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A COMPARATIVE STUDY TO FIND THE OPTIMAL ORDERING QUANTITY OF THE RISK-NEUTRAL AND RISK-AVERSE NEWSVENDOR

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Abstract: In the classical newsvendor problem, it is considered that the newsvendor is risk-neutral and the Optimal Ordering Quantity (OOQ) was found which maximizes the newsvendor expected profit. In the real world, different investors have different attitudes towards risk. Accordingly, this paper considers the utility function which is commonly being used to model the attitude of the investor who makes the investment decision to maximize his expected utility instead of expected profit. This study considers the quadratic utility function and demonstrates that it can be used to describe the risk-averse as well as a risk-neutral investor with some conditions. Finally, by considering the risk profile of the investor at different levels of investor initial wealth, we have developed a method to determine the OOQ which maximizes the expected utility. From the numerical examples, it is clear that the OOQ and hence the expected utility varies depending on an investor's attitude toward the risk and investment decision. At different levels of initial wealth, the attitudes toward the risk of the investors are different and the sensitivity analysis demonstrates how an investor can choose the initial wealth and OOQ to maximize his expected utility.

Keywords: Optimal ordering quantity, risk-neutral, risk-averse, expected utility, initial wealth, salvage value.

MSC: 90B05.

1. INTRODUCTION

In the present scenario, the market is experiencing competitive situations and investors expect high returns against investments. This paper discusses the situation by considering the attitude of the investor, who makes the investment decision to maximize his expected utility instead of expected profit. Effective risk management policy can play a vital role to achieve maximum expected utility. Under the traditional single-period newsvendor type problem, the newsvendor buys the newspaper at the start of the day and sells it during the day. The profit for the newsvendor can be estimated by calculating the difference between the selling price and the purchase cost. According to the newsvendor model based on the fixed initial wealth, the maximum profit and so the maximum utility will be obtained when the news vendor exactly matches the market demand and the ordering quantity. But the market demand is volatile and it is not possible to predict exactly what will be the demand in the future. So the basic problem arises as to what should be the OOQ to meet the volatile demand to maximize the expected utility and hence the expected profit Fathi and Nuttle [1]. Newsvendor problem has been studied extensively by many researchers and various inventory models were developed under certain assumptions to optimize cost, profit, and order quantities. Lau [2] modified the classical newsvendor model with multiple objectives to maximize the expected utility instead of the expected profit and maximizing the probability of achieving a target profit. Haung [3] presented an economic ordering quantity model for deteriorating items under random demand and optimized the order quantity to maximize expected profits. Anwari [4] reviewed the applicability of market evaluation models in stochastic inventory problems and presented a comparison with classically expected profit maximization using the capital asset pricing model. He also discussed the consideration of the relevant risk of an investor in the model. Chung [5] extended Anwari [4] paper and analyzes the newsvendor problem using the capital asset pricing model and highlighted the effect of covariance risk on optimal inventory policy. Considering demand in a form of fuzzy number Mahata [6] discusses the single-period inventory model in which the retailer has the opportunity to reorder once during the period. In real-life situations, a business unit might sell several newsvendor-type products and also have various constraints. Mohan et al.[7] consider the linear demand in a single period inventory model. Sindhuja et al.[8] discussed a quadratic demand in their economic ordering quantity model. Kumar et al. [9]discussed a pharmaceutical inventory model with price sensitive demand. Sangal et al.[10] proposed a joint inventory model with reliability and carbon emission. Jauhari et al.[11] discussed price dependent demand in their integrated inventory model. Jauhari and Saga[12] consider set up cost and service level constraints in their joint economic lot sizing problem. Jauhari [13] proposed an inventory model with idea of defective products and inspection error he assumed stochastic demand in his model. Khanna et.al [14] emphasised on pricing decision for imperfect production system. The multi-product newsvendor problem is one of the most realistic extensions which have been studied extensively by many researchers. Su and Pearn [15]consider the demand to follow a normal distribution and extended the newsvendor model to find the optimal ordering quantity to achieve a target profit. Li et al. [16] extended the single product newsvendor problem to two product newsvendor problems with independent exponential demands and suggested an analytical solution procedure. Many researchers have incorporated the reservation strategy into the newsvendor problems, like the newsvendor problem with simple reservation strategy, Chen and Chen [17], multiple item budget constraint with reservation policy Chen and Chen[18], and reservation policy with fuzzy demand Chen and Chen 2010[19]. Khouja and Mehrez[20] solved the multiproduct newsvendor problem with the consideration of budget constraints and multiple discounts. In his paper, he demonstrated that the multiple discount system motivates a higher demand and leads to higher expected profit. Chung and Flynn [21] introduced the idea of reactive production to the classic newsvendor problem and conclude that costs under the reactive production model can be considerably less than that of the costs under the classic newsvendor problem. Balki and Benkherouf [22] generalized the algorithm for linear demand rate functions and included the deteriorating item with time and stock-dependent demand rates and suggested a procedure to find the optimal replenishment schedule. Khouja [23] has discussed this classical single-period newsvendor model and suggested some extensions to the existing model. Giria and Chakraborty[24] develop a supply chain coordination model with a single vendor and a single buyer and through numerical illustrations shown that channel coordination earns significant cost savings over the non-coordinated policy. Ibarra-Salazar [25] developed a newsvendor model to analyse the qualitative effects of increases in the demand riskiness and of changes in fixed costs. They have shown that with the increase in the risk, the risk-averse newsvendor reduces the optimal ordering quantity. Bouakiz and Sobel[26] studied the inventory control under the effect of an exponential utility function. They explained the finite horizon and infinite horizon problem and concluded that OOQ is given by a sequence of critical numbers if the setup costs are linear and the penalty and carrying costs are convex. Zhang et al. [27]consider the newsvendor problem with a range of information and proposed a competitive ratio analysis to manage risk and reward of newsvendor problems which can help the newsvendor in choosing the optimal reward/risk order strategies. Ota et al.[28] used the log utility function for developing a method to maximize the expected utility by determining the OOQ based on the different levels of the investor's initial wealth. Ota et al. [29] considered the risk faces by the manufacturer to meet the volatile demand in their model. They assume that the production rate in the short run is constant and found the optimal initial stock requirement to meet the peak period demand. The remainder of this paper is presented as follows. Section 2, introduces utility theory with the utility function for the risk-neutral and risk-averse investor. In addition to this, the notation that is being used in the model is included in this section. Section 3 considers the profit model and developed the solution methodology for the proposed model. In Section 4 the numerical examples are illustrated and the sensitivity analysis is performed by considering the crucial parameters. At the end Section 5 deals with the concluding remarks of the results and suggestions for future work.

