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RELIABILITY OF RANDOM NUMBER TABLES

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Abstract: The reliability of the tables containing *Random Numbers* (#RAN or RN) is checked. Two kinds of tests are proposed in order to check the reliability of series of λ-digit numbers obtained from a table of *Random (one-digit) Numbers*, where $\lambda = 1, 2, 3, 4$. The first test is based on the *Central Limit Theorem* and the *Uniform Distribution Function* of the involved *Random Variables*, and the second one is the X^2-test. A *Random Numbers Table* with 3500 digits is checked with very good results.

Keywords: Test, reliability, random numbers.

1. INTRODUCTION

Random Numbers Tables (RNT) are $m \times k$, $m, k \in \mathbb{N}$, matrices with elements one-digit integers introduced by any random process, e.g. by a computer routine called *Random Numbers Generator* (RNG). RNT are used in order to generate series of *λ-digit Random Numbers* (λ-RN), $\lambda = 1, 2, 3, 4, \ldots$ useful in several sampling processes.

2. GENERATING λ-RN FROM RNT $m \times k$

We choose randomly an integer $i \in \{1, 2, 3, \ldots, m\}$ and an other one $j \in \{1, 2, 3, \ldots, k\}$. We start from the (i, j)-element of the RNT and we read its λ-digit spaces as λ-RN moving from the left to the right (or in the opposite direction) going from one row to the next till a set of n λ-RN be generated. The number n is the size of our sample coming from a population of $N \geq n$ members. If the rows of the used RNT are exhausted we establish the (1, 1)-element as the next one of (m, k)-element of RNT and we go on in the same style.

We can also work on the columns of a RNT as we do with its rows. We can also move from the left to the right in one row and from the right to the left in the next one

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(voustrofidon) and so on. In all the above cases we generate a series of λ-RN based on the reliability of the RNT. This reliability we are going to check by two statistical tests.

3. TESTING THE RELIABILITY OF A RNT

There are several tests testing the reliability of a RNT. We present here two of them :

A) ***t*-test**

We take a sample of n λ-RN from our RNT. We figure X_i the Random Variable (RV) with values

$x_{ij} \in \mathbb{N}$ and $0 \leq x_{ij} \leq (10^{\lambda}-1), i = 1, 2, 3, \dots, n.$

The value x_{ij} is the i-th member of a λ-RN sample with n elements during the j-th repeated creation of this sample. Every number in the set $\{0, 1, 2, 3, \dots, (10^{\lambda}-1)\}$, must have the same probability to be chosen for this sample as the rows (columns) of the RNT are swept. So the related *Probability Density Function* (PDF) of the RV X_i, $\forall i = 1, 2, 3, \dots, n$ is

$$f(x_{ij}) = \begin{cases} \dfrac{1}{10^{\lambda}} & \text{iff} \quad x_{ij} \in \{0,1,2,\dots,(10^{\lambda}-1)\} \\ 0 & \text{else} \end{cases} \tag{1}$$

and (see Farmakis, 1992, p 131)

$$\mathrm{E}\,X_i = \mu_i = \mu = \frac{10^{\lambda}-1}{2}, \quad i = 1,2,3,\dots,n \tag{2}$$

$$\mathrm{Var}\,X_i = \sigma_i^2 = \sigma^2 = \frac{1}{12}\cdot 10^{\lambda}\cdot(10^{\lambda}+1), \quad i = 1,2,3,\dots,n \tag{3}$$

Also we have according to the *Strong Law of Large Numbers of Kolmogorov* and the *Central Limit Theorem* (CLT) we have

$$\frac{S_n}{n} >\!\!-\!\!- \mu, \qquad \text{where } S_n = X_1 + X_2 + X_3 + \dots + X_n \tag{4}$$

and especially if (at least) $n \geq 30$ then

$$\frac{S_n}{n} \sim N(\mu, \frac{\sigma^2}{n}) \quad \text{(Normal Distribution)} \tag{5}$$

(see Kounias et al., 1985), i.e. if we have h samples with $n \geq 30$ the statistics $\bar{x}_i$ and s_i^2 have to be inside the related confidence intervals in the $h \cdot (1-a)$ of the h cases at least, $a \in [0, 1]$ and a is near zero. For the mean value $\bar{x}_i$ the confidence interval will be

$$\left\{\mu \pm z_{a/2}\cdot\frac{\sigma}{\sqrt{n}}\cdot\sqrt{1-f}\right\}, \quad f = \frac{n}{m\cdot k} \tag{6}$$

and for the variance s_i^2

$$\left\{\frac{(n-1)\cdot\sigma^2}{X^2_{(n-1);a/2}},\ \frac{(n-1)\cdot\sigma^2}{X^2_{(n-1);1-a/2}}\right\},\quad i=1,2,3,\dots,n \tag{7}$$

$X^2_{k;a}$ is the critical value of X^2 for k degrees of freedom and significance $a \in [0, 1]$.

