Yugoslav Journal of Operations Research
33 (2023), Number 3, 367–387
DOI: https://doi.org/10.2298/YJOR220315017S

CHOOSING THE BEST OBSERVATION CHANNEL PARAMETERS FOR MEASURING QUANTITATIVE CHARACTERISTICS OF OBJECTS IN MCDM-PROBLEMS AND UNCERTAINTY CONDITIONS

Sergey SVESHNIKOV, Victor BOCHARNIKOV, Vladimir PENKOVSKY, Elena DERGILEVA
Consulting group INEX-FT, Kyiv, Ukraine
sv367@ukr.net, bocharnikovvp@gmail.com, pvi0272vip@ukr.net, edergileva3@gmail.com

Received: March 2022 / Accepted: September 2022

Abstract: The solution of most MCDM-problems involves measuring the characteristics of a research object, converting the estimations into a confidence distribution specified on a set of qualitative gradations and aggregating the estimations in accordance with the structure of the criteria system. The quality of the problems solution as a whole directly depends on the quality of measuring the characteristics of a research object. Data for obtaining estimations of the characteristics are often inaccurate, incomplete, approximate. Modern researches either fragmentarily touch on the questions of measurement quality, or focus on other questions. Our goal is to choose such parameters for converting the value of the quantitative characteristic of a research object into a confidence distribution, which provide the best measurement quality. Based on the observation channel (OC) concept proposed by G. Klir, we refined the measurement quality criteria, determined the composition of the OC parameters, developed an algorithm for calculating the measurement quality criteria and choosing the best OC for the most common MCDM-problems. As calculations have shown, in the most common MCDM-problems, the best is OC, which is built on the basis of a bell-shaped membership function and has a scale of seven blocks. The obtained result will allow researchers to justify the choice of OC parameters from the view-point of the maximum quality of measuring the quantitative characteristics of a research object in MCDM-problems and uncertainty conditions.

Keywords: Observation channel, uncertainty, membership function, multi-criteria decision making, quantitative characteristics, measurement quality.

MSC: 28E10, 90B50, 91B06.

1. INTRODUCTION

MCDM (MultiCriteria Decision Making) problems today are widely widespread in many branches of science, including technology, sociology, and medicine. The following generalized procedure describes the sequence of solving most MCDM-problems: obtaining input estimations for each of the characteristics (properties, attributes) of research objects; transformation of these estimations to a form compatible with the criteria system; generalization of estimations in the criteria system and calculation of the resulting estimations. Data for obtaining estimations of the characteristics is often inaccurate, incomplete, approximate, and cannot always be described by probability. To generalize estimations in MCDM-problems, researchers use various composition methods, for example, arithmetic convolution [1], fuzzy set theory methods [2], fuzzy-integral calculus methods [3], and others. Let's consider several statements that describe the problem of our research and the accepted limitations.

1. The first two stages of the mentioned procedure form the initial data for further processing and, from the view-point of content, are a measurement of the characteristics of research objects. It is well known that the quality of the result of solving any problem is largely determined by the quality of the initial data. Therefore, we are interested in the execution of the first two stages of the procedure from the view-point of ensuring the quality of the measurement.

2. The generally accepted criteria for the quality of measurements are precision, reliability, accuracy, convergence, reproducibility, error. In applied problems, as a rule, not all of the listed criteria are used. The composition of relevant criteria is determined by the specifics of a particular problem. A discussion of the most commonly used measurement quality criteria can be found in [4, 5]. Note that most of the listed criteria are focused on the use of statistical data, that is, the calculation of these criteria requires several measurements. However, in many MCDM-problems, such as those in sociology, it is often very expensive to perform measurements multiple times. Therefore, below we will consider the case of a single measurement and, in accordance with this, choose the measurement quality criteria.

3. Depending on the concept of solving the MCDM-problem, the estimations of the characteristics of the research objects can be measured using quantitative or qualitative scales. The first two stages of the mentioned procedure should ensure the obtaining of characteristics estimates presented as a confidence distribution, since most often the use of this distribution ensures the compatibility of the estimates with the criteria system and the execution of the third stage of the procedure. Most of the well-known algorithms (for example, [6]) for estimating characteristics presented in a qualitative scale build a confidence distribution without any intermediate transformations. Therefore, for such characteristics, the measurement quality criterion is trivial: the more scale values, the better, but at the same time more difficult. But in the case of measurement of the characteristics

presented in a quantitative scale, it is necessary to transform the numerical estimation to a confidence distribution with values in the interval $[0, 1]$. The parameters of this transformation can greatly affect the quality of the measurement. Therefore, we are interested in such parameter values that provide the best measurement quality.

Thus, our goal is to choose such transformation parameters of the value of the quantitative characteristic of a research object into a confidence distribution, which provide the best measurement quality.

For this transformation, researchers most often use linguistic variables defined on the set of real numbers or its subset. However, the use of linguistic variables is most often motivated by the need to simplify the evaluation procedure. The authors motivate the use of linguistic variables by the fact that it is more convenient for an expert to estimate a quantitative characteristic using a qualitative scale. In other words, researchers focus on the convenience of converting a qualitative assessment into a fuzzy set defined on the set of real numbers or its subset. In addition, modern research only fragmentarily considers questions related to the choice of the best parameters of linguistic variables from the view-point of measurement quality.

Therefore, to achieve the goal of our research, we proposed to use the concept of an observation channel. This concept was proposed by G. Klir [7] for the study of complex systems. OC consists of a measurement device with a scale and a procedure for mapping value of the characteristic to this scale. In this case, the solution of our problem envisages the choice of such OC parameters that provide the best estimates of the measurement quality criteria. In the framework of this research, based on the concept of OC, we must solve the following subproblems:

- refinement of measurement quality criteria;
- determination of OC parameters that affect the quality of measurements;
- development of the procedure for calculating the measurement quality criteria based on the determined OC parameters;
- development of the algorithm for choosing the best OC;
- choosing the best OC for the most common MCDM-problems.

The obtained results will allow researchers to justify the choice of OC parameters (shape of membership functions, discreteness of the scale, etc.) from the view-point of the maximum quality of measuring the quantitative characteristics of a research object in MCDM-problems and conditions of non-statistical uncertainty.

2. LITERATURE REVIEW

Well-known studies as a whole not often use the OC concept.

In the study [8] Aljaafreh and Dong use several of the same type OC to identify targets based on the “Majority Voting” rule. Silantyev et al used a combination of the main OC and several additional OCs to build traces of moving objects in radiolocation [9]. The study [10] solves the problem of filtering a signal (variable value) under complex non-stationary interference using continuous OC. Bosov and Pankov consider the problem of observing the value of a variable also in the presence of noise, but with the help of several OCs [11]. Kleptsyna considers discrete signal filtering using linear OC [12].

Yuksel and Linder have determined the optimal OC in a stochastic control problem [13]. Kim and Ha solved a similar problem [14]. Andrievsky and Fradkov investigated the case of limited OC bandwidth [15].

