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OPTIMUM DESIGN OF STEEL TRUSSES

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Abstract: A procedure is presented for optimum design of steel trusses with constraints on stresses, displacements and design variables. Buckling is considered in the compression stresses constraints. The procedure intends to solve practical problems and it complies with the Albanian Steel Structural Design Code KTP-10-78. Optimum design of trusses is formulated as a problem of nonlinear mathematical programming and it is solved by the optimization method of Sequential Linear Programming with Move Limits. The difficulties introduced by the buckling constraints are successfully overcome. Due to the use of some approximation concepts, the procedure is presented computationally as an efficient one. Based on the results of some examples, useful recommendations are given for a favourite initial design of the procedure and also for the move limits parameter.

Keywords: Steel trusses, non-linear programming, optimum design.

1. INTRODUCTION

Based on the fact of not having a single best optimization method for steel trusses [4], it would be of interest to have a package of some optimum design procedures by the most efficient optimization methods.

The presented procedure intends to enter such a package solving practical problems and complying with the Albanian Code KTP-10-78 [6]. The procedure presents the optimum design of steel trusses formulated as a nonlinear programming problem with constraints on stresses (including buckling), displacements and design variables, and solved by the optimization method of sequential linear programming with move limits [1], [4], [10], [11].

Trusses are assumed to be with fixed topology and geometry, and subjected to constant joint loads.

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2. DESIGN VARIABLES

The design variables in the presented procedure of the optimum design of steel trusses are the cross-sectional areas of truss members. Each cross section is assumed to be described by a single design variable.

In fact, considering buckling, the cross section should be described at least by two design variables, namely, the area and the radius of gyration corresponding to the maximum slenderness ratio of the member. But, because of doubling the design space dimension, the use of these two kinds of design variables is computationally inconvenient.

So, to be efficient from the standpoint of computational effort through the use of a single design variable for each cross section and to consider buckling too, it is assumed that the area and the radius of gyration are two dependent properties of a cross section. Such a dependence can be described by empirical formulae established from discrete data of standard steel sections [2], [3], [4], [7].

The empirical formula used here is

$$r_{\min} = \beta\sqrt{A} \tag{1}$$

where $r_{\min}$ is the cross-sectional minimum radius of gyration; A is the cross-sectional area; β is a parameter determined by the least squares method. Equation (1) can be used even when the radius of gyration corresponding to the maximum slenderness ratio is not minimum. This is possible due to the use of the modified buckling length l_{om} [2],

$$l_{om} = \max\left(K_x l_{ox}, K_y l_{oy}\right) \tag{2}$$

that determines the maximum slenderness ratio λ,

$$\lambda = \frac{l_{om}}{r_{\min}} \tag{3}$$

where l_{ox}, l_{oy} are the buckling lengths of the truss member in the planes of buckling having as neutral axis X and Y principal axes of the cross section, respectively; K_x, K_y are parameters relating the radii of gyration of the cross section r_x, r_y (Figure 1) and $r_{\min}$ as follows:

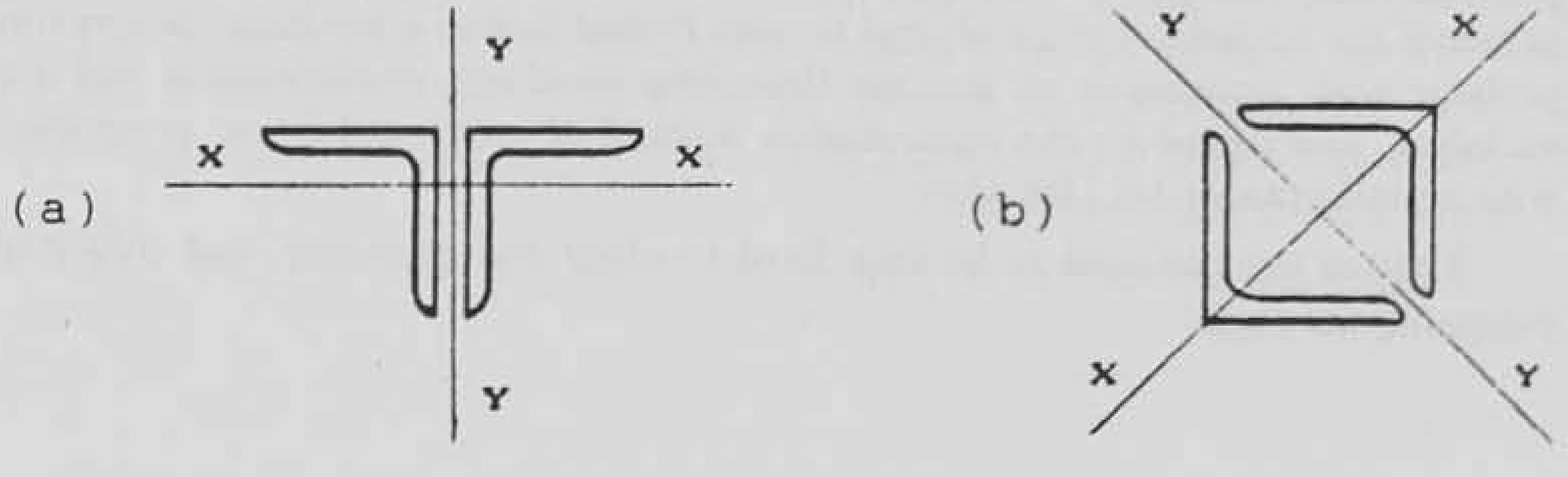


Figure 1. Cross sections of truss members

$$r_x = \frac{r_{\min}}{K_x} \quad \text{and} \quad r_y = \frac{r_{\min}}{K_y} \tag{4}$$

K_x in the case when $r_{\min} = r_y$ and K_y when $r_{\min} = r_x$ are determined from the discrete data of standard steel sections as the mean value of the quantities $r_{\min} / r_x$ and $r_{\min} / r_y$, respectively [2].

3. FORMULATION OF OPTIMUM DESIGN PROBLEM

The optimum design problem of trusses with constraints on stresses (including buckling), displacements and design variables is formulated as follows:

Problem P

Find $\{ X \}$ such that

$$W = \gamma \{ l \}^{\mathrm{T}} \{ X \} \longrightarrow \min \tag{5}$$

subject to

$$\{ D^L \} \le \{ D_k \} \le \{ D^U \}, k = 1, \ldots, K \tag{6}$$

$$\{ \sigma^L \} \le \{ \sigma_k \} \le \{ \sigma^U \}, k = 1, \ldots, K \tag{7}$$

$$\{ X^L \} \le \{ X \} \le \{ X^U \} \tag{8}$$

where $\{ X \}$ is the vector of design variables, cross-sectional areas; W is the objective function representing the truss weight; γ is steel density; $\{ l \}$ is the vector of members lengths; $\{ D_k \}$, $\{ \sigma_k \}$ are the vectors of constrained joint displacements and members stresses under the k-th load condition, respectively, $\{ D^L \}$, $\{ D^U \}$, $\{ \sigma^L \}$, $\{ \sigma^U \}$, $\{ X^L \}$, $\{ X^U \}$ are the vectors of lower and upper bounds on constrained joint displacements, members stresses and design variables, respectively; K is the number of load conditions.

The elements of $\{ D_k \}$ and $\{ \sigma_k \}$ are usually implicit nonlinear functions of $\{ X \}$, resulting with implicit nonlinear constraints for structural behavior (Equations (6) and (7)). For any given value of $\{ X \}$ the corresponding $\{ D_k \}$ and $\{ \sigma_k \}$ are assumed to be computed using the displacement method of structural analysis.

