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# REGRESSION MODELS IN PLANNING OF EXPERIMENTS

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**Abstract:** The paper deals with the results of an experiment investigating the influence of nitrogen, phosphorus and potassium on the sugar beet yield. Relationships between yield and the major factors of yield were based on the analysis of the multiple regression model with three independent variables: the used quantities of nitrogen, phosphorus and potassium.

The adequacy of the model was established by selecting the variables to be included in its final form. The potassium, the third independent variable, was insignificant statistically and therefore, this variable and linear interactions of initial inputs were eliminated from the final model.

There was a positive influence of nitrogen and phosphorus on the growth of sugar beet yield in the observed period of 25 years (1966-1990). This was shown through the positive values of linear and the negative values of quadratic expressions of independent variables.

These relations are illustrated graphically by response surface of increasing yield with the diminishing amount of growth.

Keywords: Planning of experiment, input-output relationships, response surface, adequacy of the model, isoquants.

## 1. INTRODUCTION

The methodology by which it is possible to examine the relationships between one or more results (outputs) and a number of independent variables (inputs) permits obtaining the responses to a series of problems primarily concerned with the problem of getting the optimum. In that context a response surface is used [5, 8].

There are many papers referring to the theory and application of response surface. Many of them are related to crop and livestock production, to chemistry as well as other technological disciplines. A significant number of these investigations are closely associated with the analysis of experiments and regression analysis. The literature sources which treat this problem are numerous, but this paper points only to some of them [2, 5, 1, 10, 6, 3, 12].

The methodology of response surface spreads on those empirical investigations where the accent is on the examination of the influence of the combination of factors and their effects on the result. This, depending on the purpose of examination, can be expressed as the maximum or the minimum level. In many empirical investigations the main purpose is to find the levels of those input factors for which it is possible to obtain optimum level of the dependent variable. In using the model of response surface, estimated on the basis of experimental results, the main idea is to find the stationary point (minimum, maximum or saddle point) and to examine the environs of that point.

As for the approximation of the relationships of output ($Y$) and one or more inputs ($X$), linear models are often applied and represent the linear function of unknown parameters. The estimation of unknown parameters is normally based on the method of least squares [1]. Concerning the class of regression models, multiple linear and quadratic regression models are very often used.

If the examination of the influence of one input, say $X_1$, is in question, the relationship of an output is expressed by a response curve. If the problem concerns two inputs, a three dimensional space is formed and the relationship between output $Y$ and inputs, say $X_1$ and $X_2$, is expressed by a response surface. If the examination includes $p$ independent variables (inputs), or when $p > 2$, then it concerns a response surface in a $(p + 1)$ dimensional space [2].

In this kind of analysis the equation of the second degree often has a significant place: it enables the choice of adequate form of the model and the choice of alternative solutions of the treatment combinations which will bring the response close to the optimum solution [8]. The widely spread second degree curve is very suitable because of symmetrical increase and decrease around the point of maximum [3].

## 2. METHODS

The effects of the use of different levels of nitrogen, phosphorus and potassium have been obtained in an experiment with the sugar beet yield [11]. The type of experiment is a three factorial one with four levels of each factor (variants of fertilizers): 0, 50, 100 and 150 kg/ha of N, $P_2O_5$ and $K_2O$. However, this experiment has not taken into account all combinations of factors. Only 20 selected variations , from 64 combinations, of the mentioned levels of factors have been used.

The sugar beet yield is expressed in t/ha. This examination provides the results of yield per year in five-year subperiods covering 25 years (1966 - 1990). Separately, in five-year subperiods the following varieties of sugar beet were analyzed: in the subperiods (1966 – 1970) and (1971 – 1975) the variety NS Poly 2; in the subperiod (1976 – 1980) the variety NS Poly Mono; in the subperiod (1981 – 1985) the variety Gemo (3967) Mono Pura; and in the subperiod (1986 – 1990) the variety KW Maja. In each subperiod the influence of the same combination, that is 20, of independent variables was examined.

This examination supposed the application of second degree function with three independent variables, expressed by the following general form:

$$\hat{Y} = a + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_1^2 + b_5X_2^2 + b_6X_3^2 + b_7X_1X_2 + b_8X_1X_3 + b_9X_2X_3 + b_{10}X_1X_2X_3 \quad (1)$$

where $\hat{Y}$ denotes the fitted value of dependent variable; $a$ is the average level of dependent variable; $b_i$ $(i = 1, 2, \dots, 10)$ are the partial regression coefficients; $X_i$ $(i = 1, 2, 3)$ are independent variables.

The starting regression model given by equation (1) includes three independent variables $(N, P, K)$ and their linear and quadratic expressions and their interactions of the first and second order.

In the selection of the "best" fitted regression model the method of step-wise regression was used. This procedure enabled the selection of the variables with the most important part in the total variability of dependent variable in the final regression model. So, the starting model with three independent variables, given by equation (1), was reduced to a model with two independent variables ($N$ and $P$), with their linear and quadratic expressions. The linear interactions of nitrogen and phosphorus ($NP$), of nitrogen and potassium ($NK$) and of nitrogen, phosphorus and potassium ($NPK$) were found statistically insignificant in each case, and therefore the regression model (1) was reduced to the following form (2) which has been analysed in detail:

$$\hat{Y} = a + b_1X_1 + b_2X_2 + b_3X_1^2 + b_4X_2^2 \quad (2)$$

This model can be expressed in the following form:

$$\hat{Y} = a + b_1N_1 + b_2P + b_3N^2 + b_4P^2 \quad (3)$$

The regression model given by equation (2) was the basis for the function of total production ($TP$) obtained by changing the values of $X_i$ in the selected regression model.

Using the first partial derivatives of the function of total production for each factor separately, the equations of marginal product were derived in order to exclude the influence of one independent variable. The first partial derivatives found for each factor made possible the analysis of extremum. The first partial derivatives, based on the starting form of equation (2), were derived by applying the following form:

$$dY/dX_i = b_i + 2b_{i+2}X_i \qquad (i = 1, 2) \quad (4)$$

As resources are limited, the need for more reachable economic production is always present, so the problem of combination of used inputs is of interest. For the given level of outputs it is important to have a relevant combination of inputs.

Therefore, it is of interest to determine the isoquant function which expresses the second input ($X_2$) as the function of the first one ($X_1$) for a specified level of output $Y$. The isoquant equation shows the quantity of input $X_2$ which must be used for a given level of input $X_1$, if the dependent variable $Y$ is fixed at a specific level.

The isoquant equation, where the second independent variable ($X_2 = P$) is expressed as the function of the first independent variable ($X_1 = N$), is given by the following equation:

$$X_{(1,2)} = \frac{-b_2 \pm \sqrt{b_2^2 - 4b_4(b_3X_1^2 + b_1X_1 + a - \hat{Y})}}{2b_4} \tag{5}$$

Selected regression models were used in further analysis for the graphical interpretation of production surface in a three–dimensional space.

## 3. RESULTS

The results of regression analysis based on the fitted production model expressed as the second–degree polynomial, are discussed for particular subperiods and varieties of sugar beet as well as for the whole period of 25 years (1966 – 1990).

Average yields of varieties of sugar beet in the observed subperiods were as follows:

- I subperiod – NS Poly 2: 55.8 t/ha
- II subperiod – NS Poly 2: 49.8 t/ha
- III subperiod – NS Poly Mono: 50.5 t/ha
- IV subperiod Gemo (3967) Mono Pura: 50.4 t/ha
- V subperiod KW Maja: 35.0 t/ha

The average yield of all varieties realized in 25 years of research was 48.3 t/ha. The obtained results show that in no case potassium had significant influence on the increase of yield of sugar beet.

These results are confirmed by the analysis of variance for factorial experiment, where F–ratio for potassium is insignificant while the effects of nitrogen and phosphorus are highly significant: F–ratio for nitrogen is $F = 25.78^{**}$, F–ratio for phosphorus is $F = 7.25^{**}$.

By applying step–wise regression, variable $X_3$ is eliminated from the regression model, so the model is reduced to quadratic regression with two independent variables: nitrogen ($X_1$), and phosphorus ($X_2$). This result should be accepted as objective and interpreted so that, in this case, potassium had statistically insignificant influence on realization of the yield. But this doesn't mean that there is no effect of this fertilizer on the yield of sugar beet. The effect certainly exists, but it is based either on a small amount of requirements of the plant, or the demand for this element is satisfied by the existing quantities in the soil.

As for the obtained results, the question of model adequacy is raised. Namely, it is assumed that the regression model is "correctly" specified from the aspect of unbiased estimation, size of random error, etc. It should be remarked that the chosen model never reflects or describes the reality which is analysed. Model adequacy is very important, because further analysis of the results depends on it, just as estimates and conclusions. Harvey [9], Brook and Arnold [4] pointed to the question of the quality of model and criteria that are used for the evaluation of model adequacy. Box, Hunter and Hunter [1] considered the planning of experiments in connection with the effect of blocks and treatments on the results. They pointed out that if the model is adequate, i.e. correctly specified, there is no dependence between residuals and the estimated values of dependent variable. In addition, according to Gujarati [7], in order to be correct, the model needs to include several relevant important variables which stress essential characteristics of examined phenomenon and at the same time to include all unknown and random influences in the random variable $\varepsilon_i$.

In such investigation almost in all cases, the models estimated on the basis of least squares are characterized by relatively low values of the coefficient of multiple determination, although the corresponding values of F–test for the whole model are statistically significant. That can indicate nonadequacy of the model and insufficient adaptability to the data. The fact that partial coefficients of the regression model are almost in no case significant, is connected with the question of precision of parameter estimates. Since the data in this analysis are, at the same time, the members of time series, the first order of autocorrelation of the model residuals can be expected to be present. That is confirmed by low values of Durbin–Watson statistics. Due to the significant autocorrelation the estimates of regression coefficients are not precise enough and the value of the coefficient of multiple determination is relatively low (Table 1). The maximum–likelihood correction of estimates of the parameters has increased the precision of the parameters. The regression models after the correction are characterized by a considerably higher value of the coefficient of determination (Table 2). The corresponding $t$–values of the corrected estimates of regression coefficients are significant or highly significant, which was not the case before correction.

Starting from the regression model given by equation (6), which relates to period 1966–1990:

$$\hat{Y} = 36.077 + 0.13679N + 0.081965P - 0.00034833N^2 - 0.0002943P^2 \quad (6)$$

it can be seen that if nitrogen is increased by 1 kg/ha and the amount of phosphorus is at fixed level, the yield of sugar beet is increased by 0.13665 t/ha. Also, if phosphorus is increased by 1 kg/ha and the amount of nitrogen is constant, the yield is increased by 0.08171 t/ha. The mentioned tendencies of the change of yield, depending on the applied quantities of nitrogen and phosphorus, can be clearly seen from the geometrical graph of curvature of response surface (Figure 1).

Using equation (5) for the chosen values of the yield in the interval 37–53 t/ha and by combining the level of nitrogen in the interval 0–150 kg/ha, the equations of isoquants are shown in Figure 2.

Table 1. Estimated Parameters of Regression Models (Method of Least Squares)

| Subperiods | $a$ | Estimated Parameters of Regressors | | | | $R^2$ | $DW$ |
|---|---|---|---|---|---|---|---|
| | | $b_1$ | $b_2$ | $b_3$ | $b_4$ | | |
| 1966-1970. | 49.279 | 0.092044 (2.59**) | 0.041338 (1.17) | -0.000362 (-1.59) | -0.000111 (-0.47) | 0.22 | 0.2903 |
| 1971-1975. | 38.794 | 0.17545 (3.65**) | 0.045782 (0.96) | -0.000639 (-2.07*) | -0.000142 (-0.45) | 0.32 | 0.1343 |
| 1976-1980. | 33.963 | 0.18816 (7.48**) | 0.083476 (3.34**) | -0.000427 (-2.65**) | -0.000226 (-1.37) | 0.81 | 0.4009 |
| 1981-1985. | 31.242 | 0.20860 (5.45**) | 0.151466 (3.98**) | -0.000587 (-2.40*) | -0.000553 (-2.19*) | 0.68 | 0.5260 |
| 1986-1990. | 22.190 | 0.088529 (1.54) | 0.146211 (2.56*) | -0.000081 (-0.22) | -0.000636 (-0.68) | 0.30 | 0.2979 |
| Total Period 1966-1990. | 35.094 | 0.15056 (5.39**) | 0.093655 (3.38**) | -0.000419 (-2.35**) | -0.000334 (-1.82) | 0.27 | 1.3876 |

[ In brackets there are values of $t$ – statistics ]

Table 2. Estimated Parameters of Regression Models
(Corrected for the Influence of Autocorrelation)

| Subperiods | $a$ | Estimated Parameters of Regressors | | | | $R^2$ |
|---|---|---|---|---|---|---|
| | | $b_1$ | $b_2$ | $b_3$ | $b_4$ | |
| 1966-1970. | 49.682 | 0.064536 (3.51**) | 0.014001 (0.94) | -0.000112 (-0.77) | -0.000702 (-0.67) | 0.80 |
| 1971-1975. | 39.891 | 0.18886 (11.69**) | 0.045281 (3.49**) | -0.000833 (-6.46**) | -0.000210 (-2.29) | 0.92 |
| 1976-1980. | 34.501 | 0.19078 (12.42**) | 0.081557 (6.51**) | -0.000491 (-4.05**) | -0.000254 (-2.87**) | 0.93 |
| 1981-1985. | 34.662 | 0.19445 0(7.75**) | 0.138020 (6.73**) | -0.000639 (-3.23**) | -0.000573 (3.96**) | 0.87 |
| 1986-1990. | 23.419 | 0.079523 (2.63**) | 0.142990 (5.84**) | -0.000092 (-0.39) | -0.000678 (-3.92**) | 0.81 |
| Total Period 1966-1990. | 36.077 | 0.13679 (11.26**) | 0.081965 (8.34**) | -0.000348 (-3.64**) | -0.000294 (-4.25**) | 0.87 |