2. DEVELOPMENT OF THE MODEL

2.1. Utility theory

The utility is the satisfaction that an individual obtains from a particular course of action, such as the consumption of goods. This paper generalizes this analysis by considering the situation of the newsvendor who faces the uncertain market demand and tries to maximize his utility under the uncertainty. For a newsvendor, the utility depends on original market demand which is not certain. So the newsvendor faces the risk that he may mismatch between the order quantity and the market demand. Given the uncertainty involved, the newsvendor can no longer maximize his utility. Instead, attempt to maximize his expected utility by choosing the optimal ordering quantity.

2.2. Utility function and assumptions

A utility function is a mathematical function that ranks different alternatives according to their utility to an individual. While applying the utility theory in mathematical finance it is assumed that a numerical value called the utility can be assigned to each possible value of the investor's wealth by what is known as a preference function or utility function.

The properties of the utility function can be used to express the investor attitudes towards risk and return. Thus a utility function can be chosen from the available utility functions that are being used to model investor preferences according to whether or not the investor likes, dislikes, or is indifferent towards risk.

This paper considers that the newsvendor has a quadratic utility function of the form $U(w) = aw^2 + w$,where w is his initial wealth or next period wealth and a is a parameter that determine the shape of the utility function. Based on the value of a this utility function characterized the investor preference towards risk. The specific values of the parameter depend on the preferences of the individual and the particular good or service being consumed. Generally, economists estimate the values of this parameter using empirical data on consumer behavior.

2.3. Risk neutral investor

For, $a = 0$ the utility function becomes $U(w) = w$. Hence $U'(w) = 1$ and $U''(w) = 0$

A risk neutral investor is indifferent between a fair gamble and the status quo. The condition of the utility function is $U'(w) > 0$ and $U''(w) = 0$ for the risk neutral investor. Thus the condition for a risk-neutral investor and utility function can satisfy the condition of non-satiation over a range of w such that $w > 0$

2.4. Risk Averse investor

For,$a \neq 0$ the utility function becomes $U(w) = aw^2 + w$ Hence $U'(w) = 2aw + 1$ and $U''(w) = 2a$. A risk-averse investor values an incremental increase in wealth less highly than an incremental decrease. The condition to the utility function is $U'(w) > 0$ and $U''(w) < 0$ Thus the condition for a risk-averse investor and utility function can satisfy the condition if
$U''(w) < 0 \Rightarrow 2a < 0 \Rightarrow a < 0$
but $U'(w) > 0 \Rightarrow 2aw + 1 > 0 \Rightarrow 2aw > -1 \Rightarrow 0 < w < \left(-\frac{1}{2a}\right)$