To measure characteristics, researchers often use linguistic variables, which were proposed [16] by L. Zadeh and which are in many ways similar to the concept of the OC. In particular, Natalinova et al used linguistic variables to describe input data in the problem of evaluating the quality of library and information services [17]. Schmalzel and Johnson used linguistic variables based on triangular membership functions in pattern recognition [18].

Although the use of the OC concept is not widely covered in the scientific literature, the use of linguistic variables has continued to expand in recent times. Erick González-Caballero and other authors [19] consider not discrete linguistic variables, as most studies do, but a continuous set of linguistic variables as an new mechanism for representing fuzzy data. This set is used to represent and predict time series.

The research of Xue-Feng Ding, Hu-Chen Liu [20] is aimed at developing a method for making decisions in emergency situations based on a zero-sum game and under time pressure. Linguistic variables are used to represent payoff estimates of decision makers.

Norsyahida Zulkifli and other authors [21] proposed to represent fuzzy data based on the integration of interval-valued intuitionistic fuzzy sets and vague sets. They developed a new linguistic variable for solving MCDM-problems based on the combination of the mentioned sets.

In [22], Sidong Xian and other authors use interval 2-tuple linguistic variables to describe numeric attributes, and also propose an operator for their further aggregation. The authors limit themselves to the interval form of the membership function and do not explore other possible forms.

In a medical study [23], Elif Eyigoz et al. proposed predicting the development of Alzheimer's disease based on the early detection of cognitive impairments, which are identified by analyzing patient responses and presented as linguistic variables.

The aim of the study by Ae-Hwa Kim et al. [24] was to group readers by developmental level. One of the main tasks was to identify distinct and conceptually interpretable groups of readers. The study used linguistic variables as models to represent the parameters that characterize this level: phonological awareness, phonological memory, rapid automatized naming, vocabulary and others. To determine the best match between the number of groups and statistics, several criteria were used: Akaike's information criterion, the Bayesian information criterion, and others.

Shama Parveen and others discuss [25] the fundamental merits of representing human reasoning with fuzzy sets and linguistic variables.

Chen-Tung Chen et al. [26] propose a model for measuring Digital Capability Maturity to determine an organization's readiness to implement information technology. The authors use linguistic variables to represent experts' estimations because they simplify the estimating process and do not require experts to give precise estimates. This research is structurally very similar to the problems for which we propose to use the OC concept. However, the research does not address the questions of determining the best parameters of linguistic variables.

In [27], Ali Mohammadi et al. use linguistic variables to represent partial properties of the capital market. By varying these variables singly or collectively, the authors explored the rate and magnitude of growth in market capitalization.

Research does not often and fragmentarily touch upon the questions of determining the best parameters of membership functions that are used in linguistic variables. Some of the work is presented below.

Marylu L. Lagunes et al. [28] aim to determine the optimal values of three quantitative parameters of membership functions in a fuzzy controller. This study is close to our problem from the view-point of purpose. However, the study concerns only membership functions and ignores the question of determining the cardinality of terms-set of linguistic variables that are used for fuzzification.

A similar study by Tvoroshenko and Gorokhovatskyi [29] is tuning the parameters of the membership function in the problem of determining the state of a biophysical object, in which the authors consider only one of the many types of the membership function - interval.

Olga M. Poleshchuk et al. [30] developed an approach to creating linguistic scales based on semantic scopes to make it easier for experts to estimate the parameters of complex objects. The work is focused on determining the best number of levels of linguistic scales. This research is close to our problem, however, to describe the semantic scopes, it uses only two forms of membership functions: triangular and trapezoidal. In addition, the study uses two criteria for estimating the scales: the minimum fuzziness and the maximum consistency of estimations from different experts. Note that in practice (especially when solving complex problems) it is not always possible to find several experts in order to obtain an estimate of the second criterion.

A review of the literature shows that many modern researches are aimed at solving applied problems under uncertainty using MCDM-methods. They focus mainly on how to take into account the uncertainty conditions that are inherent in the applied problem, and thereby improve the quality of the solution. One of the main causes of uncertainty is the use of an expert as a source of initial estimates, which different research papers describe using linguistic variables. The authors propose various types of linguistic variables, which greatly simplify the work of an expert. However, modern research ignores the following question. If an expert is used as a data source, then what determines the accuracy of representation of expert estimations in the formal constructions of methods that are used for further data processing? It can be assumed that the parameters of these constructions (for linguistic variables, this is the shape, the slope of the side sections, the width of the domain of definition, and others) will determine the accuracy of the presentation of estimates and, ultimately, the quality of the problem solution. In addition, in high-dimensional problems, it is also necessary to take into account technological criteria, for example, the complexity of constructing linguistic variables. Therefore, the review of the literature emphasizes the relevance of our problem regarding the determination of the best parameters of formal constructions from the view-point of ensuring the quality of measurements.

3. RESEARCH PROBLEM AND MEASUREMENT QUALITY CRITERIA

In accordance with the postulates of G. Klir, a complex system can be considered as a set of objects and the relationships between them. Any OC has two elements: a measurement device and a measurement procedure. Each object can be described with the help of a collection of characteristics. Each characteristic z has its own image v in the measurement device. For one characteristic, the OC can be represented as a mapping:

$$o : Z \to V \tag{1}$$

where o – a mapping that describes the OC for characteristic z;
$Z = \{z_i^{`}, i = \overline{1, N}\}$– many possible manifestations of the characteristic z;
N is the number of possible manifestations of the characteristic z;
V is the set of states of the variable v.

As parameters of the OC, we consider the variable v, which is chosen to represent z, the domain of v, and the mapping properties o. Mapping (1) can be functional or non-functional (surjection or injection). The mapping properties depend on the measurement procedure and the nature of the medium through which the mapping occurs (1). In practice, mapping (1) is most often functional. Therefore, below we will consider this type of mapping.

Measurement theory uses classical criteria for evaluating the quality of measurements: validity and reliability [31]. However, depending on the subject area of research, the physical meaning of these criteria may vary. In particular, studies may use other criteria, but retain their main physical meaning – quality of measurements. As an example, we can mention the study [32], which directly addresses the understanding of validity and reliability in qualitative research.

From this view-point, in our research, sensitivity can be considered as an analogue of validity, and unambiguity – as an analogue of reliability. According to the Merriam-Webster explanatory dictionary, validity is the quality of being well-grounded, sound, or correct, and reliability is the extent to which a measurement method gives the same result when repeated trials. Why can we consider the proposed criteria as images of validity and unambiguity?

Any measurement is a determination of a figure, extent, or amount that indicates the value of a property. Validity largely depends on the choice of measurement method. In a broad sense, in order to evaluate validity, we need an ideal meter for correlating to him our measurement method. However, there is no absolutely ideal meter in nature. Therefore, validity can be understood in a narrow sense, that is, as a requirement that the measurement method ensures that the result changes when the property changes. Sensitivity reflects this requirement to some extent.

In the conventional sense, the evaluation of the reliability of a measurement requires repeating the trials. However, this is not always possible due to the high cost of trials or for other reasons. Therefore, we propose to use the criterion of measurement unambiguity. This criterion is especially convenient for measurements under conditions of uncertainty, when the measurement result is presented as a pair: value, membership and can be interpreted in two ways.

