Yugoslav Journal of Operations Research
32 (2022), Number 4, 529–541
DOI: https://doi.org/10.2298/YJOR220516026M

MULTI-FRACTIONAL FUZZY PROGRAMMING FOR SUSTAINABLE AGRICULTURAL MULTI-CROPPING BI-SEASONAL PLANNING

Umar Muhammad MODIBBO
Department of Statistics & Operations Research, Modibbo Adama University, P.M.B. 2076, Yola, Nigeria
umarmodibbo@mautech.edu.ng

Received: May 2022 / Accepted: July 2022

Abstract: The agricultural production system composes of several conflicting resources that must be combined to yield the desired product output. However, some goals are not conflicting in the system; therefore, this study presents a multi-objective optimization problem using a multi-fractional fuzzy programming concept. The objective is to optimize the profit ratio to cash expenditure and production of crops in different seasons under the restriction of fertilizer, machine, manpower, water consumption, and land utilization. The proposed model is illustrated with numerical examples for validation from an agrirean village in northern Nigeria. The result shows meaningful achievements and improvement in six crops out of the seven crops for both the dry and rainy seasons. Thus the concept is recommended for decision-makers for proper planning and productive yield in the agricultural industry for ensuring food security and the global sustainable developments.

Keywords: Agriculture, multi-production, fuzzy programming, fractional programming, sustainable planning, Nigeria.

MSC: 90B85, 90C26.

1. INTRODUCTION

Planning crop areas is crucial to the scarce resource management in agricultural sector. Different countries have varying cropping seasons, however, the most notable seasons are the rainy and dry seasons globally. Decision-makers (individual farmers, group or government) always have priority and crop preferences in every season. Apart from that, certain crops can yeild better than others depending on land and soil type, weather conditions and fertilizer. The nutrients requirement of these crops, such as water consumption level, fertilizers, maturity period, machine-hour, manpower, etc., varies from one product to another. Also, the cost and expenditure for farming a particular crop on a particular landmark might differ significantly. Additionally, the decision-maker always may wish to go for mixed-cropping system rather than a single cropping, as the former tends to generate more revenue to the latter and optimizes the land utilization. Most of the afformentioned goals can be hardly acheived as desired due to state of nature and unforeseen circumstances. As a result, there is a need for a technique that can integrate these goals visa-vee attained an aspirational level of decision-maker. Since most of the problems in agricultural sector involve fractions such as ratio of productivity to land, profit to expenditure, crops to yeild, etc., there is a need for suitable optimization approach in handling such kind of problems.

Any optimization problem that involve a ratio of two functions is reffered to as a *Fractional Programming.* In reality many systems comprises of more than one objective function which maybe conflicting in nature known as *Multiobjective Problems.* These multiobjective problems in a ratio form are known as a *Multi-Fractional programming.* Because real life decision-making environment has several uncertainty beyoung the reach of the decision-maker, the concept of fuzzy aspirations comes to play in order to address the uncertainty in the process of making a decision. Optimal solution to one objective may not satisfy all the conflicting goals at the same time. Hence, the challenging task before the decision-maker is to find a best compromising solution to all the objectives. In the literature, the problem described situation is termed as *Multiobjective Multi-Fractional Fuzzy Programming.* This article is presenting a model of this problem considering agricultural production system.

2. MANUSCRIPT PREPARATION

This article is organized as follows: Sect. 1 provides the introduction and overview of the study. The related literature on agricultural optimization problems are reviewed in Sect. 3. Sect. 4 discusses the methodology of the study. The mathematical model of the considered problem is formulated and presented in Sec. 7. Sect. 5 present the application area of the methods discussed in agricultural sector with numerical illustration. Sect. 8 present and discusses the results. Finally, the article is concluded in Sect. 9.

3. LITERATURE REVIEW

Mathematical programming have been widely used in agricultural planning problems. Over several years, researchers have been using different techniques in evaluating decision-making problems in the sector. Some authors addressed the problem using a single objective function, while others employed multiple objectives under different conditions and constraints. For instance, linear programming

(LP) has been applied to crop planning with fuzzy input parameters as an MOOP [1]. Similarly, Heady applied the LP for allocating the scarce land for aggricultural cropping. A survey of the mathematical models for planning problems in agriculture has been studied extensively and documented [2, 3, 4, 5]. A systematic review for seven years period on utilization of mathematical programming models in water resource management considering uncertainty has been carried by [6]. According to the report, research trend on water consumption/management revealed that stochastic dynamic and multistage programming are the most pronounced.

