Yugoslav Journal of Operations Research
33 (2023), Number 4, 613–642
DOI: https://doi.org/10.2298/YJOR220915010K

A STOCHASTIC INVENTORY MODEL WITH PRICE-SENSITIVE DEMAND, RESTRICTED SHORTAGE AND PROMOTIONAL EFFORTS

Uttam Kumar KHEDLEKAR
Department of Mathematics and Statistics
Dr. Harisingh Gour Vishwavidyalaya, Sagar, M.P., India-470003
(A Central University)
uvkkcm@yahoo.co.in

Lalji KUMAR
Department of Mathematics and Statistics
Dr. Harisingh Gour Vishwavidyalaya, Sagar, M.P., India-470003
(A Central University)
laljikumar07@gmail.com

Mamta KESWANI
Department of Mathematics and Statistics
Dr. Harisingh Gour Vishwavidyalaya, Sagar, M.P., India-470003
(A Central University)
mamtakeswani01@gmail.com

Received: September 2022 / Accepted: March 2023

Abstract: This paper is attempt to develop a stochastic inventory model with quadratic price-sensitive demand. Objective function is developed by incorporating promotional efforts to boost the market demand, preservation technology to reduce the rate of deterioration, proportionate shortage time and partial backloggings. The proposed work is to generalise the stochastic demand with different probability distributions and their comparisons. The objective is to find the optimal price, optimal replenishment, and optimal preservation technology investment while optimizing the total profit per unit time. In the case of partial backlogging and lost sale, we deduced the optimal replenishment schedules for respective price and preservation technology cost. Also, we shown analytically and graphically that the total profit per unit time is a concave function with respect to per unit time, price, and preservation cost. The theoretical implications have been validated by useful results and numericals. Also, we examine the impact of various parameters for the best course of action. The conclusions drawn from the assessment might be useful for managerial purposes.

Keywords: Optimal stochastic control, price-sensitive demand, deterioration, promotional efforts, preservation technology, partial backlogging.

MSC: 90B05, 90B30, 90B50, 91B70, 93E20.

1. INTRODUCTION

Accounting for the various sale seasons, comprehensive knowledge and its implementation in the form of a business strategy are required to grow the business. Some of the seasons that affect consumer demand are holidays (Christmas, Easter, and Halloween), government (tax returns, the first day of school), and fiscal timeframes (end-of-quarter and year-end reporting). The advantage of understanding how seasonality influences business sales are that it informs business owners about what they can and cannot manage. Sometimes sales will increase, and sometimes sales will decrease, but management should make a policy based on industry-based statistical data and seasonal variations, and then they may be able to control the uncertainties. Businesses may take advantage of the predictability of their factors by conducting appropriate market research. If management doesn't pay attention to such things, they risk their business by losing market share and the trust of their customers. In the proposed study, we have used statistical data from the given case to examine the demand fluctuations in non-seasonal and seasonal businesses.

According to a recent report, shares of the biggest coffee chain in the world dropped more than 4%. In this line, it also forecast a larger-than-expected loss for the quarter and a drop in revenue of more than $3 billion in 2020. A study (Sinha *et at.* [1]) revealed that the Starbucks Corporation, an American multinational chain of coffee-houses and roastery reserves, is experiencing the consequences of COVID-19, and the business struggles as sludges of economics, supply chain, food safety, and change of customer's behaviour. Hence, Starbucks was most negatively impacted during the pandemic period. Even if the business offers a wide range of shipping choices, a fewer customers are ready to employ them during the pandemic. Starbucks was able to maintain its performance throughout the year, despite it appearing that business has not increased throughout the pandemic (Trefis & Team [2]). Although this business is not one of those that can be classified as a seasonal business. Seasonal products are commonly affected on a large scale due to disruption of the supply chain, availability of spaces etc. Apart from these factors, the deterioration factor mostly affects seasonal products. Deterioration is defined as the process that makes products unsuitable for consumption and is brought on by the biochemical activities of the dominant microbial populations in the product. Seasonal products often deteriorate in unsuitable weather conditions. For example, in summer season, fruits loose their original freshness after a period. This may also change in colour, a change in texture, an unpleasant odour, or an undesirable taste and hence decrease their quality. Since, all these factors effects the demand negatively, and hence reflect in financial losses. Instead, the retailers who prevent their stock using preservation technologies and manage the lot size and time span may provide better service to their customers. There are several researchers (Shukla & Khedlekar [3], Shah *et al.* [4], Sharma *et al.* [5], Khedlekar [6], Roy & Chaudhury [7], and Shah *et al.* [8]) developed inventory models by incorporating deterioration as a mathematical parameter.

The effective use of preservation technology has become essential for the business of deteriorating products. Although the deterioration could not be completely avoided, it could be effectively reduced using specialised equipment and process changes. Product preservation varies according to the type of item and can take the form of freezing, bacterial disinfection, cleaning, drying, and so on. Researchers provided various types of parameters to bind preservation technology cost (PTC) into a single mathematical expression. Khedlekar *et al.* [9] developed an inventory model for seasonal products where investment in product preservation technology can control deteriorations. Investments in preservation technology are made for valuable businesses because it helps to slow down the rate of deterioration (Shah *et al.* [10], Shah & Naik [11]). This study incorporated stock-dependent demand as exhibiting a huge volume of commodities leads to more customers and augments the trading of the goods. Barman *et al.* [12] proposed an EPQ model by incorporating the impacts of green and preservation technology investments under the effects of the COVID-19 pandemic. Saikh *et al.* [13] explored the effects of continuous investment in preservation technology on greenhouse flower retailers. They proposed a replenishment model for non-instantaneous deteriorating products, including sustainability and preservation technology.

In the modern competitive market, firms gaining complexities to determine a sustainable pricing policy due to uncertainty. Sectors continuously face fluctuations in market forces such as demand, supply, price and many more. In recent years, the COVID-19 pandemic triggered the entire world and created a chaotic state in the market. In this scenario, acquiring a better pricing policy is always a challenge for the management. Uncertain demand and price are the crucial factors that affect any business. There are plenty of researchers who put their efforts to incorporate the realistic demand in order to deal with uncertainty. Uncertainty and unforeseen events frequently cause disruptions in the production system, which also affects demand and lowers an organisation's abet margin. Management would need to research the variation in demand patterns and system disruption in the event of a disrupted production system. Effective suggestions were made to improve managerial efficiency for inventory production system for exponential demand and disrupted due to uncertainties (Khedlekar *et al.* [14]). In terms of disruption time, reproduction time, and deterioration with the disrupted production system, this study demonstrates that the effect of exponential demand is quite different. When the system could disrupted, it is discovered that the demand parameters affects the business and thus, it force the management to take decisions accordingly for the best course of action. Shah & Vaghela [15] proposed an inventory model under the presumption that the retailer sells both new and used products to their customers. The selling price is influenced by both the demand for new products and the return rate for used ones. In this scenario, the author developed a mathematical model to improve the retailer's profit. To generalise the growing as well as declining demand pattern, one can consider the demand rate on a logarithmic scale. To examine the situation that how a logarithmic demand can affect the profitability of the organisation, Khedlekar & Shukla [16] developed a dynamic pricing model with a logarithmic demand rate by incorporating the constant deterioration of the products.

Frequently, the shop is unable to pay the account as soon as the products are received. In the given circumstance, there are numerous researchers have contributed to inventory management theory incorporating trade credit systems (Shah & Mishra [17], Shah & Jani [18], Shah & Naik [11], Shah & Shroff [19] and Nigwal *et al.* [20]). In accordance with the number of orders, a study by Jani *et al.* [21] developed an inventory policy for a product with two tiers of trade credit. They have incorporated expiry date of the product, and supplier who is willing to extend a credit period to a retailer only if the retailer's order quantity exceeds the supplier's predetermined order quantity. In such a series of problems, this trade-credit system was acknowledged by addressing environmental concerns by Paul *et al.* [22]. For a supply chain including a manufacturer and a wholesaler, shared inventory policies are required. Additionally, the producer makes investments in eco-friendly technologies to reduce carbon emissions. Also, this study is one among them that addressed the impact of the deterioration of the products.

By considering the shortage at the end of the time cycle, a deterministic inventory production model developed by Khedlekar *et al.* [23], with time-proportional demand. The results of the study suggest that production managers should aim to produce products in small batches to keep the less deterioration using preservation technology. A deterministic integrated vendor-buyer model with quadratic demand under inspection policies and preservation technology was developed by Pervin *et al.* [24]. They concluded that the vendor needed to use preservation technology to reduce the loss caused by deterioration. They illustrated that if the rate of deterioration is initially very low, investing in preservation technology will not be worthwhile. Ali *et al.* [25] proposed a multi-objective mixed-integer non-linear supply chain coordination model for uncertain environments in order to lower transportation costs, account for product waste, and compensate for losses brought on by a lack of transit and storage facilities. In order to reduce the overall cost of the organization, the study used a multi-objective mixed integer fuzzy non-linear programming model to address the issue. It found that the cost of the organization incurred at three different echelons and under multiple case scenarios was lower than the actual cost. The model is first transformed into a multi-objective mixed integer crisp non-linear model using the ranking index theory, and then the multi-objective crisp model is changed into a single-objective model using the FNLP problem. A study by Shah *et al.* [26] was conducted to obtain joint inventory policies for a supply chain involving a manufacturer and a wholesaler, taking into account the manufacturer's investment in green technology to reduce carbon units. In order to optimize the problem, Kumar & Malik [27] used an uncommon strategy and worked on the Lyapunov exponent using Euler's algorithm. Paul *et al.* [28] have developed an EPQ model that includes both deterioration and investments on green operations. The study focused on the goal of determining an ideal replenishment time and an ideal green concern level by taking profit maximization. It also took into account the products' price-dependent demand as well as their level of environmental concern. Retail investments in environmentally friendly operations are incorporated into an inventory model, along with variable holding costs Paul *et al.* [29]. In this study, demand is price-sensitive and a green concern level.

