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OPTIMAL ORDERING AND PRICING DECISION FOR ITEMS FOLLOWING PRICE SENSITIVE QUADRATIC DEMAND UNDER COMBINED PAYMENT SCHEME

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Abstract: Three types of payments confront in business dealings viz. advance, cash and credit. A cash payment is ubiquitously practiced, while a credit payment scheme escalates sales and advance payment technique is implemented to avoid cancellation of orders. These are recognized as advance-cash-credit (ACC) payment scheme which is conventional in business transactions. By implicating ACC the aim is to determine optimal pricing and ordering policies. Inventory items observes quadratic demand being sensitive to unit selling price. Promising concavity of the profit function for both selling price and cycle time is posed numerically. Managerial insights are revealed as a concluding note.

Keywords: Advance-cash-credit scheme, price sensitive quadratic demand, ordering policies, trade credit.

MSC: 90B05, 90B85, 90C26.

1. INTRODUCTION

In business deals, the players of supply chain (i.e. suppliers, retailers or customers) prefer different payment method to settle their account for goods and services. In practice, they follow either of this approach: (i) A player of supply chain pays the purchase cost of the ordered items prior to the delivery of order, termed as advance payment, (ii) Secondly, the purchase cost of the ordered items is paid when the order is delivered, known as cash payment, (iii) Last alternative is an interest free permissible delay period offered when the items are purchased and player gets a fixed time to settle the account, called credit period.

There are associated pros and cons for each of these payment option based on certain circumstances. In advance payment scheme, seller can earn interest on the advance payment received. Additionally, no risk of defaulters is involved. As a drawback, this payment option may deplete the sales. Research community has contributed several inventory models with the idea of advance payment. Zhang [1] suggested that it would be convenient for customer to pay \$60 of water bill in advance for 3 months rather than paying \$20 for every month as it would save customers time. Mateut and Zanchettin [2] indicated that payments made in advance is an indication that consumer's trust the retailer. Zhang et al. [3] suggested that by collecting advance from consumer the threat of order cancellation reduces. The advance payment scheme may further favour the consumer when seller provides discounted price while opting for advance payment.

Next, researchers have contributed several inventory models using the cash payment assumption. Lastly, the credit period option turns out to be a beneficial as it helps in getting more sales. The consumer is attracted as the buying cost is reduced from his point of view, while from seller's view it involves the risk of default. The idea of permissible delay period/credit period into inventory problems was given by Haley and Higgins [4]. The first EOQ model with credit period option was given by Goyal [5]. In this model, Mandal and Phaujdar [6, 7] assimilated the idea of earing interest on the revenue generated by making sales. Goyal's model was also extended by Aggarwal and Jaggi [8] for deteriorating items which was further extended by Jamal et al. [9] by incorporating shortages. Researchers have also developed models in which the supplier provides both credit period and cash discount at the same time, i.e., cash discount is offered to encourage retailer to make payment quicker. Few articles under with cash discount assumption are Chang [10], Chen and Chuang [11], Ouyang et al. [12] and Sana and Chaudhuri [13].

It is a common assumption in the inventory models that permissible delay period is only given to retailer by supplier. But, Huang [14] found that it is gainful for the retailer when the received credit period is passed on to end consumers. In this model, it was taken as an assumption that retailer gets a permissible delay period M and passes on period N (with $M > N$) to the consumer. This practice is also labeled as two level trade credit. Later, the assumption of $M > N$ was relaxed by Teng and Chang [15]. In order to gain more insights of credit period into inventory models, Chang et al. [16] and Soni et al. [17] can be referred. Some of the scholarly articles using this notion are Min et al. [18] & Shah et al. [19].

In economics, it is recognized that lower the price, more is the demand and vice versa. Hence, selling price of an item is an important parameter effecting its demand rate. If the selling price of an item is kept high, then there is a possibility that the consumers may opt for some alternative where they avail a better deal. In case, if it is kept too low it may incur loss to the seller. Hence, to sustain in market there is a need to determine optimal selling price. This will result increase in profit and seller would be able to maintain competitiveness in a supply chain. Urban and Baker [20] determined optimal order size and price of an item where demand rate is a multivariate function displayed stock and price. Teng and Chang [21] extended the EOQ model to an EPQ model for deteriorating items. Chang et al. [22] and Wu et al. [23] framed inventory models for items following for non-instantaneous deterioration with demand sensitive to displayed stock and price. Shaikh et al. [24] worked out an EPQ model decaying items considering the reliability factor of production system and also incorporating partial trade credit rule. In a two-warehouse system, a sustainable inventory model for perishable goods following price dependent demand was developed by Mashud et al. [25] inculcating advance and delayed payments.