Thus, the sensitivity and unambiguity of the measurement result, which are the main criteria for any measurement, will depend on the correct choice of the OC parameters. To these quality criteria, it is necessary to add the criterion of simplicity of constructing the OC, which characterizes the manufacturability of the measurement procedure.

Based on this, the problem of our research is to determine the OC parameters that would be the best in the context of the specified criteria. To solve the research problem, it is necessary to solve the following partial problems:

- determination of OC parameters that affect the quality of measurements;
- development of the procedure for calculating the measurement quality criteria based on determined OC-parameters;
- development of algorithm for choosing of the best OC;
- choosing the best OC for the most common MCDM-problems.

4. SOLVING PARTIAL RESEARCH PROBLEMS

4.1. Determination of OC parameters

The properties of the OC depend on operation principle, the internal structure and properties of the measurement device, as well as the properties of the measurement procedure. Therefore, in order to select the parameters of the OC, we will consider how the measurement is carried out using the OC. Figure 1 schematically shows the measurement process (explanations of symbols are given in the text below).

At the moment of measurement, the characteristic z has some manifestation $z_i^{'}$, which is fixed by the measurement device and mapped to the corresponding value of the variable $v_i^{`} \in V$. The set V (the scale) can be divided into blocks named $x_j, X = \{x_j, j = \overline{1, M}\}$, where M – number of blocks. Each block can be described by a characteristic function with fuzzy boundaries, that is, the membership function $\varphi_j(v) : V \rightarrow [0, 1]$. Then at the OC output we will observe a vector:

$$\mu(x) = (\mu_1(x_1)/x_1...\mu_M(x_j)/x_j...\mu_M(x_M)/x_M) : X \rightarrow [0, 1], \mu(x_j) = \varphi(v_i^{`}).$$

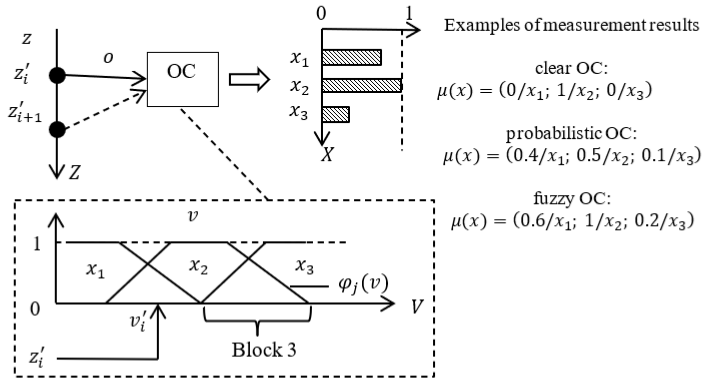

Figure 1: Measurement process of characteristic using OC

This vector is the result of mapping o for the manifestation $z_i^{'}$, of the characteristic z into a set of blocks X. The vector can be considered as a discrete membership function $\mu(x) : X \to [0, 1]$, which is defined on a set of blocks. This function describes the level of similarity of the characteristic manifestation and each block of the measurement device.

Example. We measure the temperature of some physical object with the help of a liquid thermometer. Then z is the thermal radiation of this object, the manifestation $z_i^{'}$ is the level of thermal radiation at the time of measurement. Thermal radiation passes through the medium to the thermometer, establishing the liquid level, which is described as the variable $v_i^{'}$. The set V can be identified as the length of the tube in which the liquid is placed. We compare the liquid level with the thermometer scale, which describes the temperature in degrees Celsius. The scale is divided into blocks x_j. By comparing the liquid level with the blocks of the scale $\varphi_j(v)$, we get some distribution of confidence on the set of blocks $\mu(v_i^{'})$, which is the result of the measurement.

Based on the presented OC description, the parameters that affect the measurement result are the number of blocks M and the form of membership functions $(\varphi_j(v))$.

The simplicity of OC constructing, as well as the unambiguity of the measurement result, depends on the number of blocks. The fewer the number of blocks, the easier it is to construct the OC. However, reducing the number of blocks increases their size and makes the gauge scale coarser. As a result, multiple manifestations $z_i^{'}$ will be associated with the same $v_i^{'}$, what reduces the unambiguity of the measurement result.

The OC sensitivity primarily depends on the form of the membership function $\varphi_j(v)$. The more the shape of the membership function looks like a rectangle, the less sensitive the OC. This means that two closes to each other manifestations $z_i^{'}$ and $z_{i+1}^{'}$ will be indistinguishable, that is, they will form the same function $\mu(x)$.

We can define several types of OCs depending on the form of the membership function. If each membership function $\varphi_j(v)$ is defined using a clear characteristic function and $\forall v, \varphi_j(v) = 1$, this OC can be called a clear OC. If each membership function $\varphi_j(v)$ is also determined using a clear characteristic function, but the additivity condition $\sum_{j=1}^{M} \mu(x_j) = 1$ is satisfied, this channel can be called a probabilistic OC. If the additivity condition is not satisfied $\sum_{j=1}^{M} \mu(x_j) \neq 1$, regardless of $\varphi_j(v)$, the OC can be called fuzzy. Note that a fuzzy OC is a generalization of all other OC and therefore better describes the uncertainty in measurement problems. In the following, we will use just such an OC.

4.2. Development of the procedure for calculating the measurement quality criteria

As mentioned above, the main requirements for any OC are: sensitivity, unambiguity of the measurement result, and simplicity of OC constructing. Let's consider further the order of their calculation. Suppose that we have several fuzzy OCs $\tilde{o}_r, r = \overline{1, K}$, where K is the number of OCs, among which it is necessary to choose the best OC.

The simplicity of OC constructing is inversely proportional to the number of blocks M, since each block requires the construction of its own membership function. To ensure the measurement of the criterion in the interval $[0, 1]$, we calculate the ratio of the number of blocks in the OC to the maximum possible number of blocks, which corresponds to the most complex OC:

$$J_1(\tilde{o}_r) = 1 - \frac{card(X^r)}{\max\limits_{k=\overline{1,K}} card(X^k)}, \tag{2}$$

where $card(X)$ – the power of the set of blocks of variable v in the OC $\tilde{o}_r$.

In the general case, *the sensitivity of the OC* characterizes the ability of the OC to distinguish between the adjacent manifestations of $z_i^{'}$ and $z_{i+1}^{'}$ what implies a change in the manifestations of the characteristic. However, this requires a full-scale experiment, what complicates the evaluation of sensitivity. Therefore, we propose to measure the sensitivity as the degree of difference between the resulting vectors $\mu_{\tilde{o}_r}(v_i^{'})$ and $\mu_{\tilde{o}_r}(v_{i+1}^{'})$, which can be estimated using the Euclid's distance. The sensitivity must be evaluated over the entire set V. Therefore, this set must be divided into equal intervals, and the value of the criterion must be calculated as an average value.