Products that have recently launched in the market need to ensure that their targeted customers are aware of the novelty and positioning of the products. Fashionable items or the most technologically advanced cell phones eventually become obsolete and must be replaced by a new one after a time. Due to modifications in products, such as changes in style, functionality, etc., promotional efforts are effective after a certain interval. In these cases, promotional strategies help to increase sales and liquidate overused stocks, which are more vulnerable to deterioration. Ingenious techniques for promotional activities like "Big Billion Day"on Flipkart, and "Amazon Season Sale", are such techniques used by multinational companies to clearance their stocks. Domino's and McDonald's, like the catering industry, use frequent offers and discount coupons to promote their products. Several researchers (Cardenas-Barron & Sana [30], Rajan, & Uthayakumar, [31], and Soni & Chauhan [32]) used promotional efforts to develop the inventory model and concluded that regulated marketing and advertisement are required to increase market demand and thus generate more profit. The inventory system with a finite time horizon (Tsao & Sheen [33]) included a deterioration factor with dynamic pricing, promotional efforts, and replenishment policies that allowed for payment delays. They observed demand by arbitrarily shifting promotional efforts and pricing upward and downward and concluded that optimal promotional efforts were required to generate additional profit. Soni & Shah [34] developed a continuous review inventory model that incorporates a cost function for capital investments and reduces lost sales, and further reduces ordering costs by reducing lead times. Two types of capital investment functions, namely logarithmic and power, are used in the model to lower the lost sales rate. Some authors considered the linear and logarithmic relationships between order cost and lead time. The conclusions were reached by combining various combinations of the capital investment cost function and the ordering cost lead time relationship. Since there are numerous researchers (Khedlekar *et al.* [35], Berman *et al.* [36], Fatma *et al.* [37], Poswal *et al.* [38], Kuppulakshmi *et al.* [39], Jani *et al.* [40], Akhtar *et al.* [41]) have worked to develop the deterministic inventory model by incorporating diverse demand rates and deterioration, the stochastic approach has rarely been used to contribute to this field. Table 1, illustrates a tabular comparison of the proposed research work with the available literature and demonstrates how the proposed study would fill a research gap.

Table 1: Comparison of proposed work to existing literatures

Author(s)	Demand	Deterioration	PTC	PE	PB
Shah *et al.* [4]	deterministic	✓	×	×	×
Sharma *et al.* [5]	deterministic	✓	×	×	×
Khedlekar *et al.* [6]	deterministic	✓	×	×	×
Shah & Naik *et al.* [10]	deterministic	✓	×	×	×
Barman *et al.* [12]	deterministic	✓	✓	×	✓
Saikh *et al.* [13]	deterministic	×	×	×	×
Shah & Mishra [17]	deterministic	✓	×	×	×
Paul *et al.* [22]	deterministic	✓	×	×	×
Khedlekar & Tiwari [23]	deterministic	×	×	×	✓
Pervin *et al.* [24]	deterministic	✓	✓	×	✓
Ali *et al.* [25]	deterministic	✓	×	×	×
Cardenas-Barron & Sana [30]	deterministic	×	×	✓	×
Rajan & Uthayakumar [31]	deterministic	×	×	✓	×
Tsao & Sheen [33]	deterministic	✓	×	✓	×
Barman *et al.* [36]	deterministic	✓	×	×	✓
Fatma *et al.* [37]	deterministic	✓	×	×	✓
Poswal *et al.* [38]	deterministic	✓	×	×	×
Kuppulakshmi *et al.* [39]	deterministic	✓	×	✓	×
Jani *et al.* [40]	deterministic	✓	✓	×	×
Akhtar *et al.* [41]	deterministic	✓	×	×	✓
Proposed Work	stochastic	✓	✓	✓	✓

1.1. The Problem

Seasonal products are most vulnerable to deterioration over time. Along with the higher deterioration rate of products, there is continuous economic fluctuation in the market that affect the business. Organisation uses preservation technology to reduce the deterioration rate that deals with the business of seasonal products. Along with these factors, the management employed promotional efforts to boost the market demand. Considering the fact that the promotional strategy will increase total demand, we have incorporated the promotional efforts parameter into the demand function to increase market demand. The selling price of seasonal products impacts the market demand on a large scale. Due to the availability of substitutes in the market, these seasonal products are susceptible to price elasticity. To address the elasticity of the demand rate for such products, we have employed a quadratic price-sensitive demand rate to formulate the mathematical model. Also, the study incorporates the stochastic variable with the demand by introducing a continuous random variable. This study examines optimal selling price and profit in the numerical section with the help of different probability distributions on this random variable. The proposed model also backlogged the fraction of demand during the stock-out state. More precisely, we have employed the partial backlogging of demand in the shortage period. Since plenty of uncertain factors affect the market demand, it is almost impossible to anticipate demand in any specific sale season. The market could generate high demand during the peak sale season, whereas demand may be nominal in the off-season. In this case, we have considered that the management has a choice that how long he desires to keep the shortage. For this, we have incorporated the proportional shortage time to formulate the objective function considering the constant ratio of the shortage time to the selling period. The proportional shortage could provide suitability to estimate shortage time which is proportionate to the selling period. In the empirical evaluation section, a different value of this ratio examines the selling price and the optimal profit. The proposed work develops a mathematical model incorporates the following parameters to solve above all issues:

1. Incorporated the quadratic price-sensitive demand to address the price-elastic behaviour of demand.
2. Incorporated the stochastic demand to deal with the uncertainty in the declining market,
3. Incorporated deterioration to address the natural phenomenon of deteriorating products,
4. Incorporated the preservation technology to reduce the rate of deterioration,
5. Incorporated the proportional shortage time to restrict the stock-out period proportional to the selling period,
6. Incorporated promotional efforts to increase market demand,
7. Incorporated the partial backlogging to address the possibility to backlog the fraction of demand,

In the next section, we have provided a mathematical formulation of the model and optimised the result using the profit-maximizing approach. The numerical evaluations and graphical illustrations are carried out to obtain the validation of the findings in a separate section provided at the end. Sensitivity analysis towards the model's parameters and the some useful managerial insights are given in the conclusion section.

2. COMMON NOTATIONS USED

Notations bearing usual tradition, utilizing in the subsequent discussions, are laid down as follows:

Parameters

C_0 : Unit purchasing cost,

C_1 : Holding cost per unit per unit of time,

C_3 : Set-up cost per order/cycle,

C_l : Lost cost sale per unit,

C_s : Back order cost per unit per unit of time,

C_d : Cost of deterioration per unit per unit of time,

Q : Quantity to be order at a time,

w : Maximum capital invested in preservation technology,

ρ : Variable of promotional effort, $\rho \geq 1$,

$m(\xi)$: Reduce rate of deterioration after applying preservation technology,

$I_n(t)$: Storage inventory level at time t such that $t \in [t_1, t_1 + t_2]$,

$I_p(t)$: Lot size inventory level at time t such that $t \in [0, t_1]$,

$\prod(t_1, t_2, p, \xi)$: Profit function depends on time, price and preservation technology cost,

$\prod_A(t_1, t_2, p, \xi)$: Average profit function,

Decision Variables

t_1 : Length of time at which inventory carried and reaches to zero i.e., [0, t_1],

t_2 : Length of Shortage time of inventory i.e.,[0, t_2],

p : Selling price of the product per unit,

ξ : Preservation technology cost per unit.

3. ASSUMPTIONS

Following assumptions are used to analysed this mathematical modelling, listed as follows:

1. Proposed model is a single item, and a single channel inventory model starts with stock replenishment,
2. Order size per cycle is finite while replenishment rate is infinite,
3. Lead time is considered to be zero,
4. Model begins with the replenishment of the inventory at the beginning of the cycle and declines due to demand and deterioration till the time t_1. After the time t_1, shortage occurs and it will carried out till the time t_2. Both the times, t_1 and t_2 collectively determine one cycle,
5. Shortage is partially backlogged. During shortage period $[0, t_2]$, only a fraction of demand i.e., $\beta(x)$ were fulfilled and remaining are lost. The fraction $\beta(x)$, is determined as $\beta(x) = e^{-\delta x}$, where $\delta \geq 0$.

6. Shortage time t_2 is considered proportional to selling period t_1, with constant ratio γ, i.e., $\frac{t_1}{t_2} = \frac{1}{\gamma}$.
7. Demand is considered quadratically declining with respect to selling price p, i.e., $D(p) = a - bp - cp^2$, where $a, b, c > 0$. A continuous random variable ϵ with expected value μ, is associated with demand $D(p)$ i.e., $D(p, \epsilon) = D(p) + \epsilon$ and $E(D(p, \epsilon)) = D(p) + \mu$.
8. Promotional efforts ρ, such that $\rho > 1$, is influence the market demand $D(p)$. Thus, expected demand for a unit cycle is $\rho(D(p) + \mu)$.
9. Promotional cost is function of the promotional efforts parameter ρ, and determined as,

$$PC = k(\rho - 1)^2 \left[\int_0^{t_1+t_2} (D(p) + \mu) dt \right]^\eta$$

where $k > 1$, and η is a natural number.
10. Total lot size deteriorated with constant rate of deterioration θ, where $0 \leq \theta \leq 1$, during inventory holding interval $[0, t_1]$.
11. Preservation technology is utilized to reduce the deterioration rate, $m(\xi)$ such that $0 \leq m(\xi) \leq 1$, where $m(\xi)$ is continuous, concave and increasing over the preservation cost ξ, determined by $m(\xi) = 1 - e^{-\alpha\xi}$.

4. MODEL FORMULATION

Initially, the model starts with the replenishment at the beginning of the cycle at $t = 0$. The inventory declines due to demand rate $D(p) + \mu$, where μ is stochastic factor of demand and constant deterioration θ. Total lot size will be reaches to *zero* in time t_1. After this period, shortage occur and demand is partially backlogged during stock out period $[0, t_2]$. Stochastic factor associated with the demand is responsible for the demand fluctuation and could increase or decrease the total demand based on market uncertainty. Figure 1, represents the expected demand fluctuation on the different probability distributions. This demand fluctuation would affect the profit accordingly. Promotional efforts are used to boost the market demand with the parameter ρ in such a way that total demand is directly proportional to the parameter of promotional efforts ρ. Figure 2, illustrates the impact of the different value of this parameter on total demand. Also, the effect of this parameter on the optimal results are discussed in the section of empirical evaluation.

