyzing at $z_{a*} = 1$, the L_s (average number of consumers in the system) under steady-state conditions is reached.

$$L_s = \lim_{z_{a*}\to 1} K'(z_{a*})$$

$$L_s = D_0\frac{(Dr'Nr''_a - Nr'_aDr'')}{2(Dr')^2}$$

By differentiating $H(z_{a*})$ w.r (with respect) to z_{a*} and substituting at $z_{a*} = 1$, a particular can estimate the L_q (mean number of customers in the orbit) under steady-state criteria.

$$L_q = \lim_{z_{a*}\to 1} H'(z_{a*})$$

$$L_q = D_0\frac{(Dr'Nr''_b - Nr'_bDr'')}{2(Dr')^2}$$

7. SPECIAL CASES

In this section, we take a cursory look at a few particular applications of the model that we have presented, all of which are consistent with competitively extant writings.

Case 1: There was neither balking nor bulking, any Precipitous breakdown $f = \gamma = 1$. Retrial lineups with negative arrivals are a means of minimizing this.

Case 2: There was neither balking nor bulking, without negative appearance and balking $f = 1, \delta = 0$, $\omega^*(\lambda) \to 1$. After that, we are given a SAQ (Single Arrival Queue), complete with MBV and balking.

8. NUMERICAL OUTCOMES

With the aid of the MATLAB program, the model's findings were examined to learn the consequence of performance appearances by varying the values of the system's parameters $(\delta, \lambda, f, \Upsilon_1$ and $\Upsilon_4)$. Without sacrificing generality, it has been assumed that the times for a retrial, service, vacation (modified), and repair were calculated using an exponential distribution whose density function is $w(x_{a*}) = \Psi e^{-\Psi x_{a*}}$, $x_{a*} > 0$. The parameter's rv(random values) were chosen to fulfill stability criteria and while the arbitrary values for the parameters are $\lambda = 1$, $\delta = 2$, $\Upsilon_1 = 3$, $\Upsilon_2 = 4$, $\Upsilon_3 = 2$, $\Upsilon_4 = 6$.

According to Figure 1, the probability that there will be idle time will decrease as the arrival rate and vacation time rise. Figure 2 shows that D_0 falls as the probability of balking and the retry rate rise. L_q (mean queue sizes of the orbit) and W_q (waiting time of the orbit)will grow when the retry rate rises, as seen in Figure 3. In Figure 4, we can see that as the balking rate rises, the average number of clients in the orbit and waiting time rise as well. In Figures 5 and 6, we can see that as the retrial and vacation rates rise, the probability of idle states rises as well.

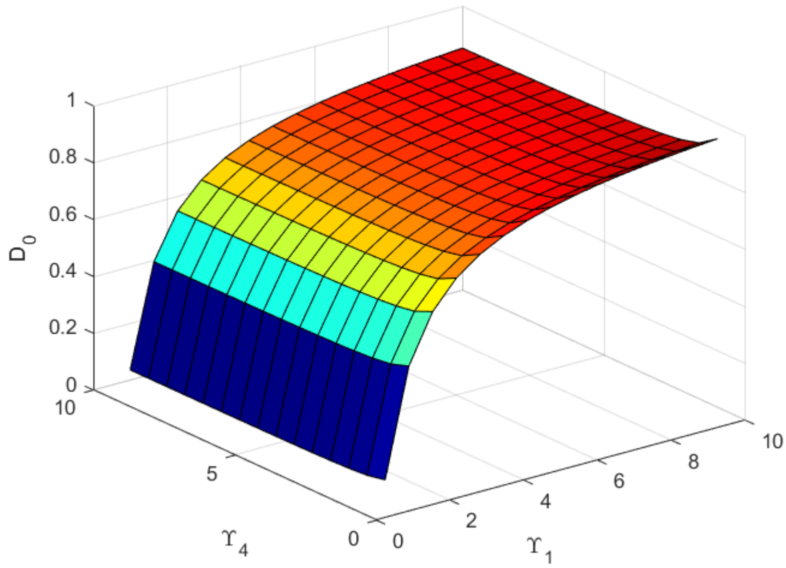

Figure 1: D_0 versus Retrial rate and Vacation rate.

