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MULTI-OBJECTIVE MATHEMATICAL MODELS TO RESOLVE PARALLEL MACHINE SCHEDULING PROBLEMS WITH MULTIPLE RESOURCES

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Abstract: Mathematical programming, and above all, the multi-objective scheduling problems stand as remarkably versatile tools, highly useful for optimizing the health care services. In this context, the present work is designed to put forward two-fold multi-objective mixed integer linear programs, simultaneously integrating the objectives of minimizing the patients’ total waiting and flow time, while minimizing the doctors' work-load variations. For this purpose, the three major health-care system intervening actors are simultaneously considered, namely, the patients, doctors and machines. To the best of our knowledge, such an issue does not seem to be actually addressed in the relevant literature. To this end, we opt for implementing an appropriate lexicographic method, whereby, effective solutions enabling to minimize the performance of two-objective functions could be used to solve randomly generated small cases. Mathematical models of our study have been resolved using the CPLEX software. Then, results have been comparatively assessed in terms of both objectives and CPU times. A real laser-treatment case study, involving a set of diabetic retinopathy patients in the ophthalmology department in Habib Bourguiba Hospital in Sfax, Tunisia, helps in illustrating the effective practicality of our advanced approach. To resolve the treated problem, we use three relevant heuristics which have been compared to the first-come first-served rule. We find that the program based on our second formulation with time-limit provided the best solution in terms of total flow time.

Keywords: Minimizing waiting time and flow time, doctors' workloads, multi-objective mixed integer linear programs, lexicographic solutions.

MSC: 90C29.

1. INTRODUCTION

Diabetic Mellitus (DM) is defined as a state of chronic hyperglycemia that follows an abnormal secretion of insulin, insulin action, or both anomalies simultaneously. Basically, this chronic disease is classified into two main categories, namely, Type I and Type II diabetes [1]. It is ranked as a severe disease, frequently predominant in the developed as well as developing countries, alike. Future prevalence estimates indicate well that diabetes is a worldwide spread disease [2]. The early detection and treatment of diabetes is mandatory, and with the existence of resource constraints in developing countries, more undiagnosed than diagnosed cases tend to persist. For this reason, this disease is followed by serious complications such us diabetic nephropathy which causes renal disease, the macro diabetic angiopathy which is a major etiology of cardiovascular diseases including gangrene and amputations lower limbs, and Diabetic Retinopathy (DR) which leads to blindness.

As a manifestation of DM complication, DR is most often a symptomatic in its early stages [3], usually affecting both eyes. If someone has DR, he might not initially witness noticeable changes in his vision. As time goes by, however, DR could seriously aggravate and bring about vision loss [4]. Consequently, care of DR affected patients stands as a highly necessary procedure for the development of the disease to be effectively restricted [3], and owing to the critical situation marking most of the DR affected patients, it is mandatory to provide them with highly efficient health care services. It is for this purpose that [5] developed an automatic computer system to maintain an effective planning of coagulation for diabetic retinopathy affected patients. As to [6], they proposed a special mathematical model enabling to determine DR screening recommendations, whose cost-effectiveness was assessed by means of a Markov-chain Monte-Carlo simulation.

Operations research is a powerful discipline enabling managers to reach the most optimal decisions regarding several application areas, particularly the healthcare domain. Indeed, it involves a wide range of problem-solving methods and frameworks designed to improve the quality of patient delivered care services. Worth citing, in this respect, is the scheduling theory, which stands as a major research area focused on retrieving the most appropriate sequences fit for optimizing relevant criteria [7]. Several models and approaches were proposed to solve the scheduling related problems, worth citing among which are the simulation techniques [8], heuristics [7, 9-10], simulation optimization algorithms [11-13], along with mathematical programming [14-16].

With respect to the health care domain, the scheduling theory has been widely applied in several known research studies, mainly, those conducted by [8-9, 12, 17-22]. In each context, appropriate algorithms and heuristics have been selected to solve the problem addressed on the basis of its complexity, the number of machines used, the scheduling system adopted and the static or dynamic nature of patient arrival.

The problem, of scheduling *n* patients on *m* machines, is considered *NP*-hard because it is a variant of the job shop problem, which has been proven to be *NP*-hard [23]. In addition, the goal of minimizing the flow time in scheduling problem of *n* tasks on one machine and with $r_j \neq 0$, (r_j is the 'release date' or 'ready time' which corresponds in our case to the date of availability for treatment of patient *i*), is an *NP*-hard problem [24]. Our problem is more complex, since its solution requires scheduling of patients not only to machines, but also to doctors.

In this paper, we present two multi-objective mathematical models. In the first model, doctors can work in only one machine while in the second model they can switch between machines. In each model, two objectives are considered in order to improve patients' satisfaction and balanced doctors' utilization and workload: The first objective function is minimizing the flow time of all the patients treated with laser machines in the ophthalmology department, thereby reducing their overall length of stay. The second objective gives an optimal schedule that levels the workload between doctors in laser photocoagulation room during the day. Lexicographic solutions are obtained for randomly generated small instances. For medium sized real case, three heuristics are compared to the existent scheduling method.

The paper is organized as follows. Section 2 involves a review of relevant literature, while section 3 is devoted to highlighting the problem addressed. Our mathematical model designs are detailed in section 4. Then, the proposed models are illustrated and compared in regard to small numerical examples in section 5. As to section 6, a comparison is established between our achieved results and a real case situation. Finally, the last section provides drawn conclusions, along with perspectives for potential research lines.