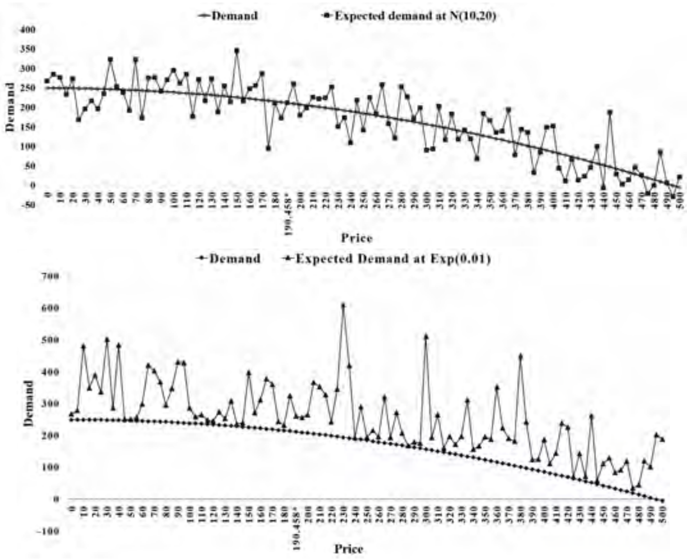

Figure 1: Expected demand on various probability distribution

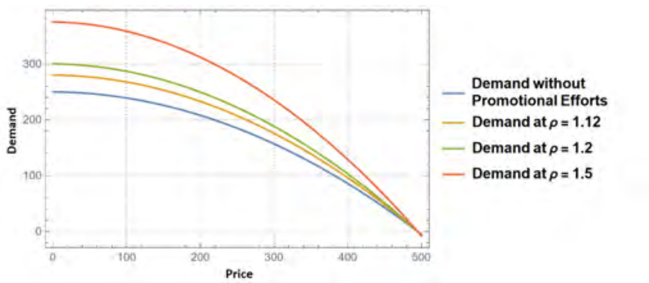

Figure 2: Effect of promotional efforts on demand

Constant rate of deterioration would affect the declining of inventory level. How the various values deterioration rate (θ) affects the net profit are discussed in results analysis. Proposed inventory model is based on the assumptions of the seasonal products. This seasonal products are highly vulnerable to deterioration. To avoid much wastage of the products due to deterioration, we incorporated the preservation technology during the interval $[0, t_1]$ with per unit of preservation cost ξ. All the aforementioned scenarios are graphically illustrated in the Figure 3.

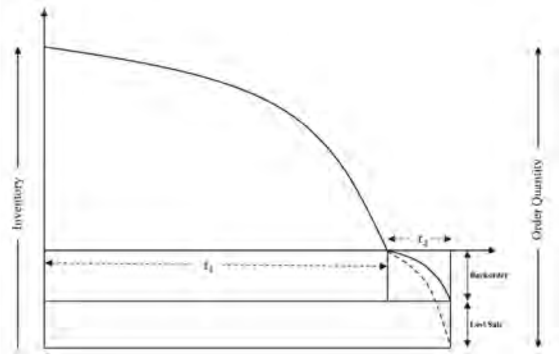

Figure 3: Inventory level

Let Q, be the total lot size including the positive inventory level $I_p(t)$, and the negative inventory level $I_n(t)$, over the duration $[0, t_1]$, and $[0, t_2]$ respectively. Positive inventory level at the time t where $t \in [0, t_1]$ is given as,

$$\frac{dI_p(t)}{dt} = -\theta\left(1 - m(\xi)\right) I_p(t) - \rho\left(D(p) + \epsilon\right), \text{ where } 0 \leq t \leq t_1, \text{ and } I_p(t_1) = 0 \quad (1)$$

or,

$$I_p(t) = \rho\left(D(p) + \epsilon\right) e^{-\theta(1-m(\xi))t} \left[\int_t^{t_1} e^{\theta(1-m(\xi))x} dx\right], \; 0 \leq t \leq t_1 \text{ and } I_p(t_1) = 0 \quad (2)$$

Consider the demand is partially backlogged in the stock-out period. This gives, only a fraction $\beta(x)$, of demand were fulfilled in the shortage time period. So, instantaneous shortage level at time $t \in [0, t_2]$ is given as,

$$\frac{dI_n(t)}{dt} = -\rho\left(D(p) + \epsilon\right) \beta(t_2 - t), \text{ with } I_n(0) = 0, \; 0 \leq t \leq t_2. \quad (3)$$

or,

$$I_n(t) = -\rho\left(D(p) + \epsilon\right) \int_0^t \beta(t_2 - x) dx, \; I_n(0) = 0, \; 0 \leq t \leq t_2 \quad (4)$$

During shortage time $[0, t_2]$, only fraction of demand $\beta(x)$, were fulfilled and remaining are lost either customers chooses another shop or any other alternative available in the market. So, let $I_l(t)$ be the lost sale quantity at time t, such that,

$$I_l(t) = \rho\left(D(p) + \epsilon\right)\left[1 - \beta(t_2 - t)\right], \; 0 \leq t \leq t_2 \quad (5)$$

Total replenish quantity (including backlogged) is,

$$Q = \{I_p(0) - I_n(t_2)\} = \rho\left(D(p) + \epsilon\right)\left\{\int_0^{t_1} e^{\theta(1-m(\xi))x} dx + \int_0^{t_2} \beta(t_2 - t) dt\right\} \quad (6)$$

Let C_l be the cost of lost sale per unit of time, therefore, expected lost cost (TLC) during time $[0, t_2]$ is,

$$TLC = E\left(C_l \int_0^{t_2} I_l(t)dt\right) = C_l\rho\left(D(p)+\mu\right)\int_0^{t_2}[1-\beta(t_2-x)]dx \tag{7}$$

If C_s is the unit backlogged cost per time then, expected backorder cost (TBC) for stock-out during the time $[0, t_2]$ is,

$$TBC = E\left(C_s \int_0^{t_2}\left[-I_n(t)\right]dt\right) = C_s\rho\left(D(p)+\mu\right)\int_0^{t_2}\left[\int_0^{t}\beta(t_2-x)dx\right]dt \tag{8}$$

Holding cost is applicable to positive inventory level in period $[0, t_1]$, with per unit carrying cost C_1 per unit of time. So, expected holding cost (THC) is,

$$\begin{aligned} THC \quad &= E\left(C_1\int_0^{t_1} I_p(t)dt\right) \\ &= C_1\rho\left(D(p)+\mu\right)\left\{\int_0^{t_1} e^{-\theta(1-m(\xi))t}\left[\int_t^{t_1} e^{\theta(1-m(\xi))x}dx\right]dt\right\} \end{aligned} \tag{9}$$

Purchasing cost of the products depends on demand level along with per unit cost price C_0. So, the expected purchasing cost (TPC) is,

$$TPC = E\left(C_0 Q\right) = C_0\rho\left(D(p)+\mu\right)\left[\int_0^{t_1} e^{\theta(1-m(\xi))x}dx + \int_0^{t_2}\beta(t_2-x)dx\right] \tag{10}$$

Expected revenue generated for both the time intervals $[0, t_1]$ and $[0, t_2]$ given by:

$$\begin{aligned} TRV \quad &= E\left(p\left\{\rho\int_0^{t_1}\left(D(p)+\epsilon\right)dt + \left(-I_n(t_2)\right)\right\}\right) \\ &= p\rho\left(D(p)+\mu\right)\left\{t_1 + \int_0^{t_2}\beta(t_2-x)dx\right\} \end{aligned} \tag{11}$$

Expected amount invested for preservation of the lot size from deterioration in period $[0, t_1]$ with preservation cost ξ per unit of time is given as,

$$PTC = t_1\xi \tag{12}$$

Total promotional cost invested during complete cycle i.e., $[0, t_1]$ and $[0, t_2]$, is given as,

$$PC = k(\rho-1)^2\left[\int_0^{t_1+t_2}\left(D(p)+\mu\right)dt\right]^{\eta} = k(\rho-1)^2\left(D(p)+\mu\right)^{\eta}\left(t_1+t_2\right)^{\eta} \tag{13}$$

where $\rho, k \geq 1$ and η is natural number.

Total deteriorated quantity with a constant deterioration rate θ per unit of time is given as,

$$\begin{aligned} Q_{det} &= \int_0^{t_1} \theta\left(I_p(t)\right) dt \\ &= \rho\theta\left(D(p)+\mu\right)\left\{\int_0^{t_1} e^{-\theta(1-m(\xi))t}\left(\int_t^{t_1} e^{\theta(1-m(\xi))x} dx\right) dt\right\} \end{aligned} \tag{14}$$

Total order quantity along with Q_{det} is,

$$\text{Total Order Quantity} = \mathrm{Q} + \mathrm{Q_{det}} \tag{15}$$

Total cost of deterioration units per unit of deterioration cost C_d is,

$$TDC = E\left(C_d \int_0^{t_1} \theta I_p(t) dt\right) \tag{16}$$

$$= C_d\rho\theta\left(D(p)+\mu\right)\left\{\int_0^{t_1} e^{-\theta(1-m(\xi))t}\left(\int_t^{t_1} e^{\theta(1-m(\xi))x} dx\right) dt\right\} \tag{17}$$

Total profit function is determined by assembling all these above costs, the proposed profit function influenced by variables t_1, t_2, p and ξ given by,

$$\Pi(t_1,t_2,p,\xi) = TRV - (C_3 + TBC + TLC + THC + TPC + PTC + PC + TDC)$$

$$\begin{aligned} \Pi(t_1,t_2,p,\xi) = \quad & p\rho\left(D(p)+\mu\right)\left\{t_1 + \int_0^{t_2} \beta(t_2-x)dx\right\} - C_3 \\ & -C_s\rho\left(D(p)+\mu\right)\int_0^{t_2}\left[\int_0^t \beta(t_2-x)dx\right] dt \\ & -C_l\rho\left(D(p)+\mu\right)\int_0^{t_2}\left(1-\beta(t_2-x)\right) dx \\ & -C_1\rho\left(D(p)+\mu\right)\left\{\int_0^{t_1} e^{-\theta(1-m(\xi))t}\left[\int_t^{t_1} e^{\theta(1-m(\xi))x} dx\right] dt\right\} \\ & -C_0\rho\left(D(p)+\mu\right)\left\{\int_0^{t_1} e^{\theta(1-m(\xi))x} dx + \int_0^{t_2} \beta(t_2-x)dx\right\} \\ & -t_1\xi - k(\rho-1)^2\left(D(p)+\mu\right)^{\eta}(t_1+t_2)^{\eta} \\ & -C_d\rho\theta\left(D(p)+\mu\right)\left\{\int_0^{t_1} e^{-\theta(1-m(\xi))t}\left(\int_t^{t_1} e^{\theta(1-m(\xi))x} dx\right) dt\right\} \end{aligned} \tag{18}$$

Subject to conditions, $C_0 \le p$, $0 \le \xi \le w$, $\frac{t_1}{t_2} = \frac{1}{\gamma}$, and $t_1, t_2 \ge 0$.