[7] proposed a multiobjective structural planning to aid decision-makers for regional crops planning. The model is based on socio-econimic and environmental supports. The model was demonstrated in a city of Iran. [8] considered profit to cash expenditure and crop production in a multiobjective fashion and used some aspect as fractional programming to solve the problem. [9] applied fuzzy goal programming to land use planning in agricultural system. [10] proposed a multiple objective mathematical model that optimized the net crop benefit and fertilizer utilization in agricultural settings. They further presented two heuristics algorithms for solving the model. [11] uses superiority and inferiority measures method to solve a linear fractional multiobjective optimization problem in agricultural planting system. [12] have proposed a new integer solution approach to a fractional linear programming model and they presented a numerial example to illustrate the approach. Other technique known as revised and column method was used in solving linear fractional programming problem [13]. Several studies conducted using mathematical programming concerning agricultural products for sustainable development of different nations in Afriac (e.g. Nigeria), Asia(e.g. India and Saudi Arabia). For more details refer to [14, 15, 16, 17, 18].

Water supply for crop consumption is one of the crucial factors in agriculture. [19] has applied mathematical programming in exermining the effect of urbanization on water consumption in agriculture, and linked to nations' economic developments. Comparing to the based-year of the study area, they concluded that 1% increase in urbanization would reduce water consumption in agriculture by $0.639mm^3$. [20] integrates three objectives related to economic, social and environmental aspects, and proposed a multiple objective model to improve agri-food production with minimum food waste through sustaianble agri-food supply chain considering fermers cropping decision. [21] studied the aspect of inbalance between demand and supply of the crop planning problems in agricultural sector under uncertain environment, and applied the concept to fresh tomato Argentinean supply chain. [22] have proposed a framework for large-scale crop planning problem integrating sustainable climate-smart crop planning and agri-food supply chain management. [23] applied the concept of robust optimization considering the crop rotation problem with uncertainties where water supply/demand and net return regarded as uncertain with variation within the allowable rotational cycle.

One can not talk of crop planning problems without inventory and warehousing, as they are central to the agricultural products. As such, several mathemathical programming models have been employed in this area. For more details, refer to [24, 25, 26, 27, 28].

4. METHODOLOGY

This section presents and discusses the general mathematical models for the MOOP, the fuzzy and the fractional programming which are subsequently used in the case study model formulation and analysis.

4.1. Multiobjective optimization (MOOP) model

Let's define a multiobjective programming problem (MOPP) with j objectives functions be given as:

$$\begin{aligned} &\textit{Optimize}\ \ [Z_1(X), Z_2(X), \ldots Z_j(X)] \\ &\textit{subject to}; \\ &g_i(x)(\leq, =, \geq)b_i,\ \ i = 1, 2, \ldots, m;\ x \geq 0. \end{aligned} \tag{1}$$

Where Z_j is the set of objectives, $g_i(x)(\leq, =, \geq)b_i$ are m set of constraints for which b_i is the i^{th} resources.

4.2. Fuzzy Goal Programming Model

Fuzzy goal programming is based on the concept of fuzzy set theory proposed by [29]. Fuzzy sets generally described imprecise goals of a decision-maker.

$$\begin{aligned} &Find \\ &X = [x_1, x_2, ..., x_n]^T \\ &\textit{such that} \\ &Z_k(X)(\succeq, \simeq, \preceq)\mathrm{g}_k,\ \ k = 1, 2, 3, ..., K. \\ &AX \leq b_i,\ \ \ \ i = 1, 2, ..., m \\ &X \geq 0 \end{aligned} \tag{2}$$

The fuzzy programming concept in solving multi-objective decision-making problems was initially proposed by [30].