3. THE MODEL

Consider that the newsvendor has an initial wealth of w and invest in short-lived products like newspaper, milk etc. When the newsvendor goes for the investment his next period wealth can be expressed as follows.

$$w^* = \begin{cases} w - pQ + sD + v(Q-D) - h(Q-D), & D \leq Q \\ w - pQ + sQ - c(D-Q) & D > Q \end{cases} \tag{1}$$

Where p is the purchase cost per unit nominal, s is selling price per unit nominal, v is salvage value per unit nominal for each unsold unit, h is the holding cost, c is the shortage cost, and Q is the is ordering quantity. Here it is considered that there is a holding cost for the unsold goods up to the time of disposal at salvage value. The demand D at any point of time is a random variable and not known in advance. It is assumed that the demand

$D \sim lognormal(\mu, \sigma^2)$ for $f(D)$ is the probability density function of D Equation (1) further can be written as:

$$w^* = \begin{cases} w - (s - v + h)D - (p - v + h)Q, & D \leq Q \\ w - cD - (p - s - c)Q, & D > Q \end{cases} \tag{2}$$

The demand for the products is uncertain and considered as a random variable. Also, the profit of the newsvendor depends on the actual quantity demanded. Thus the next period wealth of the newsvendor is also a random variable. Consider that the newsvendor has a quadratic utility function of the form $U(w) = aw^2 + w$, where w is the initial wealth and a is a suitable parameter. Hence his utility of wealth can be expressed as:

$$U(w^*) = \begin{cases} a[w + (s - v + h)D - (p - v + h)Q]^2 \\ \quad + [w + (s - v + h)D - (p - v + h)Q] \quad D \leq Q \\ a[w - cD - (p - s - c)Q]^2 + [w - cD - (p - s - c)Q] \\ \quad D > Q \end{cases} \tag{3}$$

Since the next period wealth of the newsvendor is a random variable it is not possible to maximize the newsvendor utility with certainty. However, since the functional form of the utility function is known, we can find the expected utility. Therefore the newsvendor will choose his OOQ which will maximize his expected utility. Hence the newsvendor next period expected utility of wealth will be$E(U(w^*))$ where

$$\begin{aligned} E[U(w^*)] = &\int_0^Q a[(w-(p-v+h)Q)+(s-v+h)D]^2 f(D)dD+ \\ &\int_o^Q [(w-(p-v+h)Q)+(s-v+h)D]f(D)dD+ \\ &\int_Q^\infty a[w-(p-s-c)Q)-cD]^2 f(D)dD+ \\ &\int_Q^\infty [(w-(p-s-c)Q)-cD)]\,f(D)dD \end{aligned} \tag{4}$$

Now to find the OOQ Q^* for which the expected utility of the newsvendor become maximum, differentiate the expected utility function $E[U(w^*)]$ with respect to Q and equate it to zero. Thus we have

$$\begin{aligned} \frac{\partial E[U(w^*)]}{\partial Q} = &-(p-v+h)[2a[w-(p-v+h)Q\int_0^Q f(D)dD+ \\ &2a(s-v+h)\int_0^Q Df(D)dD+\int_0^Q f(D)dD] \\ &-(p-s-c)[2a[w-(p-s-c)Q]\int_Q^\infty f(D)dD \\ &-2ac\int_Q^\infty Df(D)dD+\int_Q^\infty f(D)dD] \end{aligned} \tag{5}$$

Since the demand variable assumes to follow the lognormal distribution

$D \sim lognormal(\mu, \sigma^2)$

$$\begin{aligned}
E\left[U(w^*)\right] = a &\Big[(w-(p-v+h)Q)^2 \Phi\left(d\right) + (s-v+h)^2 e^{2\mu+2\sigma^2}\left[\Phi\left(d-2\sigma\right)\right] \\
&+2\left(w-(p-v+h)Q\right)\left(s-v+h\right)e^{\mu+\frac{1}{2}\sigma^2}\left[\Phi\left(d-\sigma\right)\right]\Big] \\
&+\left(w-(p-v+h)Q\right)\Phi(d) + (s-v+h)e^{\mu+\frac{1}{2}\sigma^2}\Phi(d-\sigma) \\
+a &\Big[\left\{w-(p-s-c)Q\right\}^2\left(1-\Phi\left(d\right)\right) + c^2 e^{2\mu+2\sigma^2}\left\{1-\Phi\left(d-2\sigma\right)\right\} \\
&-2c\left\{w-(p-s-c)Q\right\}e^{\mu+\frac{1}{2}\sigma^2}\left(1-\Phi(d-\sigma)\right)\Big] \\
&+\left(w-(p-s-c)Q\right)\left(1-\Phi(d)\right) - ce^{\mu+\frac{1}{2}\sigma^2}\left(1-\Phi(d-\sigma)\right)
\end{aligned} \tag{6}$$