Average profit function determined by,

$$\Pi_A(t_1,t_2,p,\xi) = \frac{1}{(t_1+t_2)}\Pi(t_1,t_2,p,\xi) \tag{19}$$

5. SOLUTION PROCEDURE

Now, we need to maximize the average profit function $\Pi_A(t_1, t_2, p, \xi)$, such that,

$$\max_{\xi}\left\{\max_{p}\left\{\max_{t_1,t_2}\Pi_A(t_1,t_2,p,\xi)\right\}\right\} \tag{20}$$

subject to given constraints, $C_0 \le p, 0 \le \xi \le w$, and $t_1, t_2 \ge 0$.

By putting the parameter for shortage time i.e., $t_2 = \gamma t_1$, we have reduced the total profit function (18) into three variables given as,

$$\begin{aligned}\Pi_A(t_1,p,\xi) = \quad & -\frac{1}{t_1(1+\gamma)}\left[C_3 + t_1\xi + (D(p)+\mu)\left\{C_s\rho\int_0^{t_1\gamma}\left(\int_0^t \beta(t_1\gamma - x)dx\right)dt\right.\right.\\ & +C_l\rho\int_0^{t_1\gamma}(1-\beta(t_1\gamma-x))\,dx - \frac{(C_1+C_d\theta)\,\rho\,(1+t_1\theta\,(1-m(\xi)))}{\theta^2\,(1-m(\xi))^2}\\ & +C_0\left[\int_0^{t_1\gamma}\beta(t_1\gamma-x)dx - \frac{1}{\theta\,(1-m(\xi)))}\right]\\ & -p\rho\left[t_1 + \int_0^{t_1\gamma}\beta\,(t_1\gamma-x)\,dx\right] + k\,(t_1+t_1\gamma)^{\eta}\,(\rho-1)^2\,(D(p)+\mu)^{\eta-1}\\ & \left.\left.+\frac{(C_1+C_d\theta+C_0\theta\,(1-m(\xi)))\,\rho e^{t_1\theta(1-m(\xi))}}{\theta^2\,(1-m(\xi))^2}\right\}\right]\end{aligned} \tag{21}$$

For notational convenience, consider,

$$\begin{aligned}\psi(t_1,\xi) = \quad & C_s\rho\int_0^{t_1\gamma}\left(\int_0^t\beta(t_1\gamma-x)dx\right)dt + C_l\rho\int_0^{t_1\gamma}(1-\beta(t_1\gamma-x))\,dx\\ & +C_0\left[\int_0^{t_1\gamma}\beta(t_1\gamma-x)dx - \frac{1}{\theta(1-m(\xi))}\right] - \frac{(C_1+C_d\theta)\rho(1+t_1\theta(1-m(\xi)))}{\theta^2(1-m(\xi))^2}\\ & +\frac{(C_1+C_d\theta+C_0\theta(1-m(\xi)))}{\theta^2(1-m(\xi))^2}\rho e^{t_1\theta(1-m(\xi))}\end{aligned} \tag{22}$$

Lemma 1. *For $C_s, C_l, C_0, C_d, \theta > 0$, then we have,*

a. $\frac{\partial^2\psi(t_1,\xi)}{\partial t_1^2} > 0$, *for every t_1.*

b. $\frac{\partial^2\psi(t_1,\xi)}{\partial \xi^2} > 0$, *for every ξ.*

Using (22), (21) reduced to

$$\begin{aligned}\Pi_A(t_1,p,\xi) = \quad & -\frac{1}{t_1(1+\gamma)}\left[C_3 + t_1\xi + (D(p)+\mu)\left\{\psi(t_1,\xi)\right.\right.\\ & -p\rho\left[t_1 + \int_0^{t_1\gamma}\beta(t_1\gamma-x)dx\right]\\ & \left.\left.+k\,(t_1+t_1\gamma)^{\eta}\,(\rho-1)^2\,(D(p)+\mu)^{\eta-1}\right\}\right]\end{aligned} \tag{23}$$

Theorem 1. *Profit per unit time $\Pi_A(t_1, p, \xi)$, is concave function for fixed p, and ξ, and attained its global maxima at point $t_1 = t_1^*$.*

Proof. Differentiating the above profit function (23) with respect to t_1 keeping p & ξ constant, and by equating it to zero, the obtained equation is transcendental non-linear equation. So, we have calculated the approximated solution using Taylor expansion by MATHEMATICA 13. Differentiating again it with respect to t_1, we obtained $\frac{\partial \Pi_A^2(t_1,p,\xi)}{\partial t_1^2} < 0$, for $\frac{\partial \xi^2(t_1)}{\partial t_1^2} > 0$ (Lemma 1(a)). Thus, the average profit function is concave with respect to optimal time t_1. The concavity of the profit function can also be illustrated from three-dimensional graphical representation (Figure 4).

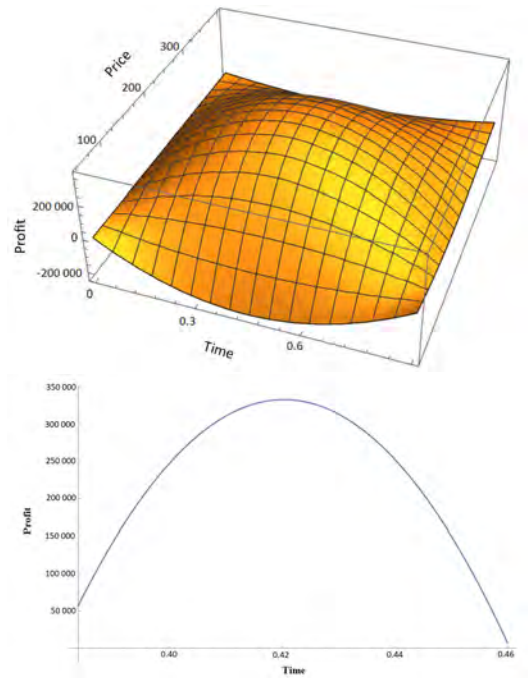

Figure 4: Concavity of profit function with respect to time

□

Above theorem proves that the unit profit function $\Pi_A(t_1, p, \xi)$, is attain maxima at the optimal solution $t_1 = t_1^*$. Now, proceeding further, we claim that the profit function is concave with respect to the optimal selling price p^*.

Theorem 2. *For the fix value of ξ, and optimal value t_1^*, average profit function $\Pi_A(t_1, p, \xi)$ attain global maxima at price $p = p^*$.*

Proof. Differentiating the profit function (23) with respect to p, keeping other decision variables constant, and by computing it to zero, we have two distinct roots from the obtained equation. These obtained roots are examined numerically on the parameters given in Example 1, and preferred the positive one (say p^*). Again differentiating the obtained equation with respect to p, on the optimal solution p^*, we have $\frac{\partial^2 \Pi_A(t_1,p,\xi)}{\partial p^2} < 0$, for the given $\frac{\partial D(p)}{\partial p} < 0$. Provided graph (Figure 5) of average profit function (23) with respect to p and ξ, illustrates the concave behaviour of objective function using the set of feasible parameters (provided in Example 1).

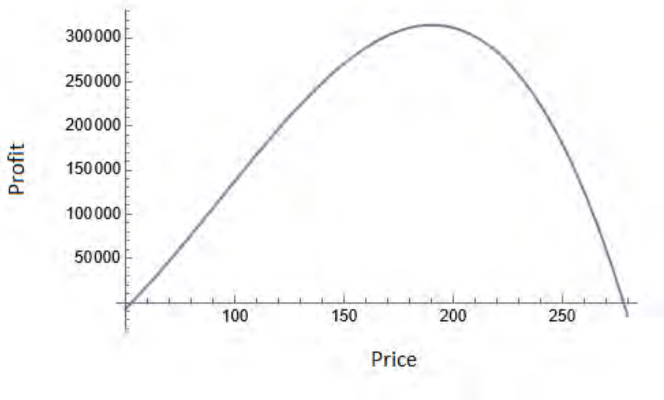

Figure 5: Concavity of profit function with respect to price.

□

By above theorem, we get the optimal value of selling price p^*. Now, we need to find optimal solution for preservation technology cost ξ, on optimal price p^*, and time t_1^*.