For a Maximization type goal, the membership function is given by Eqn. (3).

$$\mu_k(Z_k(X)) = \begin{cases} 1, & if\ \ Z_k(X) \geq \mathrm{g}_k \\ \frac{Z_k(X) - L_k}{\mathrm{g}_k - L_k}, & if\ \ L_k \leq Z_k(X) \leq \mathrm{g}_k \\ 0, & if\ \ Z_k(X) \leq L_k \end{cases} \tag{3}$$

For a Minimization type goal, the membership function is given by Eqn. (4).

$$\mu_k(Z_k(X)) = \begin{cases} 1, & if \;\; Z_k(X) \le \mathrm{g}_k \\ \frac{U_k - Z_k(X)}{\mathrm{g}_k - L_k}, & if \;\; g_k \le Z_k(X) \le \mathrm{U}_k \\ 0, & if \;\; Z_k(X) \le U_k \end{cases} \tag{4}$$

4.3. Fractional Programming

From the Fuzzy goal Programming Eqn. (2), if some non zero goals (say i) can be optimize in ratio form leaving other k goals unchange, then Eqn. (5) becomes *Fractional Programming problem,* and can be defined as follows: let $P, Q, H_k(k = 1, 2, \ldots, m)$ be denoted as real-valued functions which are defined on the set C of the n-dimensional Euclidean space $\mathcal{R}^n$. Now, consider

$$\begin{aligned} &Optimize \;\; F(x) = \frac{P(x)}{Q(x)}, \\ &over \; the \;\; set \\ &S = \{x \in C\colon \;\; H_k(x) \le 0, k = 1, 2, \ldots, m\}. \end{aligned} \tag{5}$$

Here, it is assume that $Q(x)$ is positive on the set S. Whereas if $Q(x)$ is negative, then $F(x) = \frac{(-P(x))}{(-Q(x))}$ could be used instead. The nonlinear program

$$Program \quad (P) \qquad \sup\{F(x)\colon x \in S\} \tag{6}$$

is called a *single-ratio fractional programming.* Since most of the real-life applications have more than one ratio as objective function, in such situations the minimum over the program (P) is sorted as follows

$$\sup\left\{\min_{1 \le i \le r} \; F_i(x)\colon x \in S\right\} \tag{7}$$

and

$$\sup\left\{\sum_{i=1}^{r} \; F_i(x)\colon x \in S\right\} \tag{8}$$

Where $F_i(x) = \frac{P_i(x)}{Q_i(x)}, \;\; Q_i(x) > 0$. According to [31], Eqn. (7) is a *generalized fractional programming model.* Also, both Eqns. (7) and (8) are called *multiobjective fractional programs, denoted as in Eqn.* (9).

$$\max\{(F_1(x), F_2(x), F_3(x) \ldots, F_r(x)) : x \in S\} \tag{9}$$

If the numerator function P and denominator function Q in Eqn. (5) are linear plus a constant (i.e. affine), and the set S is a convex polyhedron, then Eqn. (6)

is called a *linear fractional programming model* which can take the form in Eqn. (10).

$$\sup\left\{\frac{c^T x+\beta}{d^T x+\gamma}: \qquad Ax \leq b, \qquad x \geq 0\right\} \tag{10}$$

where $c,\ d\ \in \mathcal{R}^n,\ \beta,\ \gamma \in \mathcal{R}$, the superscript T is the transpose A is an $m \times n$ matrix, and $b \in \mathcal{R}^m$. Generally, Eqn. (6) is known as *quadratic fractional programming model*, if functions P and Q are quadratic and the set S being a covex polyhedron.
Equation (6) is regarded as *concave fractional programming model* if P is concave on Q and C, and h_k are convex on C, whereby C is a convex set. Additionally, $P(x)$ is assumed to be nonnegative on the set S if Q is unaffine. It is of interest to mention that, on ageneral note, the obejective function of Eqn. (6) being concave does not render the function as concave, rather, it comprises of both concave and covex functions. Therefore, fractional programming models are nonconcave models generally, and thus are interested for global optimization.

5. APPLICATION OF THE METHOD IN AGRICULTURAL SECTOR

Agricultural crops are produce in different seasons of the year. Every season and each crop may require different resources. A decision-maker need to optimize the use of these resources, so that the overall goal of the farming system is achieved. Let's consider the information given in the following Tables for the purpose of illustrating the methodology.

Table 1: The seasonal crops and associated variables

Seasons	**Crops**	**Decision Variables**
Dry Season (1)	(1) Rice	x_{11}
	(2) Sugarcane	x_{21}
	(3) Wheat	x_{31}
Rainy Season (2)	(4) Potato	x_{42}
	(5) Maize	x_{52}
	(6) Millet	x_{62}
	(7) Guineacorn	x_{72}

Table 2: The Resources (Manpower, Machine hours, & Water consumption)

Decision Variables	Manpower (days/ha)	Machine-hour/ha	Water consumption (inch/ha)
x_{11}	67.01	112	70
x_{21}	38.89	187	35
x_{31}	40.02	28	25
x_{42}	37.05	25	10
x_{52}	42.25	40	11
x_{62}	36.72	40	13
x_{72}	36.72	40	10

Table 3: Other Resources (Fertilizer, Production, Expenditure & Price)

Decision Variables	N P K	Total Production (kg/ha)	Expenditure (N/ha)	Price (N/measure)
x_{11}	30 30 30	2742	6785.27	1500
x_{21}	150 120 120	72324	42322.50	500
x_{31}	100 50 34	2412	23526.70	980
x_{42}	120 70 75	22295	24320	350
x_{52}	70 30 30	895	5242.60	620
x_{62}	80 40 40	1250	4892.50	520
x_{72}	115 75 75	1300	6200.10	800

There are restrictions that in both seasons the land to be use for the farming is 287572 ha, machine hours between 40,125.23 and 31429.80, Manpowers in days 48212.33, Water consumption in rainy seaon 2272.48, dry season 1280.60 inch/ha. Fertilizer nitrogen concentration 40.80, phosphate 20.70, and potashium 17.67 in metric tons. Maximum allowable cash to be spent is N5252875.20.

Note: Profit is a product of selling price and the total produce and the fuzzy aspiration amount on expenditure is given to be N500,000 in million atmost.

The minimum aspiration level of each product at each season is given in Table 4.

Table 4: The seasonal crops and associated variables

Seasons	Crops	Aspiration level (tonnes)
Dry Season (1)	(1) Rice	68000
	(2) Sugarcane	15200
	(3) Wheat	11500
Rainy Season (2)	(4) Potato	7000
	(5) Maize	18000
	(6) Millet	9200
	(7) Guineacorn	8000

6. STUDY AREA

The above information are in the case of an agricultral village of Nigeria called Wurodole in Girei district. The village is located on the geographical coordinates: $9°22^{'} North \quad and \quad 12°33^{'} East$ with mostly dwellers as farmers. The primary occupation of the people in the area is farming and cattle rearing. The major crops during the two seasons are shown on Table 4. and the map is shown in Fig. 1.

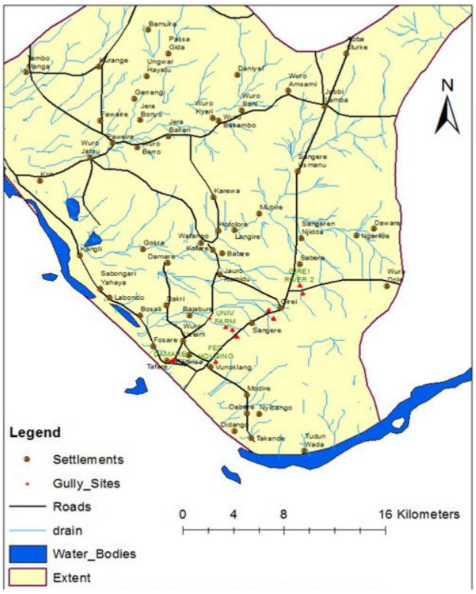

Figure 1: The Girei District Map Showing Study Areas

7. MATHEMATICAL MODEL FORMULATION

This section presents the various conflicting objectives/goals of the problem under consideration alongside its several constarints as follows:

7.1. Goals of the Problem

The problem is made up of three conflicting goals, related to ratio of profit-to-expenditure, and crop productivity at each season. The individual formulation of the goals are presented below:

Goal 1: This objective is to optimize profit to expenditure ratio.

$$z_1 = \frac{4113000x_{11} + 36162000x_{21} + 2363760x_{31} + 7803250x_{42} + 554900x_{52} + 650000x_{62} + 780000x_{72}}{6785.27x_{11} + 42322.50x_{21} + 23526.70x_{31} + 24320x_{42} + 5242.60x_{52} + 4892.50x_{62} + 62000.10x_{72}} \quad (11)$$

Goal 2: This objective is to optimize each crop productivity in the rainy season.

$$Z_2 = \frac{(2742x_{11} + 72324x_{21} + 2412x_{31})}{\sum_{i=1}^{3} x_{i1}} \tag{12}$$

Goal 3: This objective is to optimize each crop productivity in the dry season.

$$Z_3 = \frac{= (22295x_{42} + 895x_{52} + 1250x_{62} + 1300x_{72})}{\sum} {}^{7}_{i=4} x_{i2} \tag{13}$$

7.2. Constraints of the problem

The constraints are comprise of several items related to manpower per days per hectare, machine hour per hectares, water consumption per crop type per hectare, expenditure in Naira (Nigerian currency) per crop per hectare, productivity in kilogram per crop per hectare, and fertilizers (in different proportions of nitrogen, phospate and potashium) also per crop per hectare. The complete formulation of these constraints are given in (14).