[ In brackets there are values of $t$ – statistics ]

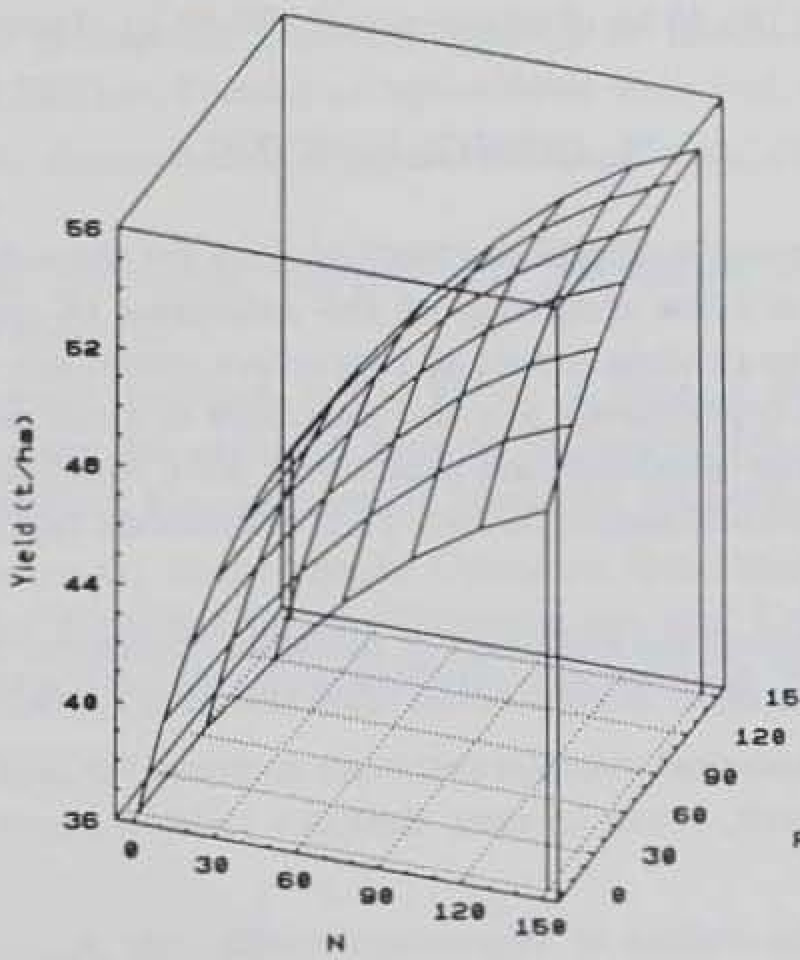


Figure 1. Change in sugar beet yield with different combinations of nitrogen and phosphorus consumption