2. LITERATURE REVIEW

It is worth recalling that a parallel machine scheduling problem consists in a set of tasks to be processed via several identical machines over a given time period. Each task is exclusively assigned to a single machine, and each machine can process just a unique task at a time. Hence, a feasible schedule should highlight and determine the sequence of tasks relevant to each machine. In this respect, parallel machine-scheduling problem is designed to find an appropriate schedule fit for optimizing one or more objective functions [25]. The parallel machine scheduling problem plays an important role in decision making in a wide range of production manufacturing and industries [26]. The parallel machine scheduling problem is used by several practitioners in many fields [27]. It is applied in social science [28, 29], computer science [30] and health care systems [9]. In parallel machine scheduling problems, the objective functions are generally related to the completion times or the due dates of different jobs. We can cite for example the total flow time [31-33], the makespan [27, 34], the maximum tardiness [35, 36]. In [37], the authors addressed the problem of bounded single machine scheduling with release dates and rejections. Their objective was to minimize the sum of the makespan of the accepted jobs and the total penalty of rejected jobs. They developed a polynomial time algorithm for solving the problem. In [38], the authors examined the single machine scheduling problem with earliness and tardiness penalties. To resolve the problem, they used an algorithm in order to obtain the upper bound of the problem which was efficiently integrated with the branch and bound search algorithm. With respect to [39], they developed a two-phase non-linear integer programming formulation for scheduling *n* jobs on two identical parallel machines with the objective of minimizing weighted total flow time. To this end, an optimization algorithm was constructed to deal with small problems, and a heuristic to resolve large problems, in order to find optimal or quasi-optimal solutions. As for [40], they investigated unrelated parallel machine scheduling with renewable constraints. Initially, they proposed an efficient mixed-integer linear programming model to solve the two-machine related problem. Then, for problem with more than two machines, they suggested implementing a two-stage heuristic. Similarly, [41] studied an order acceptance and scheduling problem regarding unrelated parallel machines enabling to maximize the total net revenue of accepted orders. For this purpose, they formulated two mixed-integer programming models, then developed enhancement techniques to improve performance of the proposed models. In a final stage, they developed a branch-and-bound algorithm to handle complex instances. Concerning [42], they investigated the unrelated-parallel machines problem with machine eligibility and sequence-dependent setup times. In order to minimize total tardiness, they put forward a mixed integer linear programming model. They managed to solve small instances enabling to assess the performance of two already-existent heuristics' adapted versions along with two newly suggested ones.

Regarding [43], they attempted to solve the problem of scheduling n jobs on m identical parallel machines in order to minimize total tardiness, by proposing a special branch and bound algorithm. As to [44], they studied a problem with unrestricted idle time by means of least-process-time as well as adjusted-short-process-time algorithms fit for treating large problem instances. Worth citing, in this respect, also, are the works of [45], that developed a polynomial lower bound scheme with job fragmentation or relaxation of release date constraints, and [46] that provided special set-up time constraints heuristics. Similarly, [47] developed a linear programming approach with due-time constraints relaxation by considering identical parallel machine scheduling problem with release due-date and equal-processing time constraints to be resolved via a polynomial algorithm. With respect to [25], and on investigating a distributional robust scheduling problem on identical parallel machines, they minimized the worst-case expected total flow time, and optimized the inner maximization sub-problem, to reduce their min-max formulation into an integer second-order cone program. They highlighted their algorithm high efficiency through demonstrating its ability to optimize instances involving a remarkably high number of jobs within a few seconds. In turn, [48] studied the parallel machine scheduling problem with time constraints on machine qualifications. Their objective was to minimize the flow time and the number of disqualified job families on machines, while [49] developed a mathematical model allowing to resolve the identical parallel machine scheduling problem with the aim of minimizing total flow time by means of a special heuristic algorithm.

On studying the problem of parallel identical machines scheduling, [50] aimed to minimize the makespan by devising a special O(nlogn) algorithm. To test their achieved results, they compare them to those reached via other state-of-the-art algorithms available in the relevant literature. The makespan minimization objective was also considered by [51], through parallel-uniform machines problem with a single preemption. In their work, they highlighted the difference between two distinct cases: the case of two uniform machines, solvable in polynomial time, and the case where the number of machines exceeds two, for which they established a global tight bound. As regards to [52], they considered treating a special case of job sequencing problem with tool requirements. Their objective function was specifically designed to minimize the makespan. Similarly, [53] proposed an identical parallel-machine scheduling problem that serves to minimize the jobs' completion time. To determine the optimum schedule, they considered adopting a heuristic and proposing a new lower bound, and subsequently, a branch and bound algorithm. On treating the problem of uniform parallel machines scheduling, In [54], the authors considered minimizing the total weighted completion time. To solve this problem, they made appeal to two hybrid meta-heuristics, and tested their methods on a large set of instances. As for [55], who studied the parallel machine scheduling problem, they set as an objective function the minimization of completion time. To this end, they made use of an ejection chain algorithm that yielded more effective results than those proposed in the literature. Worth recalling in this respect, also, is the [56] advanced architecture, designed to incorporate a backward scheduling framework, along with a backward algorithm.

Actually, multi-objective programming still remains a major research area that continues to draw the attention of researchers and academics, alike, owing mainly to the versatile real-world applications they provide. For instance, [57] devised a mixed integer-linear programming model useful for implementation in the pharmaceutical industry. They made use of a multi-criteria decision analysis for supplier selection purposes. In turn, and on studying the interval multi-objective linear programming models, [58] put forward expected Value and variance operators coupled with a Monte-Carlo simulation to reach an efficient solution to their problem. In effect, as an effective measure fit for coping with non-linear programming related problems, multi-objective optimization has been demonstrated and widely recognized to provide efficient solutions. In this regard, [59] suggested appealing to sequential optimality conditions as appropriate options for coping with non-linear multi-objective optimization problems.

In general, workload balancing was most often addressed as part of the parallel-machine scheduling problems. In this regard, [60] set up a linear mixed integer program to help minimize the machines related workload discrepancies, implemented to resolve a number of persistent problems prevailing in the literature. In turn, [61] considered a special modeling design serving to analyze transshipment collaboration of multiple couriers with flexible time periods, in order to minimize both of the workload and waiting time conflicting objectives. As for [62], they investigated the non-identical parallel machines problem via a special objective function designed to minimize the manufacturing systems' various products workload, through investigating an iterated min/max procedure.

In effect, the workload minimization objective function stands as an important undertaking in the process of solving health care problems, and remains subject of interest for several authors, scholars and practitioners. Worth citing in this respect, are the works elaborated by [63-68]. In this regard, also, [69] proposed an integer linear program useful for assigning patients to nurses, while maintaining the objective of balancing the nurses' workloads. To this end, they adopted a nurse-zone based heuristic. As to [70], they devised a new Chemotherapy treatment targeted linear program, modeled to schedule the physicians' working period. This program has been used for balancing their workload, and accounting for the patients' treatment protocols, beds' capacity and physicians' planning related constraints. With regard to [71], they addressed the patients' scheduling problem, aiming at minimizing a Cancer Clinic nurses' workload using a mathematical model. Concerning [72], they set up a stochastic mixed integer linear programming model to optimize the staff allocation problem in respect of multiple radiotherapy operations following a set of patients in flow scenarios, under uncertain demand. A real case study was considered in the radiotherapy ward of the Netherlands Cancer Institute. Similarly, [73] initiated a number of physicians' workload minimizing methods relevant to the capture and analysis of electronic health records during outpatient treatment. In this respect, also, [74] conducted a whole year follow-up study focused on 72 primary home-care health teams operating in Catalonia of patients aged over 64 years. Their objective was to identify the chronic patients' characteristics and living environment to predict and determine the nursing workload required one year following their inclusion in a home care program.