The only nonconstant bound vector of *problem P* is $\{ \sigma^L \}$ and is defined according to the Code KTP-10-78 as follows,

$$\sigma_e^L = -\varphi_e \, m \, R; \quad e = 1, \ldots, E \tag{9}$$

where σ_e^L is the e-th element of $\{ \sigma^L \}$; φ_e is the buckling factor of the e-th member; m is the factor of service conditions; R is the design strength of steel; E is the number of truss members.

φ_e is determined for any given steel in terms of the maximum slenderness ratio of the e-th member λ_e by means of formulae established from data of the respective tables of the Code KTP-10-78. Based on Equations (1) and (2), λ_e is taken as

$$\lambda_e = \frac{l_{om-e}}{\beta\sqrt{X_e}} \tag{10}$$

where l_{om-e} is the modified buckling length of the e–th member; X_e is the cross–sectional area of the e–th member.

$\{\sigma^U\}$ is defined as

$$\sigma_e^U = R; \quad e = 1, \dots, E \tag{11}$$

where σ_e^U is the e–th element of $\{\sigma^U\}$.

To comply with the following requirement of the Code KTP–10–78

$$\lambda_e \leq [\lambda]; \; e = 1, \dots, E \tag{12}$$

and based on Equations (1) and (2), $\{X^L\}$ is defined as follows:

$$X_e^L = \max(A_{\min}, A_\lambda); \quad e = 1, \dots, E \tag{13}$$

$$A_\lambda = \left(\frac{l_{om-e}}{\beta[\lambda]}\right)^2 \tag{14}$$

where X_e^L is the e–th element of $\{X^L\}$; $A_{\min}$ is the cross–sectional minimum area; $[\lambda]$ is the limiting slenderness ratio for compression or tension member by the above Code.

Problem P of Equation (5) through Equation (8) is presented as a nonlinear programming problem.

4. OPTIMIZATION METHOD

The optimization method of sequential linear programming with move limits [1], [4], [10], [11], used here to solve *problem P*, consists in replacing this problem by a sequence of linear programming *problems PA* whose solutions converge to that of *problem P*. The formulation of *problem PA* is based on:

1. The use of explicit linear approximation of structural behavior constraints, based on the first order Taylor series expansion of a function $f(\{X\})$ about a given point $\{\overline{\overline{X}}\}$.
2. The use of move limits.
3. The use of the constant vector of lower bounds on members stresses $\{\overline{\overline{\sigma}}^L\}$ computed as $\{\sigma^L\}$ at the point $\{\overline{\overline{X}}\}$.

So, *problem PA* for a given point $\{\overline{\overline{X}}\}$ is formulated as follows:

Problem PA

Find { X } such that

$$W = \gamma\{l\}^{T}\{X\} \longrightarrow \min \tag{15}$$

subject to:

$$\{D^L\} \le \{\overline{\overline{D}}_k\} + [\nabla \overline{\overline{D}}_k](\{X\} - \{\overline{\overline{X}}\}) \le \{D^U\}; \quad k = 1,\ldots,K \tag{16}$$

$$\{\overline{\overline{\sigma}}^L\} \le \{\overline{\overline{\sigma}}_k\} + [\nabla \overline{\overline{\sigma}}_k](\{X\} - \{\overline{\overline{X}}\}) \le \{\sigma^U\}; \quad k = 1,\ldots,K \tag{17}$$

$$\{X^L\} \le \{X\} \le \{X^U\} \tag{18}$$

$$\{\overline{\overline{X}}\} - \{\Delta \overline{\overline{X}}\} \le \{X\} \le \{\overline{\overline{X}}\} + \{\Delta \overline{\overline{X}}\} \tag{19}$$

