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OPTIMAL DISTRIBUTION CENTERS INVOLVING SUPPLY CAPACITY, DEMANDS AND BUDGET RESTRICTIO

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Abstract: Let $G = (V, E)$ be a connected graph expressing a distribution network. The elements of $D \subseteq V$ represent demand centres, while $S \subseteq V$ contains the candidate supply centers. To each node $v_i \in D$, we associate a demand d_i and to each element v_j of S the couple (e_j, c_j), where e_j and c_j are the set up cost and the capacity of v_j respectively. Furthermore, the distance for every arc $(v_i, v_j) \in E$ and the transportation cost of the product unit are given. In this paper an algorithm is developed which determines a subset of S in order to satisfy the demands with a minimum distribution cost, so that the total set up cost of supply centers does not exceed a given budget.

Keywords: Transportation, warehouse location, combinatorial optimization.

1. INTRODUCTION

The Location-allocation problems are connected with economic activities, therefore this class of problems has a great importance in real life cases of applications. Often the notion of transportation is involved with this type of problems.

The Transportation - Location problem has been formulated by Cooper [1], [2]. The subject presented in this paper belongs to the family of transportation - location problems [3], [5]. Specifically, we confront the problem of selecting a subset of locations to install supply centers in order to minimise the total transportation of product cost from a subset of supply centers to the given demand locations taking into account the following three restrictions:

i) the total setup cost of supply centers must not exceed a given budget

ii) the capacity of the candidate supply centers

iii) satisfaction of the required demands

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2. DEFINITIONS - NOTATIONS

The distribution network is expressed by a connected graph $G = (V, E)$, where $V = \{ v_1, v_2, ..., v_n \}$ contains the nodes of the graph, which corresponds to the demand and candidate supply centers, while E comprises the edges of G, that are the pairs (v_i, v_j) of elements of V which are directly connected.

The subsets of V which contains the demand and candidate supply centers are denoted by D and S respectively. The cardinality of D is symbolised by r and this of S by s.

To every demand node $v_i \in D$ corresponds a demand quantity d_i and to every element $v_j \in S$ the pair (c_j, e_j), where c_j expresses the product quantity capacity of $v_j \in S$ and e_j the corresponding setup cost.

The distance of an edge $(v_i, v_j) \in E$ and the corresponding transportation cost of a product unit using this edge are symbolised by a_{ij} and t_{ij} respectively. Finally, let B be the disposed capital in order to establish supply centers.

Let $X = [x_1, x_2, ... ,x_n]$ is a bivalent vector , where $x_i \in \{0, 1\}$. A subset $Q(X) \subseteq S$ is associated to vector X such that

$$\begin{cases} v_j \in Q & \text{if and only if} \quad x_j = 1 \\ v_j \notin Q & \text{if and only if} \quad x_j = 0 \end{cases}$$

Let $T(X)$ represents the distribution transportation cost which corresponds to the subset Q associated with X, taking always into consideration the posed restrictions. Thus, our problem can be formulated as follows

$$\text{minimise} \quad z = T(X), X \in \mathcal{X} \tag{1}$$

under

$$\sum_{i=1}^{s} e_i x_i \le B \tag{2}$$

$$\sum_{i=1}^{s} c_i x_i \ge \sum_{i=1}^{r} d_i \tag{3}$$

$\mathcal{X}$ represents the family of all vectors which satisfies restrictions (2) and (3)

3. THE ALGORITHM

The presented algorithm is a tree search implicit enumeration procedure [8]. Prior to its establishment, two important remarks must be noticed.

i) A vector X is minimal with respect to (3), if and only if its associated subset $Q(X)$ is not contained in any subset $Q'(X') \subseteq S$ for which X' satisfies relation (3).

A minimal vector X of relation (3), which satisfies relation (2), may not give an optimal solution to objective function (1).

ii) A vector X is maximal [4] with respect to relation (2), if and only if its associated subset $Q(X)$ does not contain any subset $Q'(X') \subseteq Q(X)$ for which X' satisfies relation (2).

An optimal solution is a maximal vector X of relation (2), which satisfies (3).

The above reasoning comes from the fact that, if the optimal solution is a subset $Q'(X') \subseteq Q(X)$, this solution will be detected after the application of a procedure which solves the transportation problem [6], since a node $v_j \in Q(X)$,which is not in the optimal solution, will result to a null quantity of supply at this node.

Thus, the proposed algorithm first generates a maximal vector X [4] with respect to relation (2), then it examines if X satisfies relation (3) and subsequently it computes the corresponding value of $T(X)$. The algorithm retains during the process the minimum value found so far of the distribution cost $T(X)$ and its associated vector, yielding the optimal solution at its termination.

The previous discussion is incorporated in the steps of the algorithm presented after the interpretation of its basic items.

S_k : Subset of V, the elements of which are not contained in a partial solution.

An element $s_j \notin S_k$ is comprised in a partial solution of the tree search.

k : Cardinality of S_k

ESk : Sum of the setup costs of a partial solution, i.e. $ESk = \sum_{s_j \notin S_k} e_j$

CSk : Sum of capacities of a partial solution i.e. $CSk = \sum_{s_j \notin S_k} c_j$

A : