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OPTIMIZATION MODEL FOR A MINE HOISTING SYSTEM CYCLE

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Abstract: A model of optimum operational regime of a mine hoisting system in a vertical shaft with the criterion of minimizing electric energy consumption subject to kinematic constraints, is analyzed and discussed in the paper. Solutions for an optimum three-period cycle covering unbalanced, balanced and overweight hoisting, restricted by velocity, acceleration, time of hoisting at a constant speed and the third derivative of distance per time, are proposed.

Key words: Hoisting system, vertical shaft, unbalanced hoisting, balanced hoisting, overweight hoisting.

1. INTRODUCTION

Selection of the equipment and operational regime of the hoisting plant in vertical shafts represents a complex engineering task for which there are no generally accepted calculation methods and therefore it can be performed on the basis of recommendations from different references.

Operational regime of the hoisting system is defined by static forces during movement of the hoisting system, by change of rates within one hoisting cycle and by a dynamic force resulting from a change in movement velocity .

When designing a hoisting plant it is indispensable to select a hoisting system and its kinematic diagram that will provide, under given conditions, minimum energy consumption in the cycle of the hoisting system.

The kinematic diagram of a hoisting cycle has to satisfy the following requirements:

$$\begin{aligned}
&\int_0^T \dot{X}(t)dt=H && (1.a)\\
&\dot{X}(0)=\dot{X}(T)=0 && (1.b)\\
&\dot{X}(t)=v(t)\le v_{dop} && (1.c)\\
&\ddot{X}(t)=\frac{dv(t)}{dt}\le a_{dop} && (1.d)\\
&\dddot{X}(t)\le \rho_{dop} && (1.e)\\
&\frac{t_2}{T}\ge \mu && (1.f)
\end{aligned} \qquad (1)$$

where: H-hoisting depth, [m]; T-total time of shaft vehicle movement, [s]; t_2 - time of movement by at a constant rate, [s]; μ - the ratio between the time of movement at a constant rate and total time of movement, according to recommendation $\mu \geq 0.6$; v_{dop} - a maximum permitted rate, [m/s]; a_{dop} - a maximum permitted acceleration-deceleration, [m/s^2];ρ_{dop} - a maximum permitted value for the third derivative of a distance per time, [m/s^3].

In expression (1), the conditions (1.a) and (1.b) define the requirements that the run distance should be equal to the hoisting depth within a cycle and the rate at the beginning and the end of cycle should be zero. The conditions (1.c) and (1.d) are defined by technical norms and refer to a maximum permitted constant rate and to maximum values of acceleration-deceleration. The condition (1.e) that, in fact, defines the maximum rate of change of force, has been frequently used when elaborating the control of the hoisting system. Condition (1.f) defines the minimum time of movement at a constant rate with the aim to secure the minimum request for cooling of electric drive by its own ventilation during the hoisting cycle.

When a system with a constant radius of winding up the hoisting rope is concerned, the force at the periphery of the drum or Koepe wheel is given by the following expression:

$$F=g[kQ_t-(q-p)(H-2X)]+M\ddot{X} \qquad (2)$$

where: g-gravity acceleration,[m/s^2]; k-coefficient that takes resistance to movement into account; Q_t-mass of the useful load, [kg]; p-a mass of hoisting rope per meter, [kg/m]; q-a mass of balance rope per meter, [kg/m]; M-referred mass of the system, [kg].

2. OPTIMIZATION CRITERION

In case of hoisting systems with a constant radius of winding up the hoisting rope, the quantity of separated heat q_t, within one cycle, is represented by the following equation:

$$q_t = c\int_0^T F^2 dt \tag{3}$$

where: c-coefficient.

As a criterion on the basis of which the consumed electric power for a process under investigation can be minimized, the following functional can be written:

$$J = \int_0^T \Phi(X,\ddot{X})dt \tag{4}$$

where:

$$\Phi = \{ g[kQ_t - (q-p)(H-2X)] + M\ddot{X} \}^2$$

3. INTEGRAL CURVES OF A FUNCTIONAL (4)

Establishment of the function of the law on motion X(t), regarding to the fact that integral (4) represents a function of a higher order than the first derivative, [1]-[3], can be done through the Euler-Poisson's equation:

$$\frac{\partial\Phi}{\partial X} - \frac{d}{dt}\left(\frac{\partial\Phi}{\partial\dot{X}}\right) + \frac{d}{dt^2}\left(\frac{\partial\Phi}{\partial\ddot{X}}\right) = 0 \tag{5}$$

On the basis of expression (5) and having in mind the expression (4) we can obtain:

$$\frac{d^4X}{dt^4} + a_o\frac{d^2X}{dt^2} + \frac{a_o^2}{4}X = b_o \tag{6}$$

where:

$$a_o = 4\frac{\Delta}{M} \;;\; b_o = \frac{\Delta(\Delta H - Q)}{M^2} \;;\; Q = gkQ_t \;;\; \Delta = g(q-p)$$