where $\{\overline{\overline{D}}_k\}$, $\{\overline{\overline{\sigma}}_k\}$ are the values of $\{D_k\}$ and $\{\sigma_k\}$ at the point $\{\overline{\overline{X}}\}$; $[\nabla \overline{\overline{D}}_k]$, $[\nabla \overline{\overline{\sigma}}_k]$, are the matrices of first derivates of displacements and stresses under the k-th load condition, respectively, with respect to design variables computed at the point $\{\overline{\overline{X}}\}$; $\{\Delta \overline{\overline{X}}\}$ is the move limits vector at the point $\{\overline{\overline{X}}\}$, defined as

$$\{\Delta \overline{\overline{X}}\} = C\{\overline{\overline{X}}\} \tag{20}$$

where C is the move limits parameter taken as $C = 0.2$ or $C = 0.1$.

The additional constraints on design variables of equation (19) and the above values of parameter C are used to ensure the assumed approximations about structural behavior constraints and the vector $\{\sigma^L\}$ to be adequate.

In addition, to solve efficiently *problem PA* the following approximation concepts [2], [4], [5], [8] are used:

1. Design variable linking to reduce the design space dimension.
2. Noncritical constraints deletion.

5. OPTIMUM DESIGN PROCEDURE

According to preceding discussions, an iterative optimum design procedure of trusses is constructed (Figure 2).

The initial design $\{\overline{X}\}$ of each iteration cycle can be or not a feasible design of *problem P*. Scaling of the design [2], [4], [5] leads to a constrained feasible design $\{\overline{\overline{X}}\}$ which is the best design on the line through the points $\{O\}$ and $\{\overline{X}\}$ called the design line, and is defined as,

$$\{\overline{\overline{X}}\} = S\{\overline{X}\}; \quad S > 0 \tag{21}$$

where S is the scaling factor $\{\overline{X}\}$ which will be discussed later.

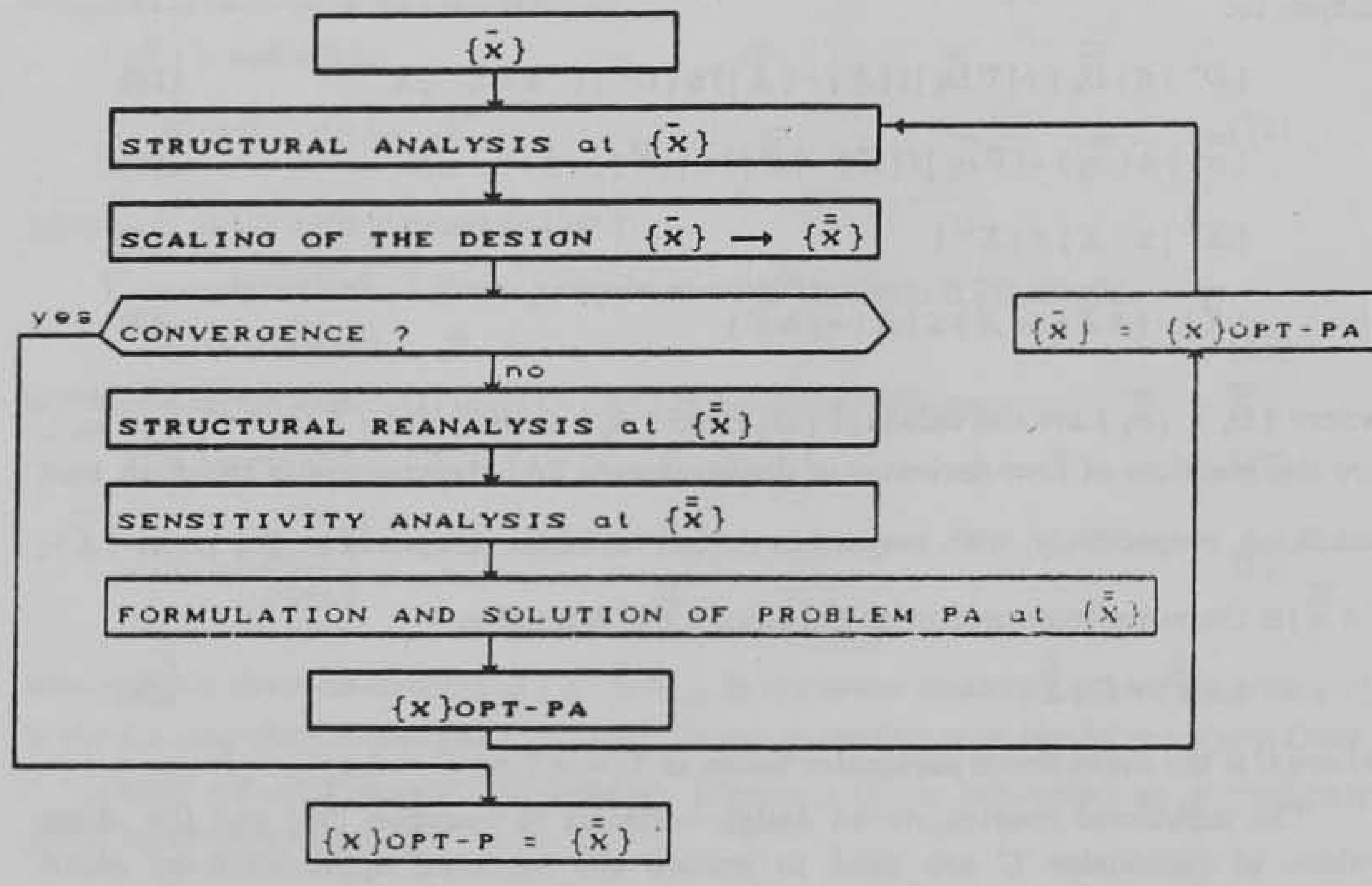


Figure 2. General flow chart of optimum design procedure

Based on truss properties, the structural reanalysis at $\{\overline{\overline{X}}\}$ is efficiently carried out as follows,

$$\{\overline{\overline{D}}_k\} = \frac{1}{S}\{\overline{D}_k\}; \quad \{\overline{\overline{\sigma}}_k\} = \frac{1}{S}\{\overline{\sigma}_k\}; \qquad k = 1,\ldots,K \tag{22}$$

where $\{\overline{D}_k\}$, $\{\overline{\overline{D}}_k\}$, $\{\overline{\sigma}_k\}$, $\{\overline{\overline{\sigma}}_k\}$ are the values of $\{D_k\}$ and $\{\sigma_k\}$ at $\{\overline{X}\}$ and $\{\overline{\overline{X}}\}$, respectively.

The sensitivity analysis at $\{\overline{\overline{X}}\}$, used to determine the matrices $[\nabla\overline{\overline{D}}_k]$ and $[\nabla\overline{\overline{\sigma}}_k]$, is carried out by the behavior space approach [3], [4].

The convergence criterion used here is:

$$\frac{|W_t - W_{t-1}|}{W_t} \le \varepsilon \quad \text{or} \quad t \le t^U \tag{23}$$

where t is the index of the current cycle; W_t, W_{t-1} are the truss weight at the t-th and $(t-1)$-th cycles, respectively; ε is a predetermined tolerance taken here as $\varepsilon = 0.01$; t^U is the upper bound on the cycles number taken as $t^U = 20$.

6. SCALING FACTOR

The scaling factor of $\{\overline{X}\}$ for all the constraints of *problem P* is defined as

$$S = \max(S_i) \quad \text{for all } i \tag{24}$$

where S_i is the scaling factor of $\{\overline{X}\}$ for the constraints on such a quantity as a joint displacement, a member stress, or a design variable.

While it is simple to compute the above scaling factors of $\{\overline{X}\}$ for the constraints bounds [2], [4], [5], the difficulty presented for compression stresses constraints (buckling constraints) is overcome as subsequently.

Scaling the design $\{\overline{X}\}$ only for the constraint on the compression stress σ_{ek} (the e-th element of $\{\sigma_k\}$) by means of an iterative search procedure, the obtained point $\{\overline{\overline{X}}\}$, where the above constraint becomes active, gives

$$S(\sigma_{ek}) = \frac{\overline{\overline{X}}_e}{\overline{X}_e} \tag{25}$$

where $S(\sigma_{ek})$ is the scaling factor of $\{\overline{X}\}$ for the constraint on σ_{ek}; $\overline{X}_e$, $\overline{\overline{X}}_e$ are the e-th elements of $\{\overline{X}\}$ and $\{\overline{\overline{X}}\}$.

As it is certified, $\{\overline{\overline{X}}\}$ lies on the design line of $\{\overline{X}\}$ in the interval of the points $\{\overline{X}\}$ and $\{\overline{\overline{X}}\}_f$, where the last point is

$$\{\overline{\overline{X}}\}_f = S_f\,\{\overline{X}\}; \quad S_f = \frac{\overline{\sigma}_{ek}}{\overline{\sigma}^L_e} \tag{26}$$

where $\overline{\sigma}_{ek}$, $\overline{\sigma}^L_e$ are the e-th elements of $\{\sigma_k\}$ and $\{\sigma^L\}$ at $\{\overline{X}\}$.

The iterative search procedure used to find $\{\overline{\overline{X}}\}$ at a given cycle tests for being $\{\overline{\overline{X}}\}$ the point $\{\hat{X}\}$ in the middle of the interval of the points $\{X_1\}$ and $\{X_2\}$. This interval has the following properties:

1. It contains $\{\overline{\overline{X}}\}$ and lies in the interval of the points $\{\overline{X}\}$ and $\{\overline{\overline{X}}\}_f$, where the last interval is that of the points $\{X_1\}$ and $\{X_2\}$ at the first cycle.

2. Unless the first cycle, the distance between the points $\{X_1\}$ and $\{X_2\}$ at a given cycle is half of that distance at the preceding cycle.

The condition for being $\{\overline{\overline{X}}\}$ the point $\{\hat{X}\}$ is as follows:

$$\frac{\hat{\sigma}_{ek}}{\hat{\sigma}_e^L}=1\pm\varepsilon;\quad \hat{\sigma}_{ek}=\frac{\overline{X}_e}{\hat{X}_e}\,\overline{\sigma}_{ek} \tag{27}$$

where $\hat{\sigma}_{ek}$, $\hat{\sigma}_e^L$ are the e–th elements of $\{\sigma_k\}$ and $\{\sigma^L\}$ at $\{\hat{X}\}$; $\hat{X}_e$ is the e–th element of $\{\hat{X}\}$; ε is a predetermined tolerance taken as $\varepsilon = 0.05$.

The above procedure is computationally efficient one.

7. EXAMPLES

The examples of $T1$ and $T2$ trusses (Figure 3) were solved using the computer program ST–OPT, an implementation of the optimum design procedure presented here.

Two kinds of the initial design of the procedure are considered

$$\{\overline{X}\}=\{A_{\min}\}\quad\text{and}\quad\{\overline{X}\}=\{X\}_{\text{ST-AD}} \tag{28}$$

where $\{A_{\min}\}$ is the vector of cross–sectional minimum areas of truss members; $\{X\}_{\text{ST-AD}}$ is the optimum design by the optimization criterion of the feasible most stressed state and/or the feasible biggest slenderness ratio state, an optimum design obtained using the computer program ST–AD [9].

While the move limits parameter C is assumed to be in the beginning of the procedure $C = 0.2$ and then $C = 0.1$, four cases of the number of cycles where $C = 0.2$ are considered, taking this number as 0, 2, 5, and 10. For each truss example, from the results taken for the above cases, only the results of the case of minimum weight truss are reported here.