3. PROBLEM DESCRIPTION

In a previously conducted work, [75] applied discrete event simulation to study three specific models for diabetic patients with Retinopathy signs. They computed the total time for a DR affected patient to develop blindness via their ARENA software incorporated models. The average time was discovered to be estimated at around thirteen years in case of no treatment, around twenty-three years when a vitrectomy treatment is implemented, and around 46 years when the patient is treated with Laser photocoagulation. They, then, concluded that Laser treatment turns out to be the most effective in terms of blindness development extended time for Retinopathy affected patients.

Hence, given the importance of Laser treatment for DR affected patients, and the high expenses necessary to get treated in the private sector, we undertake to develop two multi-objective mathematical models, designed to maintain an effective scheduling of the laser-treatment requiring patients in the ophthalmology department in Habib Bourguiba hospital of Sfax in Tunisia. Despite the great deal of research dealing with the health-care sector dedicated scheduling theory, almost no research work has been discovered to deal simultaneously with scheduling the doctors' staff and machine resources, useful for treating these patients, while considering to minimize the patients' treatment stay span and the doctors' workload variations. Our choice of the doctor-workload variations' objective is owed to the fact that the number of doctors exceeds the number of laser machines in our real study case.

4. MODEL FORMULATIONS

At hospital, and particularly in the Laser treatment room, patients are usually serviced on a First Come First Served (FCFS) basis. Yet, this method does not necessarily help minimize the patients' total stay time. Once the patients' arrivals order is ignored, the total cumulative stay time might be reduced for all patients, yielding longer waiting time for some patients, having long processing time [76]. Hence, by introducing a maximum limit on the patients' flow time, long waiting time spans could be noticeably reduced.

In our models, two main objectives are independently considered. While the first objective is aimed to minimize the Total Flow Time for all patients (TFT), the second is targeted to equitably distributing the total Workload among doctors (WL).

4.1. Formulation of the first mathematical model

It is worth noting that the patients' arrival for treatment is synonymous with the case of accepting n different tasks to be served on m parallel machines, whose processing would be executed via different machines. In this context, we assume that each task can be processed on one single machine, and that the task interruption is not allowed [76]. In our study case, tasks denote patients, while machines denote the Laser machines. If, for instance, a patient started his/her treatment via a particular machine, he/she will pursue treatment via the same machine until completion.

Our first bi-objective patients' scheduling model is quite similar to the birth-allocation problem mathematical model used by [77], but includes doctors as supplement resources, an extra objective function and extra decision variables.

Actually, our model maintains the following assumptions:

- Every machine is able to process only one patient at a time.
- Every patient can be assigned to only one machine.
- Each doctor is to be assigned no more than a single machine at a time.
- Each doctor can treat only a single patient at a time.
- The processing time of patient *i* remains unchanged for all machines.
- The processing time of patient *i* depends on the disease severity level.
- If patient *i* is assigned to doctor *l*, he/she will continue to be treated by him/her till the end of the treatment process.
- The planning process is considered as either dynamic (i.e., all patients might arrive at different times) or static (i.e., all patients would arrive at the same timing: time zero).

Notations

i: the patients related index, $i = 1,..., I$, $i \in P$.

j: the available machines relevant index, $j = 1,..., J$, $j \in M$.

l: the available doctors associated index, $l = 1,..., L$, $l \in D$.

k: the patients service order relevant index, $k = 1,..., I$, $k \in P$.

n: the index of patients order for each doctor, $n = 1..., I$, $n \in P$.

Sets and parameters

- D: the set of available doctors.
- P: the patients' set.
- M: the set of available machines.
- P_i: patient *i* processing time.
- R_i: patient *i* release date (ready time).
- S_l: doctor *l* availability date.
- N: a large positive constant.

Decision variables

$$X_{ijk} = \begin{cases} 1 \text{ if patient i is assigned to machine } j \text{ in order } k \\ 0 \text{ otherwise} \end{cases}$$

$$Y_{ijln} = \begin{cases} 1 \text{ if patient } i \text{ is assigned to doctor } l \text{ on machine } j \text{ in order } n \\ 0 \text{ otherwise} \end{cases}$$

F_{ijk}: flow time of patient *i* assigned to machine *j* in order *k*.

C_{ijk}: 'Completion time', which corresponds to patient *i* treatment completion date on machine *j* in order *k*, according to the formula: $C_{ijk} = F_{ijk} + (R_i * X_{ijk})$.

$Z_{ijk} = F_{ijk} * X_{ijk}$.

4.1.1. Mathematical model

The analytical form corresponding to the first model is shown through Program 1.

Program 1:

$$\text{TFT} = \text{Min} \sum_{i\in P}\sum_{j\in M}\sum_{k\in O} Z_{ijk}$$

$$\text{WL} = \text{Min} \sum_{l\in D}\left|\sum_{i\in P}\sum_{j\in M}\sum_{n\in N} Y_{ijln} P_i - \frac{\sum_{i\in P} P_i}{card(D)}\right|$$

Subject to the following constraints:

$$\sum_{l\in D}\sum_{n\in P} Y_{ijln} - \sum_{(k\in P)\neq 0} X_{ijk} = 0, \quad \forall i \in P, j \in M \tag{1}$$

$$\sum_{i\in P}\sum_{j\in M} Y_{ijln} \leq 1, \quad \forall l \in D, n \in P \tag{2}$$

$$\sum_{i\in P}\sum_{j\in M} Y_{ijln} \leq 1, \quad \forall j \in M, k \in P \tag{3}$$

$$\sum_{j\in M}\sum_{k\in P} X_{ijk} = 1, \quad \forall i \in P \tag{4}$$

$$\sum_{j\in M}\sum_{l\in D}\sum_{j\in M} Y_{ijln} = 1, \quad \forall i \in P \tag{5}$$

$$F_{ijk} \geq C_{tj(k-1)} - (r_i \times X_{ijk}) + (P_i \times X_{ijk}),$$
$$\forall i \in P, k \in P \text{ and } k \neq 0, t \in P \text{ and } t \neq i, j \in M \tag{6}$$

$$F_{ijk} \geq (P_i \times X_{ijk}), \quad \forall i \in P, j \in M, k \in P \tag{7}$$

$$F_{ijk} = C_{ijk} - (r_i \times X_{ijk}), \quad \forall i \in P, j \in M, k \in P \tag{8}$$

$$C_{tj0} = \sum_{i\in P}\sum_{l\in D}(Y_{ijl1} \times S_l), \quad \forall t \in P, j \in M \tag{9}$$

$$F_{ijk} + N(1 - X_{ijk}) \geq Z_{ijk}, \quad \forall i \in P, j \in M, k \in P \tag{10}$$

$$N(1 - X_{ijk}) + Z_{ijk} \geq F_{ijk}, \quad \forall i \in P, j \in M, k \in P \tag{11}$$

$$N(X_{ijk}) \geq Z_{ijk}, \quad \forall i \in P, j \in M, k \in P \tag{12}$$

$$\sum_{i'\in P}\sum_{n'\in P}\sum_{(j'\in M)\neq j} Y_{i'j'ln'} \leq N(1 - Y_{ijln}), \quad \forall i \in P, j \in M, l \in D, n \in P \tag{13}$$

$$N(1 - w2_l) + \sum_{i\in P}\sum_{j\in M}\sum_{n\in P} Y_{ijln} P_i - \frac{\sum_{i\in P} P_i}{card(D)} \geq 0, \quad \forall l \in D \tag{14}$$

$$w1_l + N(1 - w2_l) \geq \sum_{i\in P}\sum_{j\in M}\sum_{n\in P} Y_{ijln} P_i - \frac{\sum_{i\in P} P_i}{card(D)}, \quad \forall l \in D \tag{15}$$

$$w1_l \leq \sum_{i\in P}\sum_{j\in M}\sum_{n\in P} Y_{ijln} P_i - \frac{\sum_{i\in P} P_i}{card(D)} + N(1 - w2_l), \quad \forall l \in D \tag{16}$$

$$N\, w2_l - \left(\sum_{i\in P}\sum_{j\in M}\sum_{n\in P} Y_{ijln} P_i - \frac{\sum_{i\in P} P_i}{card(D)}\right) \geq 0, \quad \forall l \in D \tag{17}$$

$$w1_l + N\, w2_l \geq -\left(\sum_{i\in P}\sum_{j\in M}\sum_{n\in P} Y_{ijln} P_i - \frac{\sum_{i\in P} P_i}{card(D)}\right), \quad \forall l \in D \tag{18}$$

$$w1_l \leq -\left(\sum_{i\in P}\sum_{j\in M}\sum_{n\in P} Y_{ijln} P_i - \frac{\sum_{i\in P} P_i}{card(D)}\right) + N\, w2_l, \quad \forall l \in D \tag{19}$$

$X_{ijk} \in \{0,1\}$, $Y_{ijln} \in \{0,1\}$, $Z_{ijk} \geq 0$, N: a large constant, $w2_l \in \{0,1\}$, $w1_l \geq 0$, $\forall\, i, k, n \in P, j \in M, l \in D$ (20)

The constraints' respective descriptions are as follows:
Constraints (1) guarantee that if $X_{ijk} = 0$, then $Y_{ijln} = 0$, and if $X_{ijk} = 1$, $Y_{ijln} = 1$. Constraints (2) guarantee that each doctor is to be simultaneously assigned just a single machine and a single patient at a time. Constraints (3) guarantee that each machine is to be used by at most one patient at a time. Constraints (4) guarantee that all patients will be served by one machine in a given service order. Constraints (5) guarantee that all patients will be served by one doctor in a given service order. Constraints (6) give the flow time value of patient i on machine j according to the order k, where $C_{tj(k\text{-}1)}$ is greater than r_i. Constraints (7) show the flow time value of the patient i on machine j according to the order k, when r_i is greater than $C_{tj(k\text{-}1)}$. Constraints (8) highlight the relation between flow time and completion time. Constraints (9) show the initial value of the completion time C_{jt0}, which is equal to S_l. Constraints (10) and (11) ensure that if $X_{ijk} = 1$ then $Z_{ijk} = F_{ijk}$. Constraints (12) guarantee that if $X_{ijk} = 0$ then $Z_{ijk} = 0$. Constraints (13) guarantee that each doctor works only on one machine. Constraints (14), (15) and (16) guarantee that if $\sum_{i\in P}\sum_{j\in M}\sum_{n\in N} Y_{ijln} P_i - \frac{\sum_{i\in P} P_i}{card(D)} \geq 0$ then $w1_l = \sum_{i\in P}\sum_{j\in M}\sum_{n\in N} Y_{ijln} P_i - \frac{\sum_{i\in P} P_i}{card(D)}$. Constraints (17), (18) and (19) guarantee that if $-(\sum_{i\in P}\sum_{j\in M}\sum_{n\in N} Y_{ijln} P_i - \frac{\sum_{i\in P} P_i}{card(D)}) \geq 0$, then: $w1_l = -(\sum_{i\in P}\sum_{j\in M}\sum_{n\in N} Y_{ijln} P_i - \frac{\sum_{i\in P} P_i}{card(D)})$. Constraints (20) define the decision variables.

4.1.2. Lexicographic solution

This subsection is devoted to presenting our model's two suggested formulations. While the first formulation is designed to minimize the patients' total flow time (Program 2), the second is targeted to minimize the doctors' workloads variations (Program 3).

A. Total Flow time objective

Our first mathematical model is aimed to fulfill the Total Flow Time (TFT) minimization objective. The model's analytical formulation is presented as follows:

Program 2: TFT* = $\text{Min} \sum_{i\in P}\sum_{j\in M}\sum_{k\in O} Z_{ijk}$

Subject to constraints (3), (4), (6), (7), (8), (9), (13), (20).

Since $Z_{ijk} = (F_{ijk} \times X_{ijk})$ is non-linear, we consider introducing the constraints (10), (11) and (12) for linearization purpose.

B. The Doctor-workload variations objective

The workload need be distributed as uniformly as possible among doctors scheduled for the shift. The following mathematical model should serve to attain a sequence enabling to minimize the workload discrepancy among doctors in service. The optimal workload variation value, WL*, is then obtained in the following way:

Program 3: WL* = $\text{Min} \sum_{l\in D}\left|\sum_{i\in P}\sum_{j\in M}\sum_{n\in N} Y_{ijln} P_i - \frac{\sum_{i\in P} P_i}{card(D)}\right|$

Subject to the constraints (1), (2), (3), (4), (5), (13), (20).

Constraints (14), (15), (16), (17), (18) and (19) are used to maintain linearization of this objective function absolute value to resolve Program 3.

4.2. Notation and formulation of the second mathematical model

Our second bi-objective model, relevant to scheduling patients to treatment, is similar to [40] developed mixed-integer linear program, to help in scheduling jobs on unrelated parallel machines with a renewable resource constraint (UPMR). In their established model, [40] attempted achieving both of the TFT and make span minimization objectives. As to our constructed model, it is designed to account for both of the TFT and doctor-workload variations minimization objectives, as the makespan minimization objective is not subject of the present work. In the first formulation, adopted from [77] published work, a doctor is only allowed to work on one single machine, as determined through our real case study of Habib Bourguiba hospital. In our current formulation, however, doctors are allowed to work on different machines.