Constraints of the Model

$$\begin{aligned}
&\textstyle\sum_{i=1}^{3} x_{i1} \leq 287.527;\\
&\textstyle\sum_{i=4}^{7} x_{i2} \leq 287.527;\\
&67.01x11 + 38.89x_{21} + 40.02x_{31} + 37.05x_{42} + 42.25x_{52} + 36.72(x_{52} + x_{72}) \geq 40125.23;\\
&112x_{11} + 187x_{21} + 28x_{31} + 25x_{42} + 40(\textstyle\sum_{i=4}^{7} x_{i2}) \geq 48212.33;\\
&70x_{11} + 35x_{21} + 25x_{31} \geq 2272.48;\\
&10(x_{42} + x_{72}) + 11x_{52} + 13x_{62} \geq 1280.60;\\
&30x_{11} + 150x_{21} + 100x_{31} + 120x_{42} + 70x_{52} + 80_x62 + 115x_{72} \geq 40800;\\
&30(x_{11} + x_{52}) + 120x_{21} + 50x_{31} + 70x_{42} + 40x_{62} + 75x_{72} \geq 20700;\\
&30(x_{11} + x_{52}) + 120x_{21} + 40(x_{31} + x_{62}) + 75(x_{42} + x_{72}) \geq 17670;\\
&6785.27x_{11} + 42322.50x_{21} + 23526.70x_{31} + 24320x_{42} + 5242.60x_{52} + 4892.50x_{62}\\
&+62000.10x_{72} \leq 5252875.20;\\
&2742x_{11} \geq 68000;\\
&72324x_{21} \geq 15200;\\
&2412x_{31} \geq 11500;\\
&22295x_{42} \geq 7000;\\
&895x_{52} \geq 18000;\\
&1250x62 \geq 9200;\\
&1300x72 \geq 8000;\\
&4113000x_{11} + 36162000x_{21} + 2363760x_{31} + 7803250x_{42} + 554900x_{52} + 650000x_{62}\\
&+780000x_{72} \geq 500000;\\
&\textstyle\sum_{i=1}^{7} x_{ij} >= 0; \quad j = 1, 2.
\end{aligned} \tag{14}$$

The productivity values of various crops in the farming seasons are shown in Table 5.

Table 5: The seasonal crops and associated productivity

Seasons	**Crops**	**Productivity (tonnes)**
Dry Season (1)	(1) Rice	70000
	(2) Sugarcane	14200
	(3) Wheat	11800
Rainy Season (2)	(4) Potato	85000
	(5) Maize	20000
	(6) Millet	11200
	(7) Guineacorn	9500

8. RESULTS ANALYSIS AND DISCUSSION

After solving the mathematical formulation of the problem in Eqn. (11)-(14), using the optimization package Lingo version 21.0, the compromised solutions has been found to be $Z_1 = 364.1681054$, $Z_2 = 129937.75kg/ha$, $Z_3 = 154282.0207kg/ha$ Also, the various productivity yeilds has been optimized and presented in Table 5. From the above results anyalysis, it can be seen that the crops in both seasons have been optimized as follows:

In the dry season, the productivity of rice has improved from sixty-eight thousand to seventy thousand (68000-70000) tonnes, wheat from elevn thousand five hundred to eleven thousand eight hundred (11500-11800) tonnes while the sugarcane has drop slightly from fifteen thousand two hundred to fourteen thiusand two hundred (15200-14200) tonnes respectively.

However, in the rainy season, all the four crops yield are improved. The productivity of potato has been optimized from seventy thousand to eighty five thousand (7000-85000) tonnes per hectare, the maize crop is improved from eighteen thousand to twenty thousand (18000-20000) tonnes per hectare, the millet increased from ninety thousand two hundred to eleven thousan two hundred (9200-11200) tonnes per hectare, and the guineacorn from eight thousand to nine thousand five hundred (8000-9500) tonnes per hectare.

Furthermore, these improvement can be achieved with just an expenditure **364.168** million naira as against the proposed amount which is **500** million naira, with the total productivity of **129937.75kg/ha, and 154282.0207kg/ha** for both dry and rainy seasons. Therefore, the results of this study is promising for which every farmer will admire to go for it. Since, it cut cost and yeild higher crops productivity.