The number of intervals P can be determined based on the Kotelnikov theorem [33] and the number of blocks M. Since the block sizes are equal, it is possible to identify M with frequency, that is, with the number of repetitions within the domain of the variable v. Therefore, based on the Kotelnikov theorem, the number of intervals P must be greater than M by at least 2 times.

Suppose we divided the set into intervals: $V = \bigcup\limits_{i=1}^{P} |v_i; v_{i+1}|, P > 2M$, where P is the number of intervals. Then the OC sensitivity can be calculated by the expression:

$$J_2(\tilde{o}_r) = \frac{1}{P}\sum_{i=1}^{P} d(\mu_{\tilde{o}_r}(v_i^{`}), \mu_{\tilde{o}_r}(v_{i+1}^{`})), \quad (3)$$

where $d(\mu_{\tilde{o}_r}(v_i^{`}), \mu_{\tilde{o}_r}(v_{i+1}^{`}))$ - Euclid's distance between two membership functions.

The unambiguity of the OC measurement result can be characterized by the crispness of the membership function $\mu(x)$, which describes the measurement result at the OC output. The crispness of $\mu(x)$ depends on its form and the number of blocks M. To evaluate this criterion, we used the R. Yager specificity measure [34], which characterizes the granularity (accuracy) of a sorted by descending order fuzzy set: $\forall j = \overline{1, M}, \mu(x_j) \geq \mu(x_{j+1})$. To obtain a more accurate estimate of the criterion, we will use the average value of the Yager-measure in every point of the interval $V = \bigcup_{i=1}^{P} |v_i; v_{i+1}|, P > 2M$ into which the set of values of the variable v is divided in accordance with the Kotelnikov theorem:

$$J_3(\tilde{o}_r) = \frac{1}{P+1}\sum_{i=1}^{P+1} g^{Y}_{[v_i;v_{i+1}]}, \quad (4)$$

where Yager-measure defined as: $g^{Y}_{[v_i;v_{i+1}]} = \sum_{j-1}^{M-1} \frac{1}{j}(\mu(x_j) - \mu(x_{j+1}))$.

Generalized criterion for OC evaluating. As mentioned above, the proposed criteria are contradictory and interdependent. Therefore, to select the best OC, it is necessary to use a generalized criterion for evaluation. We propose to use the simplest criterion – the weighted average of the partial criteria. By denoting the importance of the partial criteria as w_c, we calculate the weighted average using the expression:

$$J(\tilde{o}_r) = \sum_{c=\overline{1,3}} (w_c) \cdot J_c(\tilde{o}_r). \quad (5)$$

4.3. Algorithm for choosing the best OC

Above, we determined the parameters of the OCs, on which the quality of measuring the property of the research object depends. These are the number of blocks M and the form of membership functions $\varphi_j(v)$. By their nature, these OC parameters are discrete. In this case, according to the theory of discrete optimization [35], one of the methods for solving discrete problems of the best choice is the branch and bound method [36], which reduces the computational complexity of the algorithm by discarding obviously inefficient variants. However, in the case of small problem dimensions, the computational complexity of this algorithm approaches the complexity of exhaustive enumeration. Since our problem has a small dimension, further we will analyze the generalized measurement quality criterion, considering all combinations of OC parameters. The algorithm for choosing the best OC consists of the following steps.

Step 1. Determination of the number of OC, from which we will choose the best one.

Let us assume that the form of all block membership functions in the OC is the same. As we found out above, the quality of the measurement is affected by the form of the membership functions of the blocks and the number of blocks. Therefore, the number K of considered OCs will be determined by the product of the number S of possible forms of membership functions and the number M of blocks: $K = S \cdot M$, and OCs parameters will be determined by the full set of possible combinations of the selected forms and the number of blocks. All these combinations must meet the requirements of research problems.

Step 2. Calculation of estimations of measurement quality criteria.

The algorithm performs the calculation according to formulas (2)-(4) for each of the combinations $\{(f_s, M_t)_r\}, r = \overline{1, K}$.

Step 3. Calculation of the estimation of the generalized criterion. The algorithm performs the calculation according to formula (5) for each of the combinations $\{(f_s, M_t)_r\}, r = \overline{1, K}$.

Step 4. Choosing the best OC. The algorithm chooses the OC with the best estimation by iterating through all OCs according to the following formula:

$$\tilde{o}^* = arg \max_{r=\overline{1,K}} J(\tilde{o}_r). \tag{6}$$

4.4. An example of calculating measurement quality criteria by OC constructed using three triangular membership functions

Let's consider an example of calculation of measurement quality criteria using OC, which is built on the basis of three ($M = 3$) identical triangular membership functions. We have used triangular membership functions as an example because they are the simplest and most visual. Let's us establish that in the calculation of the OC sensitivity criterion (see expression 3), in accordance with the Kotelnikov theorem, the number of intervals is $P = 20$. Almost sevenfold excess over the number of blocks will ensure the accuracy of the estimate for the sensitivity criterion. Table 1 shows the results of the criteria calculations. In calculations, we assumed that $\max_{k=\overline{1,K}} card(X^k) = 9$. Explanations for the established assumptions are discussed below.

Table 1: Example of calculating measurement quality estimates

$v_i^{`}$	$\varphi_1(vi^{`})$	$\varphi_2(vi^{`})$	$\varphi_3(vi^{`})$	$d(\mu_{\tilde{o}_r}(v_i^{`}), \mu_{\tilde{o}_r}(v_{i+1}^{`}))$	$g^Y_{[v_i;v_{i+1}]}$	w_c
1	1	0.2	0	0.113137	0.9	
2	0.92	0.28	0	0.113137	0.78	
3	0.84	0.36	0	0.113137	0.66	
4	0.76	0.44	0	0.113137	0.54	
5	0.68	0.52	0	0.113137	0.42	
6	0.6	0.6	0	0.113137	0.3	
7	0.52	0.68	0	0.113137	0.42	
8	0.44	0.76	0	0.12	0.54	
9	0.36	0.84	0.04	0.138564	0.65(3)	
10	0.28	0.92	0.12	0.138564	0.76	
11	0.2	1	0.2	0.138564	0.8(6)	
12	0.12	0.92	0.28	0.138564	0.76	
13	0.04	0.84	0.36	0.12	0.65(3)	
14	0	0.76	0.44	0.113137	0.54	
15	0	0.68	0.52	0.113137	0.42	
16	0	0.6	0.6	0.113137	0.3	
17	0	0.52	0.68	0.113137	0.42	
18	0	0.44	0.76	0.113137	0.54	
19	0	0.36	0.84	0.113137	0.66	
20	0	0.28	0.92	0.113137	0.78	
21	0	0.2	1		0.9	
J_1			0.667			0.1
J_2				0.1189		0.63
J_3					0.6102	0.27
J						**0.3063**

The OC construction simplicity criterion J_1 is calculated in accordance with expression (2), based on the ratio of the number of blocks in this example and the maximum possible number of blocks.

The OC sensitivity criterion J_2 is calculated in accordance with expression (3) as the average Euclid's distance between two neighboring membership functions that correspond to the points v_i and v_{i+1} from the domain of definition V of variable v.

The OC unambiguity criterion J_3 is calculated in accordance with expression (4) as the average value of the Yager measure in the points v_i from the domain of definition V of variable v.

The generalized measurement quality criterion J is calculated as a weighted average of the three mentioned criteria, taking into account their importance.

Similarly, the algorithm calculates the quality criteria for other OCs.

4.5. Choosing the best OC for the most common MCDM-problems

Let us specify our assumptions in the calculations. 1. We assumed that the membership functions of blocks $\varphi_i(v) : V \to [0, 1]$ within the same OC should have the same form. In practice, researchers rarely use membership functions of different forms, since this complicates their tuning, although it allows constructing better OCs. An analysis of the literature has shown that researchers most often use triangular and trapezoidal membership functions. The bell-shaped (Gaussian) form of the membership function is used less frequently. Other forms are practically not used. We will evaluate the quality of OCs constructed using the three indicated forms of membership functions.

2. We assumed that the maximum possible number of blocks in the OC will be more than 2, but will not exceed 9. As a rule, in practice, researchers do not use two blocks, since this makes minor the variety of estimates, that is, the meaning of the measurement disappears. On the other hand, ten or more blocks significantly complicate the construction of the OC and make neighboring values of property hardly distinguishable. This statement correlates well with Saaty's well-known relative scale [37], who limited the maximum preference to 9 due to the possible increase in rating inconsistency. We will evaluate the measurement quality criteria for three OCs that are constructed using three, five, and seven blocks. We assume that these three variants are enough to see patterns in the change in the quality of measurements.

3. To ensure the accuracy of the evaluation of the measurement quality criteria, we proposed to use the Kotelnikov theorem, which establishes that the sampling frequency should be at least twice as high as the signal change frequency. In our case, this means that $P > 2M$. We have set $P = 20$. Since in the calculations the maximum value is $M = 7$, then $P \approx 3M$, which ensures the fulfillment of the Kotelnikov theorem.

4. We have determined the importance of the criteria based on the following considerations. In our opinion, the criterion of sensitivity should be of the greatest importance. In accordance with the physical meaning, it comes closest to the main requirement for any measurement – adequacy (validity). The criterion for the unambiguity of the measurement result should be somewhat less important. This criterion is also related to the adequacy of the measurement. The simplicity criterion should be the least important because it have a technological nature. The OC is construct once and then reused for measurements. Based on these considerations we experimentally determined the importance of the criteria to ensure the maximum difference in the generalized estimations of OCs. As a result, we have determined the following values for the importance of the criteria: $w_1 = 0.1, w_2 = 0.63, w_3 = 0.27$.

As we can see, the assumptions made in the calculations correspond to both the main theoretical provisions and the practice of solving research problems.

We have formed several variants of OC constructing as a Cartesian product of three variants for the membership function and three variants for the number of blocks. For each combination of these variants, we calculated the criteria for the sensitivity and unambiguity of the measurement result.

The calculated estimates of partial measurement quality criteria for all researched OCs are shown in Table 2. The generalized OCs estimates are shown in Table 3.

Table 2: Estimates of partial criteria

Number of blocks		Triangular membership		Trapezoidal membership		Bell-shaped membership	
M	$J_1(\tilde{o}_r)$	$J_2(\tilde{o}_r)$	$J_3(\tilde{o}_r)$	$J_2(\tilde{o}_r)$	$J_3(\tilde{o}_r)$	$J_2(\tilde{o}_r)$	$J_3(\tilde{o}_r)$
$M = 3$	0.667	0.1189	0.6102	0.148	0.7657	0.1322	0.7198
$M = 5$	0.444	0.2426	0.619	0.2686	0.7729	0.259	0.7052
$M = 7$	0.222	0.396	0.6365	0.3536	0.7043	0.4106	0.701

Table 3: Estimates of the generalized criterion

Number of blocks	Triangular membership	Trapezoidal membership	Bell-shaped membership
$M = 3$	0.3063	0.366	0.3443
$M = 5$	0.3644	0.4221	0.3981
$M = 7$	0.4436	0.4351	0.4701

As follows from the table 3, all researched OCs can be divided into three groups: the best OC ($J > 0.4$), satisfactory OC ($0.35 < J < 0.4$), and the worst OC ($J \leq 0.35$).

The first group consists of OCs, which are based on the bell-shaped and triangular membership functions with $M = 7$, as well as on the trapezoidal membership function with $M = \{5; 7\}$.

The second group consists of OCs, which are based on the bell-shaped and triangular membership function with $M = 5$, as well as on the trapezoidal membership function with $M = 3$.

The third group consists of OCs, which are based on the bell-shaped and triangular membership function with $M = 3$.

Based on the estimates of the generalized measurement quality criterion, the best OC is constructed on the basis of a bell-shaped membership function with seven blocks. But in some research problems, it is necessary to use simpler membership functions. In this case, it is better to use an OC constructed using a triangular or trapezoidal membership function and seven blocks. If in a research problem the number of blocks is question of principle, we recommend using an OC with a trapezoidal membership function and five blocks. Other combinations of OC parameters will degrade the quality of the measurement.

5. DISCUSSION

In this section, we discuss the quality of the results obtained, which is determined by the methods used for the calculations. In particular, these are:

method (5) for generalizing the estimates of partial criteria;

methods (3) and (4) for calculating the most important partial criteria.

The method of generalizing the estimates of partial criteria.

The method of generalizing the estimates of partial criteria is based on the calculation of the weighted average. This method is widely used in modern research, however, it allows obtaining acceptable results if the estimates of partial criteria for the evaluated objects are evenly distributed over a wide range. Otherwise, the resulting estimates become poorly distinguishable and unsuitable for decision making. In addition, criteria with high estimates suppress criteria with low estimates, what also distorts the data for decision making. In such cases, researchers use non-linear methods of generalizing estimates.

As the analysis of Table 2 shows, the estimates of partial criteria for the analyzed OCs are located in a wide range with the size 0.654, what gives us reason to assert that the method (5) is used correctly. However, partial criteria estimates are not completely evenly distributed. Figure 2 shows the estimates of the most important criteria J_2 and J_3 on the numeric axis. As you can see, the values of J_2 are concentrated in the range of $[0.1, \approx 0.4]$, and the values of J_3 are concentrated in the range of $[0.6, 0.8]$. This distribution of estimates means that in the case of using the weighted average (5) to generalize the criteria, large estimates of J_3 can compensate for the low importance of this criterion ($w_3 = 0.27$). Conversely, small estimates of J_2 can compensate for the high importance of this criterion ($w_2 = 0.63$).

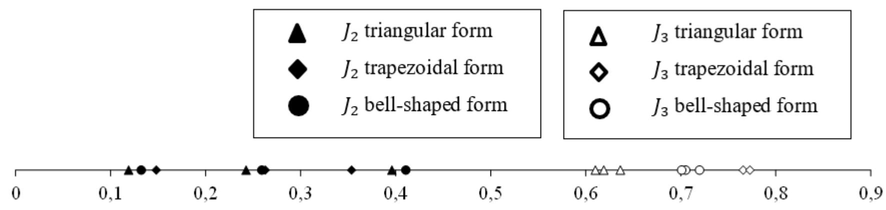

Figure 2: Estimates of partial criteria

This compensation effect leads to a decrease in the range of the resulting estimates. As follows from Table 3, the values of the generalized criterion change in the range of 0.164. In our opinion, this range remains quite acceptable for decision making.