Notations

i: the patients' index, $i=1,\ldots, I$, $i \in P$.
j: the index of machines available, $j = 1,\ldots, J$, $j \in M$.
l: the index of doctors, available $l = 1,\ldots, L$, $l \in D$.

Sets and parameters

- D: the set of available doctors;
- P: the set of patients;
- M: the set of available machines;
- S_i: processing time of patient i;
- R_i: patient i availability date;
- A_l: doctor l availability date;
- B_j: machine j availability date;
- N: a large positive constant.

Decision variables

$$X_{ij} = \begin{cases} 1 \text{ if patient } i \text{ is assigned to machine } j \\ 0 \text{ otherwise} \end{cases}$$

$$Y_{il} = \begin{cases} 1 \text{ if patient } i \text{ is assigned to doctor } l \\ 0 \text{ otherwise} \end{cases}$$

Q_{ik}: auxiliary variable;
F_i: flow time of patient i;
$waiting_i$: the waiting time of patient i;
CS_i: 'Completion time', which corresponds to the treatment end date of patient i;
$Time_l = \sum_{i \in P}(Y_{il} \times \sum_{r \in M} X_{ir} \times S_i)$: the working time executed by each doctor.

4.2.1. Mathematical modeling

Our envisioned model involves the achievement of two independent objectives: on the one hand, it targets minimizing the total flowtime (TFT), while attempting, on the other hand, to equally distribute the total workload among doctors (WL). The model's analytical form is depicted through Program 4.

Program 4:

$$TFT = Min \sum_{i \in P} F_i$$

$$WL = Min \sum_{l \in D} \left| \sum_{i \in P} Y_{il} \times S_i - \left(\frac{\sum_{i \in P} S_i}{card(D)} \right) \right|$$

Subject to the following constraints:

$$\sum_{j \in M} X_{ij} = 1, \ \forall \, i \in P \tag{1}$$

$$\sum_{l \in D} Y_{il} = 1, \ \forall \, i \in P \tag{2}$$

$$CS_i - \sum_{j \in M} X_{ij} \times S_i \geq 0, \ \forall \, i \in P \tag{3}$$

$$CS_i - C_k - \sum_{r \in M} X_{ir} \times S_i + N \times (2 - X_{ij} - X_{kj} + Q_{ik}) \geq 0, \ \forall \, i \in P, k \in P, i < k, j \in M \tag{4}$$

$$CS_k - C_i - \sum_{r \in M} X_{kr} \times S_k + N \times (2 - X_{ij} - X_{kj} + 1 - Q_{ik}) \geq 0, \ \forall \, i \in P, k \in P, i < k, j \in M \tag{5}$$

$$CS_i - CS_k - \sum_{r \in M} X_{ir} \times S_i + N \times (2 - Y_{il} - Y_{kl} + Q_{ik}) \geq 0 \ \ \forall \, i \in P, k \in P, i < k, l \in D \tag{6}$$

$$CS_k - CS_i - \sum_{r \in M} X_{kr} \times S_k + N \times (2 - Y_{il} - Y_{kl} + 1 - Q_{ik}) \geq 0 \ \ \forall \, i \in P, k \in P, i < k, l \in D \tag{7}$$

$$waiting_i - CS_i + \sum_{r \in M} X_{ir} \times S_i + R_i \geq 0, \ \forall \, i \in P \tag{8}$$

$$F_i - CS_i + R_i \geq 0, \ \forall \, i \in P \tag{9}$$

$$cs_i - S_i - A_l + N \times (1 - Y_{il}) \geq 0, \ \forall \, i \in P, l \in D \tag{10}$$

$$cs_i - S_i - R_i + N \times (1 - X_{ij}) \geq 0, \ \forall \, i \in P, j \in M \tag{11}$$

$$cs_i - S_i - B_j + N \times (1 - X_{ij}) \geq 0, \ \forall \, i \in P, j \in M \tag{12}$$

$$N(1 - w2_l) + \sum_{i \in P} Y_{il} \times S_i - \left(\frac{\sum_{i \in P} S_i}{card(D)} \right) \geq 0, \ \forall \, l \in D \tag{13}$$

$$w1_l + N(1 - w2_l) \geq \sum_{i \in P} Y_{il} \times S_i - \left(\frac{\sum_{i \in P} S_i}{card(D)} \right), \ \forall \, l \in D \tag{14}$$

$$w1_l \leq \sum_{i \in P} Y_{il} \times S_i - \left(\frac{\sum_{i \in P} S_i}{card(D)} \right) + N(1 - w2_l), \ \forall \, l \in D \tag{15}$$

$$Nw2_l - \sum_{i \in P} Y_{il} \times S_i - \left(\frac{\sum_{i \in P} S_i}{card(D)} \right) \geq 0, \ \forall \, l \in D \tag{16}$$

$$w1_l + N\ w2_l \geq -\ \sum_{i\in P} Y_{il} \times S_i - \left(\frac{\sum_{i\in P} S_i}{card(D)}\right),\ \ \forall\ l \in D \quad (17)$$

$$w1_l \leq -(\sum_{i\in P} Y_{il} \times S_i - \left(\frac{\sum_{i\in P} S_i}{card(D)}\right)) + Nw2_l,\ \ \forall\ l \in D \quad (18)$$

$X_{ij} \in \{0,1\}$, $Y_{il} \in \{0,1\}$, N:a large positive constant, $w1_l \geq 0, w2_l \in \{0,1\}$

$$\forall i \in P, j \in M, l \in D \quad (19)$$

The constraints respective descriptions turn out to be:

Constraints (1) guarantee that each patient i is exclusively assigned to a single machine j; Constraints (2) guarantee that each patient i is exactly assigned to a single doctor l. Constraints (3) guarantee that the completion time of patient i is greater than its processing time. Constraints (4) and (5) guarantee that there is no overlapping among the set of patients assigned to the same machine. Constraints (6) and (7) guarantee that there is no overlapping among the patients assigned to the same doctor. Constraints (8) provide the value of waiting time of each patient i. Constraints (9) provide the value of each patient i flow time. Constraints (10) ensure that any patient's treatment starting time cannot by any means precede the doctor's availability. Constraints (11) guarantee that each patient's treatment starting time cannot precede the patient associated release date. Constraints (12) guarantee that any patient's starting treatment time cannot precede the machine's availability. Constraints (13), (14) and (15) guarantee that if $\sum_{i\in P} Y_{il} \times S_i - \left(\frac{\sum_{i\in P} S_i}{card(D)}\right) \geq 0$ then $w1_l = \sum_{i\in P} Y_{il} \times S_i - \left(\frac{\sum_{i\in P} S_i}{card(D)}\right)$. Constraints (16), (17) and (18) guarantee that if $-\left(\sum_{i\in P} Y_{il} \times S_i - \left(\frac{\sum_{i\in P} S_i}{card(D)}\right)\right) \geq 0$ then $w1_l = -\sum_{i\in P} Y_{il} \times S_i - \left(\frac{\sum_{i\in P} S_i}{card(D)}\right)$. Constraints (19) define the decision variables.