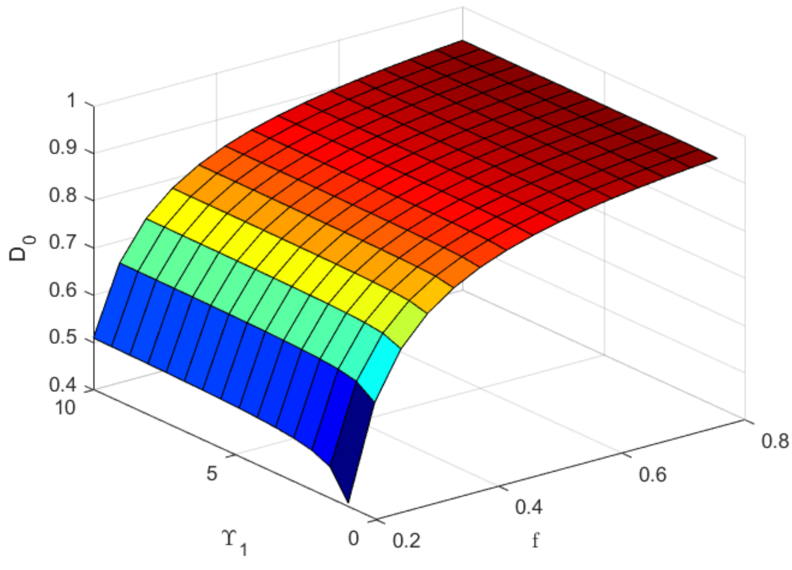

Figure 2: D_0 versus Retrial rate and balking.

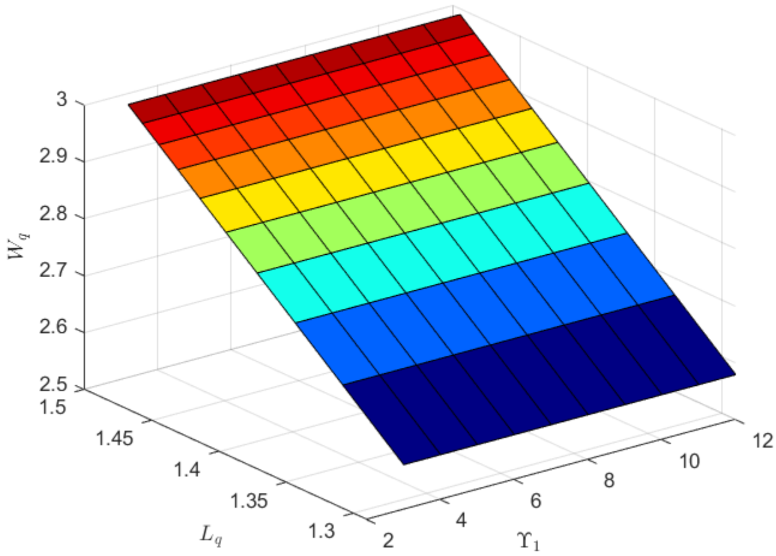

Figure 3: Retrial versus L_q versus W_q.

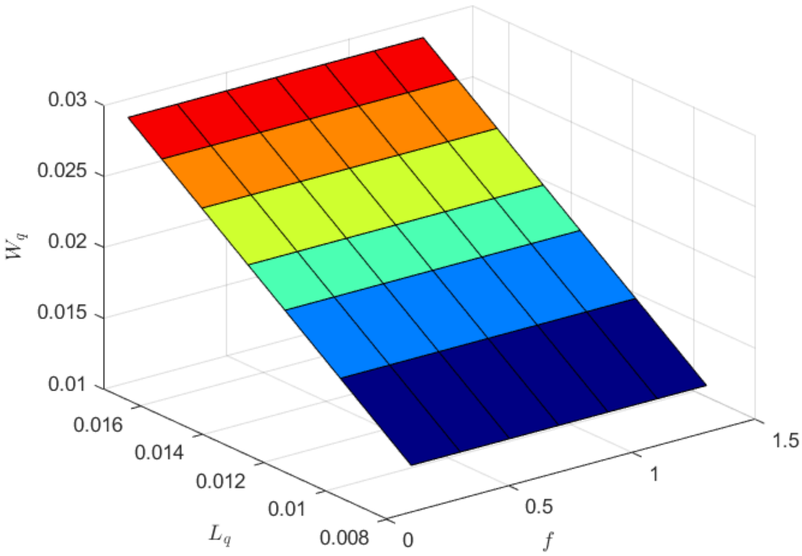

Figure 4: Balking versus L_q and W_q.