As in expression (6) the difference Δ, between the weights of the hoisting rope per meter and the balance rope per meter may vary, thus consequently three characteristic cases can be distinguished when solving equation (6): case a. $\Delta = g(q-p) < 0$ when roots are real values; case b. $\Delta = g(q-p) > 0$ when roots are imaginary values; case c. $\Delta = g(q-p) = 0$. In the last case the following equation for defining the requested function X(t) can be obtained on the basis of equation (4),and from expression (5):

$$\frac{d^4X}{dt^4}=0 \tag{7}$$

On the grounds of the aforesaid, we shall consider the solutions, [4], for all the three cases of balanced hoisting that can be encountered in practice:

3.1. Case $\Delta=0$, namely BALANCED HOISTING. The solution of equation (7) and the expression for velocity, acceleration and the third derivative for a distance per time are represented by the following expression:

$$\begin{aligned} X&=C_1+C_2t+C_3t^2+C_4t^3 \\ \dot{X}&=C_2+2C_3t+3C_4t^2 \\ \ddot{X}&=2C_3+6C_4t \\ \dddot{X}&=6C_4 \end{aligned} \tag{8}$$

3.2. Case $\Delta<0$, namely UNBALANCED HOISTING. The solution of equation (6), the rate, acceleration and the third derivative of a distance per time read as follows:

$$\begin{aligned} X&=e^{\alpha t}(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta \\ \dot{X}&=C_1\alpha e^{\alpha t}+C_2e^{\alpha t}(1+\alpha t)-C_3\alpha e^{-\alpha t}+C_4e^{-\alpha t}(1-\alpha t) \\ \ddot{X}&=C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2) \\ \dddot{X}&=\alpha^3e^{\alpha t}(C_1+C_2t)+3C_2\alpha^2e^{\alpha t}-\alpha^3e^{-\alpha t}(C_3+C_4t)+3C_4\alpha^2e^{-\alpha t} \end{aligned} \tag{9}$$

where:

$$\alpha=\sqrt{\frac{-a_o}{2}}\ ;\ \beta=4\frac{b_o}{a_o^2}$$

3.3. Case $\Delta>0$, namely OVERWEIGHT HOISTING. The solution of equation (6), the rate, acceleration and the third derivative of a distance per time can be written as follows:

$$
\begin{aligned}
X&=\cos\alpha t(C_1+C_2t)+\sin\alpha t(C_3+C_4t)+\beta\\
\dot{X}&=\sin\alpha t(-C_1\alpha-C_2\alpha t+C_4)+\cos\alpha t(C_2+C_3\alpha+C_4\alpha t)\\
\ddot{X}&=\cos\alpha t(-C_1\alpha^2-C_2\alpha^2 t+2C_4\alpha)-\sin\alpha t(2C_2\alpha+C_3\alpha^2+C_4\alpha^2 t)\\
\dddot{X}&=\alpha^3(C_1+C_2t)\sin\alpha t-\alpha^3(C_3+C_4t)\cos\alpha t-3C_2\alpha^2\cos\alpha t-3C_4\alpha^2\sin\alpha t
\end{aligned}
\qquad (10)
$$

where:

$$
\alpha=\sqrt{\left|\frac{-a_o}{2}\right|}
$$

4. OPTIMIZATION MODEL

Having in mind the expression (3), integral curves of functional (4) and the conditions (1), the mathematical model of optimization of a three-period hoisting cycle, using the expression for equivalent force F_e, [5]-[6], can be written in the following form:

$$
F_e=\sqrt{\frac{\int_0^T F^2 dt}{wT}} \rightarrow \min \qquad (11)
$$

conditions at the beginning and at the end of cycle:

$$
\begin{gathered}
X(0)=0\ ;\ X(T)=H\\
\dot{X}(0)=v(0)=0\ ;\ \dot{X}(T)=v(T)=0
\end{gathered}
$$

for the conditions per periods:

in the first period:

$$
\begin{gathered}
0\le t\le t_1\\
0\le X(t)\le S_1\\
0\le \dot{X}(t)\le V_{max}\\
a_{1,min}\le \ddot{X}(t)\le a_{1,max}\\
\dddot{X}(t)\le |\rho_{1,max}|\\
X(0)=0\ ;\ X(t_1)=S_1\\
\dot{X}(0)=v(0)=0\ ;\ \dot{X}(t_1)=v(t_1)=V_{max}
\end{gathered}
\qquad (12)
$$

in the second period:

$$t_1 \le t \le t_1 + t_2$$
$$S_1 \le X(t) \le S_1 + S_2$$
$$\dot{X}(t) = V_{max} = const \tag{13}$$
$$\ddot{X}(t) = 0$$
$$X(t_1) = S_1 \ ; \ X(t_1 + t_2) = S_1 + S_2$$

in the third period:

$$t_1 + t_2 \le t \le T$$
$$S_1 + S_2 \le X(t) \le H$$
$$0 \le \dot{X}(t) \le V_{max}$$
$$|a_{3,min}| \le \ddot{X}(t) \le |a_{3,max}| \tag{14}$$
$$\dddot{X}(t) \le |\rho_{3,max}|$$
$$X(t_1 + t_2) = S_1 + S_2 \ ; \ X(T) = H$$
$$\dot{X}(t_1 + t_2) = v(t_1 + t_2) = V_{max} \ ; \ \dot{X}(T) = v(T) = 0$$

where: w - a coefficient, takes into consideration the aggravation of cooling the motor within the period of both acceleration and deceleration; V_{max} - constant velocity of vehicle movement; a_i -restriction of acceleration (deceleration); ρi - restriction of the third derivative per distance; S_i, t_i - distances and time of the corresponding periods, i=1,2,3.

The solution for the model of optimization of the hoisting system subject to (1.a) and (1.b) from (1) is given in [4].

5. ILLUSTRATIVE EXAMPLE

The example presented here will enable us the practical presentation of the developed model covering investigation of the degree of influence of static balanced hoisting on the optimum hoisting cycle. With this aim we shall try to use an almost equal referred mass of the hoisting system in all studied variants. We shall not use, in this example, the third derivative of the distance per time, as a restriction factor, but we shall present its value.

Interpretation of the suggested model will be exemplified by the hoisting plant having the following properties:

- Hoisting system - frictional, Koepe system; hoisting depth 1000 m; height of a headgear 60 m;
- Type of shaft vehicle-Cage; Mass of empty cage 4775 kg; number of trams in a cage: 1; Mass of one empty tram 2940 kg; Total mass of useful load is 5880 kg;
- Number of hoisting ropes 1; Number of balance ropes 1;
- Diameter of Koepe wheel 6.44 m; flywheel moment of wheel 273000 kgm^2; diameter of sheave at the headgear 5 m; flywheel moment of sheave 96000 kgm^2; flywheel moment of motor rotor 213000 kgm^2; system engine without reduction gears;

- Diagram of winding three-period; total time of wind is 100 s; constant velocity $V_{max}=12$ m/s; minimum ratio between a period with a constant rate and time of movement $\mu=0.6$; interval value for acceleration and deceleration $0.6\leq|a|\leq1.2$ m/s^2.

Optimum solutions obtained by computer program CIKOPT under development, are given in first approximation with the time step of 1 s and the distance step of 1 m. The results and elements of calculation at w=1 are presented in Table 1.

TABLE 1. The elements of calculation and results

$\Delta=g(q-p)$, N	-9.81	0	9.81
p, kg/m	11.10	10.60	10.10
q, kg/m	10.10	10.60	11.10
M, kg	62877.30	62862.30	62847.30
α	0.0177	/	0.0177
β	4028	/	-3028.00
C_1	-1513.57	0	3028.00
C_2	18.69	0	-10.51
C_3	-2514.43	0.3	594.39
C_4	-36.37	0	44.03
S_1, m	116	120	114
t_1, s	19	20	19
$a_{1,max}$, m/s^2	0.6884	0.60	0.6421
$a_{1,min}$, m/s^2	0.6121	0.60	0.6106
$\|\rho_{1,max}\|$,m/s^3	0.011	0	0.0066
t_2, s	62	61	62
S_2, m	772	766	771
t_2/T	0.62	0.61	0.62
C_1	-398.74	-2157.90	1108.35
C_2	1.19	63.16	-17.44
C_3	-6549.68	-0.32	4053.04
C_4	-16.05	0	-0.73
S_3	112	114	115
t_3, s	19	19	19
$a_{3,max}$, m/s^2	-0.6011	-0.6316	-0.6149
$a_{3,min}$, m/s^2	-0.6678	-0.6316	-0.648
$\|\rho_{3,max}\|$,m/s^3	0.0045	0	0.2037
$F_{e,min}$, kN	75.47	73.32	71.75

6. CONCLUSION

This paper presents the optimization model for a three-period cycle of a hoisting system with the criterion of minimum electric energy consumption during the cycle, subject to the constraints given by expression (1). The presented model allows optimization of the parameters of hoisting kinematics, expression (1), and the parameters of force at the border of the drum, expression (2).

Definition of optimization model of hoisting cycle, expressions from (11) to (14), was carried out by using calculus of variation with a functional given by the expression (4).

The functions of optimum laws on movement of the hoisting vehicles were obtained from equation (6) and for three characteristic cases: The case of unbalanced hoisting, the solution of which are given by expression (9); the case of overweight hoisting, the solution of which are given by expression (10); the case of balanced hoisting, equation (7), the solutions are given by expression (8).

Recognition of the optimum law on movement of hoisting vehicle can represent a basis for a correct selection of concrete parameters of hoisting cycle, i.e. a basis for selecting a real hoisting cycle that is almost equal to the optimum one.

7. REFERENCES

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