4.2.2. Lexicographic solution

In this subsection, two mathematical programs are put forward, whereby, each of the model objectives associated optimal solution could be determined. Thus, by optimizing Program 5, the total flow time objective (TFT*) relating optimal solution could be achieved. Then, for the doctors' workloads variation associated objective (WL*) to be reached, Program 6 should be resolved.

A. The Total Flow time objective

For the TFT to be effectively minimized, we consider the following mathematical model, whose analytical form looks as follows:

Program 5: TFT* = $\min \sum_{i\in P} F_i$

Subject to constraints (1), (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (19).

B. The Doctors' workloads variation objective

The workload should be distributed as uniformly and equitably as possible among the shift scheduled doctors. The following mathematical model is designed to help attain an appropriate sequence fit for minimizing the workload discrepancy among doctors. The relating optimal solution (WL*) is obtained as follows:

Program 6: $\text{WL*} = \text{Min} \sum_{k \in D} \left| \sum_{i \in P} Y_{ik} \times S_i - \left(\frac{\sum_{i \in P} S_i}{card(D)} \right) \right|$

Subject to the constraints: (2), (13), (14), (15), (16), (17), (18), (19).

In the subsequent section, a small numerical example is provided to illustrate the implementation of our two multi-objective models, as depicted through the Programs 2, 3, 5 and 6. Then, a number of varying instances are applied to compare results in term of values and computational times.

5. NUMERICAL EXAMPLES

5.1. Small example

Let us consider the example of scheduling eight patients i=1,...,8 on three machines j = 1,...,3 and four doctors l = 1,...,4 to be processed for the purpose of optimizing the patients' total flow time and the doctors' workload variations, simultaneously. The patients, their processing time (in minutes) as well as availability time (in minutes) are presented in Table 1, below. As for the Tables 2 and 3, they respectively illustrate the machines and doctors' availability in minutes.

Table 1: Patients related values

Patient	Processing time	Ready time
P1	15	10
P2	20	5
P3	15	10
P4	30	15
P5	25	5
P6	25	5
P7	10	15
P8	15	5

Table 2: Machines' availability

Machine	Availability
M1	0
M2	0
M3	0

Table 3: Doctors' availability

Doctor	Availability
D1	0
D2	0
D3	0
D4	50

The scheduling sequences, allowing to optimize the second mathematical model, are achieved by implementing the following steps:

Step 1: Considering the first total-flow-time objective function, the optimal solution is obtained through Program 5, after eight minutes and twenty three seconds of running time. TFT* = 245 minutes and WL= 77.5 minutes. The patients' schedule associated sequence is depicted through the Gantt chart diagram, below.

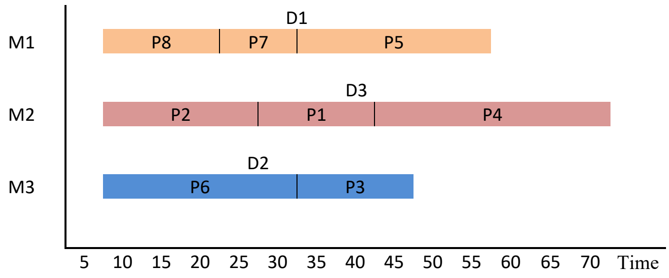

Figure 1: Gantt chart diagram using Program 5 on a small-scale sample

Step 2: Applying Program 6, the optimal solution enabling to minimize the doctors' workloads variations is attained within two seconds. This solution leads to achieving the scheduling sequence displayed on the below figuring Gantt chart diagram, with WL* = 7.5 minutes and TFT=285 minutes.

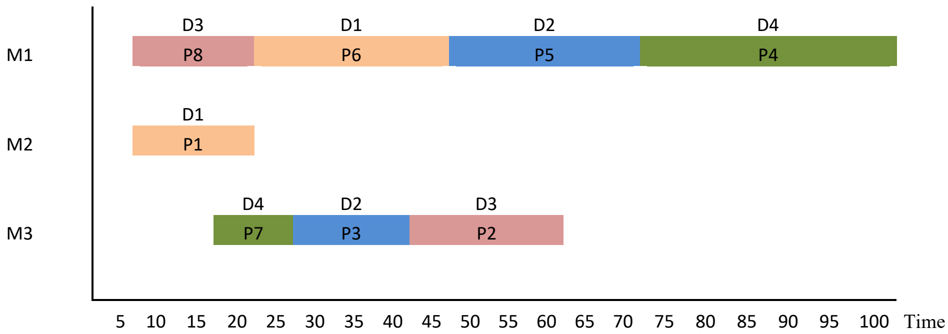

Figure 2: Gantt chart diagram using Program 6 on a small-scale sample

In effect, the first mathematical model appears to take greater time to resolve the eight-patient related problem of TFT minimization objective. So, to compare both of the Programs 2 and 5 illustrated mathematical formulations, we end up opting for considering five, six and seven patient involving samples to fulfill the TFT objective. As to the WL variation objective, the second model turns out to provide an optimal solution with a number of patients ranging up to seventeen. Still, our initially designed mathematical model turns out to require greater CPU time in relation to the second. Then, on comparing Programs 3 and 6, we end up settling for the five, six, seven and eight patient involving samples for WL variation objective to be effectively achieved.

5.2. Comparing the two developed models

After several execution attempts, the mathematical models have been discovered to differ noticeably in terms of values of optimal solutions and CPU times, for the optimal TFT and WL variation solutions.

5.2.1. Flow time minimization

To minimize the objective function associated total flow time, we consider assessing our two developed models' respective performance and comparing their achieved outputs on executing and processing ten problems involving five, six and seven patients as modeled via Programs 5 and 2, respectively. Table 4, below, displays the results reached to solve our test instances by means of three machines and four doctors, following implementation of the Program 2 and Program 5 defined models.