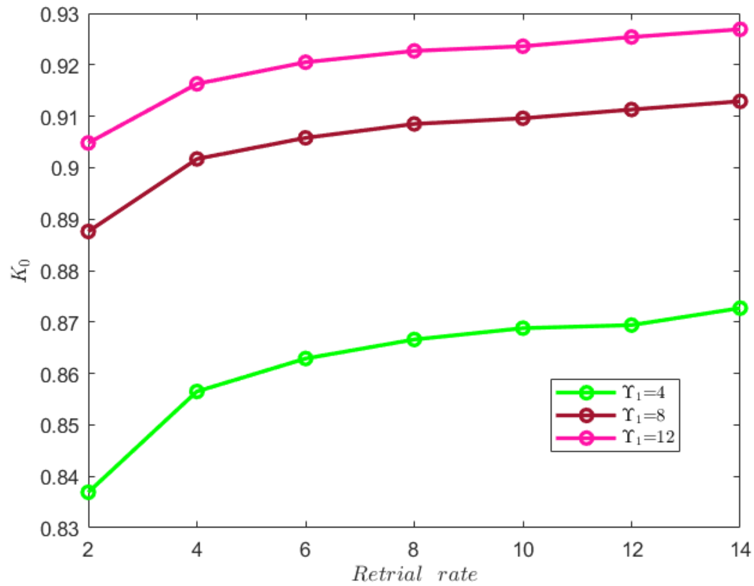

Figure 5: D_0 versus Retrial rate.

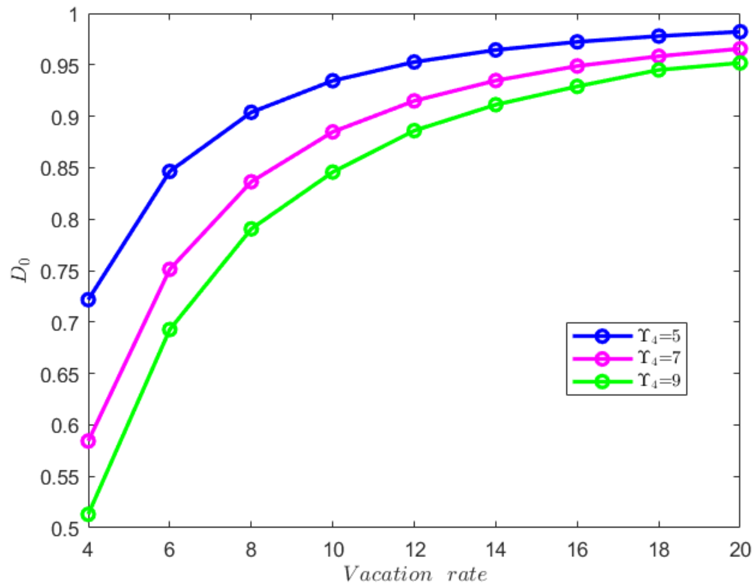

Figure 6: Vacation rate versus D_0.

9. CONCLUSION

We have examined a single server retrial queue with negative consumers and positive consumers in this paper. In addition, we have assumed that the server will accept MBV after the service has been completed. This model will be significant for system administrators who organize capacity and other aspects of the system, as well as for those who use mobile conversations applicable to production industries and the emergency medical field. Furthermore, we plan to extend the multi-stage services and vacation queueing system.

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REFERENCES

[1] T. Van Do, "Bibliography on g-networks, negative customers and applications," *Mathematical and Computer Modelling*, vol. 53, no. 1, pp. 205–212, 2011. doi: 10.1016/j.mcm.2010.08.006

[2] B. K. Kim and D. H. Lee, "The m/g/1 queue with disasters and working breakdowns," *Applied Mathematical Modelling*, vol. 38, no. 5, pp. 1788–1798, 2014. doi: 10.1016/j.apm.2013.09.016

[3] S. P. Madheswari and P. Suganthi, "An m/g/1 retrial queue with second optional service and unreliable server under single exhaustive vacation," *Nonlinear Studies*, vol. 24, no. 2, 2017.

[4] C. Ancker and A. V. Gafarian, "Some queuing problems with balking and reneging," *Operations Research*, vol. 11, no. 1, pp. 88–100, 1963. doi: 10.1287/opre.11.1.88

[5] A. Choudhury and P. Medhi, "Balking and reneging in multiserver markovian queuing system," *International Journal of Mathematics in Operational Research*, vol. 3, no. 4, pp. 377–394, 2011. doi: 10.1504/IJMOR.2011.040874

[6] J. Radha, K. Indhira, and V. Chandrasekaran, "A group arrival retrial g-queue with multi optional stages of service, orbital search and server breakdown," vol. 263, no. 4, 2017. doi: 10.1088/1757-899X/263/4/042150

[7] K. Madan, "A bulk input queue with a stand-by," *South African Statistical Journal*, vol. 29, no. 1, pp. 1–7, 1995.

