Yugoslav Journal of Operations Research
32 (2022), Number 2, 203–218
DOI: https://doi.org/10.2298/YJOR210219021N

KNAPSACK PROBLEM IN FUZZY NATURE: DIFFERENT MODELS BASED ON CREDIBILITY RANKING METHOD

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Received: February 2021 / Accepted: August 2021

Abstract: This paper deals with knapsack problem in fuzzy nature, where both the objective function and constraints are considered to be fuzzy. Three different models for fuzzy knapsack problem are proposed including, expected value model, chance-constrained model, and dependent-chance model. Credibility ranking method is applied to convert the fuzzy models into a crisp equivalent linear one considering triangular and trapezoidal fuzzy numbers. The solution of the fuzzy problem is obtained with respect to different satisfaction degrees in the objective function and constraints. Several numerical examples are given to demonstrate different models and concepts. The proposed approaches are applied to model and to solve a fuzzy pre-disaster investment decision problem.

Keywords: Fuzzy Knapsack Problem, Credibility Ranking Method, Expected-Value Model, Chance-Constraint Model, Pre-Disaster Investment Decision Problem.

MSC: 90Bxx, 90Cxx.

1. INTRODUCTION

The knapsack problem is one of the most important problems in mathematical programming, which has numerous applications in different areas such as various packing problem, cargo loading, stock cutting or economic planning [10]. Sahoo et al. [30] presented how the knapsack problem relates to the problem of optimal allocation of physical resource blocks. The generalization of the standard 0-1 knapsack problem as a set-union knapsack problem was suggested in [15]. Another application of the knapsack problem is the radio communication decision-making, storage, and computing problem [8]. Mengistu et al. [19] modeled the problem of placing a virtual machine in voluntary cloud computing as a constrained knapsack problem and use three initiative-based algorithms to meet the specific goals and limitations of voluntary cloud computing. The goal of knapsack problem is to maximize the total amount of profit without exceeding the maximum weight. The knapsack problem has a simple structure that allows it to be used in combinatorial optimization problems. Suppose that there are N objects where the weight of the object j is w_j and the profit of adding this object to the knapsack is p_j. The mathematical model of the continues knapsack problem is stated as follows ([7], [31], [18]):

$$\begin{array}{rl} \max & \sum_{j=1}^{n} p_j x_j \\ s.t. & \sum_{j=1}^{n} w_j x_j \leq C \\ & 0 \leq x_j \leq 1, j = 1, 2, ..., N. \end{array}$$

In the real world, because of the inherent uncertainty, the amount of weight and profit of the objects is vague [4]. Two approaches, probabilistic and fuzzy, are used to consider uncertainty in a knapsack problem. In the probabilistic approach, the values of the parameters of the problem are considered as random variables [26]. Since calculations in probabilistic space are very complex and also the varies knapsack problems deal with linguistic descriptions like high weight, low profit, etc, fuzzy approach is applied to deal with vague and imprecise data in knapsack problem ([1], [5]). Therefore, this approach is more popular in recent years [26]. To solve the knapsack problem using fuzzy programming, Abboud et al. modeled the multi-objective knapsack problem as a fuzzy linear programming problem (FLP) and used genetic algorithms to solve it [2]. A fuzzy hyperheuristic approach, which is a combination of a fuzzy inference system with a selection hyperheuristic is proposed in [24]. An integrated fuzzy TOPSIS-knapsack problem model for order selection in a bakery is presented in [9]. The fuzzy knapsack problem for single and bi-objective function, in which weights and profit were considered as triangular fuzzy numbers is investigated in [29]. The authors proposed dynamic programming approach using multi-stage decision making. It is applied to solve an investment problem in an imprecise environment. Uncertain multidimensional knapsack problem with different decision criteria was studied by [6]. An Integer Linear Programming Model for Binary Knapsack Problem with Dependent Item Values was demonstrated in [20]. The results of some important researches in the field of fuzzy knapsack problem is summarized in Table 1.

Table 1: Summery of researches in the subject of knapsack problem in fuzzy nature

Reference	Year	Uncertain parameters	proposed method
Abass and Abdallah [1]	2018	Capacities of the knapsack	Possibility Theory
Abboud et al. [2]	1997	Fuzzy goals	Mathematical optimization
Chen [5]	2009	Object weights	Parametric programming
Cheng et al. [6]	2017	All parameters	Chance-constrained programming
Kasperski and Kulej [13]	2007	Imprecise profits and imprecise weights of items	Bisection approach
Lin and Yao [15]	2001	Weight coefficients	Fuzzy optimization
Olivas et al. [24]	2021	-	Fuzzy hyperheuristic approach
Singh and Chakrabor [28]	2019	Profit and weights	Two-stage decision-making approach
Singh [29]	2019	Weights and profit	Dynamic programming approach

In our work, the profit and the weight of objects are considered to be fuzzy numbers. Kasperski et al. solved the 0-1 backpack problem with fuzzy data [13]. Lin et al. described the fuzzy knapsack problem (FKP). They considered $w_j,\ j = 1, ..., N$ as an interval value [15]. In our paper, different models for fuzzy knapsack problem considering impreciseness in profit and weigh parameters are proposed. The models include expected value model, chance-constrained model and dependent-chance model. The credibility ranking method is applied to obtain the crisp equivalent of the models. The proposed models are applied to model and solve a fuzzy pre-disaster investment decision problem.
The paper is organized as follows. In section 2 preliminaries from credibility measure and fuzzy credibility programming are reviewed. Different models for fuzzy knapsack problem are proposed in section 3. Numerical results are presented in section 4. The last section ends the paper with a brief conclusion and future directions.