where $d = \left(\frac{\log Q - \mu}{\sigma}\right)$
And the partial derivative becomes

$$\begin{aligned}
\frac{\partial E[U(w^*)]}{\partial Q} = &-(p-v+h)[2a\{w-(p-v+h)Q\}\Phi(d)+ \\
&2a(s-v+h)e^{\mu+\frac{1}{2}\sigma^2}\Phi\left(d-\sigma\right)+\Phi(d)] \\
&-(p-s-c)[2a\{w-(p-s-c)Q\}(1-\Phi(d))- \\
&2ace^{\mu+\frac{1}{2}\sigma^2}\{1-\Phi\left(d-\sigma\right)\}+(1-\Phi(d))]
\end{aligned} \tag{7}$$

For a risk neutral investor $a = 0$, therefore the Equation (7) reduce to

$\frac{\partial E[U(w*)]}{\partial Q} = (v-h-s-c)\Phi(d)-(p-s-c)$ and $\frac{\partial^2 E[U(w^*)]}{\partial Q^2} = \frac{1}{Q\sigma}(v-h-s-c)\Phi(d)$
Since the quantity $(v-h-s-c) < 0$ the quantity must also $\frac{1}{Q\sigma}(v-h-s-c)\Phi(d) < 0$ and the condition $\frac{\partial^2 E(U(w^*))}{\partial Q^2} < 0$ satisfies.

Thus the existence of maximal value can be verified. The maximal values of $E(U(w^*))$ occur at Q^* satisfying $\frac{\partial E(U(w^*))}{\partial Q} = 0$.

For the risk-averse investor $a < 0$. Now differentiating equation (7) we have

$$\begin{aligned}
\frac{\partial^2 E[U(w^*)]}{\partial Q^2} = &-(p-v+h)[-2a(p-v+h)\Phi(d)+ \\
&2a\{w-(p-v+h)Q\}f(d)\left(\frac{1}{Q\sigma}\right) \\
&+2a(s-v+h)e^{\mu+\frac{1}{2}\sigma^2}f\left(d-\sigma\right)\left(\frac{1}{Q\sigma}\right)+f(d)\left(\frac{1}{Q\sigma}\right)] \\
&-(p-s-c)[[-2a(p-s-c)](1-\Phi(d))+ \\
&2a\{w-(p-s-c)Q\}f(d)\left(\frac{-1}{Q\sigma}\right)+ \\
&2ace^{\mu+\frac{1}{2}\sigma^2}f\left(d-\sigma\right)\left(\frac{1}{Q\sigma}\right)-f(d)\left(\frac{1}{Q\sigma}\right)]
\end{aligned} \tag{8}$$

It can be seen that the sum of purchase cost and holding cost is higher than the salvage value and we must have $p - v + h > 0$. Also, the sum of selling price and holding cost is higher than the salvage value and we have $s - v + h > 0$. On the other hand, the sum of selling price and shortage cost must be higher than the purchase cost and therefore $p - s - c < 0$. Since $0 < w < \left(-\frac{1}{2a}\right)$ and we must have $a < 0$, and the value of a has to be sufficiently small for a wide range of initial wealth. Consequently, $E[U(w^*)]$ is a concave function of Q and hence for the specified range of values of a, the value of Q^* that obtained from Equation (4) is unique and attend its maximum. Thus the existence of maximal value can be verified. The maximal values of $E(U(w^*))$ with initial wealth occur at Q^* satisfying $\frac{\partial E(U(w^*))}{\partial Q} = 0$

$$\begin{aligned} \Rightarrow -(p-v+h)[2a\{w-(p-v+h)Q\}\Phi(d)+2a(s-v+h) \\ e^{\mu+\frac{1}{2}\sigma^2}\Phi\left(d-\sigma\right)+\Phi(d)] \\ -(p-s-c)[2a\{w-(p-s-c)Q\}(1-\Phi(d)) \\ -2ace^{\mu+\frac{1}{2}\sigma^2}\{1-\Phi\left(d-\sigma\right)\}+(1-\Phi(d))]=0 \end{aligned} \tag{9}$$

Thus, we can obtain an OOQ once the parameters of the model are specified. Subsequently, the newsvendor expected utility and hence the expected profit based on the initial wealth and quadratic utility function can be estimated.

4. NUMERICAL EXAMPLES

Feasibility of the proposed model is shown by numerical illustrations at different levels of initial wealth and considering different demand parameters. Let us consider the parameter values of the newsvendor model as ($p = \$10, s = \$25, v = \$5, h = \$1, c = \$5$). Also, it is considered that the demand $D \sim lognormal(5, 4)$. Since different investors have a different level of initial wealth and a different attitude toward risk, this paper looks for to investigate, what will be the different OOQ and the expected utility based on different levels of initial wealth of different type of investor.

4.1. The Risk neutral investor

The risk-neutral investor is indifferent between a fair gamble and the status quo and initial wealth does not affect the choice of the optimal ordering quantity. Only the change in the price of the different crucial parameters can affect the optimal ordering quantity. Therefore the OOQ is Q*=647 for the risk neutral investor irrespective of any level of initial wealth with the above value of the parameters. The expected profit is \$2,458.

4.1.1. Purchase cost

Figure 1 shows the variation in expected utility and OOQ based on the purchase cost. Considering other variable values remaining unchanged, the value of the purchase cost assigned a value in the range between \$5 to \$15, by changing in the increment of \$1 to observe the effect of it on the expected utility and OOQ. Figure 1 demonstrates that both expected utility and OOQ are having a negative correlation with the purchase price. At a lower purchase cost, the profit derived from the additional units of stock sold is higher than the shortage cost which will motivate the investor and leads to a higher OOQ and expected utility. But at a high purchase cost, the profit derived from each unit of an item sold is lesser than the shortage cost leads to a decrease in OOQ and expected utility.

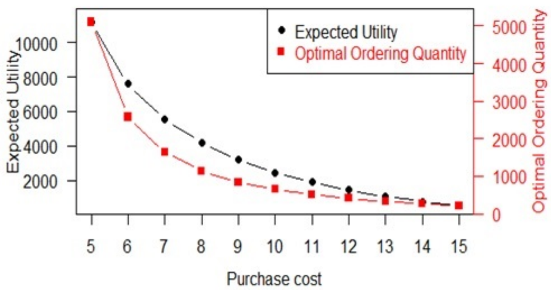

Figure 1: Purchase cost

4.1.2. Selling Price

Figure 2 shows the variation in expected utility and OOQ based on selling price. Considering other variable values remaining unchanged, the value of the selling price assigned a value in the range between \$12.5 to \$37.5, by changing in the increment of \$2.5 to observe the effect of it on the expected utility and OOQ. Figure 2 demonstrates that both expected utility and OOQ are positively correlated with the selling price. At a lower level of selling price the opportunity cost derived from each extra unit of stock sold is lesser than the opportunity cost derived from each extra unit of stock sold at a higher selling price. Therefore at a low level of selling price the investor has lower OOQ and expected utility and from the figure it can be observed that as the selling price increases the OOQ as well as expected profit increases.

Figure 2: Selling price

4.1.3. Shortage Cost

Figure 3 shows the variation in expected utility and OOQ based on shortage cost. Considering all the other variable values remaining unchanged, the value of the shortage cost assigned a value in the range between \$2.5 to \$7.5, by changing in the increment of \$.5 to estimate the effect of this to the expected utility and OOQ. It can be observed that shortage cost has a positive impact on OOQ whereas it has a negative impact on expected utility. This relation with the OOQ can be easily explained as, at a lower shortage cost, the loss incurred from the unsold stock is higher, which triggers the investor to underestimate the market demand and order lesser quantity. On the other hand at a high shortage cost, the loss incurred by the shortage cost is higher than the cost of the unsold stock. Therefore at a higher shortage cost, the OOQ is higher. However a lower shortage cost leads to higher profit and higher expected utility and as shortage cost increases, it eroded the profit and hence reduces the expected utility.