Theorem 3. *Profit per unit time $\Pi_A(t_1, p, \xi)$, found to be concave on the optimal value ξ^*, at fixed t_1^* and p^*.*

Proof. by differentiating the profit function (23), let we have an approximated optimal solution ξ^* computed using MATHEMATICA 13, and by putting $\frac{\partial \Pi_A(t_1,p,\xi)}{\partial \xi} = 0$. Again differentiating the profit function, we have,

$$\left[\frac{\partial^2 \Pi_A(t_1, p, \xi)}{\partial \xi^2}\right]_{\xi=\xi^*} = -\frac{1}{t_1(1+\gamma)}\left[t_1\left(D(p)+\mu\right)\frac{\partial^2 \psi(t_1, \xi^*)}{\partial {\xi^*}^2}\right] \tag{24}$$

For given $\frac{\partial^2 \psi(t_1,\xi)}{\partial \xi^2} > 0$ (Lemma 1(b)), we have $\frac{\partial^2 \Pi_A(t_1,p,\xi)}{\partial \xi^2} < 0$, on the optimal value $\xi = \xi^*$. Thus, the unit profit is concave with respect to the preservation cost ξ, and attains the maxima on the obtained optimal solution ξ^*. Figure 6 is drawn on the parameters discussed in Example 1, and illustrates the concavity of the profit function with respect to ξ.

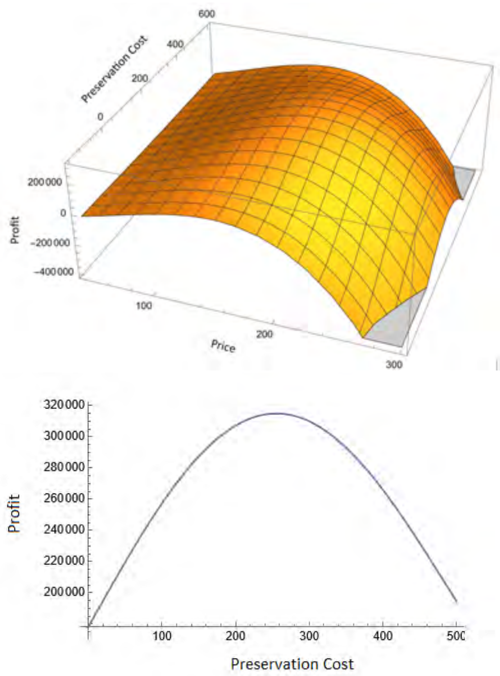

Figure 6: Concavity of profit function with respect to preservation cost

□

Thus, the unit profit function $\Pi_A(t_1, t_2, p)$ shows concavity and attain its maximum value on the point (t_1^*, p^*, ξ^*). Based on the above solution, we concluded the total replenishment size,

$$Q^* = \rho\left(a - bp - cp^2 + \mu\right)\left(\frac{1 - e^{-t_1^*\gamma\delta}}{\delta} + \frac{e^{\alpha\xi}\left(-1 + e^{t_1^*\theta e^{-\alpha\xi}}\right)}{\theta}\right) \tag{25}$$

Total deteriorating quantity per order,

$$Q_{det}^* = \frac{\rho\left(a - bp - cp^2 + \mu\right)e^{\alpha\xi}}{\theta}\left(e^{\alpha\xi}\left(-1 + e^{t_1^*\theta e^{-\alpha\xi}}\right) - t_1^*\theta\right) \tag{26}$$

and,

$$\text{Total Order Quantity} = \mathrm{Q}^* + \mathrm{Q}_{\text{det}}^* \tag{27}$$

Here are some numerical results for validation of the proposed model discussed next.

6. NUMERICAL ANALYSIS

For applicability of model and for support of our theoretical findings, we need to validate model numerically. Moreover, all the values of the parameters in proposed model chosen carefully given below:

Example 1. *A retailer purchases Q lot of quantities of a specific seasonal product, which contains 12 items per lot. It takes purchasing cost ₹122.4 for a lot with a set-up cost ₹2250 per cycle. Retailer store these products in a separate warehouse with holding cost ₹0.1 per unit. Due to the natural tendency of the seasonal product, it deteriorates at the rate of 1% per unit per unit of time. To avoid such wastage, retailers preserve it with specific preservation technology costs ₹*ξ *per unit of time. After applying the preservation cost, let the reduced rate of deterioration is* $m(\xi) = 1 - e^{-0.001\xi}$. *Also, retailers make promotional efforts to boost the demand in the market with the promotional efforts parameters* $\rho = 1.12, k = 4$ *and* $\eta = 1$. *Let the total demand of this product is assumed to be price-dependent and given as* $D(p) = 250 - 0.01p - 0.001p^2$. *If retailer allow the shortage only 15% to the total holding time in which demands will be partially backlogged with the fractional value* $\beta(x) = e^{(-10x)}$. *Other parameters of the model are taken as follows: shortage cost,* $C_s =$₹0.6 *per unit, cost of a lost sale,* $C_l =$₹0.2 *per unit, deterioration cost,* $C_d =$₹0.1 *per unit, and stochastic factor of the demand,* $\mu = 10$.
Based on these data, we have computed the following numerical results using Wolfram MATHEMATICA 13, given as follows:

- *Inventory will be carried out till the time* $t_1 = 0.4303$ *month,*
- *Shortage should be allowed till the time* $t_2 = 0.0645$ *month,*
- *Optimal price should be ₹190.46 per lot,*
- *Total quantity will be sold* 118.82 *lots as per the given demand and total* 0.229 *lots deteriorated during holding time. Thus, total optimum order quantity should be* 119.076 *lots,*
- *Optimum preservation cost is* ₹255.45 *per unit time,*
- *Total profit will be ₹3,14,897 after end of one cycle.*

The above example discussed a numerical illustration of the proposed model based on the feasible set of parameters. The optimal result of the above example provides the optimal value of selling price, shortage and holding time, optimal preservation cost, and optimal profit. In Example 1, we have considered the value of the promotional effort parameter $\rho = 1.12$, for $k = 4$. The proposed work assumes that total demand is directly proportional to the promotional effort's parameter ρ. According to this assumption, increasing the value of ρ will provide a higher overall demand. Symbolically it can be concluded that, if management makes extra efforts to boost the market demand through advertisement and companionship make the product more viable to customers and demand will increase.

Increasing demand will provide the opportunity to rise per unit selling price of the product and generate higher profit (Table 2, and Figure 7). The higher advertisement efforts encourage the customers to choose their product and end the stock level in short time. Therefore, increasing the value ρ provides a higher unit selling price, higher optimal profit per cycle, and decreases the optimal holding time. Due to proportion between the holding and shortage time, the total shortage time will also decreases accordingly.

Table 2: Effect of promotional effort (ρ)

		t_1^*	t_2^*	Price (p^*)	Profit(Π_A^*)
	1.10	0.4519	0.0678	154.54	149684
	1.11	0.4453	0.0668	170.68	214518
	1.12	0.4301	0.0645	190.46	314897
ρ	1.13	0.4256	0.0638	206.64	416874
	1.14	0.4103	0.0615	228.28	582937
	1.15	0.4047	0.0607	245.86	747025
	1.16	0.3978	0.0597	264.48	951470
	4	0.4301	0.0645	190.46	314897
	5	0.4571	0.0669	184.54	282810
	6	0.4647	0.0697	177.93	249479
k	7	0.4787	0.0718	173.40	228164
	8	0.4814	0.0722	172.56	224363
	9	0.4889	0.0733	170.28	214166
	10	0.4924	0.0738	169.24	209635

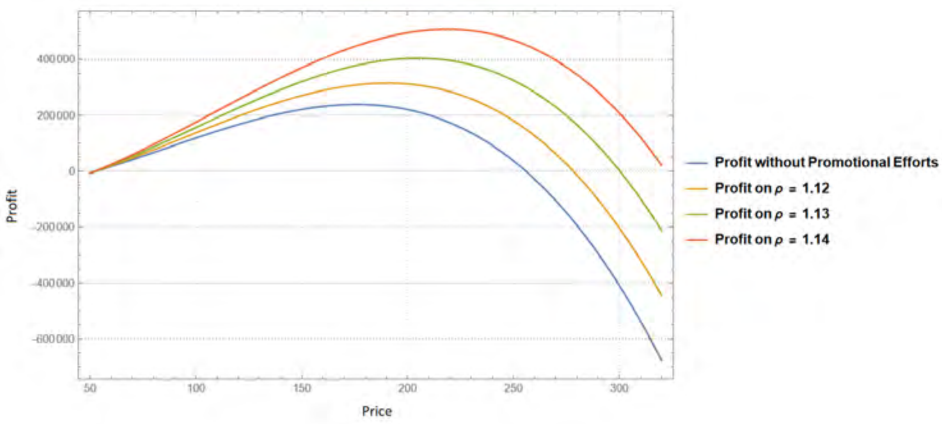

Figure 7: Effect of promotional efforts (ρ) on optimal profit

The model based on the realistic situations is obviously incorporate the price sensitive demand, because the product's demand is more often influenced by the selling price of the products. That makes the selling price most effective factor to determines the demand and hence profit. Most of the products follow the inverse relation of demand with the selling price. If we look in the market, we realize that there are most of the products that demanded less as the selling price goes higher. That type of the demand is termed as the declining demand. Proposed model has incorporated the declining price-sensitive demand for the product which consist the scaling parameter (a), and parameter of elasticity (b, c). Symbolically, we can conclude that scaling parameter is purchasing capacities of any specific product in the market, while elastic parameter (b and c) are responsible to increase or decrease the market demand. This increasing demand due to increasing value of 'a' will provide an opportunity to raise the selling price, and thus, the optimal profit is increase (Table 3, Figure 8). So, in this line, management need to increase the lot size accordingly.