2. PRELIMINARIES

In this section, credibility measure for ranking fuzzy numbers are illustrated ([3], [11], [12], [17], [22]).
Definition 1. Let $\tilde{A}$ be a fuzzy subset of X with the membership function $\mu_{\tilde{A}} : X \to [0, 1]$. The α-cut of $\tilde{A}$ is denoted with $[\tilde{A}]_\alpha$ and it is defined as follows:

$$[\tilde{A}]_\alpha = \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}.$$

To compare a couple of fuzzy numbers $\widetilde{a}$ and $\widetilde{b}$, concepts of possibility and necessity relations are defined as the following

Definition 2. Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy quantities with the membership functions $\mu_{\tilde{a}}$ and $\mu_{\tilde{b}}$, respectively. The possibility relation and the necessity relation can be defined as the following:

$$Pos(\tilde{a}\preceq\tilde{b}) = \sup\left\{\min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y))|x \in \tilde{a}, y \in \tilde{b}, x \leq y\right\}$$

$$Nec(\tilde{a}\preceq\tilde{b}) = \inf\left\{\max(1-\mu_{\tilde{a}}(x), 1-\mu_{\tilde{b}}(y))|x \in \tilde{a}, y \in \tilde{b}, x > y\right\}$$

Possibility relation is very optimistic, but necessity relation is very pessimistic. To exhibit with risk-neutral personality, it is more acceptable to use the following credibility relation as the average of possibility relation and necessity relation:

$$Cr(\widetilde{a}\preceq\widetilde{b}) = 0.5\{Pos(\widetilde{a}\preceq\widetilde{b}) + Nec(\widetilde{a}\preceq\widetilde{b})\}.$$

Generally, the credibility of a fuzzy variable is not linear. When the fuzzy variables are special fuzzy numbers, such as triangular fuzzy numbers and trapezoidal fuzzy numbers, which are widely used in modeling fuzziness, this ranking method is easier to apply. The use of fuzzy credibility programming in fuzzy mathematical programming problems whose parameters are triangular or trapezoidal fuzzy numbers has been studied by a number of researchers ([14],[25], [28]).

Proposition 1. Considering a triangular fuzzy number $\tilde{A} = (\underline{A}, A, \bar{A})$with membership function

$$\mu_{\tilde{A}}(a) = \begin{cases} \frac{a-A}{A-\underline{A}} & \underline{A} \leq a \leq A \\ \frac{a-\bar{A}}{A-\bar{A}} & A \leq a \leq \bar{A} \\ 0 & o.w. \end{cases}$$

For this special membership function, the credibility of the fuzzy event $\tilde{A}\succeq\omega$ is computed as

$$Cr(\tilde{A}\succeq\omega) = \begin{cases} 1 & \omega \leq \underline{A} \\ \frac{2A-\underline{A}-\omega}{2(A-\underline{A})} & \underline{A} \leq \omega \leq A \\ \frac{\omega-\bar{A}}{2(A-\bar{A})} & A \leq \omega \leq \bar{A} \\ 0 & o.w. \end{cases}$$

Figure 1 shows the values of $Cr(\tilde{A}\succeq\omega)$ in terms of different values of ω.

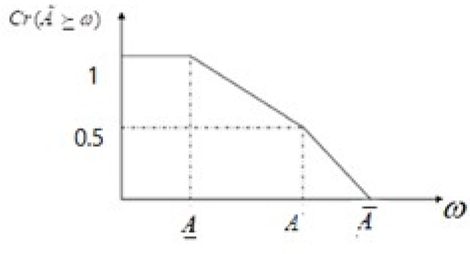

Figure 1: The values of $Cr(\tilde{A}\succeq\omega)$ in terms of different values of ω.

3. MODELS for KNAPSACK PROBLEM in FUZZY NATURE

Research on fuzzy logic provides the tools needed to model complex problems, which, along with other methods, contribute to better results [27]. In practice, many backpack problems involve items whose weight or benefit is not precisely known. In these cases, the decision maker (DM) has only vague knowledge of objects that uses approximate estimation and fuzzy linguistics, or decimal truncation for problem weight and coefficients [21]. Therefore, the weight and profit of each object in the knapsack problem are considered to be imprecise. In this section, three models including the expected value model, the chance-constrained programming model, and the dependent-chance programming model for the fuzzy knapsack problem are developed. One of the most well-known method for ranking fuzzy variables, which is based on credibilistic mappings is the expected value of a fuzzy variable defined by the weighted average of all possible values. The expected value model for modeling fuzzy/vague phenomena in mathematical programming was proposed by Liu and Liu, based on the concept of expected value, had been applied in many real-life applications [16]. The model optimizes the expected value of the objective function subject to some constraints. In this way, if the decision maker prefers finding a knapsack with the maximum expected value of the objective function, the following expected value model for the fuzzy knapsack problem is proposed:

$$\begin{array}{rl} EVK : \max & \mathrm{E}(\sum_{j=1}^{n} \tilde{p}_j x_j) \\ s.t. & \sum_{j=1}^{n} \tilde{w}_j x_j \preceq C \\ & 0 \le x_j \le 1, j = 1, 2, ..., N \end{array}$$

This modal states that from all the items that can be selected to fill the knapsack, the items associated with the maximum expected profit are preferred. In summary, this problem is called EVK problem. Therefore $x^* = (x_1^*, x_2^*, ..., x_n^*)$is the optimal solution of EVK problem if and only if $\mathrm{E}(\sum_{j=1}^{n} \tilde{p}_j x_j) \le \mathrm{E}(\sum_{j=1}^{n} \tilde{p}_j {x^*}_j)$ in which $x = (x_1, x_2, ..., x_n)$is a feasible solution. The expected value of the fuzzy variable $\tilde{A}$ is defined as follows [10]:

$$E(\tilde{A}) = \int_0^{\infty} cr(\tilde{A}\succeq r)dr - \int_{-\infty}^{0} cr(\tilde{A}\succeq r)dr$$

in which at least one of the two integrals is finite.

Since, the expected value of a fuzzy variable has a linear property, the EVK problem is equivalent to:

$$\begin{array}{rl} EEVK : \max & \sum_{j=1}^{n} E(\tilde{p}_j) x_j \\ s.t. & \sum_{j=1}^{n} \tilde{w}_j x_j \preceq C \\ & 0 \le x_j \le 1, j = 1, 2, ..., N \end{array}$$

In addition, if the decision maker considers the satisfaction level for the constraint, the EEVK problem can be written using credibility measure as follows, which is named a credibility equivalent expected valued knapsack problem (CEEVK):

$$\begin{array}{lll} CEEVK: \max & \sum_{j=1}^{n} E(\tilde{p}_j)x_j \\ \quad s.t. & cr\left(\sum_{j=1}^{n} \tilde{w}_j x_j \preceq C\right) \geq \beta \\ & 0 \leq x_j \leq 1, j = 1, 2, ..., N \end{array}$$

There are many methods for ranking fuzzy numbers, which is an important issue in fuzzy set theory. The selection of the appropriate method is highly dependent on the decision maker's preferences. Based on this, several more practical versions of the knapsack problem are presented. If the decision maker's preference is to optimize the value of the fuzzy objective function with certain confidence level α, the chance-constrained model of the fuzzy knapsack problem can be stated as follows:

$$\begin{array}{lll} CCK: \max & \omega \\ \quad s.t. & cr\left(\sum_{j=1}^{n} \tilde{p}_j x_j \succeq \omega\right) \geq \alpha \\ & \sum_{j=1}^{n} \tilde{w}_j x_j \preceq C \\ & 0 \leq x_j \leq 1, j = 1, 2, ..., N \end{array}$$

Again, considering the satisfaction level β for the constraint of the problem, the following credibility chance-constrained knapsack (CCCK) problem is proposed.

$$\begin{array}{lll} CCCK: \max & \omega \\ \quad s.t. & cr\left(\sum_{j=1}^{n} \tilde{p}_j x_j \succeq \omega\right) \geq \alpha \\ & cr\left(\sum_{j=1}^{n} \tilde{w}_j x_j \preceq C\right) \geq \beta \\ & 0 \leq x_j \leq 1, j = 1, 2, ..., N \end{array}$$

The advantage of this model compared to the previous expected value model is that if the membership functions of the fuzzy parameters are triangular or trapezoidal, the crisp equivalent model can be constructed as the following, which also retains its linearity. Therefore, a solution for the fuzzy knapsack problem under credibility ranking method can be generated by solving the crisp model. Based on the proposition 1, if $\tilde{p}_j$ and $\tilde{w}_j,\ j = 1, ..., N$ be triangular fuzzy numbers, $cr\left(\sum_{j=1}^{n} \tilde{p}_j x_j \succeq \omega\right) \geq \alpha$can be rewritten as:

$$(2 - 2\alpha)\sum_{j=1}^{n} p_j x_j + (2\alpha - 1)\sum_{j=1}^{n} \underline{p}_j x_j \geq \omega$$

in which $\tilde{p}_j = \left(\underline{p}_j, p_j, \bar{p}_j\right)$ and $\tilde{w}_j = \left(\underline{w}_j, w_j, \bar{w}_j\right)$. On the other hand, $cr(\tilde{A} \preceq r) = 1 - cr(\tilde{A} \succeq r)$. Therefore, $cr\left(\sum_{j=1}^{n} \tilde{w}_j x_j \preceq C\right) \geq \beta$ is equivalent to $cr\left(\sum_{j=1}^{n} \tilde{w}_j x_j \succeq C\right) \leq$

$1-\beta$ and $\tilde{w}_j = \left(\underline{w}_j, w_j, \bar{w}_j\right)$can be rewritten as:

$$(2-2\beta)\sum_{j=1}^{n} w_j x_j + (2\beta-1)\sum_{j=1}^{n} \bar{w}_j x_j \leq C$$

Substituting the last two constraints in CCCK problem, the crisp equivalent model for the fuzzy knapsack problem is obtained. The solution of the crisp model is an (α,β)-optimal solution for the fuzzy chance-constraint knapsack problem under credibility ranking method. Based on the decision making preferences, the satisfaction levels α and β are usually greater than 0.5.

If the decision maker prefers that the objective function value is not lower than ω, in which ω is a preferred minimum, the dependent-chance model can be obtained as follows:

$$\begin{array}{rl} DCK : \max & cr\left(\sum_{j=1}^{n} \tilde{p}_j x_j \succeq \omega\right) \\ s.t. & \sum_{j=1}^{n} \tilde{w}_j x_j \preceq C \\ & 0 \leq x_j \leq 1, j = 1,2,...,N \end{array}$$

Using credibility measure and the satisfaction degree β for the constraints, the following credibility dependent-chance knapsack problem is presented:

$$\begin{array}{rl} CDCK : \max & cr\left(\sum_{j=1}^{n} \tilde{p}_j x_j \succeq \omega\right) \\ s.t. & \mathrm{cr}\left(\sum_{j=1}^{n} \tilde{w}_j x_j \preceq C\right) \geq \beta \\ & 0 \leq x_j \leq 1, j = 1,2,...,N \end{array}$$

The CDCK is equivalent to:

$$\begin{array}{rl} CDCK : \max & \alpha \\ s.t. & cr\left(\sum_{j=1}^{n} \tilde{p}_j x_j \succeq \omega\right) \geq \alpha \\ & cr\left(\sum_{j=1}^{n} \tilde{w}_j x_j \preceq C\right) \geq \beta \\ & 0 \leq x_j \leq 1, j = 1,2,...,N \end{array}$$

in which $\alpha \in [0,1]$. Two cases should be considered.
Case 1: if $0 \leq \alpha \leq 0.5$

$$\begin{array}{rl} CDCK(1) : \max & \alpha \\ s.t. & 2\alpha \sum_{j=1}^{n} p_j x_j + (1-2\alpha)\sum_{j=1}^{n} \bar{p}_j x_j \geq \omega \\ & (2-2\beta)\sum_{j=1}^{n} w_j x_j + (2\beta-1)\sum_{j=1}^{n} \bar{w}_j x_j \leq C \\ & 0 \leq x_j \leq 1, j = 1,2,...,N, 0 \leq \alpha \leq 0.5 \end{array}$$

Case 2: if $0.5 \leq \alpha \leq 1$

$$
\begin{array}{lll}
CDCK(2): \max & & \alpha \\
& & (2\alpha - 1)\sum_{j=1}^{n} \underline{p}_j x_j + (2 - 2\alpha)\sum_{j=1}^{n} p_j x_j \geq \omega \\
s.t. & & (2 - 2\beta)\sum_{j=1}^{n} w_j x_j + (2\beta - 1)\sum_{j=1}^{n} \bar{w}_j x_j \leq C \\
& & 0 \leq x_j \leq 1, j = 1, 2, ..., N, 0.5 \leq \alpha \leq 1
\end{array}
$$

4. COMPUTATIONAL EXPERIMENTS

To illustrate the potential application of the proposed models, a fuzzy knapsack problem is modeled by different approaches and comparison analysis are given. Consider a fuzzy knapsack problem with 6 objects and $C = 86$. The fuzzy values of weights and profits are given in Table 2. A review of the available literature shows that the best form of membership functions in practical applications modeled as a form of fuzzy mathematical programming problems is the linear membership functions such as triangular or trapezoidal fuzzy numbers [22]. The most important advantage of linear membership functions is the production of linear models that use efficient and simple algorithms to solve them. Therefore, fuzzy numbers are considered to be triangular and trapezoidal. After estimating the boundaries of fuzzy numbers, it is enough to determine the core values of fuzzy numbers based on the opinion of the experts.

Table 2: The fuzzy values of weights and profits of objects.

J	$\tilde{p}_j$	$\tilde{w}_j$
1	(7,12,13)	(7.8,8.1,9)
2	(13,15,17)	(11.6,12.1,13.8)
3	(18,21,22)	(12.4,13.2,13.8)
4	(10,12,14)	(63,64.2,64.6)
5	(16,19,20)	(21.3,22.2,23.3)
6	(23,25,27)	(40,41.2,41.6)

The fuzzy knapsack problem is formulated as follows:

$$
\begin{array}{ll}
\max & (7, 12, 13)x_1 + (13, 15, 17)x_2 + (18, 21, 22)x_3 \\
& + (10, 12, 14)x_4 + (16, 19, 20)x_5 + (23, 25, 27)x_6 \\
s.t. & (7.8, 8.1, 9)x_1 + (11.6, 12.1, 13.8)x_2 + (12.4, 13.2, 13.8)x_3 \\
& +(63, 64.2, 64.6)x_4 + (21.3, 22.2, 23.3)x_5 + (40, 41.2, 41.6)x_6 \leq 86 \\
& 0 \leq x_j \leq 1, j = 1, 2, ..., N.
\end{array}
$$

According to the results obtained in the previous section, the credibility chance-constrained knapsack (CCCK) problem model can be formulated as:

$$
\begin{array}{ll}
\max & \omega \\
s.t. & (17 - 10\alpha)x_1 + (17 - 4\alpha)x_2 + (24 - 6\alpha)x_3 \\
& +(14 - 4\alpha)x_4 + (22 - 2\alpha)x_5 + (27 - 4\alpha)x_6 \geq \omega \\
& (7.2 + 1.8\beta)x_1 + (10.4 + 3.4\beta)x_2 + (12.6 + 1.2\beta)x_3 \\
& (63.8 + 0.8\beta)x_4 + (21.1 + 2.2\beta)x_5 + (40.8 + 0.8\beta)x_6 \leq 86 \\
& 0 \leq x_j \leq 1, j = 1, 2, ..., N
\end{array}
$$

The above model is solved with AMPL (A Mathematical Programming Languages). The solution of the problem for $\alpha = 0.7$and $\beta = 0.8$ is:

$$x_1 = 1,\ \mathrm{x}_2 = 1,\ x_3 = 1,\ x_4 = 0,\ x_5 = 1,\ x_6 = 0.67$$

$$\omega = 80.84$$

Also, the relation between parameters α, β and the objective function value ω is demonstrated in Figure 2. As one can see, increasing the parameter α and β decreases the objective function value ω. In fact, the least value for ω is obtained by $\alpha = 0.9$ and $\beta = 0.9$ and the highest value is gained by $\alpha = 0.5$and $\beta = 0.5$. The reason is that, by increasing the parameters α and β, the degree of violation of the constraints decreases. In fact, the reduction in profit is a cost that must be paid for more certainty about the constraints of the problem.

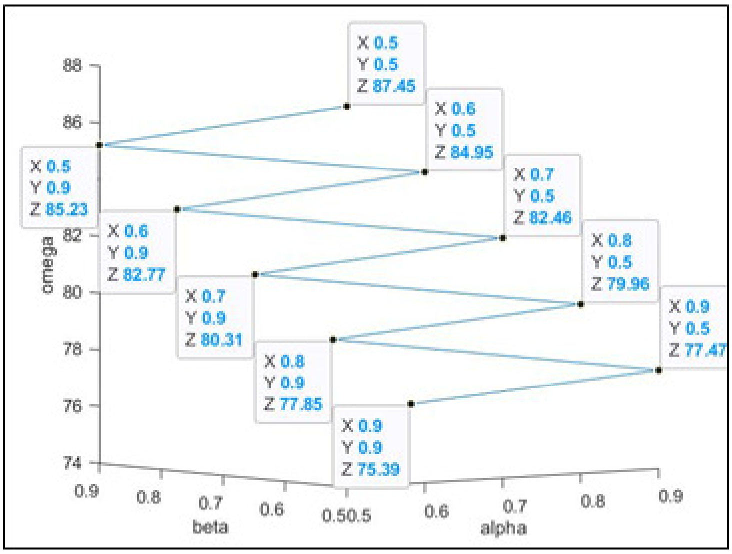

Figure 2: The relation between parameters α, β and ω

Furthermore, the dependent-chance model for the fuzzy knapsack problem with $\omega = 78$ can be formulated as follows:

$$\begin{array}{ll} \max & \mathrm{Cr}((7,12,13)\mathrm{x}_1 + (13,15,17)\mathrm{x}_2 + (18,21,22)\mathrm{x}_3 + (10,12,14)\mathrm{x}_4 \\ & +(16,19,20)\mathrm{x}_5 + (23,25,27)\mathrm{x}_6) \geq 78 \\ s.t. & (7.8,8.1,9)\mathrm{x}_1 + (11.6,12.1,13.8)\mathrm{x}_2 + (12.4,13.2,13.8)\mathrm{x}_3 \\ & +(63,\ 64.2,64.6)\mathrm{x}_4 + (21.3,22.2,23.3)\mathrm{x}_5 + (40,41.2,41.6)\mathrm{x}_6 \ \leq 86 \\ & 0 \leq x_j \leq 1, j = 1,2,...,N. \end{array}$$

According to the transformations described is section 3, the following equivalent models can be obtained.

Case 1: $0.5 \le \alpha < 1$

$$\begin{array}{ll} \max & \alpha \\ s.t. & (17-10\alpha)\,x_1 + (17-4\alpha)x_2 + (24-6\alpha)x_3 \\ & +(14-4\alpha)\mathrm{x}_4 + (22-2\alpha)x_5 + (27-4\alpha)x_6 \ge 78 \\ & (7.2+1.8\beta)\mathrm{x}_1 + (10.4+3.4\beta)x_2 + (12.6+1.2\beta)x_3 \\ & (63.8+0.8\beta)\mathrm{x}_4 + (21.1+2.2\beta)x_5 + (40.8+0.8\beta)x_6 \le 86 \\ & 0 \le x_j \le 1, j = 1,2,...,N, \;\; 0.5 \le \alpha \le 1. \end{array}$$

Case 2: $0 < \alpha < 0.5$

$$\begin{array}{ll} \max & \alpha \\ s.t. & (13-2\alpha)\,x_1 + (17-4\alpha)x_2 + (22-2\alpha)x_3 \\ & +(14-4\alpha)\mathrm{x}_4 + (20-2\alpha)x_5 + (27-4\alpha)x_6 \ge 78 \\ & (7.2+1.8\beta)\mathrm{x}_1 + (10.4+3.4\beta)x_2 + (12.6+1.2\beta)x_3 \\ & (63.8+0.8\beta)\mathrm{x}_4 + (21.1+2.2\beta)x_5 + (40.8+0.8\beta)x_6 \le 86 \\ & 0 \le x_j \le 1, j = 1,2,...,N, \;\; 0.5 \le \alpha \le 1. \end{array}$$

Considering $\beta = 0.8$, the problem is feasible when$0.5 \le \alpha < 1$ and the optimal solution is

$$x_1 = 1,\; x_2 = 1,\; x_3 = 1,\; x_4 = 0,\; x_5 = 1,\; x_6 = 0.6713,\; \alpha = 0.8153$$

Again, the relation between parameters α and β is demonstrated in Figure 3. This figure shows that by increasing the parameter β, the objective function value of the dependent-chance model –equal to α- decreases due to higher confidence level that the constraints will be satisfied.

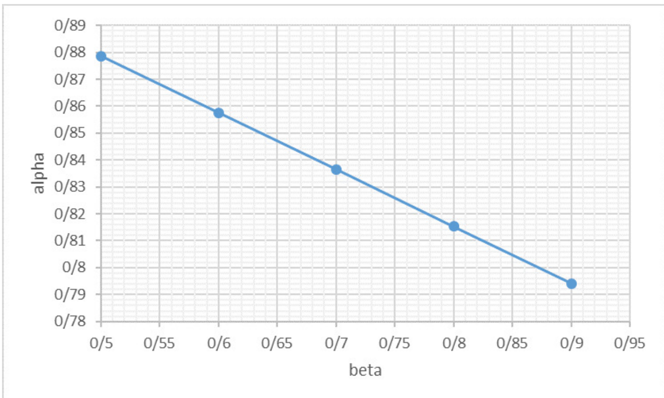

Figure 3: The relation between parameters α and β in dependent-chance model.

4.1. Application to Pre-Disaster Investment Decision Problem

Knapsack problem has many applications in real world problems such as production planning, energy management, resource management, power allocation, workflow mapping and so on. In this section, the application of the fuzzy knapsack problem is investigated in a pre-disaster investment decision problem. Natural or man-made hazards can significantly affect the performance of civil systems, such as transportation networks, water, energy and communications. Continuity of this infrastructure is essential for effective crisis management and post-crisis response. The first stage of crisis management is pre-crisis preparedness, which is made possible by long-term decisions such as investing in the infrastructure of these networks to strengthen the network and increase their likelihood of survival in the event of a crisis [22]. Of course, strengthening all network links requires a huge budget, which in many cases is impossible, so decisions must be made to select the subset of network links for investment to provide the most factor of network stability.

Let $G = (N, A)$is the designed logistic network for post-disaster transportation problem, in which N is the set of nodes and A is the set of links. Let $\tilde{p}_{i,j}$is the survival factor of the link (i, j). The survival factor of the link (i, j) can be increased to $\tilde{q}_{i,j}$by investing an amount equal to $\tilde{c}_{i,j}$. The total amount of budget for increasing the survival factor of the network is bounded by B. The investment decision variable is denoted by $z = (z_{i,j})$, in which $z_{i,j} = 1$ if link (i, j) is selected for the investment and $z_{i,j} = 0$, otherwise. The model of the pre-disaster investment decision problem is formulated as follows:

$$\begin{array}{rl} PDIDP: \max & \sum_{(i,j)\in A}(\tilde{q}_{i,j} - \tilde{p}_{i,j})z_{i,j} \\ s.t. & \sum_{(i,j)\in A} \tilde{c}_{i,j} z_{i,j} \preceq B \\ & z_{i,j} \in \{0, 1\}, (i, j) \in A \end{array}$$

Accordingly, the chance-constraint model of the fuzzy pre-disaster investment decision problem can be stated as follows:

$$\begin{array}{rl} \max & \omega \\ s.t. & cr\left(\sum_{(i,j)\in A}(\tilde{q}_{i,j} - \tilde{p}_{i,j})z_{i,j} \succeq \omega\right) \geq \alpha \\ & cr\left(\sum_{(i,j)\in A} \tilde{c}_{i,j} z_{i,j} \preceq B\right) \geq \beta \\ & z_{i,j} \in \{0, 1\}, (i, j) \in A \end{array}$$

The computational results are based on a highway network with 30 links depicted in Figure 4 and adapted from [23].

The total budget for investment in all links is 2640 units. Also the values of parameters $\tilde{p}_{i,j}$, $\tilde{q}_{i,j}$ and $\tilde{c}_{i,j}$for all $(i, j) \in A$ are reported in Table 3. The transformed problem is solved by GLPK solver. Table 4 shows the solution of the fuzzy pre-disaster investment decision problem for different values of the budget amount. For each value of the budget, links with a value of one are selected for investment. The last row of the table shows the total increase in the survival factor of the network for different values of the budget. In all cases it is assumed that α=β=0.8. Also, Figure 5 shows the relation between budget and the increase in the total survival factor of the network. The results show that increasing the budget increases the total survival factor of the network. Of course, for higher budget amounts, the intensity of increase in the survival factor is reduced.

Table 3: The values of parameters of the fuzzy pre-disaster investment decision problem

Link number	$\tilde{p}_{i,j}$	$\tilde{q}_{i,j}$	$\tilde{q}_{i,j} - \tilde{p}_{i,j}$	$\tilde{c}_{i,j}$
1	(0.3,0.4,0.5)	(0.7,0.8,1)	(0.2,0.4,0.7)	(75,80,85)
2	(0.3,0.4,0.5)	(0.7,0.8,1)	(0.2,0.4,0.7)	(75,80,85)
3	(0.3,0.4,0.5)	(0.7,0.8,1)	(0.2,0.4,0.7)	(310,320,330)
4	(0.2,0.3,0.5)	(0.6,0.7,1)	(0.1,0.4,0.8)	(255,260,265)
5	(0.3,0.4,0.5)	(0.7,0.8,1)	(0.2,0.4,0.7)	(155,160,165)
6	(0.1,0.3,0.4)	(0.5,0.7,1)	(0.1,0.4,0.9)	(410,420,430)
7	(0.3,0.4,0.5)	(0.7,0.8,1)	(0.2,0.4,0.7)	(155,160,165)
8	(0.1,0.3,0.4)	(0.5,0.7,1)	(0.1,0.4,0.9)	(610,620,630)
9	(0.3,0.4,0.5)	(0.7,0.8,1)	(0.2,0.4,0.7)	(115,120,125)
10	(0.2,0.3,0.5)	(0.6,0.7,1)	(0.1,0.4,0.8)	(335,340,345)
11	(0.1,0.3,0.4)	(0.5,0.7,1)	(0.1,0.4,0.9)	(930,940,950)
12	(0.2,0.3,0.5)	(0.6,0.7,1)	(0.1,0.4,0.8)	(155,160,165)
13	(0.1,0.3,0.4)	(0.5,0.7,1)	(0.1,0.4,0.9)	(610,620,630)
14	(0.1,0.3,0.4)	(0.5,0.7,1)	(0.1,0.4,0.9)	(1170,1180,1190)
15	(0.1,0.3,0.4)	(0.5,0.7,1)	(0.1,0.4,0.9)	(35,40,45)
16	(0.1,0.3,0.4)	(0.5,0.7,1)	(0.1,0.4,0.9)	(930,940,950)
17	(0.2,0.3,0.5)	(0.6,0.7,1)	(0.1,0.4,0.8)	(295,300,305)
18	(0.1,0.3,0.4)	(0.5,0.7,1)	(0.1,0.4,0.9)	(510,520,530)
19	(0.3,0.4,0.5)	(0.7,0.8,1)	(0.2,0.4,0.7)	(35,40,45)
20	(0.1,0.3,0.4)	(0.5,0.7,1)	(0.1,0.4,0.9)	(790,800,810)
21	(0.3,0.4,0.5)	(0.7,0.8,1)	(0.2,0.4,0.7)	(35,40,45)
22	(0.2,0.3,0.5)	(0.6,0.7,1)	(0.1,0.4,0.8)	(155,160,165)
23	(0.3,0.4,0.5)	(0.7,0.8,1)	(0.2,0.4,0.7)	(35,40,45)
24	(0.1,0.3,0.4)	(0.5,0.7,1)	(0.1,0.4,0.9)	(610,620,630)
25	(0.2,0.3,0.5)	(0.6,0.7,1)	(0.1,0.4,0.8)	(255,260,265)
26	(0.1,0.3,0.4)	(0.5,0.7,1)	(0.1,0.4,0.9)	(770,780,790)
27	(0.1,0.3,0.4)	(0.5,0.7,1)	(0.1,0.4,0.9)	(790,800,810)
28	(0.3,0.4,0.5)	(0.7,0.8,1)	(0.2,0.4,0.7)	(115,120,125)
29	(0.2,0.3,0.5)	(0.6,0.7,1)	(0.1,0.4,0.8)	(215,220,225)
30	(0.2,0.3,0.5)	(0.6,0.7,1)	(0.1,0.4,0.8)	(490,500,510)