B) **X^2 -test**

We divide the interval $[-1/2, 10^\lambda - 1/2]$ into g subintervals $[(-1/2)+(i-1)\cdot r, (-1/2)+i\cdot r]$, $r = 10^\lambda/g$ and $i = 1, 2, 3, \dots g$. After this we take a sample of λ-RN from our RNT with sample size $n > 5\cdot g$ and let o_i and e_i are – respectively – the set sizes of observed and expected cases of λ-RN in the i-th subinterval, $i = 1, 2, 3, \dots g$. The uniform distribution of the numbers $0, 1, 2, 3, \dots, 10^\lambda-1$ in the interval $[-1/2, 10^\lambda - 1/2]$ gives us $e_i = n/g$, $i = 1, 2, 3, \dots g$.

Now the next X^2 -test is organized :

$H_0 : e_1 = e_2 = e_3 = \dots = e_g = n/g$

H_1 : negative of H_0

Significance : $a \in [0,1]$ (usually $a = 0.05$ or $a = 0.01$)

Rejection area $R = \{\, X^2 > X^2_{g-1;a},\ X^2 = \sum_{i=1}^{g} \frac{o_i^2}{e_i} - n \,\}$.

4. EXAMPLES

We are going to give two examples based on the same RNT of 3500 RN, with $m = 70$ and $k = 50$, Table 1.

A) **t-test example**

$\lambda = 3$, $n = 50$ and obviously $\mu = 499.5$, $\sigma^2 = 83416.667$, $\sigma = 288.819$. In order to get the sample we start from the (11, 1)-element of the RNT (randomly selected) and the sample was:

A = { 032,895,467,368,795,214,017,963,148,793,105,964,127,883,221,
044,732,315,687,496,123,698,764,296,542,631,493,605,760,152,
303,124,770,863,014,789,563,250.505,024,865,174,419,397,908,
078,985,785,946,913 }.

From the above sample we get :

$\bar{x} = 500.1$, $s^2 = 103711.969$, $s = 322.04$ and $f = 50/3500 = 0.0143$

and a 95% confidence interval for $\bar{x}$ is

95% c.i. = $\{\, 499.5 \pm 79.5 \,\} = I_\mu$ and $\bar{x} \in I_\mu$

Table 1.

This Table contains 3500 Random Numbers					
	1234567890	1234567890	1234567890	1234567890	1234567890
1	1598740212	0291018498	7789067534	8525572461	0542814842
2	2587698500	2568801434	6684345636	4553456965	4426732902
3	5877419087	0011132435	3463752045	5067780139	6412139013
4	6930117627	1891096302	5801478950	4916548237	8720052351
5	3693159346	0178022699	3367899372	6928294397	3697876781
6	3575823961	4874598887	3014720047	3858845626	0607889911
7	9512566348	5247490014	8522113673	3546795588	3071207699
8	8764786210	3902587977	1027327553	2478997014	4517996523
9	6669442846	5789210305	6046302136	8952103004	8964302311
10	1353069832	2014589745	5336942214	1120047899	6731264679
11	0328954673	6879521401	7963148793	1059641278	8322104473
2	2315687496	1236987642	9654263149	3605760152	3031247708
3	6301478956	3250505024	8651744193	9790807898	5785946913
4	6213485975	3018402925	0596878369	7481020576	8043784075
5	4125869398	9364239714	2117413525	9987123465	0586063012
6	7898584623	2560297471	1207250003	9877457912	4638392197
7	4258963987	1253036544	1203114989	1465056871	2335420245
8	1369879650	9648272301	8791807882	8971456827	3014647831
9	9899652039	9180746715	0372105469	7896633101	4564646546
20	8521573014	8927250146	6303023103	5890795456	3280347264
21	5789142390	4063012029	8741236012	8974001245	2415053339
2	1365289701	3698703369	8741562102	0457798986	6810124633
3	7893298741	5591098391	4668723020	8092927360	0941598763
4	5698745732	0298741368	7459857564	3834440757	5160192027
5	2589631456	7964586320	1564784876	4657120155	3545381122
6	1543542321	1245310312	4351273100	7068798462	6528363548
7	3598456844	0045968173	6126849836	4520531680	8453302473
8	2789641656	7210578987	6411098890	0489894984	8048721662
9	1253695147	9721635979	2050602347	3970690978	7701559523
30	8723159036	9416982712	8520369741	3527905636	2790150714
31	1285297539	4132890360	5048407080	1036485912	0489087038
2	9954238308	7090954063	5632105859	8712834457	1202832574
3	8956413275	9613484932	2594110369	8102587451	3214977962
4	3578974128	5899620147	4561210014	5666313232	3126548567
35	6652369877	1706977467	6126987337	7621498630	6970654050

Table 1. (continued)

This Table contains 3500 Random Numbers					
	1234567890	1234567890	1234567890	1234567890	1234567890
36	7536987412	5789319877	3126068640	0312398799	4368743324
7	6203794168	2641230985	4789502584	1020381856	1236911158
8	2361159786	1370203025	1595197149	3691022307	1947893799
9	6549484845	3023010702	0555810072	6667789345	7896423647
40	5698521871	0250506327	5203874258	4394678912	0456464465
41	5896814238	5108010987	4780909871	8182891785	8487141094
2	2852658702	8296321085	9631789460	5086425381	5737756941
3	4652297024	0036974156	3320009874	6421295456	7136101511
4	9523697150	4297536528	4695147233	3253040334	9727412542
5	3312569796	8783963070	2430369789	7741341564	5606569632
6	2236974692	2685436982	8269874401	8000033336	5682695155
7	7589302964	7581203737	9564747417	7569780214	8339170791
8	1230356998	0823647123	4595711679	8372269987	0251547314
9	2563980369	0502303032	5987413154	1065481576	6112120082
50	7062309098	6113689484	8215454561	6447404175	9988754208
51	0012358987	7945384681	0124501893	9506377328	7690780432
2	1045698789	5235023632	6456960174	9970788241	0641232916
3	6485142837	9456243854	0223764685	0441789802	7503199163
4	9026456312	0289997875	5126056087	1287536758	7211012456
5	4331005452	4625799789	6951643897	0397183143	7354063211
6	4276678418	6786176562	1615112403	1649070748	5567261534
7	9012500023	3563909603	0093135645	2528799789	8426542335
8	2315994097	8451246779	4954844421	1023698775	4913131653
9	6952632178	5239023120	4897810938	7041707807	4781415638
60	5936506960	2135589875	0321326987	0239770808	8248681945
61	1878487810	0035688157	2431563484	9628746814	0400545040
2	2360213659	7741025978	4175025897	8786100433	1357534857
3	0231590640	3698787996	5132132665	3987462325	8122002124
4	6932162591	6131095162	7444490997	7898767527	6487123935
5	7539908220	1036549695	3489739312	0959748645	1012865623
6	0303032105	8900123465	0398740256	3210258996	0125374040
7	1023799689	1096787987	3612154923	9769550588	7135478499
8	0269876314	8960178954	3122704557	7937863999	3413474701
9	6023159876	6321587974	0258856418	0001245488	8567454112
70	5236412987	6382365512	3686764756	3210369871	5260124244

and for σ^2 is

$$95\% \text{ c.i. } = \{ \frac{49 \cdot 83416.667}{X^2_{49;0.025}}, \frac{49 \cdot 834165.667}{X^2_{49;0.975}} \}$$

$$= \{ 58215.36,\ 129492.05 \} = I_\upsilon \quad \text{and} \quad s^2 \in I_\upsilon .$$

We repeated this process for more than 20 times and the result was $\bar{x} \in I_\mu$ for all the cases and $s^2 \in I_\upsilon$ for all the cases except one case.

B) **X^2 -test example**

$\lambda = 3, n = 50, g = 5$ and obviously $e_i = 10, i = 1, 2, 3, 4, 5$.

Using the sample we used in the above case for t–test we get:

$$o_1 = 13 \quad o_2 = 8 \quad o_3 = 7 \quad o_4 = 12 \quad o_5 = 10$$

and $e_i = 10,\ a = 0.05,\ X^2 = 2.60$ (8)

$H_0 : e_i = 10,\ i = 1, 2, 3, 4, 5$

H_1 : negative of H_0

Rejection area $R = \{ X^2 > X^2_{4;0.05} = 9.488,\ X^2 = \sum_{i=1}^{g} \frac{o_i^2}{e_i} - n \}$. (9)

From (8) and (9) we get that $X^2 = 2.6$ and so H_0 is valid for significance level $a = 0.05$. This sampling process repeated for more than 20 times gave the same result except one case.

5. RESULTS AND CONCLUSIONS

The main conclusion is that the examined *Number Table* is a really a RNT under the criterion of t–test and X^2–test. Many other tests are also valid for our purpose. We used the above mentioned two tests because they are well known. The over 20 repetitions of sampling were of several ways of reading numbers on the RNT (from left to the right, from right to the left, voustrofidon, etc. etc.).Same examinations with $\lambda = 2$ and $\lambda = 4$ gave also very good results.

REFERENCES

Farmakis,N. "Introduction to sampling", C. Christodoulidis, Thessaloniki–Greece, (in Greek), 1992.

Kounias,S., Kolyva–Mahera,F., Bagiatis,C., and Bora–Senta,E., *Introduction to Statistics,* Aristotle University of Thessaloniki Publishing Services, Thessaloniki, Greece (in Greek), 1985.

Walpole, R. *Introduction to Statistics*, MacMillan Publishing Co., Inc., New York 1974.