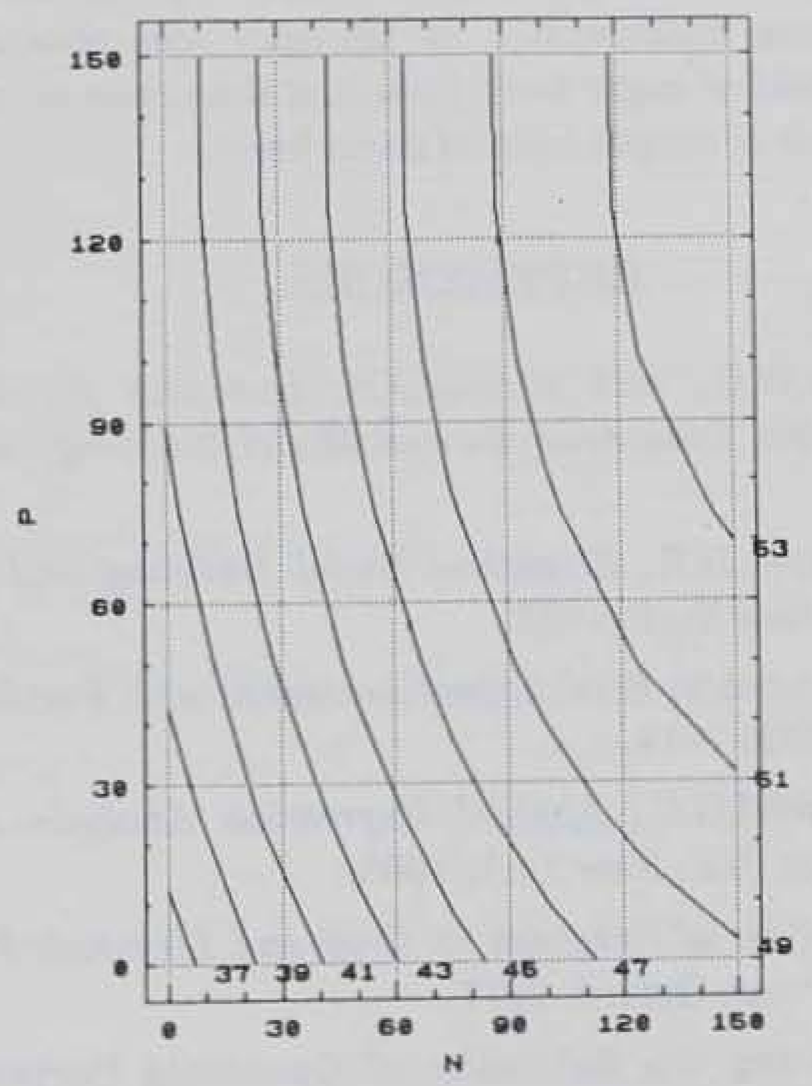


Figure 2. Isoquants in the production of sugar beet depending on the use of nitrogen and phosphorus

The potential maximum yield (technical optimum) of sugar beet of 55.21 t/ha is realized with the use of 196.35 kg of nitrogen and 160.49 kg of phosphorus per hectare.

## 4. CONCLUSIONS

The focus in this investigation is the methodology for the selection of variables in the regression model and the obtaining of the estimates of regression parameters when assumptions of the random errors of regression model are not satisfied. As the results of several-year experiments are the members of time series, the models are estimated by pooling cross-sectional and time series data. Accepting such an approach, the response surfaces are estimated for five-year subperiods separately and the whole period as well.

As a result of analysing the adequacy of multiple regression model, potassium, as the third independent variable, is eliminated from the final regression model.

Starting from the accepted average response surfaces for particular time intervals considered in this research, it can be seen that the yield of sugar beet increases with the decrease of growth.

On the basis of the values of regression coefficients it can be concluded that nitrogen influences the increase of the yield more than phosphorus.

A potential maximum yield (technical optimum) of about 55 t/ha is realized by the use of about 196 kg/ha of nitrogen and about 160 kg/ha of phosphorus.

The obtained regression model, i.e. the average response surface, helps finding the economically optimum consumption of nitrogen and phosphorus, as well as economically optimum yield of sugar beet, including the prices of input (nitrogen and phosphorus) and the prices of output (root of sugar beet).

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# OPTIMIZATION MODEL FOR A MINE HOISTING SYSTEM CYCLE

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**Abstract:** A model of optimum operational regime of a mine hoisting system in a vertical shaft with the criterion of minimizing electric energy consumption subject to kinematic constraints, is analyzed and discussed in the paper. Solutions for an optimum three-period cycle covering unbalanced, balanced and overweight hoisting, restricted by velocity, acceleration, time of hoisting at a constant speed and the third derivative of distance per time, are proposed.

**Key words:** Hoisting system, vertical shaft, unbalanced hoisting, balanced hoisting, overweight hoisting.

## 1. INTRODUCTION

Selection of the equipment and operational regime of the hoisting plant in vertical shafts represents a complex engineering task for which there are no generally accepted calculation methods and therefore it can be performed on the basis of recommendations from different references.

Operational regime of the hoisting system is defined by static forces during movement of the hoisting system, by change of rates within one hoisting cycle and by a dynamic force resulting from a change in movement velocity .

When designing a hoisting plant it is indispensable to select a hoisting system and its kinematic diagram that will provide, under given conditions, minimum energy consumption in the cycle of the hoisting system.

The kinematic diagram of a hoisting cycle has to satisfy the following requirements:

$$
\begin{aligned}
&\int_0^T \dot{X}(t)dt=H && (1.a)\\
&\dot{X}(0)=\dot{X}(T)=0 && (1.b)\\
&\dot{X}(t)=v(t)\le v_{dop} && (1.c)\\
&\ddot{X}(t)=\frac{dv(t)}{dt}\le a_{dop} && (1.d)\\
&\dddot{X}(t)\le \rho_{dop} && (1.e)\\
&\frac{t_2}{T}\ge\mu && (1.f)
\end{aligned}
\qquad (1)
$$

where: H-hoisting depth, [m]; T-total time of shaft vehicle movement, [s]; $t_2$ - time of movement by at a constant rate, [s]; $\mu$ - the ratio between the time of movement at a constant rate and total time of movement, according to recommendation $\mu \geq 0.6$; $v_{dop}$ - a maximum permitted rate, [m/s]; $a_{dop}$ - a maximum permitted acceleration-deceleration, [m/s$^2$];$\rho_{dop}$ - a maximum permitted value for the third derivative of a distance per time, [m/s$^3$].