Table 4: The total increase in the survival factor of the highway network for different values of the budget amount

Links	Budget amount		
	2640	1900	3000
1	1	1	1
2	1	1	1
3	1	1	1
4	1	0	1
5	1	1	1
6	0	0	0
7	1	1	1
8	0	0	0
9	1	1	1
10	0	0	1
11	0	0	0
12	1	1	1
13	0	0	0
14	0	0	0
15	1	1	1
16	0	0	0
17	1	1	1
18	0	0	0
19	1	1	1
20	0	0	0
21	1	1	1
22	1	1	1
23	1	1	1
24	0	0	0
25	1	0	1
26	0	0	0
27	0	0	0
28	1	1	1
29	1	0	1
30	0	0	0
Total increase in survival factor	43.4	36.8	45.6

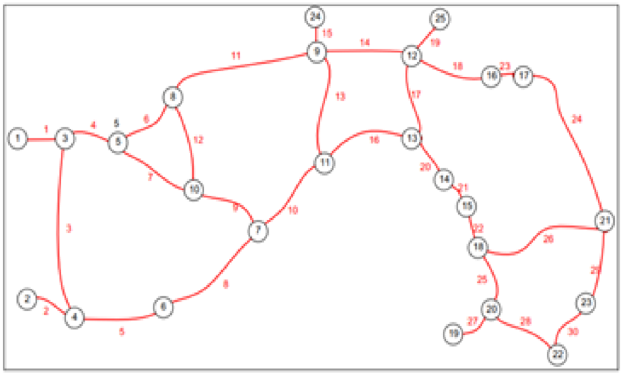

Figure 4: Highway network with 30 links

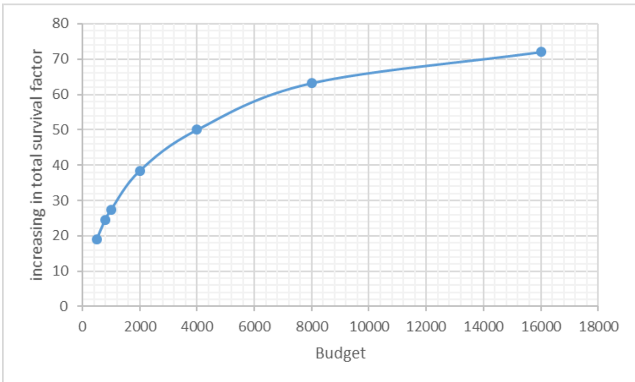

Figure 5: The relation between budget and total survival factor of the network.

5. CONCLUSIONS and SUGGESTIONS

This paper discuses knapsack problem in fuzzy nature, considering fuzziness in the profit and weight parameters. Therefore, both the objective function and constraints of the problem are considered to be fuzzy. Three models are proposed which include expected value model, chance-constrained model and dependent-chance model. When the membership function of the fuzzy parameters are assumed to be triangular or trapezoidal, credibility ranking method is used to convert the fuzzy model into a crisp linear programming model in which the solution of the crisp model is an (α, β)-optimal solution for the fuzzy problem. In this way, based on the preferences of the decision maker on the selection of the satisfaction degrees for the objective functions and the constraints, an appropriate solution is proposed. Numerical examples are solved to demonstrate the proposed models. The proposed models are applied to formulate and solve fuzzy pre-disaster investment decision problem. In the future, providing efficient approaches to solve the fuzzy proposed models will be pursued.

Acknowledgement: The authors express their sincere thanks to the editor and the anonymous reviewers for their valuable and constructive comments and suggestions.

REFERENCES

[1] Abass, S. A., and Abdallah, A. S., "Stability of Multiple Knapsack Problems with Interval Capacities", *Journal of Advances in Mathematics and Computer Science*, 2018, 1–11.

[2] Abboud, N. J., Sakawa, M., and Inuiguchi, M., "A fuzzy programming approach to multiobjective multidimensional 0–1 knapsack problems", *Fuzzy sets and systems*, 86 (1) (1997) 1–14.

[3] Attari, H., and Nasseri, S. H, "New concepts of feasibility and efficiency of solutions in fuzzy mathematical programming problems", *Fuzzy Information and Engineering*, 6 (2) (2014) 203–221.

[4] Buchheim, C., and Kurtz, J., "Robust combinatorial optimization under convex and discrete cost uncertainty", *EURO Journal on Computational Optimization*, 6 (3) (2018) 211–238.

[5] Chen, S. P., "Analysis of maximum total return in the continuous knapsack problem with fuzzy object weights", *Applied Mathematical Modelling*, 33 (7) (2009) 2927–2933.