It has been observed that when a new product is launched in the market its demand increases with time and reaches a peak after which it starts falling down, this pattern of demand is known as quadratic demand. Shah and Chaudhari [26] articulated a supply chain model with three different players for units possessing fixed lifetime and credit period sensitive quadratic demand. An integrated model having 3 players was also developed by Shah [27] for deteriorating items following quadratic demand. Shah et al. [28] studied a defective manufacturing structure in an inflationary environment for items following quadratic demand. Shaikh and Mishra [29] formulated an EOQ model with two level trade credit for deteriorating items following quadratic demand. Further, for deteriorating items with demand being quadratic and also dependent on selling price, Shaikh and Mishra [30] framed a model with deterioration being controlled by spending on preventive measures.

The above cited articles have either of the payment policy implemented into their model. Some researchers have provided inventory models combining advance, credit and cash payment option in different ways. Teng [31] used cash credit payment option where proportion of purchase amount is to be paid while placing an order and the remaining is paid after delay period. An advance cash payment scheme was used by Taleizadeh [32] wherein a gas supplier asks his customer to prepay proportion of purchase cost while placing an order while the remaining is to be paid upon the delivery of product. Zhang et al. [3] used the advance credit scheme where customer prepay proportion of purchase cost which reduces the risk of withdrawing order and on the remaining amount a permissible credit period is granted to boost the sales. A very few researchers have utilized the idea of ACC payment, which is a widely used practice in day to day life. In literature, Wu et al. [33] studied an inventory model with shortages using ACC payment scheme. Li et al. [34, 35] used ACC payment scheme and formulated an EOQ model without shortages. Chang et al. [36] developed an EPQ model for depreciating units using

ACC payment scheme. Mishra et al. [37] determined the optimal payment option and cycle time for the retailer. Feng et al. [38] determined optimal selling price, cycle time and payment option for the seller. Shi et al. [39] studied replenishment policies for deteriorating units under carbon tax regulation using ACC scheme. Shi et al. [40] also presented refilling plan for items following increasing demand. Li et al. [41] formulated an eoq to study pricing and credit decision for customers. Feng et al. [42] framed pricing and lot sizing policies for items with demand dependent on price, showcased goods and its age.

Investigating the existing literature and to the best of our knowledge. It has been observed that there is a gap and need for the proposed model. Here, an EOQ model is constructed where (i) the present value of retailer's total profit is to be maximized by determining the best selling price and cycle time, (ii) the retailer ask the supplier to pay in ACC payment scheme while cash-credit payment scheme is offered to the customer and (iii) we use discounted cash-flow approach to get the time value of money.

The remaining part of this paper is structured as: Section 2 defines the notations used and assumptions made for this model. Section 3 provides the developed mathematical model with different cases. In section 4, methodology to find the optimal values of decision variables is presented satisfying the necessary and sufficient conditions. In section 5, numerical examples pertaining to different cases are provided. Further, inventory parameters are analyzed to provide managerial insights in section 6. Lastly, conclusion is drawn in section 7.

2. NOTATION and ASSUMPTIONS

The following notation are used throughout the paper.

2.1. Notation

C	unit purchase cost, $C > 0$
α	fraction of procurement amount to be paid before items are delivered, $0 \leq \alpha \leq 1$
β	fraction of procurement amount to be paid when items are delivered, $0 \leq \beta \leq 1$
γ	fraction of procurement amount to be paid when the delay period offered by the supplier ends, $0 \leq \gamma \leq 1$ and $\alpha + \beta + \gamma = 1$
M	permissible time period acquired from the supplier, $M \geq 0$
N	permissible time period offered to the customer by retailer, $N \geq 0$
I_e	interest earned rate
I_c	interest charged rate
ρ	fraction of sale amount on which retailer offers credit period, $0 \leq \rho \leq 1$
r	compound interest rate per \$ per year
l	time in year at which advance payment is to be done, $l > 0$
O	ordering cost per order

A	total procurement cost of quantity to be ordered at time $-l$
h	holding cost per unit time excluding interest charges
$I(t)$	inventory level at time t, $0 \leq t \leq T$
Decision Variables	
S	retailer unit selling price, $S > C$
T	replenishment cycle time
Q	procurement quantity per cycle
Functions	
$TP(S,T)$	retailer's total profit per unit time
$D(S,t)$	$a(1+bt-ct^2)S^{-\eta}$; Price sensitive quadratic demand rate; with $a>0,\ b>0,\ 0<c<1$ expressing scale demand, linear and quadratic rate of demand while mark up for selling price is expressed by $\eta > 1$

The objective of this developed model is given below:

Maximize $TP(S,T)$
Subject to $S > C$ and $S,\ T \geq 0$

Note that the asterisk symbol on a parameter represents the optimal value of the parameter. For example, S^* is the optimal value of S.

Next, to formulate the model, following assumptions are made.

