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AN ALGORITHM FOR FINDING A CYCLE OF FIELDS IN A MATRIX

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Abstract: The note proposes a simple algorithm for finding a cycle of fields in a matrix. This combinatorial problem arises in the potential method for transportation problem.

Keywords: Transportation problem, potential method, combinatorial optimization.

1. INTRODUCTION

An ordered pair (i,j) such that $i \in \{1,2,\dots,m\}$, $j \in \{1,2,\dots,n\}$ is called a *field* of an $m \times n$ matrix. A finite sequence $F_1, F_2, \dots, F_{2k}$ forms a *cycle* if, F_i and F_{i+1} are in the same row for odd numbers i and F_i and F_{i+1} are in the same column for even numbers i as well as F_{2k} and F_1 are in the same column.

The following combinatorial problem (P) arises in the potential method for classical transportation problem ([2]):

Given a family S of fields of an $m \times n$ matrix and a field $F_0 \notin S$. (P)

Find a cycle (*) $F_0, F_1, F_2, \dots, F_k$ with $F_1, F_2, \dots, F_k \in S$.

In the potential method for Hitchcocks transportation problem, S is the set of fields corresponding to old basic variables, F_0 is a field corresponding to the entering variable ([2], [4]). It is shown ex. in [4], that there is a unique cycle of the form (*), which is used to find the variable that leaves the base, but an algorithm for finding this cycle is not given. The problem is solved for more general optimization problems on networks (see [1], [3]), but it is still of interest to find a simple algorithm without referring to graphs. The aim of this note is to show such an algorithm.

2. AN ALGORITHM FOR PROBLEM (P)

Step 0. Let $k = 0$ and give the index 1 to every $F \in S \setminus \{F_0\}$ that lies in the same row as F_0.

Step 1. Replace k by $k+1$.

Step 2. If k is an odd number, give the index $k+1$ to every nonindexed field $F \in S$ that lies in the same column as a field indexed by k.

If k is an even number, give the index $k+1$ to every nonindexed field $F \in S$ that lies in the same row as a field indexed by k.

If there is nothing to index, stop with the message "cycle does not exist".

Step 3. If F_0 is indexed by $k+1$, go to Step 4; otherwise return to Step 1.

Step 4. In the column of the field F_0 find a field F_k indexed by k; in the row of F_k find a field F_{k-1} indexed by $k-1$, etc. until F_1 indexed by 1 is found. Stop with the message " cycle is $F_0, F_1, \dots, F_k$.

THEOREM. The previous algorithm solves the combinatorial problem (P).

PROOF. If there is a cycle of the form (*), then the field F_0 obtains an index equal to the number of fields in the shortest cycle of the form (*).

If, in the flow of the algorithm, the field F_0 obtains an index, finding a cycle in Step 4 is possible.

The algorithm is obviously polynomial. The number of indexations is less than $m \cdot n$.

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