[8] J. Keilson and L. Servi, "Oscillating random walk models for gi/g/1 vacation systems with bernoulli schedules," *Journal of applied Probability*, vol. 23, no. 3, pp. 790–802, 1986. doi: 10.2307/3214016

[9] "The m/g/1 retrial queue with bernoulli schedules and general retrial times," *Computers and Mathematics with Applications*, vol. 43, no. 1-2, pp. 15–30, 2002.

[10] R. Tripathi, "Optimal ordering policy under two stage trade credits financing for deteriorating items using discounted cash flow approach," *International Journal of Process Management and Benchmarking*, vol. 7, no. 1, pp. 120–140, 2017. doi: 10.1504/IJPMB.2017.080942

[11] J. E. A. Bagyam and K. U. Chandrika, "Multi-stage retrial queueing system with bernoulli feedback," *International Journal of Scientific and Engineering Research*, vol. 4, no. 9, pp. 496–499, 2013.

[12] M. Baruah, K. C. Madan, and T. Eldabi, "Balking and re-service in a vacation queue with batch arrival and two types of heterogeneous service," *Journal of Mathematics Research*, vol. 4, no. 4, pp. 114–124, 2012.

[13] H. Li and T. Yang, "Geo/g/1 discrete time retrial queue with bernoulli schedule," *European Journal of Operational Research*, vol. 111, no. 3, pp. 629–649, 1998. doi: 10.1016/S0377-2217(97)90357-X

[14] S. Gao, J. Wang, and T. Van Do, "A repairable retrial queue under bernoulli schedule and general retrial policy," *Annals of Operations Research*, vol. 247, no. 1, pp. 169–192, 2016. doi: 10.1007/s10479-015-1885-6

[15] E. Gelenbe, “Product-form queueing networks with negative and positive customers,” *Journal of applied probability*, vol. 28, no. 3, pp. 656–663, 1991. doi: 10.2307/3214499

[16] E. Gelenbe, P. Glynn, and K. Sigman, “Queues with negative arrivals,” *Journal of applied probability*, vol. 28, no. 1, pp. 245–250, 1991. doi: 10.2307/3214756

[17] A. Begum and G. Choudhury, “Analysis of an m/g 1 g 2/1 queue with bernoulli vacation and server breakdown,” *International Journal of Applied and Computational Mathematics*, vol. 9, no. 1, 2023.

[18] L. Lakaour, D. Aissani, K. Adel-Aissanou, K. Barkaoui, and S. Ziani, “An unreliable single server retrial queue with collisions and transmission errors,” *Communications in Statistics-Theory and Methods*, vol. 51, no. 4, pp. 1085–1109, 2022.

[19] G. Malik, S. Upadhyaya, and R. Sharma, “Particle swarm optimization and maximum entropy results for mx/g/1 retrial g-queue with delayed repair,” *International Journal of Mathematical, Engineering and Management Sciences*, vol. 6, no. 2, 2021. doi: 10.33889/IJMEMS.2021.6.2.033

[20] W. Xu, L. Liu, L. Li, Z. Wang, and S. Wittevrongel, “Analysis of a collision-affected m/gi/1//n retrial queuing system considering negative customers and transmission errors,” *Mathematics*, vol. 11, no. 16, p. 3532, 2023.

[21] S. I. Ammar and P. Rajadurai, “Performance analysis of preemptive priority retrial queueing system with disaster under working breakdown services,” *Symmetry*, vol. 11, no. 3, 2019. doi: 10.3390/sym11030419

[22] C. J. Singh, S. Kaur, and M. Jain, “Unreliable server retrial g-queue with bulk arrival, optional additional service and delayed repair,” *International Journal of Operational Research*, vol. 38, no. 1, pp. 82–111, 2020. doi: 10.1504/IJOR.2020.106362

[23] Z. Nesrine, P. Spiteri, and N. Djellab, “Numerical solution for the performance characteristics of the m/m/c/k retrial queue with negative customers and exponential abandonments by using value extrapolation method,” *RAIRO-Operations Research*, vol. 53, no. 3, pp. 767–786, 2019. doi: 10.1051/ro/2017059