Table 3: Effect of scaling parameter (a), and elastic parameter (b)

		t_1^*	t_2^*	Price (p^*)	Profit(Π_A^*)
	250	0.4301	0.0645	190.46	314897
	300	0.4248	0.0637	193.36	321810
a	350	0.4201	0.0630	196.09	328026
	400	0.4156	0.0623	198.76	334171
	450	0.4100	0.06215	201.91	343566
	0.01	0.4301	0.0645	190.46	314897
	0.02	0.4457	0.0668	184.52	282933
b	0.03	0.4498	0.0675	183.02	275421
	0.04	0.4517	0.0677	182.33	272118
	0.05	0.4568	0.0685	180.57	263180

Market potential, elastic parameters, and selling price are known factors that influence businesses. These parameters are subject to business requirements and can be adjusted based on research findings. Thus, these factors can not be the reason of organisation's instability. The main cause of market instability is unknown factors. In the economic world, these unknown factors are referred to as "market uncertainty."There is always an element of uncertainty in the market economy associated with the business that responsible to change statistically or have a different value of known parameters than the estimation. In the proposed model, we have incorporated the stochastic demand to address the uncertainty. The parameter of uncertainty is responsible for the fluctuation of overall demand and, consequently, optimal profit. Table 4, and Figure 9, provide the theoretical aspect of these fluctuations and illustrate that the expected profit on the

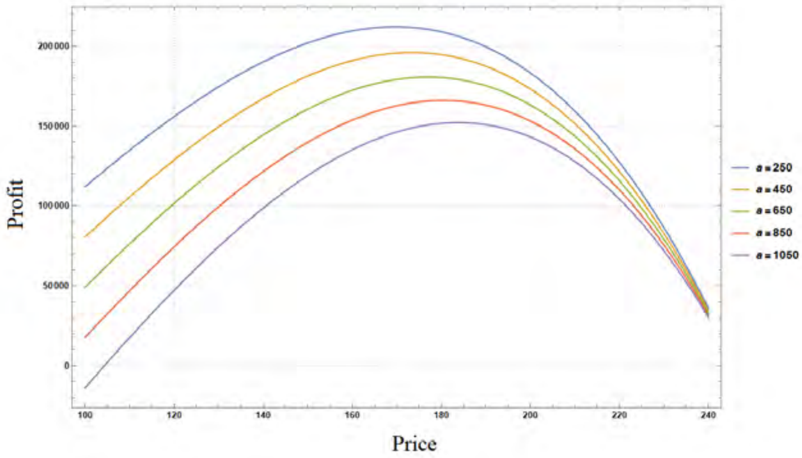

Figure 8: Impact of the scaling parameter (a) on optimal profit

various probability distributions is quite different from the deterministic value of profit. Taking demand into consideration, the analysis reveals that the market uncertainty may result in fluctuating profits due to demand. To overcome the impact of uncertainty, the retailer could employ a short-term replenishment strategy that allows for more frequent price adjustments. Except for rare instances of pandemics, market uncertainty cannot impact the diverse market economy in the same way. There are a variety of challenges and complexity that a business faces in diverse marketplaces. As far as we are concerned, the world is rich in cultural and economic diversity. This diversity can influence the profitability of businesses operating in these places. So, market uncertainty can challenge the validity of an efficient pricing strategy in these diverse markets. Nevertheless, the demand pattern may be interpreted in the context of the current market demand and price. To deal with this type of uncertainty, the retailer could employ a multi-pricing strategy for different markets via multiple outlets. Taking into account the past demand pattern, this approach enables the retailer to select the most optimal price strategy. Consequently, the demand could not be estimated in a realistic market environment, but a retailer might determine the optimal replenishment schedule or pricing strategy to maximise benefit and profit by accounting for the demand pattern.

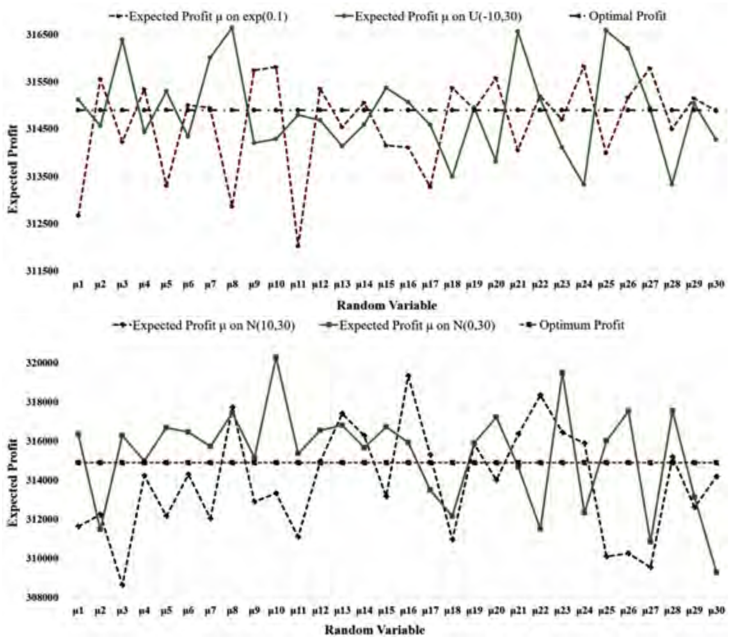

Figure 9: Expected profit influenced by different probability distribution

Table 4: Effect of proportionate parameter (γ)

		t_1^*	t_2^*	Price (p^*)	Profit(Π^*)
	0.15	0.4301	0.0645	190.46	314897
	0.20	0.4301	0.086	188.51	294865
	0.25	0.4301	0.1075	188.27	282243
γ	0.30	0.4301	0.1290	189.71	276106
	0.35	0.4301	0.1505	192.91	276133
	0.40	0.4301	0.1720	198.09	282576
	0.45	0.4301	0.1935	205.59	296360

Most of the organizations experience different seasonal sales patterns for different products. Often, specific months exhibit a rise when compared to the rest of the year. However, the period of these spikes varies by industry and even by the product within an industry. By the end of any specific sale season, the products in the lot size become obsolete due to low demand. As a result, the retailer must finish this inventory before the end of the season. In this procedure, retailers must strike a balance between shortage time and holding time. In this line, we have incorporated the proportionate time to deal with such situation. Consider that the ratio of holding time to shortage time is constant $\left(\gamma = \frac{t_2}{t_1}\right)$. As a result, this provides the capability of determining shortage time proportional to holding time.

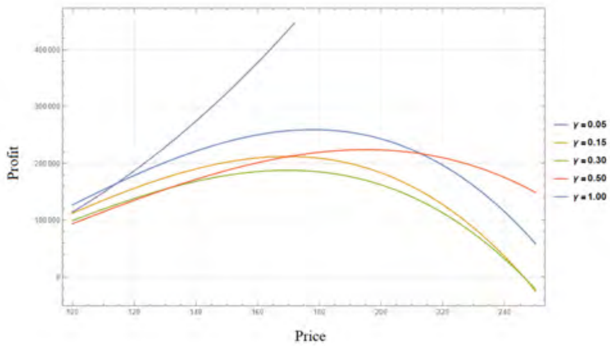

Figure 10: Effect of (γ) on Optimal Profit

When the value of γ is increased, the value of the shortage time for a fixed holding time changes. Increasing shortage time indicates that the available inventory level is not enough to satisfy the complete demand. These rising demands shows the interest of customers towards the product's. So, in this situation, management need to increase the selling price in order to get befit of increasing demand and thus, profit will increase accordingly (Table 4, and Figure 10).

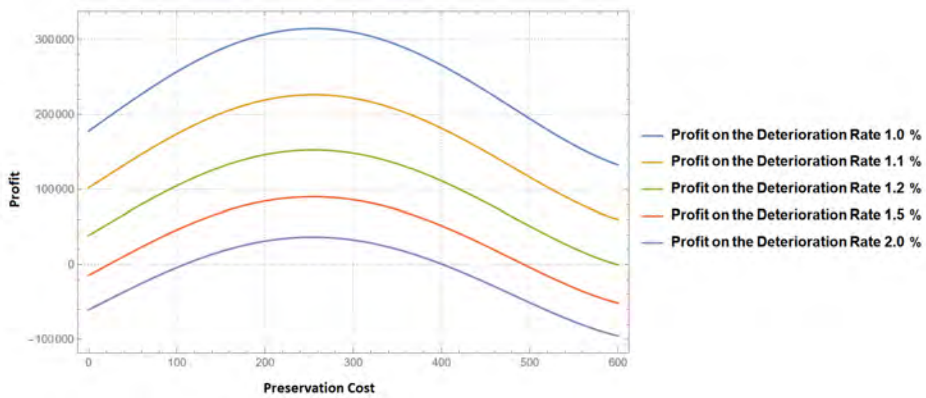

Figure 11: Effect of deterioration rate (θ) on Optimal Profit

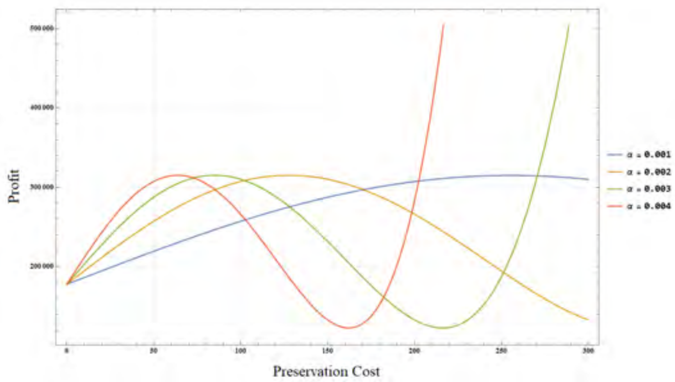

Figure 12: Effect of preservation efforts parameter (α) on optimal profit

Preservation of the product involves different material processing steps to maintain the product's quality at the desired level, so that the maximum utility can be achieved. The key objective of preservation technology is to overcome the inappropriate degradation of substances to produce value-added products. Seasonal products usually consist of food items such as fruits, bakery products, and dairy products. As far as we are concerned, seasonal products are most vulnerable to deterioration with time. In high season, products are held in large quantities due to higher demand and need extra preservation efforts. We considered $m(\xi)$, a reduced rate of deterioration with parameters for preservation cost ξ per unit of time and promotional efforts parameter α. Making additional efforts (increasing the value of α) will result in a higher profit by lowering the preservation cost ξ per unit of time (Figure 12). Increasing the value of the deterioration rate (θ) on a constant preservation cost per unit of time will also result in more products degrading per unit of time. Thus, due to this wastage, optimal profit will decrease (Figure 11). Thus, management need to balance between the promotional efforts and promotional cost to get maximum profit.