2.2. Assumptions

1. The supply chain consists of single supplier and single retailer dealing with single item.
2. The demand function $D(S,t)$ is selling price sensitive and also quadratic function of time. Note that $D(S,t)$ or D will be used interchangeably for notational convenience.
3. When the purchase cost of the ordered quantity is more, the retailer may ask the supplier to pay in ACC scheme. Wherein, the retailer will pay fraction α of the procurement amount A (i.e. αA) l year before the delivery is made. Next, pay fraction β of procurement amount A (i.e. βA) when the items are delivered. Lastly, to pay the remaining γ proportion of the purchase cost A (i.e. γA) retailer avails an interest free delay period M.
   If the supplier do not wish to get paid in advance then, $\alpha = 0$. Likewise, at delivery time if supplier do not wish to take revenue then, $\beta = 0$. Lastly, if supplier do not wish to provide delay period then, $\gamma = 0$. Hence, the ACC term is a broad structure including several payment options as distinct cases.
4. In case, when the retailer gets an upstream credit period from the supplier, he is bound to settle the account as soon as the allowed time period M ends. If he does not have adequate fund, then he has to bare the interest charged by supplier at the rate of I_c. In addition, to have gains from generated revenue, retailer can put up the revenue in a secure interest earning account for a period of $[0,M]$ and earn interest at the rate of I_e.

5. In a similar manner, customers are allowed a time period N on fraction ρ of sales amount from retailer while the remaining fraction $(1-\rho)$ is paid in cash. As a result, ρ proportion of revenue inflows in the period $[N, T+N]$. Observe that when $\rho = 1$, no amount is to be paid in cash and the customers get a full downstream credit period. When $\rho = 0$, the entire amount of sales is to be paid in cash by the customer.
6. Shortages are not allowed. Planning horizon is infinite.

3. MATHEMATICAL MODEL

For the proposed model, inventory level with the retailer is assumed to be Q at time $t = 0$ (i.e. $I(0) = Q$), which depletes to 0 at the end of cycle time T (i.e. $I(T) = 0$) because of the price sensitive quadratic demand. Therefore, the replication of change in retailer's inventory level at time t is given by differential equation (1).

$$\frac{dI(t)}{dt} = -a(1 + bt - ct^2)S^{-\eta}, \quad 0 \le t \le T \tag{1}$$

The solution to (1) using the condition $I(T) = 0$ is

$$I(t) = -aS^{-\eta}\left(-1/3\,t^3c + 1/2\,t^2b + t\right) + aS^{-\eta}\left(-1/3\,T^3c + 1/2\,T^2b + T\right) \tag{2}$$

Next, by using the assumption $Q = I(0)$ the procurement quantity is

$$Q = aS^{-\eta}\left(-1/3\,T^3c + 1/2\,T^2b + T\right) \tag{3}$$

The procurement units are ordered by the retailer l years before the delivery time i.e. $(t = 0)$. Hence, the current worth of ordering cost (OC) is

$$OC = Oe^{rl} \tag{4}$$

The customers get a delay period N on fraction ρ of sales amount from retailer (i.e. at time t customer gets the item and pays the ρ proportion of selling amount at time $t+N$), the remaining $(1-\rho)$ is paid cash. Therefore, the present value of sales amount (SR) for retailer is given by

$$SR = S\rho \int_{N}^{T+N} De^{-rt}dt + S(1-\rho)\int_{0}^{T} De^{-rt}dt \tag{5}$$

The present value of holding cost (HC) for stocking up the inventory items is given by

$$HC = h\int_{0}^{T} I(t)e^{-rt}dt \tag{6}$$

Next, procurement cost of inventory items not considering the time value of money is given by

$$A = CQ \tag{7}$$

According to the assumption, the procurement amount of the items is to be paid in 3 parts; (i) The advance payment in which α proportion of the procurement amount A (i.e. αA) is to be paid l year before the delivery is made, (ii) The cash payment in which β proportion of the procurement amount A (i.e. βA) is paid at the delivery time and (iii) The credit payment in which the remaining γ proportion of the procurement amount A (i.e. γA) is paid at time M. Hence, for retailer the current value of procurement amount (PC) is

$$PC = \alpha A e^{rl} + \beta A + \gamma A e^{-rM} \tag{8}$$

Further, for the retailer, amount of interest earned and interest charged depends on the values of (N and M) downstream and upstream credit period. Based on these values, either of the two cases will occur (i) $M \geq N$ or (ii) $M < N$. Next, we discuss each of this case separately in detail.

3.1. Case 1 $M \geq N$

There will be two sub cases depending on the values of $T + N$ and M (i.e. the time when last payment is received by retailer and credit period).