The future scope of this research is enermous. (i) One can compare different aspect of these combinations, by alternating the crops in different seasons. (ii) Another comparision can be made acording the amount of fertilizer used (iii) Also, a situation where some or all parameters assume certain probability distribtutions can be explore. The researcher hope to explore all the aforementioned and several other aspects of this work as an extension to benefit policymakers (farmers/government) in this regrads.

9. CONCLUSION AND FUTURE SCOPE

Decision-making is an integral part of human endeavours. Food production is bedrock of human existence, without sufficient and affordable food security, no meaningful achievement can be made in any sector of life. Therefore, agriculutre has a paramount important for sustainable development goals of every nations. Any decision-making tools that can foster the yeild and growth of the agricultural sector are the must to do by the policymakers and researchers. This study formulated a multiobjective fractional optimization problem and applied to the agricultural crop planning and yeild improvement under fuzzy environment. The application of this method can help the decision-makers (farmers and government) to make a proper plans and have better farm produce with reasonable profit using the scarce resources.

The concept is useful in real-life applications and can be extended to different sectors other than Agriculture.

Aknowledgement. The author wish to aknowledge the editor-in-chief and his team, the guest editors as well as the anonymous reviewers for their valuable suggestions which improved the quality and presentation of this work.

Conflict of interest. The author have no known conflict of interest regarding the data sources and publication of this paper.

Funding. This research received no funding from any organization or agency.

REFERENCES

[1] X. Zeng, S. Kang, F. Li, L. Zhang, and P. Guo, “Fuzzy multi-objective linear programming applying to crop area planning,” *Agricultural Water Management*, vol. 98, no. 1, pp. 134–142, 2010.

[2] J. J. Glen, “Mathematical models in farm planning: A survey,” *Operations Research*, vol. 35, no. 5, pp. 641–666, 1987.

[3] J. R. Black and J. Hlubik, “Basics of coputerized linear programs for ration formulation,” *Journal of Dairy science*, vol. 63, no. 8, pp. 1366–1378, 1980.

[4] K. Dhawan and A. Kahlon, “Some methodological issues in using linear programming technique in agriculture,” *Indian Journal of Agricultural Economics*, vol. 32, no. 902-2018-1342, pp. 147–159, 1977.

[5] J. Nix, “Farm management: the state of the art (or science),” *Journal of Agricultural Economics*, vol. 30, no. 3, pp. 277–292, 1979.

[6] T. W. Archibald and S. E. Marshall, “Review of mathematical programming applications in water resource management under uncertainty,” *Environmental Modeling & Assessment*, vol. 23, no. 6, pp. 753–777, 2018.

[7] M. M. Najafabadi, S. Ziaee, A. Nikouei, and M. A. Borazjani, “Mathematical programming model (mmp) for optimization of regional cropping patterns decisions: A case study,” *Agricultural Systems*, vol. 173, pp. 218–232, 2019.

[8] B. Mishra, A. K. Nishad, and S. Singh, "Fuzzy multi-fractional programming for land use planning in agricultural production system," *Fuzzy Information and Engineering*, vol. 6, no. 2, pp. 245–262, 2014.

[9] A. Biswas and B. B. Pal, "Application of fuzzy goal programming technique to land use planning in agricultural system," *Omega*, vol. 33, no. 5, pp. 391–398, 2005.

[10] S. Jain, D. Ramesh, and D. Bhattacharya, "A multi-objective algorithm for crop pattern optimization in agriculture," *Applied Soft Computing*, vol. 112, p. 107772, 2021.

[11] G. Yang, X. Li, L. Huo, and Q. Liu, "A solving approach for fuzzy multi-objective linear fractional programming and application to an agricultural planting structure optimization problem," *Chaos, Solitons & Fractals*, vol. 141, p. 110352, 2020.

[12] S. Gupta and A. A. Raina, "A new integer solution approach for fractional linear programming problem," *Annals of Optimization Theory and Practice*, vol. 4, no. 2, pp. 1–12, 2021.

[13] A. A. Raina, S. Gupta, I. Ali, and J. Iqbal, "Solving linear fractional programming problem using revised and column simplex method," in *Applications of Advanced Optimization Techniques in Industrial Engineering.* CRC Press, 2022, pp. 35–51.