Table 4: The TFT objective function reached results

Treated cases	With 5 patients	With 6 patients	With 7 patients
Number of instances	10	10	10
Number of instances in which Pr5 gives better solution	2	1	3
Number of instances in which Pr5 is faster	10	8	8
Number of instances in which Pr2 is faster	0	2	2
Average CPU for Pr5 (in seconds)	1.9	298.8	451.5
Average CPU for Pr2 (in seconds)	43.7	309	1090

As could be noticed, Program 5 turns out to record shorter CPU time with respect to the entirety of the five patient associated problems, and to most of the six and seven patients involving problems.

5.2.2. Workload minimization deviation

Regarding workload variation minimization objective, noticeable differences have been identified in regard to computation times. For comparison purposes, we consider processing ten problem instances involving five, six, seven and eight patients to assess both of the Program 6 and Program 3 defined models. The number of instances solved through the WL deviation minimization objective proves to exceed those processed through the TFT minimization objective. Table 5, below, displays the results achieved on considering to resolve our test instances by means of three machines and four doctors on implementing the Programs 3 and 6 respective models.

Table 5: The WL deviation objective function achieved results

Treated cases	With 5 patients	With 6 patients	With 7 patients	With 8 patients
Number of treated cases	10	10	10	10
Number of instances in which Pr6 is faster	10	10	10	10
Number of instances in which Pr3 is faster	0	0	0	0
Average CPU for Pr6 (in seconds)	0.2	1.2	1.4	1.5
Average CPU for Pr3 (in seconds)	0.7	6.1	54.1	200

In light of these examples, one might well note that Program 6 appears to record lower CPU execution time as compared to Program 3, with respect to the entirety of the problems involving five, six, seven as well as eight patients, respectively.

In the following section, our mathematical models will be put to test regarding a real case problem of assigning patients to machines and doctors.

6. REAL CASE

The Habib Bourguiba hospital ophthalmology department is equipped with three laser photocoagulation machines in the laser treatment room, frequently used by four Doctors (3 senior doctors and a resident). Every day, a P number of patients need be scheduled for laser photocoagulation treatment. This number is selected in advance in conformity with each machine's daily capacity. In this context, a real sample of laser photocoagulation-machine treated patients is selected. Fifteen patients are usually treated on a daily basis. Our study case will therefore include fifteen patients i =1,...,15 to be scheduled on three machines j = 1,...,3 and four doctors l = 1,...,4 in order to optimize the total flow time and the doctors' workloads variation. The patients, their processing time (in minutes) along with their availability time (in minutes) are displayed in Table 6, while Tables 7 and 8 respectively depict the machines' and doctors' availabilities in minutes.

Table 6: Patient values for a real study case

Patient	Processing time	Ready time
P1	19	0
P2	10	0
P3	11	3
P4	13	5
P5	16	5
P6	13	8
P7	3	10
P8	11	10
P9	8	11
P10	23	12
P11	15	13
P12	18	14
P13	21	15
P14	10	15
P15	19	16

Table 7: Machines' availabilities for a real study case

Machine	Availability
M1	0
M2	0
M3	0

Table 8: Doctors' availabilities for a real study case

Doctor	Availability
D1	0
D2	0
D3	0
D4	50

6.1. The first heuristic achieved results

For easier resolution purposes, we consider subdividing our study case patients' set into two subsets, in conformity to the below stated steps.

Step 1: Consists in running the first subset of eight patients, figuring on table 6, using the tables 7 and 8 provided availabilities. Considering the first total flow time related objective function, the first patients' subset relevant ideal solution is reached by implementing Program 5, after twenty-six minutes of running time. Thus, $TFT_1* = 138$ minutes, and $WL_1= 48$ minutes. Then, 3.7 seconds following Program 6 execution, the effective doctor-workload variations minimizing solution is achieved, which turns out to be: $WL_1* = 4$ minutes, and $TFT_1 =208$ minutes.

Step 2: The first subset reached results are used as input for the second subset figuring on table 6. Regarding the first total-flow-time associated objective function, the second patients' subset relating optimal solution, reached following execution of Program 5, is achieved seven minutes following running time; accordingly, $TFT_2* = 302$ minutes, and $WL_2= 40$ minutes. Then, just 0.992 seconds following Program 6 implementation, we can obtain the most optimal doctor-workload variations minimizing solution, specifically: $WL_2* = 11$ minutes, and $TFT_2 = 394$ minutes. As regards the entire study case patients' set relevant TFT, it is achieved through implementation of the heuristic (TFT^{h1}), attained by summation of the two proposed subsets of patients' flow time, thereby, $TFT_1* = 138$ minutes, and $TFT_2* = 302$ minutes. As to the entire fifteen patients' set, the reached $TFT^{h1} = 440$ minutes. The patients' schedule ensuing sequence is represented through the Gantt chart diagram, below.

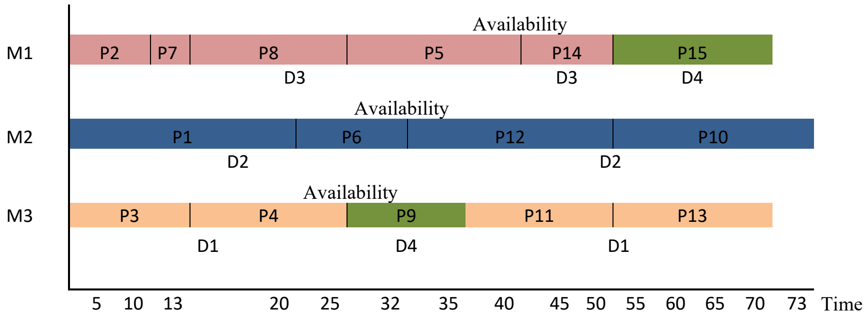

Figure 3: Study case relevant Gantt chart diagram using Program 5

6.2. The second heuristic reached results

At hospital, particularly in the laser room, patients are usually serviced following the First Come First Served (FCFS) rule. Thus, patients with minimum ready times are most often the first to be scheduled. Considering the total-flow-time related objective function, and on implementing Program 5, the associated FCFS flow time turns out to be: $TFT^{h2} = 468$ minutes. The resultant patients' schedule sequence is depicted through the Gantt chart diagram below.