3.1.1. Subcase 1.1 $M > T + N$

For each cycle, the interest charged for making prepayment and cash payment at present time is given by (9) and it is shown in Figure 1.

$$\begin{aligned} IC_a = \alpha A I_c &\left[\int_{-l}^{N} e^{-rt} dt + \int_{N}^{T+N} \left(\frac{T+N-t}{T} \right) e^{-rt} dt \right] \\ +\beta A I_c &\left[\int_{0}^{N} e^{-rt} dt + \int_{N}^{T+N} \left(\frac{T+N-t}{T} \right) e^{-rt} dt \right] \end{aligned} \tag{9}$$

For this subcase, the retailer will receive the entire sales amount at the time $T + N$ and there will sufficient fund to settle the account before the allowed time period M ends. Hence, zero interest is charged in this case.

$$IC_1 = 0 \tag{10}$$

Next, at present time the interest earned by retailer on cash and credit period

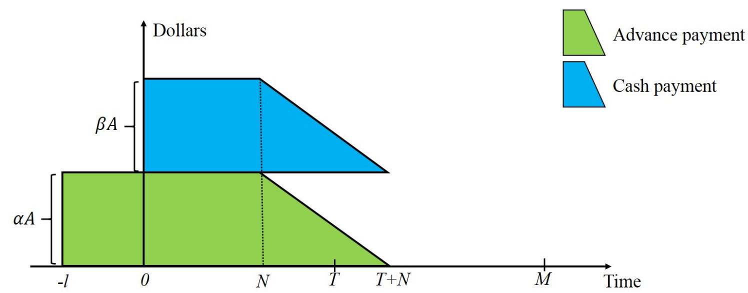

Figure 1: Interest charged trends

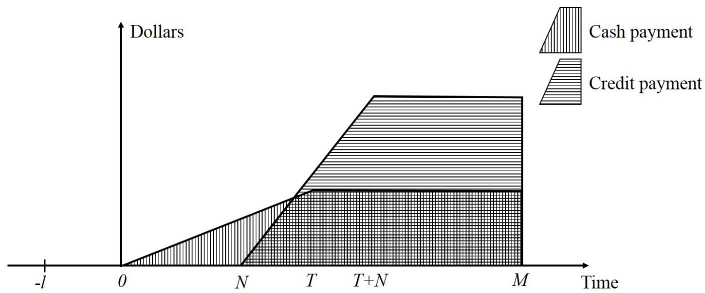

Figure 2: Interest earned on cash and credit payment

is given by (11) and it is shown in Figure 2.

$$IE_1 = SI_e\rho \left[\int_{N}^{T+N} D(T+N-t)e^{-rt}dt + \int_{T+N}^{M} D(T)e^{-rt}dt \right] + SI_e(1-\rho) \left[\int_{0}^{T} D(T-t)e^{-rt}dt + \int_{T}^{M} D(T)e^{-rt}dt \right] \tag{11}$$

Therefore, the current value of retailer's profit is given by

$$TP_1(S,T) = \frac{1}{T}\left(SR - OC - HC - PC - IC_a - IC_1 + IE_1\right) \tag{12}$$

In the next section, we discuss the other sub case.

3.1.2. Subcase 1.2 $M \leq T+N$

For each cycle, the interest charged for making prepayment and cash payment at present time is shown in Figure 3, and it is same as for subcase 3.1.1. Therefore, it is given by (9).

Next, at present time for cash and credit period the interest charged by supplier is given by (13) and it is shown in Figure 4.

$$IC_2 = \gamma A I_c \left[\rho \int_M^{T+N} I(t-N)e^{-rt}dt + (1-\rho) \int_M^{T} I(t)e^{-rt}dt \right] \tag{13}$$

Lastly, at present time for cash and credit period the interest earned by retailer is given by (14) and it is shown in Figure 5.

$$IE_2 = S I_e \left[\rho \int_N^{M} D(t-N)e^{-rt}dt + (1-\rho) \int_0^{M} D(t)e^{-rt}dt \right] \tag{14}$$

Hence, the current value of retailer's profit is given by

$$TP_2(S,T) = \frac{1}{T}\left(SR - OC - HC - PC - IC_a - IC_2 + IE_2\right) \tag{15}$$

At last, we discuss the other case in which the scenario of $M < N$ is discussed.

3.2. Case 2 $M < N$

For each cycle,the interest charged for making prepayment and cash payment at present time is shown in Figure 6 and it is same as for subcase 3.1.1. Therefore, it is given by (9).

At present time for cash and credit period the interest charged by supplier is given by (16) and it is shown in Figure 7.

$$IC_3 = \gamma A I_c \left[\rho \int_M^{N} Q e^{-rt}dt + \rho \int_N^{T+N} I(t)e^{-rt}dt + (1-\rho) \int_M^{T} I(t)e^{-rt}dt \right] \tag{16}$$

The retailer earns interest only for the cash period in this case. Hence, at present time the interest earned is given by (17) and it is shown in Figure 8.

$$IE_3 = S I_e (1-\rho) \left[\int_0^{M} D(M-t)e^{-rt}dt \right] \tag{17}$$

Therefore, the current value of retailer's profit is given is given by

$$TP_3(S,T) = \frac{1}{T}\left(SR - OC - HC - PC - IC_a - IC_3 + IE_3\right) \tag{18}$$