Truss members consisting of two equal leg angles (Figure 1 (a)) and the material of Steel 3 with R = 2100 daN/cm^2 are used.

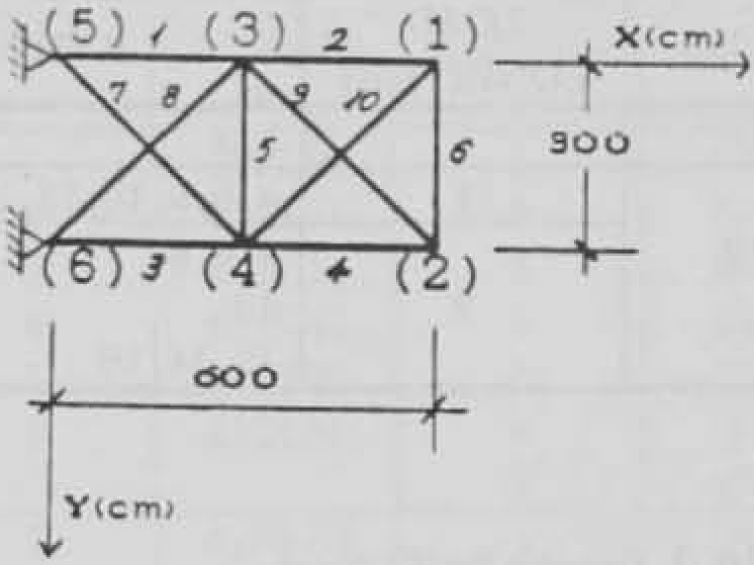
In addition, unless the joints 7 and 11 of $T2$ truss, all the other joints of $T1$ and $T2$ trusses are assumed to be tied out of the truss plane.

T1 Truss

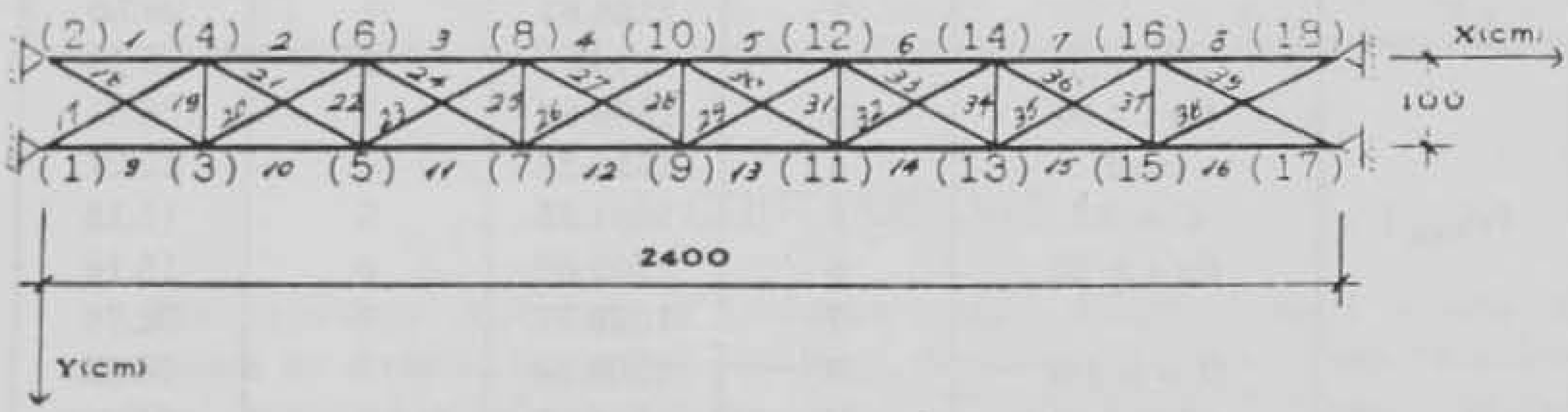
Bounds on displacements in X and Y directions at the joints 1 to 4: ±1cm