[14] A. AlArjani, U. M. Modibbo, I. Ali, and B. Sarkar, "A new framework for the sustainable development goals of saudi arabia," *Journal of King Saud University-Science*, vol. 33, no. 6, p. 101477, 2021.

[15] I. Ali, U. M. Modibbo, J. Chauhan, and M. Meraj, "An integrated multi-objective optimization modelling for sustainable development goals of india," *Environment, Development and Sustainability*, vol. 23, no. 3, pp. 3811–3831, 2021.

[16] U. M. Modibbo, E. D. Heman, and R. Hafisu, "Multi-criteria decision analysis for malaria control strategies using analytic hierarchy process: A case of yola north local government area, adamawa state nigeria," *Amity Journal of Computational Sciences*, vol. 3, no. 2, pp. 43–50, 2019.

[17] U. M. Modibbo, I. Ali, and A. Ahmed, "Multi-objective optimization modelling for analysing sustainable development goals of nigeria: Agenda 2030," *Environment, Development and Sustainability*, vol. 23, no. 6, pp. 9529–9563, 2021.

[18] A. Haq, U. M. Modibbo, A. Ahmed, and I. Ali, "Mathematical modeling of sustainable development goals of india agenda 2030: a neutrosophic programming approach," *Environment, Development and Sustainability*, pp. 1–28, 2021.

[19] S. Avazdahandeh and S. Khalilian, "The effect of urbanization on agricultural water consumption and production: the extended positive mathematical programming approach," *Environmental Geochemistry and Health*, vol. 43, no. 1, pp. 247–258, 2021.

[20] A. Esteso, M. Alemany, A. Ortiz, and S. Liu, "Optimization model to support sustainable crop planning for reducing unfairness among farmers," *Central European Journal of Operations Research*, pp. 1–27, 2021.

[21] M. Alemany, A. Esteso, Á. Ortiz, and M. del Pino, "Centralized and distributed optimization models for the multi-farmer crop planning problem under uncertainty: Application to a fresh tomato argentinean supply chain case study," *Computers & Industrial Engineering*, vol. 153, p. 107048, 2021.

[22] A. Hajimirzajan, M. Vahdat, A. Sadegheih, E. Shadkam, and H. El Bilali, "An integrated strategic framework for large-scale crop planning: sustainable climate-smart crop planning and agri-food supply chain management," *Sustainable Production and Consumption*, vol. 26, pp. 709–732, 2021.

[23] I. Fikry, A. Eltawil, and M. Gheith, "A robust crop rotation optimization model with water scarcity and net return uncertainty considerations," *IEEE Access*, vol. 9, pp. 128 938–128 950, 2021.

[24] A. K. Malik and H. Garg, "An improved fuzzy inventory model under two warehouses," *Journal of Artificial Intelligence and Systems*, vol. 3, no. 1, pp. 115–129, 2021.

[25] T. Tyagi, S. Kumar, A. K. Malik, and V. Vashisth, "A novel neuro-optimization technique for inventory models in manufacturing sectors," *Journal of Computational and Cognitive Engineering*, 2022, https://doi.org/10.47852/bonviewJCCE2202184.

[26] A. K. Malik, A. Sharma, M. K. Pandey, and K. K. Bansal, "Study of inventory model for decaying items through stock-dependent demand under permissible delay in payments,"

Annals of Optimization Theory and Practice, 2021.

[27] I. Ali, S. Gupta, and A. Ahmed, "Multi-objective linear fractional inventory problem under intuitionistic fuzzy environment," *International Journal of System Assurance Engineering and Management*, vol. 10, no. 2, pp. 173–189, 2019.

[28] A. A. H. Ahmadini, U. M. Modibbo, A. A. Shaikh, and I. Ali, "Multi-objective optimization modelling of sustainable green supply chain in inventory and production management," *Alexandria Engineering Journal*, vol. 60, no. 6, pp. 5129–5146, 2021.

[29] L. A. Zadeh, "Fuzzy sets," in *Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers by Lotfi A Zadeh*. World Scientific, 1965, pp. 338–353.

[30] H.-J. Zimmermann, "Fuzzy programming and linear programming with several objective functions," *Fuzzy sets and systems*, vol. 1, no. 1, pp. 45–55, 1978.

[31] J.-P. Crouzeix, J. A. Ferland, and S. Schaible, "Duality in generalized linear fractional programming," *Mathematical Programming*, vol. 27, no. 3, pp. 342–354, 1983.