However, as an alternative to the weighted average (5), we can consider the use of nonlinear generalization methods, in particular, the Sugeno [3] or Choquet [38] fuzzy integrals, which can provide various logics for generalizing partial criteria.

Table 4 shows the estimates of the generalized criterion calculated using the Sugeno fuzzy integral (the importance of partial criteria is presented as a fuzzy confidence measure ($w_1 = 0.031, w_1 = 0.333, w_1 = 0.076$).

Table 4: The values of the generalized criterion calculated with the help of Sugeno fuzzy integral

Number of blocks	Triangular membership	Trapezoidal membership	Bell-shaped membership
$M = 3$	0.135	0.148	0.135
$M = 5$	0.243	0.269	0.259
$M = 7$	0.396	0.354	0.411

As we can see, in the case of using the fuzzy Sugeno integral, the size of the range of estimates increased from 0.164 to 0.276, what improves the data for decision making. However, the composition of the best OCs has not changed. This confirms the convergence of our results with the results of other approaches, as well as the stability of the proposed solutions regarding the choice of the best OC.

Methods for calculating partial criteria.
In the calculation of the partial criteria J_2 and J_3, the main questions are:

- the influence of the slope of the side faces of membership functions in the OC blocks;
- use of alternative methods for calculating partial criteria estimates.

Influence of slope of the side faces of membership functions in the OC blocks.
The estimates of criteria J_2 and J_3 presented in Table 2 are calculated for membership functions that are built using the following empirical rule: the membership functions of neighboring blocks must intersect at a level from the range $[0.6, 0.7]$ (see the example in Table 1). This rule follows from the conclusions [39], the main of which is that it is necessary to choose the set with the minimum degree of fuzziness as the best one. If the rule is observed, then the number of blocks and the level of intersection of neighboring membership functions will almost unambiguously determine the slope of the side faces. In other words, we are not in a position to determine the best slope here, since it is already defined by the empirical rule. Nevertheless, let us find out how the deviation from the rule will affect the estimates of the criteria J_2 and J_3.

Table 5 shows the results of calculating criteria J_2 and J_3 for three variants of the triangular membership function with three blocks: accepted in the calculations (observance of the rule), with shallow side faces, with steep side faces.

Table 5: Estimates of criteria J_2 and J_3 for different slopes of the side faces of membership functions in the OC blocks

	Accepted in the calculations			Shallow side faces			Steep side faces		
$v_i^{'}$	$\varphi_1(v_i^{'})$	$\varphi_2(v_i^{'})$	$\varphi_3(v_i^{'})$	$\varphi_1(v_i^{'})$	$\varphi_2(v_i^{'})$	$\varphi_3(v_i^{'})$	$\varphi_1(v_i^{'})$	$\varphi_2(v_i^{'})$	$\varphi_3(v_i^{'})$
1	1	0.2	0	1	0.5	0	1	0	0
2	0.92	0.28	0	0.95	0.55	0.05	0.83	0	0
3	0.84	0.36	0	0.9	0.6	0.1	0.67	0	0
4	0.76	0.44	0	0.85	0.65	0.15	0.5	0	0
5	0.68	0.52	0	0.8	0.7	0.2	0.33	0	0
6	0.6	0.6	0	0.75	0.75	0.25	0.14	0.14	0
7	0.52	0.68	0	0.7	0.8	0.3	0	0.33	0
8	0.44	0.76	0	0.65	0.85	0.35	0	0.5	0
9	0.36	0.84	0.04	0.6	0.9	0.4	0	0.67	0
10	0.28	0.92	0.12	0.55	0.95	0.45	0	0.83	0
11	0.2	1	0.2	0.5	1	0.5	0	1	0
12	0.12	0.92	0.28	0.45	0.95	0.55	0	0.83	0
13	0.04	0.84	0.36	0.4	0.9	0.6	0	0.67	0
14	0	0.76	0.44	0.35	0.85	0.65	0	0.5	0
15	0	0.68	0.52	0.3	0.8	0.7	0	0.33	0
16	0	0.6	0.6	0.25	0.75	0.75	0	0.14	0.14
17	0	0.52	0.68	0.2	0.7	0.8	0	0	0.33
18	0	0.44	0.76	0.15	0.65	0.85	0	0	0.5
19	0	0.36	0.84	0.1	0.6	0.9	0	0	0.67
20	0	0.28	0.92	0.05	0.55	0.95	0	0	0.83
21	0	0.2	1	0	0.5	1	0	0	1
J_1		0.667			0.667			0.667	
J_2		0.1189			0.0866			0.1812	
J_3		0.6102			0.5325			0.5933	
J		0.3063			0.269			0.3441	

As the analysis of Table 3 shows, a decrease in the steepness of the faces increases the level of intersection of the membership functions of neighboring blocks, and also reduces both the sensitivity and the unambiguity of the OC measurement. An increase in the steepness of the edges increases the sensitivity of the OC, but reduces the unambiguity of the measurement. Therefore, calculations confirm the rationality of using the mentioned empirical rule.

Alternative methods for calculating partial criteria estimates.

In practice, various mathematical methods can be used to evaluate measurement quality criteria. First, instead of the Euclidean distance in (3), we will consider using the relative Hamming distance. Second, instead of the Yager specificity measure in (4), we will consider the use of Higashi-Klir uncertainty measure [40]. Table 6 shows the estimates of criteria J_2 and J_3 in the case of using these methods. Table 7 shows the estimates of the generalized quality criterion.

Table 6: Estimates of criteria J_2 and J_3 in the case of using alternative calculation methods

Number of blocks		Triangular membership		Trapezoidal membership		Bell-shaped membership	
M	$J_1(\tilde{o}_r)$	$J_2(\tilde{o}_r)$	$J_3(\tilde{o}_r)$	$J_2(\tilde{o}_r)$	$J_3(\tilde{o}_r)$	$J_2(\tilde{o}_r)$	$J_3(\tilde{o}_r)$
$M = 3$	0.667	0.06	0.829	0.065	0.867	0.064	0.848
$M = 5$	0.444	0.076	0.828	0.07	0.861	0.078	0.847
$M = 7$	0.222	0.088	0.843	0.071	0.84	0.089	0.853

Table 7: Estimates of the generalized measurement quality criterion in the case of using alternative calculation methods

Number of blocks	Triangular membership	Trapezoidal membership	Bell-shaped membership
$M = 3$	0.3283	0.3418	0.3364
$M = 5$	0.3159	0.3213	0.322
$M = 7$	0.3054	0.2941	0.3087

The analysis of Table 6 shows that due to the adopted changes, the ranges of estimates of criteria J_2 and J_3 narrowed. In addition, the range of estimates of the J_2 criterion has shifted to 0, and the range of estimates of the J_3 criterion has shifted to 1. As a result, the conditions for the correct use of the generalization method (5), which we have showed above, were violated.