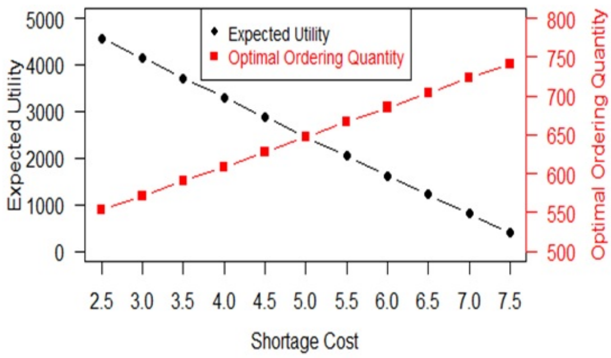

Figure 3: Shortage cost

4.1.4. Demand Growth rate

Figure 4 shows the variation in expected utility and OOQ based on the drift parameter. Considering other variable values remaining unchanged, the value of the demand growth rate assigned a value in the range between 2.5 to 5.5, by changing in the increment of 0.5 to estimate the effect of this to the expected utility and OOQ. From figure 4, it can be observed that the demand growth rate is positively correlated with the OOQ whereas it is negatively correlated with the expected utility. Since in this model as the demand growth rate increase both the mean demand and the variance of the demand increases which leads to highly volatile demand. Because of this highly volatile demand, the OOQ is high but the expected utility is less at a higher demand growth rate.

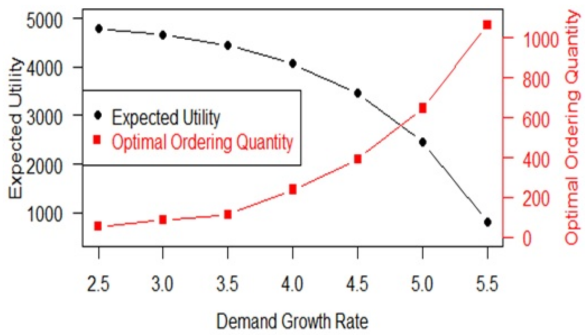

Figure 4: Demand growth rate

4.1.5. Demand volatility rate

Figure 5 shows the variation in expected utility and OOQ based on diffusion parameters. Considering other variable values remaining unchanged, the value of the demand volatility rate assigned a value in the range between 1 to 2, by changing in the increment of 0.25 to estimate the effect of this to the expected utility and OOQ. Figure 5, demonstrates that the demand volatility rate is positively correlated with the OOQ whereas it is negatively correlated with the expected utility. Since in this model as the demand volatility rate increase both the mean demand and the variance of the demand increases which leads to high volatile demand. Because of this highly volatile demand, the OOQ is high but the expected utility is less at a higher demand volatility rate.

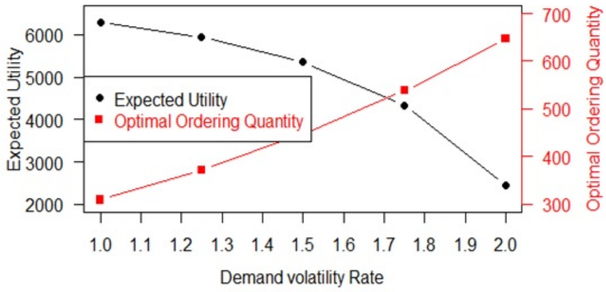

Figure 5: Demand volatility rate

4.1.6. Salvage Value

Figure 6 shows the variation in expected utility and OOQ based on Salvage values. Considering other variable values remaining unchanged, the value of the salvage value assigned a value in the range between $2.5 to 7.5$, by changing in the increment of 0.5 to estimate the effect of this to the expected utility and OOQ. From figure 6, it can be observed that both OOQ and expected utility are positively correlated with the salvage value. This correlation is easily explained. Presumably, a lesser salvage value leads to a higher loss for the disposable items where a higher salvage value reduces the loss of overestimated stocks. Accordingly, at a lower level of salvage value, both OOQ, as well as expected utility, is less and the salvage value increases both OOQ and expected utility increases.

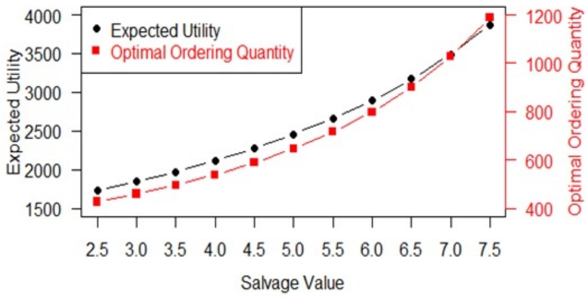

Figure 6: Salvage value

4.2. The risk Averse Investor

The risk-averse investors value an incremental increase in wealth less highly than an incremental decrease in wealth and reject a fair gamble. For a risk-averse investor, not only the change in the price of the different crucial parameter affect the OOQ but also the initial level of wealth play an important role and affect on the choice of the optimal ordering quantity.