In expression (1), the conditions (1.a) and (1.b) define the requirements that the run distance should be equal to the hoisting depth within a cycle and the rate at the beginning and the end of cycle should be zero. The conditions (1.c) and (1.d) are defined by technical norms and refer to a maximum permitted constant rate and to maximum values of acceleration-deceleration. The condition (1.e) that, in fact, defines the maximum rate of change of force, has been frequently used when elaborating the control of the hoisting system. Condition (1.f) defines the minimum time of movement at a constant rate with the aim to secure the minimum request for cooling of electric drive by its own ventilation during the hoisting cycle.

When a system with a constant radius of winding up the hoisting rope is concerned, the force at the periphery of the drum or Koepe wheel is given by the following expression:

$$F=g[kQ_t-(q-p)(H-2X)]+M\ddot{X} \qquad (2)$$

where: g-gravity acceleration,[m/s$^2$]; k-coefficient that takes resistance to movement into account; $Q_t$-mass of the useful load, [kg]; p-a mass of hoisting rope per meter, [kg/m]; q-a mass of balance rope per meter, [kg/m]; M-referred mass of the system, [kg].

## 2. OPTIMIZATION CRITERION

In case of hoisting systems with a constant radius of winding up the hoisting rope, the quantity of separated heat $q_t$, within one cycle, is represented by the following equation:

$$q_t = c\int_0^T F^2 dt \tag{3}$$

where: c-coefficient.

As a criterion on the basis of which the consumed electric power for a process under investigation can be minimized, the following functional can be written:

$$J = \int_0^T \Phi(X, \ddot{X})dt \tag{4}$$

where:

$$\Phi = \{ g[kQ_t - (q-p)(H-2X)] + M\ddot{X} \}^2$$

## 3. INTEGRAL CURVES OF A FUNCTIONAL (4)

Establishment of the function of the law on motion X(t), regarding to the fact that integral (4) represents a function of a higher order than the first derivative, [1]-[3], can be done through the Euler-Poisson's equation:

$$\frac{\partial\Phi}{\partial X} - \frac{d}{dt}\left(\frac{\partial\Phi}{\partial\dot{X}}\right) + \frac{d}{dt^2}\left(\frac{\partial\Phi}{\partial\ddot{X}}\right) = 0 \tag{5}$$

On the basis of expression (5) and having in mind the expression (4) we can obtain:

$$\frac{d^4X}{dt^4} + a_o\frac{d^2X}{dt^2} + \frac{a_o^2}{4}X = b_o \tag{6}$$

where:

$$a_o = 4\frac{\Delta}{M}\ ;\ b_o = \frac{\Delta(\Delta H - Q)}{M^2}\ ;\ Q = gkQ_t\ ;\ \Delta = g(q-p)$$

As in expression (6) the difference $\Delta$, between the weights of the hoisting rope per meter and the balance rope per meter may vary, thus consequently three characteristic cases can be distinguished when solving equation (6): case a. $\Delta = g(q-p) < 0$ when roots are real values; case b. $\Delta = g(q-p) > 0$ when roots are imaginary values; case c. $\Delta = g(q-p) = 0$. In the last case the following equation for defining the requested function X(t) can be obtained on the basis of equation (4), and from expression (5):

$$\frac{d^4X}{dt^4}=0 \tag{7}$$

On the grounds of the aforesaid, we shall consider the solutions, [4], for all the three cases of balanced hoisting that can be encountered in practice:

3.1. Case $\Delta=0$, namely BALANCED HOISTING. The solution of equation (7) and the expression for velocity, acceleration and the third derivative for a distance per time are represented by the following expression:

$$\begin{aligned} &X=C_1+C_2t+C_3t^2+C_4t^3\\ &\dot{X}=C_2+2C_3t+3C_4t^2\\ &\ddot{X}=2C_3+6C_4t\\ &\dddot{X}=6C_4 \end{aligned} \tag{8}$$