The total profit function is a multivariate function of cycle time and unit selling price. Hence, depending upon the values of cycle time, credit period availed and credit period offered the total profit function is given by

$$TP(S,T) = \begin{cases} TP_1(S,T) & M \geq N,\ M > T+N \\ TP_2(S,T) & M \geq N,\ M \leq T+N \\ TP_3(S,T) & M < N \end{cases} \tag{19}$$

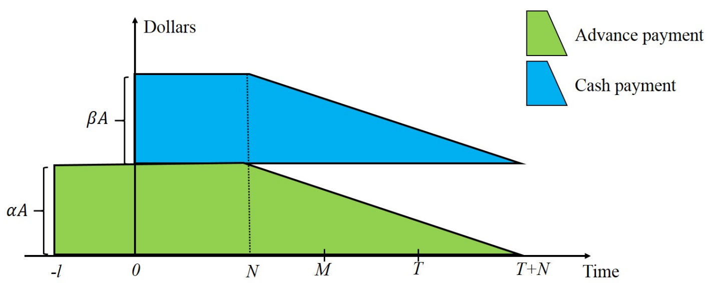

Figure 3: Interest charged trends

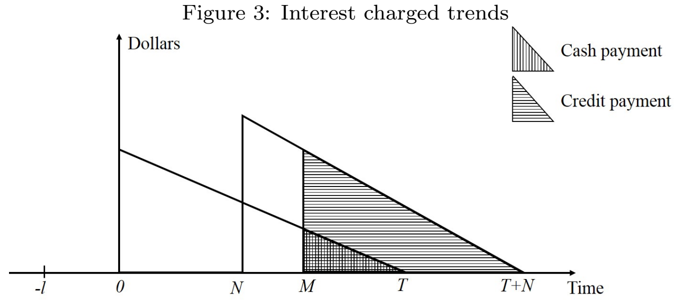

Figure 4: Interest imposed for the credit period

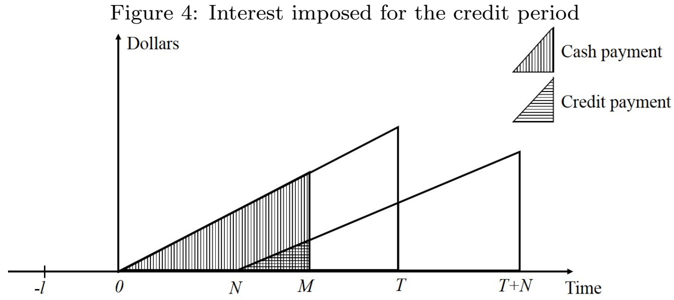

Figure 5: Interest earned on cash and credit payment

The objective is to yield maximum profit for the formulated profit function by determining the best selling price and cycle time for the retailer. In the next section, we propose the algorithm to find the optimal values of the decision variables S and T.

4. ALGORITHM

To maximize the profit function of retailer the necessary condition is to solve simultaneously $\frac{\partial TP_i}{\partial S} = 0$ and $\frac{\partial TP_i}{\partial T} = 0$ for S and T. Then follow the below mentioned steps to decide the optimal policy for the retailer.