4.2.1. Initial Wealth

Since we have kept the initial wealth to maximum level of 50,000, we need our parameter value of $a \leq -10^{-5}$, since $0 < w < (-1/2a)$. In this numerical calculation we have consider the value of $a = -10^{-5}$. Figure 7 shows the results of sensitivity analysis involving the initial wealth and reveals a clear divergence between OOQ and expected utility in response to changes in initial wealth. Expected utility and OOQ increas with the increase in the initial wealth and thus initial wealth significantly and positively influence the OOQ and expected utility. With the increase in initial wealth, OOQ increases exponentially whereas expected utility increases logarithmically. Initial wealth plays a very important role in decision making and at a higher initial wealth the investor can take more risk leads to higher OOQ and expected utility.

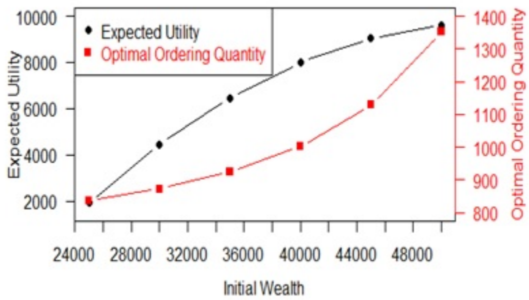

Figure 7: Initial wealth

4.2.2. Purchase cost

Figure 8 demonstrates the variation in expected utility and OOQbased on purchase cost at an initial wealth of \$ 50,000. Considering other variable values remaining unchanged, the value of the purchase cost assigned a value in the range between \$7 to \$13, by changing in the increment of 1 to estimate the effect of this to the expected utility and OOQ. From figure 8, it can be seen that both OOQ and expected utility negatively correlated with the purchase cost. This can be easily explained as a lesser purchase cost increases the profit whereas a higher purchase cost reduces the profit. So at a lower purchase cost the OOQ, as well as the expected utility, is higher whereas the purchase cost increases both the OOQ and expected utility gradually decreases.

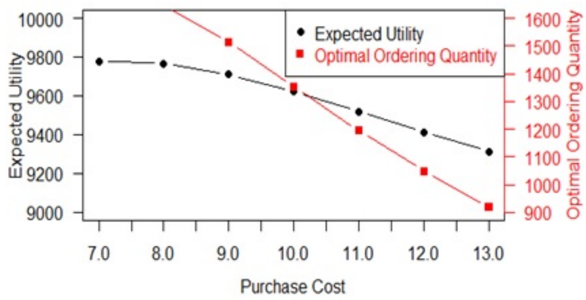

Figure 8: Purchase cost

4.2.3. Shortage Cost

Figure 9 shows the variation in expected utility and OOQ based on shortage cost at an initial wealth of $ 50,000. Considering all the other variable values remaining unchanged, the value of the shortage cost assigned a value in the range between $2.5 to $6, by changing in the increment of $.5 to estimate the effect of this to the expected utility and OOQ. From figure 9, it can be seen that shortage cost has a positive impact on OOQ whereas it has a negative impact on expected utility. The shortage cost has the same impact on both the risk-neutral and risk-seeking investor. At a higher shortage cost, the investor is forced to order a higher quantity to offset the shortage and due to that, the expected utility reduces. On the other hand, lower shortage cost encourages the investor to underestimate the market demand order lower quantity.

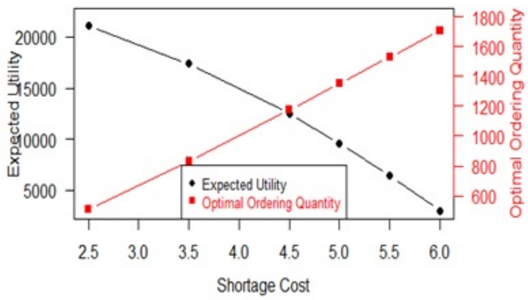

Figure 9: Shortage cost

4.2.4. Demand growth rate

Figure 10 shows the variation in expected utility and OOQ based on the drift parameter at an initial wealth of $ 50,000. Considering other variable values remaining unchanged, the value of the demand growth rate assigned a value in the range between 2.5 to 5.5, by changing in the increment of 0.5 to estimate the effect of this to the expected utility and OOQ. From figure 10, it can be observed that the demand growth rate is positively correlated with the OOQ whereas it is negatively correlated with the expected utility. In this model, an increase in demand growth rate, increase both the mean demand and the variance of the demand, which leads to a highly volatile demand at a high demand growth level. Because of this highly volatile demand, the OOQ is high but the expected utility is less at a higher demand growth rate.