3.2. Case $\Delta<0$, namely UNBALANCED HOISTING. The solution of equation (6), the rate, acceleration and the third derivative of a distance per time read as follows:

$$\begin{aligned} &X=e^{\alpha t}(C_1+C_2t)+e^{-\alpha t}(C_3+C_4t)+\beta\\ &\dot{X}=C_1\alpha e^{\alpha t}+C_2e^{\alpha t}(1+\alpha t)-C_3\alpha e^{-\alpha t}+C_4e^{-\alpha t}(1-\alpha t)\\ &\ddot{X}=C_1\alpha^2e^{\alpha t}+C_2\alpha e^{\alpha t}(2+\alpha t)+C_3\alpha^2e^{-\alpha t}+C_4\alpha e^{-\alpha t}(\alpha t-2)\\ &\dddot{X}=\alpha^3e^{\alpha t}(C_1+C_2t)+3C_2\alpha^2e^{\alpha t}-\alpha^3e^{-\alpha t}(C_3+C_4t)+3C_4\alpha^2e^{-\alpha t} \end{aligned} \tag{9}$$

where:

$$\alpha=\sqrt{\frac{-a_o}{2}}\ ;\ \beta=4\frac{b_o}{a_o^2}$$

3.3. Case $\Delta>0$, namely OVERWEIGHT HOISTING. The solution of equation (6), the rate, acceleration and the third derivative of a distance per time can be written as follows:

$$\begin{aligned}
X &= \cos\alpha t(C_1+C_2t)+\sin\alpha t(C_3+C_4t)+\beta \\
\dot{X} &= \sin\alpha t(-C_1\alpha-C_2\alpha t+C_4)+\cos\alpha t(C_2+C_3\alpha+C_4\alpha t) \\
\ddot{X} &= \cos\alpha t(-C_1\alpha^2-C_2\alpha^2 t+2C_4\alpha)-\sin\alpha t(2C_2\alpha+C_3\alpha^2+C_4\alpha^2 t) \\
\dddot{X} &= \alpha^3(C_1+C_2t)\sin\alpha t-\alpha^3(C_3+C_4t)\cos\alpha t-3C_2\alpha^2\cos\alpha t-3C_4\alpha^2\sin\alpha t
\end{aligned} \tag{10}$$

where:

$$\alpha=\sqrt{\left|\frac{-a_o}{2}\right|}$$

## 4. OPTIMIZATION MODEL

Having in mind the expression (3), integral curves of functional (4) and the conditions (1), the mathematical model of optimization of a three-period hoisting cycle, using the expression for equivalent force $F_e$, [5]-[6], can be written in the following form:

$$F_e=\sqrt{\frac{\int_0^T F^2 dt}{wT}} \rightarrow \min \tag{11}$$

conditions at the beginning and at the end of cycle:

$$X(0)=0 \ ; \ X(T)=H$$
$$\dot{X}(0)=v(0)=0 \ ; \ \dot{X}(T)=v(T)=0$$

for the conditions per periods:

in the first period:

$$\begin{gathered}
0 \le t \le t_1 \\
0 \le X(t) \le S_1 \\
0 \le \dot{X}(t) \le V_{max} \\
a_{1,min} \le \ddot{X}(t) \le a_{1,max} \\
\dddot{X}(t) \le |\rho_{1,max}| \\
X(0)=0 \ ; \ X(t_1)=S_1 \\
\dot{X}(0)=v(0)=0 \ ; \ \dot{X}(t_1)=v(t_1)=V_{max}
\end{gathered} \tag{12}$$

in the second period:

$$
\begin{aligned}
& t_1 \le t \le t_1 + t_2 \\
& S_1 \le X(t) \le S_1 + S_2 \\
& \dot{X}(t) = V_{max} = const \\
& \ddot{X}(t) = 0 \\
& X(t_1) = S_1 \ ; \ X(t_1 + t_2) = S_1 + S_2
\end{aligned}
\tag{13}
$$

in the third period:

$$
\begin{gathered}
t_1 + t_2 \le t \le T \\
S_1 + S_2 \le X(t) \le H \\
0 \le \dot{X}(t) \le V_{max} \\
|a_{3,min}| \le \ddot{X}(t) \le |a_{3,max}| \\
\dddot{X}(t) \le |\rho_{3,max}| \\
X(t_1 + t_2) = S_1 + S_2 \ ; \ X(T) = H \\
\dot{X}(t_1 + t_2) = v(t_1 + t_2) = V_{max} \ ; \ \dot{X}(T) = v(T) = 0
\end{gathered}
\tag{14}
$$

where: w - a coefficient, takes into consideration the aggravation of cooling the motor within the period of both acceleration and deceleration; $V_{max}$ - constant velocity of vehicle movement; $a_i$ -restriction of acceleration (deceleration); $\rho i$ - restriction of the third derivative per distance; $S_i$,$t_i$ - distances and time of the corresponding periods, i=1,2,3.

The solution for the model of optimization of the hoisting system subject to (1.a) and (1.b) from (1) is given in [4].