1. Except for decision variables, put in values for remaining parameters.

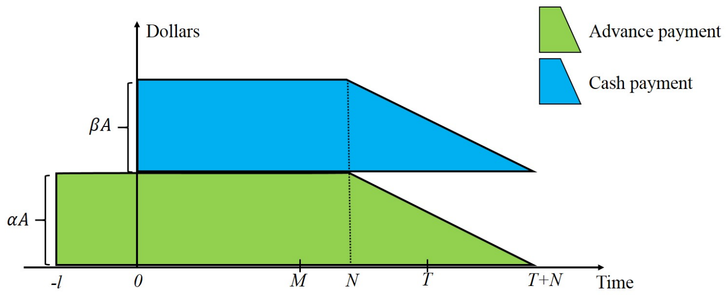

Figure 6: Interest charged trends

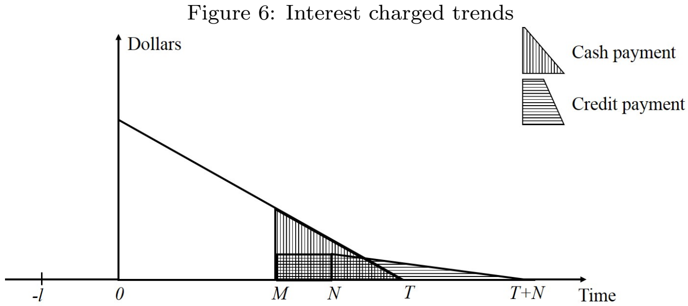

Figure 7: Interest imposed for the credit period

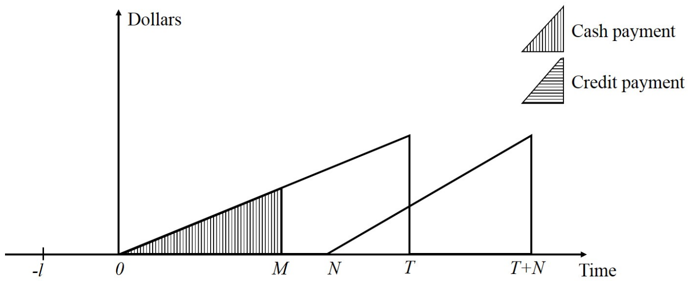

Figure 8: Interest earned on cash payment

2. When $M \geq N$, solve $\frac{\partial TP_1}{\partial S} = 0$, $\frac{\partial TP_1}{\partial T} = 0$ for S and T
   For the obtained values,
   If $M > T + N$, then retailer's profit can be evaluated by substituting the value S^* and T^* in $TP_1(S, T)$.
   Else, solve $\frac{\partial TP_2}{\partial S} = 0$ and $\frac{\partial TP_2}{\partial T} = 0$ for S and T. Using the obtained value of S and T evaluate the total profit from $TP_2(S, T)$ by substituting the value S^* and T^*.
3. For $M < N$, the optimal value of S and T can be obtained by solving $\frac{\partial TP_3}{\partial S} = 0$ and $\frac{\partial TP_3}{\partial T} = 0$ simultaneously. The total profit per unit time is evaluated by substituting the value of S^* and T^* in $TP_3(S, T)$.

After getting the optimal value of S^* and T^*, the optimal ordering quantity (Q) is obtained by substituting these values in (3).

In either case, for S^* and T^* to be the optimal solution. The sufficiency conditions $\frac{\partial^2 TP_i}{\partial S^2} < 0, \frac{\partial^2 TP_i}{\partial T^2} < 0$ and $\begin{vmatrix} \frac{\partial^2 TP_i}{\partial S^2} & \frac{\partial^2 TP_i}{\partial S \partial T} \\ \frac{\partial^2 TP_i}{\partial S \partial T} & \frac{\partial^2 TP_i}{\partial T^2} \end{vmatrix} > 0$ for obtained solution to be optimal are verified.

In the next section, suitable values are assigned to inventory parameters and examples are presented to demonstrate the algorithm and validate the formulated model.

5. NUMERICAL EXAMPLES

Example 1: Consider $a = 5000, b = 0.01, c = 0.05, \eta = 1.5, C = \$5$ per unit, $O =$ \$60 per order, $h = \$2$ per unit per year, $r = 6\%, l = 0.1$ year, $\alpha = 0.2, \beta = 0.2, \gamma = 0.6, \rho = 0.4, M = 0.8$ year, $N = 0.2$ year, $I_e = 8\%$ per year and $I_c = 11\%$ per year

Here $M > N$, solving $\frac{\partial TP_1}{\partial S} = 0$ and $\frac{\partial TP_1}{\partial T} = 0$ simultaneously for S and T we get $S = \$17.92$ and $T = 0.5873$ year. It can be observed that $M > T + N$. Next, for the obtained values of S and T, $\frac{\partial^2 TP_1}{\partial S^2} = -1.68 < 0, \frac{\partial^2 TP_1}{\partial T^2} = -607.38 < 0$ and $\begin{vmatrix} \frac{\partial^2 TP_1}{\partial S^2} & \frac{\partial^2 TP_1}{\partial S \partial T} \\ \frac{\partial^2 TP_1}{\partial S \partial T} & \frac{\partial^2 TP_1}{\partial T^2} \end{vmatrix} = 980.30 > 0$. Therefore, the values obtained for S and T are optimal and concavity of profit function is shown in Figure 9. The total profit per unit time in this case is \$723.79 and the optimal order quantity is 38.49 units.

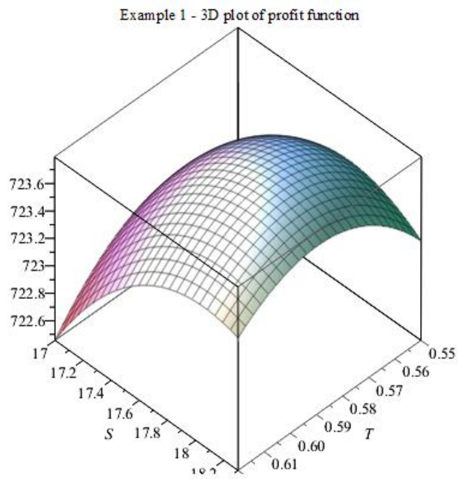

Figure 9: Profit function plot in 3D

Example 2: Consider $N = 0.4$ year and remaining inventory parameters value same as in example 1.

Here also $M > N$, solving $\frac{\partial TP_1}{\partial S} = 0$ and $\frac{\partial TP_1}{\partial T} = 0$ simultaneously for S and T we get $S = \$18.35$ and $T = 0.5845$ year. It can be observed that $M \ngtr T + N$.