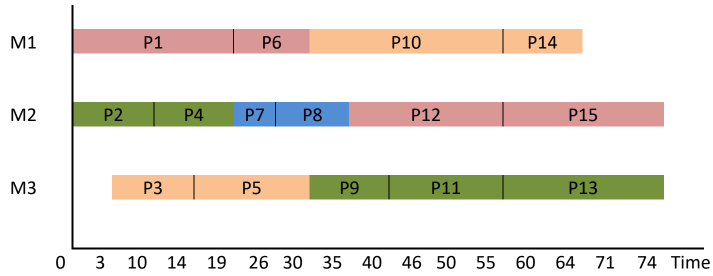

Figure 4: Gantt chart diagram of real case using FCFS rule

Given the NP hard nature of our problem, as our study case involves I = 15, we are required to use the time limit on running the Programs 2 and 5 to attain schedules that give approximate solutions. Hence, the time limit is fixed at thirty-three minutes.

6.3. The third heuristic results

The approximate solution reached through Program 2 and Cplex software thirty three minutes following running time turns out to be: TFT^{h3} = 454 minutes, and WL= 89 minutes. The resultant patients' schedule sequence is illustrated through the Gantt chart diagram (Figure 5).

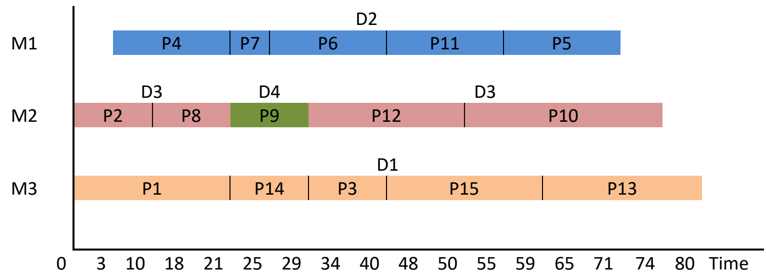

Figure 5: Study case associated Gantt chart diagram using Program 2 with time limit

6.4. The fourth heuristic reached results

Considering the second mathematical model, the approximate solution obtained through Program 5 and Cplex software following thirty three minutes of running time turns out to be: TFT^{h4} = 429 minutes and, WL= 68 minutes. The resultant patients' schedule relating sequence is presented in Figure 6.

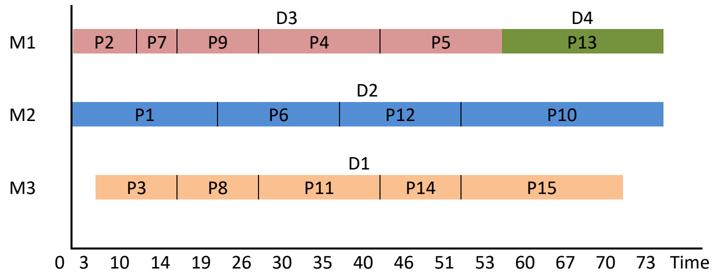

Figure 6: Study case related Gantt chart diagram using Program 5 with time limit

6.5. Comparing the four proposed methods

An examination of the various results regarding the real study case obtained through our proposed methods, one could well note that the subdivision heuristic turns out to provide the most effective TFT solution as compared to the FCFS rule and Program 2 with time limit. Still, Program 5 with time limit proves to yield the most appropriately fit and efficient solution over all the assessed methods. Regarding the WL variation objective, however, the entirety of the suggested methods proved to yield quasi similar WL values equal to two. The difference between the four administered methods achieved values is presented in Figure 7.

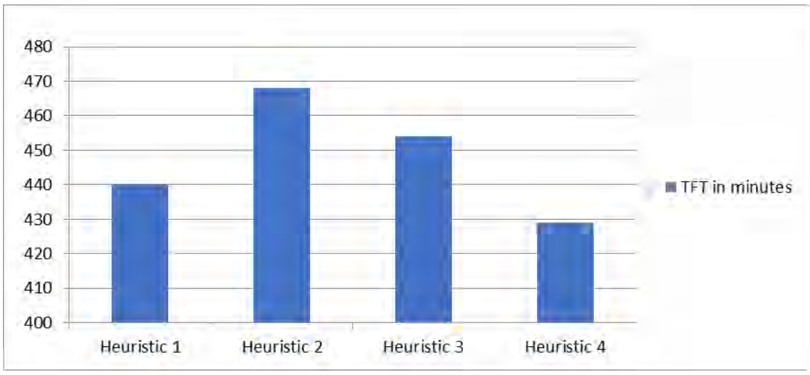

Figure 7: Comparison between the different methods' patient TFT

Hence, it seems rather appropriately useful to deploy the subdivision heuristic to get an effective patients' schedule that efficiently accounts for the problem's dynamic nature. Indeed, the ready and processing times might well vary noticeably over time. Hence, by appealing to the subdivision heuristic, one could always opt for a range of efficiently operable schedules rather than just a single schedule. More specifically, this particular method is liable to ensure remarkable performance, owing mainly to the significant flexibility and practicality it demonstrates in catering for the patients' dynamic nature.

7. CONCLUSION AND PERSPECTIVES

The present work is predominantly focused on maintaining an effective scheduling framework, whereby, patients requiring laser photocoagulation treatment by qualified doctors using special machines could be equitably and efficiently treated. To this end, two distinguishable mathematical models have been proposed. While the first mathematical model enables doctors to work exclusively on a single machine throughout the schedule the second allows doctors to utilize several machines. In this context, we construct two novel bi-objectives models, and compare their performance in terms of reliability and CPU time.

A major outcome and contribution provided through the present study lie mainly in the fact that the same problem could be addressed and processed in different ways through different modeling frameworks. This allows us to opt for the most effective design, whereby, both accurate and approximate solutions could be efficiently maintained. Actually, the [40] modeling based framework proves to demonstrate higher performance over the [77] based strategy, as confirmed by the real case study achieved results. Given the NP-hard nature of the problem, the developed models turn out to require greater computational time for the program to be optimally applicable on a larger number of patients, machines and doctors. To surmount this difficulty, appeal could be made to a meta-heuristic, such as genetic algorithm, or variable neighborhood search. In this respect, pinpointing the most appropriate formulation fit for the implementation of the meta-heuristic remains an issue that requires further investigation.

As a matter of fact, we have been able to achieve and provide two fold lexicographic solutions. The first should serve to minimize the total flow time of patients with the aim of minimizing the doctors' workloads variation by means of a second objective function (while simultaneously maintaining the first objective). As for the second solution, it is designed to help optimize the doctors' workload variations, while considering the total flow-time minimization objective. We could, therefore, opt for introducing manager preferences, whereby, a compromise solution enabling to minimize the deviations between the achievement level of each objective and its ideal value could be reached to simultaneously consider both of the patient and doctor satisfaction objectives.

Ultimately, the developed models could serve to solve other health care related scheduling problems, and might even fit for implementation in other application domains, such as the industrial sector, wherein, doctors could be substituted by workers and patients by jobs.

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