## 5. ILLUSTRATIVE EXAMPLE

The example presented here will enable us the practical presentation of the developed model covering investigation of the degree of influence of static balanced hoisting on the optimum hoisting cycle. With this aim we shall try to use an almost equal referred mass of the hoisting system in all studied variants. We shall not use, in this example, the third derivative of the distance per time, as a restriction factor, but we shall present its value.

Interpretation of the suggested model will be exemplified by the hoisting plant having the following properties:

- Hoisting system - frictional, Koepe system; hoisting depth 1000 m; height of a headgear 60 m;
- Type of shaft vehicle-Cage; Mass of empty cage 4775 kg; number of trams in a cage: 1; Mass of one empty tram 2940 kg; Total mass of useful load is 5880 kg;
- Number of hoisting ropes 1; Number of balance ropes 1;
- Diameter of Koepe wheel 6.44 m; flywheel moment of wheel 273000 kgm$^2$; diameter of sheave at the headgear 5 m; flywheel moment of sheave 96000 kgm$^2$; flywheel moment of motor rotor 213000 kgm$^2$; system engine without reduction gears;

- Diagram of winding three-period; total time of wind is 100 s; constant velocity $V_{max}=12$ m/s; minimum ratio between a period with a constant rate and time of movement $\mu=0.6$; interval value for acceleration and deceleration $0.6\leq|a|\leq 1.2$ m/s$^2$.

Optimum solutions obtained by computer program CIKOPT under development, are given in first approximation with the time step of 1 s and the distance step of 1 m. The results and elements of calculation at w=1 are presented in Table 1.

**TABLE 1. The elements of calculation and results**

| $\Delta=g(q-p)$, N | -9.81 | 0 | 9.81 |
|---|---|---|---|
| p, kg/m | 11.10 | 10.60 | 10.10 |
| q, kg/m | 10.10 | 10.60 | 11.10 |
| M, kg | 62877.30 | 62862.30 | 62847.30 |
| $\alpha$ | 0.0177 | / | 0.0177 |
| $\beta$ | 4028 | / | -3028.00 |
| $C_1$ | -1513.57 | 0 | 3028.00 |
| $C_2$ | 18.69 | 0 | -10.51 |
| $C_3$ | -2514.43 | 0.3 | 594.39 |
| $C_4$ | -36.37 | 0 | 44.03 |
| $S_1$, m | 116 | 120 | 114 |
| $t_1$, s | 19 | 20 | 19 |
| $a_{1,max}$, m/s$^2$ | 0.6884 | 0.60 | 0.6421 |
| $a_{1,min}$, m/s$^2$ | 0.6121 | 0.60 | 0.6106 |
| $\|\rho_{1,max}\|$, m/s$^3$ | 0.011 | 0 | 0.0066 |
| $t_2$, s | 62 | 61 | 62 |
| $S_2$, m | 772 | 766 | 771 |
| $t_2/T$ | 0.62 | 0.61 | 0.62 |
| $C_1$ | -398.74 | -2157.90 | 1108.35 |
| $C_2$ | 1.19 | 63.16 | -17.44 |
| $C_3$ | -6549.68 | -0.32 | 4053.04 |
| $C_4$ | -16.05 | 0 | -0.73 |
| $S_3$ | 112 | 114 | 115 |
| $t_3$, s | 19 | 19 | 19 |
| $a_{3,max}$, m/s$^2$ | -0.6011 | -0.6316 | -0.6149 |
| $a_{3,min}$, m/s$^2$ | -0.6678 | -0.6316 | -0.648 |
| $\|\rho_{3,max}\|$, m/s$^3$ | 0.0045 | 0 | 0.2037 |
| $F_{e,min}$, kN | 75.47 | 73.32 | 71.75 |

## 6. CONCLUSION

This paper presents the optimization model for a three-period cycle of a hoisting system with the criterion of minimum electric energy consumption during the cycle, subject to the constraints given by expression (1). The presented model allows optimization of the parameters of hoisting kinematics, expression (1), and the parameters of force at the border of the drum, expression (2).

Definition of optimization model of hoisting cycle, expressions from (11) to (14), was carried out by using calculus of variation with a functional given by the expression (4).

The functions of optimum laws on movement of the hoisting vehicles were obtained from equation (6) and for three characteristic cases: The case of unbalanced hoisting, the solution of which are given by expression (9); the case of overweight hoisting, the solution of which are given by expression (10); the case of balanced hoisting, equation (7), the solutions are given by expression (8).

Recognition of the optimum law on movement of hoisting vehicle can represent a basis for a correct selection of concrete parameters of hoisting cycle, i.e. a basis for selecting a real hoisting cycle that is almost equal to the optimum one.

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