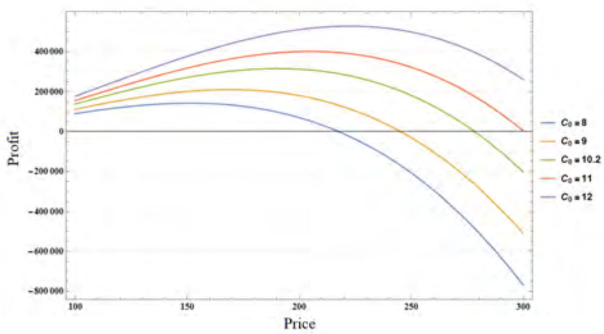

Figure 13: Effect of unit purchasing cost C_0 on the optimal profit

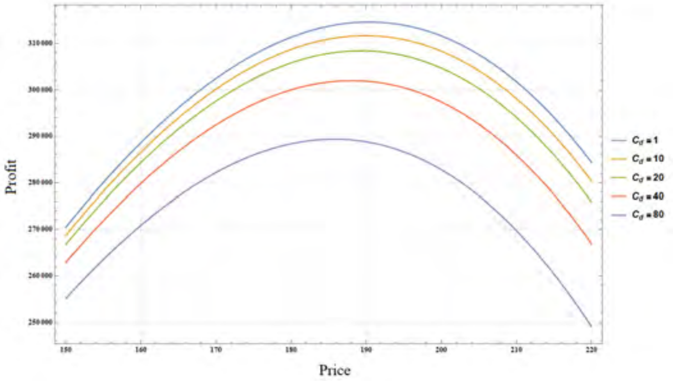

Figure 14: Effect of unit deterioration cost (C_d) on the optimal profit

Table 5: Effect of unit purchasing cost (C_0)

		t_1^*	t_2^*	p^*	Π^*
	7	0.4273	0.0639	129.56	111568
	8	0.4281	0.0642	151.98	143731
	9	0.4295	0.0644	169.26	210523
C_0	10.2	0.4301	0.0645	190.46	314897
	11	0.4318	0.0647	204.01	395843
	12	0.4324	0.0648	221.61	519120
	13	0.4336	0.0651	238.91	661833

Cost factors involved in the objective function, such as purchasing cost, deteriorating cost, holding cost, and cost of lost sales, will also have a huge impact on optimal profit. The cost of per-unit deterioration is the loss incurred by the retailer as a result of qualitative or quantitative deterioration of the products. The increasing value of the per-unit deterioration cost will be responsible for less profit (Figure 14). As a result of the additional preservation (increasing the value of α), retailers could protect more products from deterioration and thus earn a higher profit. During peak selling seasons, retailers should use the bulk ordering approach, which has a low per-unit purchasing cost and take advantage of additional discounts to generate more profit (Figure 13). Therefore, the retailer should take all the essential measures that decrease these costs per unit and hence keep less incurred costs.

7. CONCLUSION

A decision-making policy for deteriorating items in a declining market is developed in this paper by considering stochastic price-sensitive demand and proportional shortage time. Additionally, the proposed model discussed the effect of promotional efforts on seasonal products. In the global marketing environment, firms face difficulties to compete with others while serving the deteriorating products. In response to this situation, we incorporated preservation technology to reduce the rate of deterioration. Delivering goods to a specific market far from the manufacturing site while they are still fresh is a difficult task in itself. In addition to deterioration, there are several additional challenges to inventory control, such as shortages, unpredictable market demand, off-trend products, etc. Our model incorporated partial backlogging, stochastic demand and promotional efforts, preservation technology, and restricted shortages, which collectively made the scope of the application broader. In addition, an examination of the objective function with an exponential partial backlog and quadratic price-sensitive demand shows a concave function. A graphic representation has been given to establish that the total profit function attains maximum value. In addition, the model is validated numerically, and a sensitivity analysis of the parameters has been conducted. Research revealed, with the help of analysis, that variations in promotional effort, purchasing cost, lost sale cost, market potential demand, constant parameter, scaling parameter, and a random variable have significant effects on order quantity and price, and thus the model is sensitive to total profit. Based on sensitivity analysis, it reveals that:

- According to the findings, there is a certain selling price that corresponds to each demand which allows for the maximum overall profit (Theorem 2). This reveals that even increases in the value of the selling price would not necessarily result in an increase in profit. As a result of the price-sensitiveness, an increase in the selling price will lead to a decrease in the overall demand, whereas a lowering in the selling price will not result in a significant profit. So, based on the analysis, management could set the selling price accordingly, taking into account in line of demand, sale season, product availability, etc.
- The profit rises as market potential parameter is increased. This phenomenon works in favour of management. As a result, managers are encouraged to use tools like campaigns and advertisements to promote recently launched products to customers in order to increase the potential parameter (a), of the product in order to increase earnings.
- On decreasing the parameter 'b' (linear coefficient of price), the demand increases accordingly and hence profit increases. This means a product with a high value of the parameter 'b' beneficiary to management. Also, the suggestions nullify the negative impact associated with business that is responsible for its decline. So, management could take all essential measures to avoid the negative factors responsible in declining demand.
- According to the analysis, increasing promotional efforts (increasing value of ρ) increases demand and profits but lowers consumption period. This reveals that management intensified promotional efforts to raise overall demand, and as consumption decreased, total holding costs decreased.
- A higher rate of deterioration will have a negative impact on profits. Therefore, optimal preservation might minimise the rate of deterioration and in-

crease the optimal profit. Management needs to balance between preservation cost as per the findings.

- The preservation investment cost is the expense incurred by the retailer or shopkeeper to preserve the products. This cost for a particular product varies with the different sale seasons as well as the rate of deterioration. Analysis reveals that there exists a unique preservation cost for each inventory cycle, taking into account the sale season, deterioration rate, and total holding time at which overall profit is at its maximum. Thus, based on the analysis, the retailer could choose the optimal investment strategy for the preservation of products, taking into account the various sale seasons and the susceptibility of the products to deterioration.
- Examining the uncertainty factor associated with market demand demonstrates that fluctuating demand is an inherent characteristic of the business. However, the dealer must employ the short-term replenishment strategy, which reduces optimal holding time and proportional shortage time by employing the ratio. This pricing method provides the flexibility to alter pricing strategies more frequently in accordance with the uncertainty factor.
- Our world is full of cultural and economic diversity. This demographic variety influences all businesses. Those enterprises that depend on a single pricing approach will experience higher uncertainty. Businesses operating in a variety of geographies should employ a dynamic pricing approach. In this approach, companies may implement a multi-outlet and dynamic pricing strategy rather than a single-outlet and static pricing strategy. It will allow the retailer to employ diverse strategies based on geographic location to deal with market uncertainties.
- The stochastic factor of the proposed model may be responsible for the demand fluctuations. The market clearly exhibited these fluctuations in demand throughout the sale season. The fluctuation in demand could be the reason for the fluctuation in profit. Consequently, the analysis reveals that incorporating the stochastic factor into model formulation is significantly more realistic than the deterministic approach.

According to the findings of the analysis, inventory managers should promote the products as much as possible with promotional effort and use product preservation to keep deterioration low with higher profitability. Also, management could take measures to reduce these fixed costs as much as they can to generate more profit.

For further research, this model can be extended by considering other factors such as including lead time by including delayed shipment, disruption risk, defective rate, varying holding cost with time, taking multi-items rather than single items, etc. An objective function model could be designed in a fuzzy environment.

Acknowledgements. Authors are thankful to reviewers and Area Editor for their constructive comments and suggestions which improved the quality of the manuscript.

Funding. This research received no external funding.

REFERENCES

[1] R. Sinha, V. Ganatra, P. Pandey, D. M. H. Kee, G. S. Lekha, N. Kumar, S. D. Kew, A. Dhupad, R. K. Nair, Y. Y. Tenh, and H. Yue, "Impact of Covid-19 on business performance: A case study of Starbucks", *International Journal of Tourism and Hospitality in Asia Pasific*, vol. 4, no. 2, pp. 13-27, 2021.

[2] T. Team, "Quality Vs. Quick Service: The Difference Between Starbucks and McDonald's", *Forbs*, 2020.

[3] D. Shukla, and U. K. Khedlekar, "An order level inventory model with three-component demand rate (TCDR) for newly launched deteriorating item", *International Journal of Operations Research*, vol. 7, no. 2, pp. 61-70, 2010.

[4] N. H. Shah, D. G. Patel, and D. B. Shah, "EPQ model for imperfect production processes with rework and random preventive machine time for deteriorating items and trended demand", *Yugoslav Journal of Operations Research*, vol. 25, no. 3, pp. 425-443, 2015. doi: https://doi.org/10.2298/YJOR130608019S

[5] A. Sharma, C. Singh, P. Verma, and A. K. Malik, "Flexible Inventory System of Imperfect Production Under Deterioration and Inflation", *Yugoslav Journal of Operations Research*, vol. 32, no. 4, pp. 515-528, 2022. doi: http://dx.doi.org/10.2298/YJOR220318025S

[6] U. K. Khedlekar, D. Shukla, and A. Namdeo, "Pricing policy for declining demand using item preservation technology", *SpringerPlus*, vol. 5, no. 1, pp. 1-11, 2016. doi: https://doi.org/10.1186/s40064-016-3627-x

[7] T. Roy, and K. S. Chaudhury, "An EPLS model for a variable production rate with stock-price sensitive demand and deterioration", *Yugoslav Journal of Operations Research*, vol. 22, no. 1, pp. 19-30, 2012. doi: 10.2298/YJOR080709003R

[8] N. H. Shah, U. Chaudhari, and M. Jani, "Inventory control policies for substitutable deteriorating items under quadratic demand", *Operations and Supply Chain Management: An International Journal*, vol. 12, no. 1, pp. 42-48, 2019. doi: http://doi.org/10.31387/oscm0360221