Also, because of this, the range of estimates of the generalized criterion was narrowed, what created conditions for the distortion of the resulting solution. As the analysis of Table 7 shows, the best OCs here are OCs with three blocks. This indicates that the values of the most important criterion J_2 are almost not taken into account in the estimate of the generalized criterion. Therefore, we can conclude that in the case of a change in the order of calculation of partial criteria, it is necessary to carefully monitor the width of the range and the uniform distribution of criteria estimates in order to satisfy the conditions for using the generalization method (5). You should choose mathematical constructions that provide the maximum range of estimates and their uniform distribution.

6. CONCLUSIONS

We proposed an approach regarding the choice of the best OC parameters for measuring the quantitative characteristics of objects in MCDM-problems based on the concept of G. Klir. Unlike well-known approaches that focus on the convenience of the expert's work when evaluating quantitative characteristics, we have focused on ensuring the quality of the measurement of these characteristics. We proposed to use a scale that is described by a set of blocks with fuzzy membership functions. This scale is analogous to a linguistic variable. We have established that the OC parameters which determine the quality of the measurement are the number of blocks and the form of the membership function. The main partial criteria for the quality of measuring are sensitivity, unambiguity and complexity of construction. We proposed to calculate sensitivity based on the Euclidean distance, unambiguity based on the Yager specificity measure, construction complexity based on the number of scale blocks. We also proposed using the weighted average to calculate the estimate of the generalized criterion.

Based on the study of the experience of solving MCDM-problems, we found out the most commonly used combinations of OC parameters and evaluated the quality of measurement for these combinations. As a result, we have identified a group of four best OCs. The measurement quality of these OCs does not differ much from each other. This is OCs, built on the basis of a bell-shaped, trapezoidal or triangular membership function and seven blocks (the most complex OC). This is also OC with five blocks and a trapezoidal membership function (the most simple OC). We investigated the proposed solution and found that it is stable in terms of both replacing the method of generalizing the estimates of partial criteria and replacing the methods for calculating them. However, in the case of using other methods, we recommend that you carefully monitor the width of the range and the even distribution of partial criteria estimates. We recommend using the proposed approach in MCDM-problems that do not require very accurate measurements. For example, these are problems from sociology. In MCDM-problems from the field of engineering (in particular, the control of technical devices), it is often necessary to consider OCs that are built using unequal membership functions. Moreover, these problems often require other quality criteria, such as the speed of measurement. The study of the measurement quality of such OCs is the main direction in the development of the proposed approach.

Funding. This research received no external funding.

REFERENCES

[1] T. L. Saaty and K. P. Kearns, *Analytic Planning: The organization of System.* New York: Pergamon Press, 1991.

[2] D. Dubois, "The role of fuzzy sets in decision sciences: Old techniques and new directions", *Fuzzy sets and Systems*, vol. 184, issue 1, pp. 3-28, Dec. 2011. Available at: https://cutt.ly/uUdNJRj. Date accessed: 17 august 2022.

[3] M. Sugeno, "Fuzzy Measure and Fuzzy Integral", *Transaction of the Society of Instrument and Control Engineers*, vol. 8, pp. 95-102, 1972.

[4] Criteria of Measurement Quality, Available at: https://cutt.ly/rZYmMbI. Date accessed: 17 august 2022.

[5] A. Schmitz, "Measurement Quality," *in Principles of sociological inquiry: Qualitative and quantitative methods*, Saylor Academy, 2012. Available at: https://cutt.ly/mZYQmTp. Date accessed: 17 august 2022.

[6] I. Derbel, N. Hachani and H. Ounelli, "Membership Functions Generation Based on Density Function," *in 2008 International Conference on Computational Intelligence and Security*, 2012, pp. 96-101, DOI: 10.1109/CIS.2008.211.

[7] G. Klir, "Systemology. Automation of solving system problems," *Radio and communications*, Moscow, 1990, pp. 544. Available at: https://cutt.ly/cUdMkou. Date accessed: 17 august 2022. [in Russian].

[8] A. Aljaafreh and L. Dong, "Cooperative detection of moving targets in wireless sensor network based on fuzzy dynamic weighted majority voting decision fusion," *in 2010 International Conference on Networking, Sensing and Control (ICNSC)*, 2010, pp. 544-548. DOI: 10.1109/ICNSC.2010.5461603.

[9] A. B. Silantyev, A. V. Golubev, C. A. G. Fernandu, J. F. Olim and A. V. Timoshenko, "Track Association in Radars with Additional Multi-Scanning Observation Channels," *in 2021 Systems of Signal Synchronization, Generating and Processing in Telecommunications SYNCHROINFO*, 2021, pp. 1-6.

[10] N. Natalinova, O. Rozhkova, E. Moldovanova and N. Ilina, "Filtering for Stochastic Systems in the Case of Continuous Observation Channels with Memory of Arbitrary Multiplicity and Anomalous Noise," *in 2016 International Siberian Conference on Control and Communications (SIBCON)*, 2016, pp. 21-26.

[11] A. Bosov and A. Pankov, "Conditionally minimax filtering of a process in a system with switching observation channels", *Automation and telemechanics*, vol 6, pp. 87-97, 1995. [in Russian].

[12] M. Kleptsyna, A. Le Bretonb and M. Viot, "Filtering with exponential criteria via linear observation channels", *Global and Stochastic Analysis*, vol. 1, no. 1, pp. 57-77, 2014.

[13] S. Yuksel and T. Linder, "Optimization and Convergence of Observation Channels in Stochastic Control", *Optimization and Control (math.OC)*, 2012. Avialable at: https://cutt.ly/dBgJcNT.

[14] J.T. Kim and S. Ha, "Observation Space Matters: Benchmark and Optimization Algorithm", *Artificial Intelligence*, 2020. Avialable at: https://cutt.ly/fBgJ20w.

[15] B. Andrievsky and A. Fradkov, "Control and Supervision through Communication Channels with Limited Bandwidth", *Gyroscopy and navigation*, vol. 4, pp. 103-114, 2009. [in Russian].

[16] L. A. Zadeh, "The Concept of linguistic variable and its applications to approximate reasoning", *Information Sciences*, pp. 301-357, 1975. Available at: https://doi.org/10.1016/0020-0255(75)90046-8. Date accessed: 17 august 2022.

[17] N. Natalinova, O. Galtsevaa, N. Xiub and A. Belana, "Qualitative and Quantitative Methods for Estimating the Library and Information Services", *European Proceedings*, 2017, (August 2022). Available at: http://dx.doi.org/10.15405/epsbs.2017.01.65. Date accessed: 17.

[18] J. L. Schmalzel and J.F. Johnson, "Pattern recognition based on fuzzy logic," *in IEEE Instrumentation and Measurement Technology Conference*, 1993, pp. 681-684, DOI: 10.1109/IMTC.1993.382557.