T2 Truss

Bounds on displacements in Y directional at the joints 5, 7, 9, 11, 13: +4cm



(a)



(b)

Figure 3. Examples: (a) $T1$ truss; (b) $T2$ truss

Table 1. Members groups for $T2$ truss

GROUP	MEMBERS
1	1–8
2	9–16
3	19, 22, 34, 37
4	25, 28, 31
5	17, 18, 38, 39
6	20, 21, 23, 24, 35, 36, 38, 39
7	26, 27, 29, 30

Table 2. Load conditions for $T1$ and $T2$ trusses

TRUSS	LOAD CONDITION	JOINT	LOADS (daN)	
			X	Y
$T1$	1	2, 4	–	50000
$T2$	1	4, 6, 8, 10, 12, 14, 16	–	7540
	2	4, 6, 8	–	7540
		10	–	6595
		12, 14, 16	–	5650

Table 3. Results for $T1$ truss

INITIAL DESIGN $\{\bar{X}\}$	MOVE LIMITS PARAMETER C	WEIGHT PROGRESSION		OPTIMUM DESIGN	
		CYCLE	WEIGHT (kg)	MEMBER	AREA (cm^2)
$\{A_{min}\}$		1	2145.81	1	98.56
	$C = 0.2$	2	1916.82	2	15.18
	for $t \leq 10$	3	1896.86	3	105.31
	and	4	1750.38	4	51.21
	$C = 0.1$	5	1681.25	5	15.18
	for $t > 10$	6	1709.88	6	15.18
		7	1620.77	7	62.76
	(t = index	8	1600.28	8	76.82
	of cycles)	9	1571.07	9	84.29
		10	15571.44	10	81.26
$\{X\}_{ST\text{-}AD}$		1	1708.75	1	120.62
		2	1606.75	2	15.51
	$C = 0.2$	3	1566.49	3	101.43
	for $t \leq 2$	4	1548.96	4	54.49
	and	5	1532.24	5	10.34
	$C = 0.1$			6	14.43
	for $t > 2$	NOTE: $\{X\}_{ST\text{-}AD}$ is taken after 9 cycles		7	61.77
				8	68.70
				9	72.84
				10	32.73

Table 4. Results for $T2$ truss

INITIAL DESIGN $\{\bar{X}\}$	MOVE LIMITS PARAMETER C	WEIGHT PROGRESSION		OPTIMUM DESIGN	
		CYCLE	WEIGHT (kg)	MEMBER	AREA (cm^2)
$\{A_{min}\}$	$C = 0.2$ for $t \leq 5$ and $C = 0.1$ for $t > 5$	1	4957.88	1	60.72
		2	4619.01	2	83.82
		3	4356.18	3	22.25
		4	4295.42	4	22.25
		5	4169.23	5	50.06
		6	4167.92	6	30.44
				7	22.25
$\{X\}_{ST-AD}$	$C = 0.1$	1	4101.07	1	74.58
		2	4050.34	2	67.77
		3	4040.31	3	12.17
		NOTE: $\{X\}_{ST-AD}$ is taken after 2 cycles		4	12.17
				5	47.08
				6	32.44
				7	18.11

8. CONCLUSIONS

The presented procedure of minimum weight design of steel trusses with constraints on stresses (including buckling), displacements and design variables, a practical procedure complying with the Albanian Code KTP–10–78, through the use of approximate concepts is computationally efficient procedure. The difficulties introduced by the buckling constraints are successfully overcome.

A favourite initial design for the presented procedure is obtained by the computer program ST–AD, leading to the minimum weight design (global optimum) with less computational effort.

The move limits parameter C is recommended being taken $C = 0.2$ in the beginning of the procedure for 5 to 10 cycles for the initial design $\{A_{min}\}$ and up to 2 cycles for $\{X\}_{ST-AD}$; and then $C = 0.1$.

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