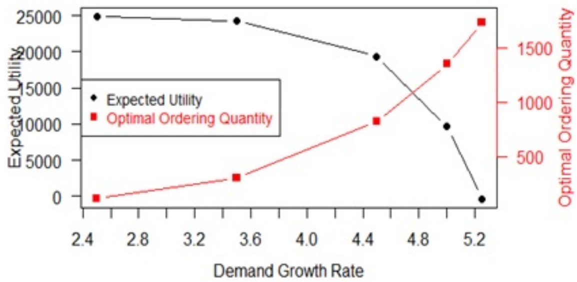

Figure 10: Demand growth rate

4.2.5. Demand volatility rate

Figure 11 shows the variation in expected utility and OOQ based on the change in demand volatility rate at an initial wealth of \$ 50,000.The standard deviation is being commonly used to measure the Demand volatility rate, which indicates uncertainty and volatility of the market demand, and this demand volatility influence significantly on the optimal order quantity and expected profits. Since high demand volatility significantly reduces the predictability of the market demand, higher volatility often adversely impacts expected profits. Consequently, considering other variable values remaining unchanged, the value of the demand volatility rate assigned a value in the range between 1 to 2, by changing in the increment of 0.25 to estimate the effect of this to the expected utility and OOQ. From figure 11 it can be observed that at a lower volatility rate the investor have lower OOQ and higher expected utility, but as the volatility increases the investor needs to order a higher quantity and meet the volatile demand and thus the expected profit reduces.

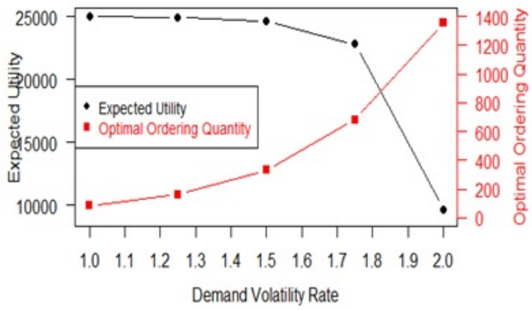

Figure 11: Demand volatility rate

4.2.6. Change in mean demand

Figure 12 shows the variation in expected utility and OOQ based on the change in mean demand keeping the variance of demand constant at an initial wealth of \$ 50,000. Considering other variable values remaining unchanged and keeping the variance of demand constant, the value of the mean demand assigned a value in the range between 500 to 700 by changing in the increment of 200 to estimate the effect of this to the expected utility and OOQ. From figure 12, it can be observed that the mean demand rate exhibit a close and positive linear relationship with both the OOQ and expected utility. So a risk-averse investor has a lower OOQ and expected utility at a lower level of demand and the OOQ and expected utility increases linearly with an increase in mean demand.

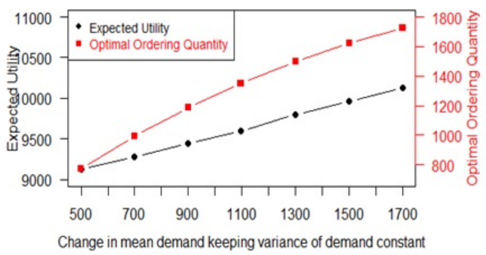

Figure 12: Change in mean demand

5. CONCLUDING REMARKS AND FUTURE WORK

To reflect the need for the different types of investors, this paper extended the classical newsvendor model to the risk-averse and risk-neutral newsvendor model by introducing the quadratic utility function to the model. The paper provides two solutions to find the OOQ by considering whether the investor is risk-averse or risk-neutral. From the illustrated examples, it can be seen that initial wealth plays an important role in deciding the OOQ for a risk-averse newsvendor whereas there is no effect of initial wealth on the OOQ for the risk-neutral investor. From the sensitivity analysis, it can be inferred that at a lower level of initial the optimal order quantity of a risk-averse investor is lower than that of a risk-neutral investor. On the other hand as the wealth of a risk-averse investor increases the OOQ increases and it is higher than that of a risk-neutral investor. The effect of the different parameters like purchase cost, selling price, demand growth rate, demand volatility, etc have the same impact for both the risk-neutral and risk-averse investor. In conclusion, the newsvendor model that is presented here and the experimental finding may be helpful to the different types of investors whose risk profiles match with that of the proposed model. For further research may include the variant demand distribution with different utility functions. In addition, the quadratic utility function which is considered in this paper, future work could develop the state depended on utility function which will illustrate the behavior of the investors at a different level of wealth. Finally, we conclude that the proposed newsvendor with utility maximization model in this paper not only gives an alternative approach to the model, but it matches the real world situations better.

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