[19] E. Gonzalez-Caballero, R.A. Espin-Andrade, W. Pedrycz et al. "Continuous Linguistic Variables and Their Applications to Data Mining and Time Series Prediction", *International Journal of Fuzzy Systems*, vol. 23, pp. 1431-1452, 2021. DOI:10.1007/s40815-020-00968-w.

[20] Xue-Feng Ding and Hu-Chen Liu, "A new approach for emergency decision-making based on zero-sum game with Pythagorean fuzzy uncertain linguistic variables", *International journal of intelligent systems*, vol. 34, issue 7, pp. 1667-1684, July 2019. DOI:10.1002/int.22113.

[21] N. Zulkifli, L. Abdullah and H. Garg, "An Integrated Interval-Valued Intuitionistic Fuzzy Vague Set and Their Linguistic Variables", *International Journal of Fuzzy Systems*, vol. 23, pp. 182-193, 2021. DOI:10.1007/s40815-020-01011-8.

[22] Sidong Xian, Zhou Liu, Xiaoling Gou and Wenhua Wan, "Interval 2-tuple Pythagorean fuzzy linguistic MULTIMOORA method with CIA and their application to MCGDM", *International journal of intelligent systems*, vol. 35, issue 4, pp. 650-681, April 2020. DOI: 10.1002/int.22221.

[23] E. Eyigoz, S. Mathur, M. Santamaria, G. Cecchi and M. Naylor, "Linguistic markers predict onset of Alzheimer's disease", *EClinicalMedicine*, vol. 28, 2020. DOI:10.1016/j.eclinm.2020.100583.

[24] A-H. Kim, UJ. Kim, JC. Kim and S. Vaughn, "Subgrouping of Korean Readers Based on Reading Achievement and the Relation of Cognitive-Linguistic Variables to the Subgroups", *Learning Disability Quarterly*, vol. 44, issue 3, pp. 197-209, 2021. DOI:

10.1177/0731948720958140.

[25] S. Parveen, S. Parveen and N. Rahman, "Fuzzy Systems: A Human Reasoning Approach Using Linguistic Variables," in *Intelligent Communication Technologies and Virtual Mobile Networks: ICICV 2019. Lecture Notes on Data Engineering and Communications Technologies*, S. Balaji, A. Rocha, YN. Chung, Eds. Springer, Cham. vol 33, 2020. DOI:10.1007/978-3-030-28364-3-55.

[26] CT. Chen, A. Ova and WZ. Hung, "A MCDM Method for Measuring Digital Capability Maturity Based on Linguistic Variables and Fuzzy Integral," in *Intelligent and Fuzzy Systems. INFUS 2022. Lecture Notes in Networks and Systems*, C. Kahraman, A.C. Tolga, S. Cevik Onar, S. Cebi, B. Oztaysi, I.U. Sari, Eds. Springer, Cham. vol. 505, 2022. DOI:10.1007/978-3-031-09176-6-66.

[27] A. Mohammadi, A.M. Shirazi, A. Abbasi and S. Akhlaghpour, "Simulating the Linguistic Variables Interactions in Capital Market Development Process Using Fuzzy Inference System in a System Dynamic Context", *Journal of Financial Management Strategy*, vol. 7, no. 25, 2019. Available at: https://cutt.ly/LZY4O3F. Date accessed: 17 august 2022.

[28] M. L. Lagunes, O. Castillo, F. Valdez, J. Soria and P. Melin, "Parameter Optimization for Membership Functions of Type-2 Fuzzy Controllers for Autonomous Mobile Robots Using the Firefly Algorithm," in *Fuzzy Information Processing. NAFIPS 2018. Communications in Computer and Information Science*, G. Barreto, R. Coelho, Eds. Springer, Cham. vol. 831, 2018. DOI:10.1007/978-3-319-95312-0-50.

[29] I. Tvoroshenko and V. Gorokhovatskyi, "Effective Tuning Of Membership Function Parameters In Fuzzy Systems Based On Multi-Valued Interval Logic", *Telecommunications and Radio Engineering*, vol. 79, issue 2, pp. 49-163, 2020. DOI: 10.1615/TelecomRadEng.v79.i2.70.

[30] O. M. Poleshchuk, "Creation of Linguistic Scales for Expert Evaluation of Parameters of Complex Objects Based on Semantic Scopes," *in 2018 International Russian Automation Conference (RusAutoCon)*, 2018, pp. 1-6. DOI: 10.1109/RUSAUTOCON.2018.8501686.

[31] C. L. Kimberlin and A. G. Winterstein, "Validity and reliability of measurement instruments used in research", *American journal of health-system pharmacy*, vol. 65, issue 23, pp. 2276-2284, 2008.

[32] N. Golafshani, "Understanding reliability and validity in qualitative research", *The qualitative report*, vol.8, issue 4, pp. 597-607, 2003. Available at: https://cutt.ly/wBgC0Lk. Date accessed: 17 august 2022.

[33] V.A. Kotelnikov, "On the carrying capacity of the ether and wire in telecommunications," *in Material for the First All-Union Conference on Questions of Communication, Moscow, Izd. Red. Upr. Svyazi RKKA, Reprint article in the journal UFN*, vol. 176, no. 7, 1933, pp. 762-770.

[34] R. Yager, "Measuring tranquility and anxiety in decision making: an application of fuzzy sets", *International Journal of General Systems*, vol. 8, issue 3, pp. 139-146, 1982. DOI: 10.1080/03081078208547443.

[35] M. Dorigo, G. Di Caro, and L. M. Gambardella, "An algorithms for discrete optimization", *Artificial life*, vol. 5, issue 2, pp. 137-172, 1999. Available at: https://cutt.ly/GZXfNbs. Date accessed: 17 august 2022.

[36] A. H. Land and A. G. Doig, "An automatic method for solving discrete programming problems," in *50 Years of Integer Programming 1958-2008*, Berlin, Springer, Heidelberg, 2010, pp. 105-132. Available at: https://cutt.ly/iZXfAZj. Date accessed: 17 august 2022.

[37] T.L. Saaty, "Relative measurement and its generalization in decision making why pairwise comparisons are central in mathematics for the measurement of intangible factors the analytic hierarchy/network process", *Rev. R. Acad. Cien. Serie A. Mat.*, vol. 102, pp. 251-318, 2008. Available at: https://cutt.ly/AXeD53j. Date accessed: 17 august 2022.

[38] G. Choquet, "Theory of capacities", *Annales de l'Institut Fourier*, no 5, 1953, pp. 131-295. Available at: https://cutt.ly/Nb8m6UQ. Date accessed: 17 august 2022.

[39] A. P. Ryzhov, "Elements of fuzzy set theory and its applications", Moscow, Publishing House of Moscow State University, vol. 2, 2003. Available at: https://cutt.ly/wXsx3gX. Date accessed: 17 august 2022. [In Russian].

[40] M. Higashi and G. J. Klir, "Measures of uncertainty and information based on possibility distributions", *International Journal of General Systems* , vol. 9, issue 1, pp. 43-58, 1982.