[6] Cheng, L., Rao, C., and Chen, L., "Multidimensional knapsack problem based on uncertain measure", *Scientia Iranica. Transaction E, Industrial Engineering*, (24) (5) (2017) 2527–2539.

[7] Coniglio, S., Furini, F., and San, S. P., "A new combinatorial branch-and-bound algorithm for the knapsack problem with conflicts", *European Journal of Operational Research*, 289 (2) (2021) 435–455.

[8] Dang, T., and Peng, M., "Joint radio communication, caching, and computing design for mobile virtual reality delivery in fog radio access networks", IEEE Journal on Selected Areas in Communications, 37 (7) (2019) 1594–1607.

[9] İç, Tansel, Y., Özel, M., and Kara, İ., "An integrated fuzzy TOPSIS-knapsack problem model for order selection in a bakery", *Arabian Journal for Science and Engineering*, 42 (12) (2017) 5321–5337.

[10] Ivanchev, D., and Radovanova, E., "Realization of knapsack problem solving algorithm and some of its applications", *Yugoslav Journal of Operations Research*, 19 (1) (2009) 113–122.

[11] Inuiguchi, M., and Ramık, J., "Possibilistic linear programming: a brief review of fuzzy mathematical programming and a comparison with stochastic programming in portfolio selection problem", *Fuzzy sets and systems*, 111 (1) (2000) 3–28.

[12] Ghatee, M., and Niksirat, M., "A Hopfield neural network applied to the fuzzy maximum cut problem under credibility measure", *Information Sciences*, 229 (2013) 77–93.

[13] Kasperski, A., and Kulej, M., "The 0-1 knapsack problem with fuzzy data", *Fuzzy Optimization and Decision Making*, 6 (2) (2007) 163–172.

[14] Khalili, F., Naseri, S. H., and Taghi-Nezhad, N. A., "A new interactive approach for solving fully fuzzy mixed integer linear programming problems", *Yugoslav journal of operations research*, 30 (1) (2019) 71–89.

[15] Lin, F. T., and Yao, J. S., "Using fuzzy numbers in knapsack problems", *European Journal of Operational Research*, 135 (1) (2001) 158–176.

[16] Liu, B., and Liu, Y. K., "Expected value of fuzzy variable and fuzzy expected value models", *IEEE transactions on Fuzzy Systems*, 10 (4) (2002) 445–450.

[17] Mahmoudi, F., and Nasseri, S. H., "A new approach to solve fully fuzzy linear programming problem", *Journal of applied research on industrial engineering*, 6 (2) (2019) 139–149.

[18] Meng, T., and Pan, Q. K., "An improved fruit fly optimization algorithm for solving the multidimensional knapsack problem", *Applied Soft Computing*, 50 (2017) 79–93.

[19] Mengistu, T., Che, D., and Lu, S., "Multi-objective resource mapping and allocation for volunteer cloud computing", 2019 IEEE 12th International Conference on Cloud Computing (CLOUD), 2019, 344–348.

[20] Mougouei, D., Powers, D. M. W., and Moeini, A., "An integer linear programming model for binary knapsack problem with dependent item values", *Australasian Joint Conference on Artificial Intelligence*, 2017, 144-154.

[21] Nasseri, S. H., Ebrahimnejad, A., and Cao, B. Y., "Fuzzy linear programming", *Fuzzy linear programming: solution techniques and applications*, 2019, 39–61.

[22] Niksirat, M., Hashemi, S. M., and Ghatee, M., "Branch-and-price algorithm for fuzzy integer programming problems with block angular structure", *Fuzzy Sets and Systems*, 296 (2016) 70–96.

[23] Niksirat, M., "Fuzzy Integer Credibility Programming for Modeling and Solving Humanitarian Relief and Transportation Problem After the Crisis Under Uncertainty", *Defensive Future Study Researches Journal*, 4 (15) (2020) 61–84.

[24] Olivas, F., Amaya, I., Ortiz-Bayliss, J. C., Conant-Pablos, S. E., and Terashima-Marín, H., "Enhancing Hyperheuristics for the Knapsack Problem through Fuzzy Logic", *Computational Intelligence and Neuroscience*, 2021, (2021)

[25] Peeta, S., and Salman, F. S., Gunnec, D., and Viswanath, K., "Pre-disaster investment decisions for strengthening a highway network", *Computers & Operations Research*, 37 (10) (2010) 1708–1719.

[26] Ren, A., Wang, Y., and Xue, X., "Interactive programming approach for solving the fully fuzzy bilevel linear programming problem", *Knowledge-Based Systems*, 99 (2016) 103–111.

[27] Saffarian, M., Niksirat, M., and Kazemi, S. M., "A Hybrid Genetic-Simulated Annealing-Auction Algorithm for a Fully Fuzzy Multi-Period Multi-Depot Vehicle Routing Problem", *International Journal of Supply and Operations Management*, 8 (2) (2021) 96–113.

[28] Singh, V. P., and Chakraborty, D., "Solving Knapsack Problem with Fuzzy Random Variable Coefficients", *International Conference on Innovation in Modern Science and Technology*, 1037–1048, 2019.

[29] Singh, V. P., "An Approach to Solve Fuzzy Knapsack Problem in Investment and Business Model", *Networked Business Models in the Circular Economy*, 154–173, 2020.

[30] Sahoo, A., Hall, T. A., and Hagwood, C., "Optimal dynamic spectrum access scheme for utilizing white space in lte systems", 2019 IEEE Wireless Communications and Networking Conference (WCNC) 1–8, 2019.

[31] Wan, S. P., and Dong, J. Y., "Possibility linear programming with trapezoidal fuzzy numbers", *Applied Mathematical Modelling*, 38 (5-6) (2014) 1660-1672.