[9] U. K. Khedlekar, "Logarithmic inventory model with shortage for deteriorating items", *Yugoslav Journal of Operations Research*, vol. 23. no. 3, pp. 431-440, 2016. doi: 10.2298/YJOR120925005K

[10] N. H. Shah, K. Rabari, and E. Patel, "Inventory and Preservation Investment for Deteriorating Inventory System with Stock-dependent Demand and Partial Backlogged Shortages", *Yugoslav Journal of Operations Research*, vol. 31, no. 2, pp. 181-192, 2020. doi: https://doi.org/10.2298/YJOR200217038S

[11] N. H. Shah, and M. K. Naik, "Imperfect inventory model for trended demand under radio frequency identification and trade credit", *Yugoslav Journal of Operations Research*, vol. 30, no. 4, pp. 499-523, 2020. doi: https://doi.org/10.2298/YJOR190315021S

[12] H. Barman, M. Pervin, and S. K. Roy, "Impacts of green and preservation technology investments on a sustainable EPQ model during COVID-19 pandemic", *RAIRO-Operations Research*, vol. 56, no. 4, pp. 2245-2275, 2022. doi: https://doi.org/10.1051/ro/2022102

[13] A. S. Saikh, P. P. Mishra, and I. Talati, "Lot sizing and pricing decision for items following price-sensitive quadratic demand under various payment schemes", *Yugoslav Journal of Operations Research*, vol. 33, no. 1, pp. 41–58, 2022. doi: https://doi.org/10.2298/YJOR210715010S

[14] U. K. Khedlekar, "A disruption production model with exponential demand", *International Journal of Industrial Engineering Computations*, vol. 3, no. 4, pp. 607-616, 2012. doi: http://dx.doi.org/10.5267/j.ijiec.2012.03.006

[15] N. H. Shah, and C. R. Vaghela, "Retailer's optimal pricing and replenishment policy for new product and optimal take-back quantity of used product", *Yugoslav Journal of Operations Research*, vol. 28, no. 3, pp. 345-353, 2018. doi: https://doi.org/10.2298/YJOR171120017S

[16] U. K. Khedlekar, and D. Shukla, "Dynamic pricing model with logarithmic demand", *Opsearch*, vol. 50, no. 1, pp. 1-13, 2013. doi: https://doi.org/10.1007/s12597-012-0093-2

[17] N. H. Shah, and P. Mishra, "An EOQ model for deteriorating items under supplier credits when demand is stock dependent", *Yugoslav Journal of Operations Research*, vol. 20, no. 1, pp. 145-156, 2010. doi: https://doi.org/10.2298/YJOR1001145S

[18] N. H. Shah, and M. Y. Jani, "Optimal Ordering for Deteriorating Items of Fixed-Life with Quadratic Demand and Two-Level Trade Credit: Optimal Ordering... Two-Level Trade Credits", *In Optimal Inventory Control and Management Techniques*, *IGI Global*, pp. 1-16, 2016.

[19] B. J. Shah, and A. Shroff, "Inventory model for sustainable operations of fixed-life products: Role of trapezoidal demand and two-level trade credit financing", *Journal of Cleaner Production*, vol. 380, no. 2, ID 135093, 2022. doi: https://doi.org/10.1016/j.jclepro.2022.135093

[20] A. Nigwal, U. K. Khedlekar, L. Sharma, and N. Gupta, "Trade Credit Policies for Supplier, Manufacturer, and Retailer: An Imperfect Production-Inventory System with Rework", *Journal of Mathematical & Fundamental Sciences*, vol. 54, no. 1, pp. 76-108, 2022.

[21] M. Y. Jani, N. H. Shah, and U. Chaudhari, "Inventory control policies for time-dependent deteriorating item with variable demand and two-level order linked trade credit", *In Optimization and Inventory Management*, pp. 55-67, 2022. https://doi.org/10.1007/$978-981-13-9698-4\_3$

[22] A. Paul, M. Pervin, S. K. Roy, G. W. Weber, and A. Mirzazadeh, "Effect of price-sensitive demand and default risk on optimal credit period and cycle time for a deteriorating inventory model", *RAIRO-Operations Research*, vol. 55, pp. S2575-S2592, 2021. doi: https://doi.org/10.1051/ro/2020108

[23] U. K. Khedlekar, A. Namdeo, and A. Nigwal, "Production inventory model with disruption considering shortage and time proportional demand", *Yugoslav Journal of Operations Research*, vol. 28, no. 1, pp. 123-139, 2018. doi: https://doi.org/10.2298/YJOR161118008K

[24] M. Pervin, S. K. Roy, and G M. Weber, "An integrated vendor-buyer model with quadratic demand under inspection policy and preservation technology", *Hacettepe Journal of Mathematics and Statistics*, vol. 49, no. 3, pp. 1168-1189, 2020. doi: https://doi.org/10.15672/hujms.476056

[25] S. S. Ali, H. Barman, R. Kaur, H. Tomaskova, and S. K. Roy, "Multi-Product Multi-Echelon Measurements of Perishable Supply Chain: Fuzzy Non-Linear Programming Approach", *Mathematics*, vol. 9, no. 17, ID 2093, 2021. https://doi.org/10.3390/math9172093

[26] N. H. Shah, P. H. Shah, and M. B. Patel, "Supply chain coordination with flexible payment policy under effect of green technology investments", *Yugoslav Journal of Operations Research*, 2022. [Online], doi: http://dx.doi.org/10.2298/YJOR220616029S

[27] M. Kumar, and A. K. Malik, "Lyapunov Exponent Using Euler's Algorithm with Applications in Optimization Problems", *Yugoslav Journal of Operations Research*, vol. 32, no. 4, pp. 503-514, 2022. doi: http://dx.doi.org/10.2298/YJOR220615024A

[28] A. Paul, M. Pervin, R. V. Pinto, S. K. Roy, N. Maculan, and G. W. Weber, "Effects of multiple prepayments and green investment on an EPQ model", *Journal of Industrial and Management Optimization*, 2022. [Online], doi: https://doi.org/10.3934/jimo.2022234

[29] A. Paul, M. Pervin, S. K. Roy, N. Maculan, and G. W. Weber, "A green inventory model with the effect of carbon taxation", *Annals of Operations Research*, vol. 309, no. 1, pp. 233-248, 2022. doi: https://doi.org/10.1007/s10479-021-04143-8

[30] L. E. Cardenas-Barron, and S. S. Sana, "Multi-item EOQ inventory model in a two-layer supply chain while demand varies with promotional effort", *Applied Mathematical Modelling*, vol. 39, no. 21, pp. 6725-6737, 2015. doi: https://doi.org/10.1016/j.apm.2015.02.004

[31] S. R. Rajan, and R. Uthayakumar, "Analysis and optimization of an EOQ inventory model with promotional efforts and back ordering under delay in payments", *Journal of Management Analytics*, vol. 4, no. 2, pp. 159-181, 2017. doi: https://doi.org/10.1080/23270012.2017.1280423

[32] H. N. Soni, and A. D. Chauhan, "Joint pricing, inventory, and preservation decisions for deteriorating items with stochastic demand and promotional efforts", *Journal of Industrial Engineering International*, vol. 14, no. 4, pp. 831-843, 2018. doi: https://doi.org/10.1007/s40092-018-0265-7

[33] Y. C. Tsao, and G. J. Sheen, "Quality Vs. Dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payments",

Computers & Operations Research, vol. 35, no. 11, pp. 3562-3580, 2008. doi: https://doi.org/10.1016/j.cor.2007.01.024

[34] H. N. Soni, and K. Shah, "Continuous Review Inventory Model with Lost Sale Reduction and Ordering Cost Dependent on Lead Time for the Mixtures of Distributions", *Yugoslav Journal of Operations Research*, vol. 31, no. 2, pp. 221-233, 2020. doi: https://doi.org/10.2298/YJOR200418006S

[35] U. K. Khedlekar, and R. K. Tiwari, "Decision makings in discount pricing policy for imperfect production system", *Yugoslav Journal of Operations Research*, vol. 29, no. 2, pp. 273-293, 2018. doi: https://doi.org/10.2298/YJOR180607029K

[36] H. Barman, M. Pervin, S. K. Roy, and G. W. Weber, "Back-ordered inventory model with inflation in a cloudy-fuzzy environment", *Journal of Industrial & Management Optimization*, vol. 17, no. 4, ID 1913, 2021. doi: https://doi.org/10.3934/jimo.2020052

[37] S. Fatma, V. K. Mishra, and R. Singh, "Inventory Model for Instantaneous Deteriorating Items With Time Sensitive Demand for Post COVID-19 Recovery", *Yugoslav Journal of Operations Research*, 2022. [Online], doi: http://dx.doi.org/10.2298/YJOR220915039F

[38] P. Poswal, A. Chauhan, Y. K. Rajoria, R. Boadh, and A. Goel, "Fuzzy optimization model of two parameter weibull deteriorating rate with quadratic demand and variable holding cost under allowable shortages", *Yugoslav Journal of Operations Research*, vol. 32, no. 4, pp. 453-470, 2022. doi: https://doi.org/10.2298/YJOR220115021P

[39] V. Kuppulakshmi, C. Sugapriya, J. Kavikumar, and D. Nagarajan, "Fuzzy Inventory Model for Imperfect Items with Price Discount and Penalty Maintenance Cost", *Mathematical Problems in Engineering*, vol. 2023, ID 1246257, 2023. doi: https://doi.org/10.1155/2023/1246257

[40] M. Y. Jani, M. R. Betheja, U. Chaudhari, and B. Sarkar, "Effect of Future Price Increase for Products with Expiry Dates and Price-Sensitive Demand under Different Payment Policies", *Mathematics*, vol. 11, no. 2, ID 263, 2023. doi: https://doi.org/10.3390/math11020263

[41] M. Akhtar, A. K. Manna, and A. K. Bhunia, "Optimization of a non-instantaneous deteriorating inventory problem with time and price dependent demand over finite time horizon via hybrid DESGO algorithm", *Expert Systems with Applications*, vol. 211, ID 118676, 2023. doi: https://doi.org/10.3390/math11020263