Therefore, we solve $\frac{\partial TP_2}{\partial S} = 0$ and $\frac{\partial TP_2}{\partial T} = 0$ simultaneously for S and T we get $S =$ \$19.53 and $T = 0.6302$ year. For the obtained values, the sufficiency conditions $\frac{\partial^2 TP_2}{\partial S^2} = -1.70 < 0$, $\frac{\partial^2 TP_2}{\partial T^2} = -1381.21 < 0$ and $\begin{vmatrix} \frac{\partial^2 TP_2}{\partial S^2} & \frac{\partial^2 TP_2}{\partial S \partial T} \\ \frac{\partial^2 TP_2}{\partial S \partial T} & \frac{\partial^2 TP_2}{\partial T^2} \end{vmatrix} = 2323.69 > 0$ are verified. Therefore, the values obtained for S and T are optimal and concavity of profit function is shown in Figure 10. The total profit per unit time in this case is \$670.52 and the optimal order quantity is 36.37 units.

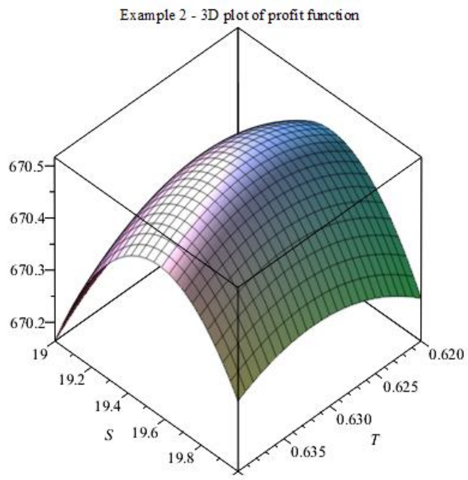

Figure 10: Profit function plot in 3D

Example 3: Consider $N = 1$ year and remaining inventory parameters value same as in example 1.

Here $M < N$, on solving $\frac{\partial TP_3}{\partial S} = 0$ and $\frac{\partial TP_3}{\partial T} = 0$ simultaneously for S and T we get $S =$ \$26.71 and $T = 0.5511$ year. Next, for the obtained values of S and T the sufficiency conditions $\frac{\partial^2 TP_3}{\partial S^2} = -0.96 < 0$, $\frac{\partial^2 TP_3}{\partial T^2} = -961.58 < 0$ and $\begin{vmatrix} \frac{\partial^2 TP_3}{\partial S^2} & \frac{\partial^2 TP_3}{\partial S \partial T} \\ \frac{\partial^2 TP_3}{\partial S \partial T} & \frac{\partial^2 TP_3}{\partial T^2} \end{vmatrix} = 819.26 > 0$ are verified. Therefore, the values obtained for S and T are optimal and concavity of profit function is shown in Figure 11. The total profit per unit time in this case is \$560.12 and the optimal order quantity is 19.91 units.

6. SENSITIVITY ANALYSIS

Now with the values of inventory parameters taken in example 2, we investigate their effect by changing one parameter at a time by -20%, -10%, 0%, 10% and 20%. The impact of changing the inventory parameters on unit selling price, cycle time and retailers profit is shown in Figure 12, Figure 13 and Figure 14, respectively. Examining the obtained results carefully we get the following managerial insights:

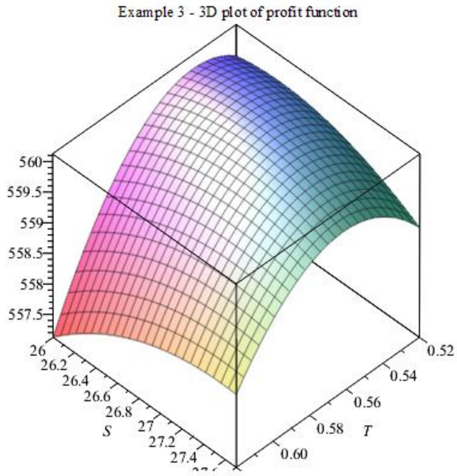

Figure 11: Profit function plot in 3D

- For a, increase in its value results to hike in S and TP but reduces the value of T. This means, when the scale demand increases the retailer must hike up the unit selling price and decrease the cycle time. Thus, there will be an increase in the present value of total profit.
- For b, increase in its value results to hike in values of T and TP while the value of S falls down. It means, when the linear rate of change of demand increases the retailer should raise the cycle time and decrease the unit selling price. Thus, there will be a hike in the present value of total profit.
- Rising the value of c or h or r or ρ induces an increase in value of S, while the values of T and TP falls down. This means, when the quadratic rate of change of demand or holding cost or interest rate or proportion of sales revenue offered permissible delay increases, the retailer should increase the selling price and reduce the cycle time to cover up the drop in total profit.
- Hike in C or I_c or O results into reduced value of TP with hike in value of S and T. Thus, when the unit purchase cost or interest charge rate or ordering cost increases the retailer should increase the cycle time and unit selling price to cover up the drop in total profit.
- The value of η is very sensitive, increase in its value results a sharp hike in value of T and a rapid fall in the values of S and TP. This means, when the mark up for selling price increases, the retailer has to elevate the cycle time and take down the unit selling price to cover up the rapid fall in total profit.
- Increase in M hikes up the values of S, T and TP. It represents that when the values of permissible credit period received by the retailer increases, the retailer can have a spike in the cycle time and unit selling price and this will lead to hike in present value of total profit.
- The value of N is very sensitive, increase in its value hikes up the values of

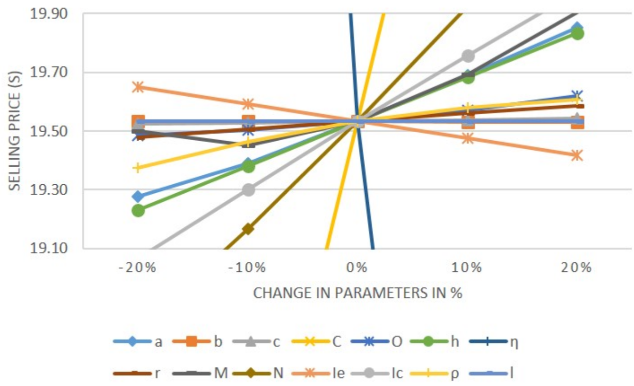

Figure 12: Parameters Vs. Selling price

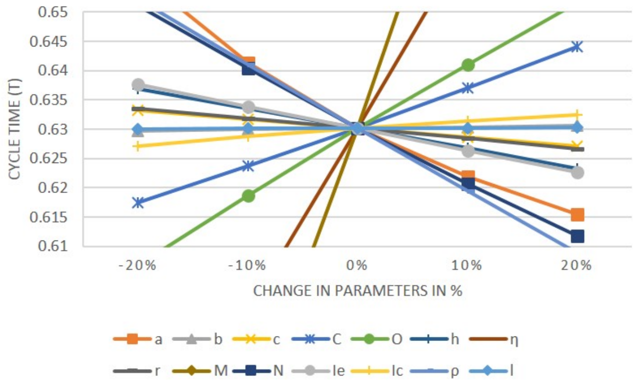

Figure 13: Parameters Vs. Cycle time

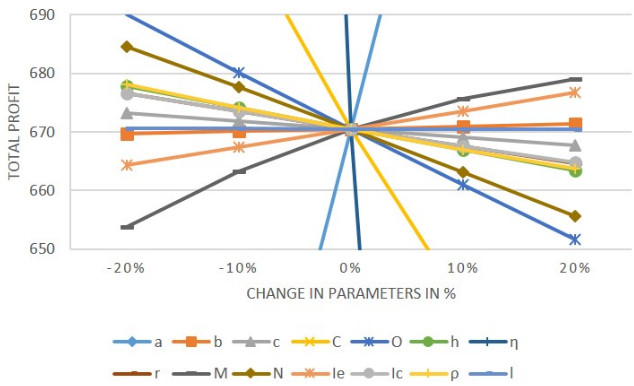

Figure 14: Parameters Vs. Total profit

S while the value of T and TP falls down. It means that when the values of permissible credit period offered by the retailer increases, the retailer should reduce the cycle time and increase the selling price to cover up the decrease in present value of total profit.

- Increase in value of I_e leads to decrease in S, T, while the profit increases. This means, on increase of interest earn rate, the retailer should reduce unit selling price and his cycle time which on other side will increase the total profit.

7. CONCLUSION

The present work sculpts an EOQ model with an objective to attain maximum profit by determining two parameters viz. optimal selling price per unit and cycle time. The retailer opts to pay supplier using ACC (advance-cash-credit) payment scheme with several payment options as distinct cases, which is further extended as cash-credit payment scheme to the end customer. A more factual nature is imposed by incorporating time value of money. An algorithm meeting the necessary and sufficient conditions of optimality is implemented to obtain optimal solution. For a better insight of the solution scheme, numerical examples are provided for validation of the established model and concavity of profit function is also shown using the 3D plots. Lastly, effect of parameters is analyzed and based on the observations made managerial insights are provided.

After having a careful inspection of parameters it has been witnessed that total profit is supersensitive to the mark up for selling price, rise in its rate bounds the retailer to levitate the cycle time and push down the selling price to cover up the brisk fall in total profit. The cycle time drops hastily when the proportion of sales revenue offered permissible delay is elevated, this causes reduction in profit and to overcome this loss the retailer has to increase the selling price. This models could serve the purpose for managers dealing with items like cotton fiber, wool, other items from textile industry. The presented model can be extended in future by incorporating realistic process of decaying/deterioration for inventory and further preservation technology can be implemented to overcome the decay.

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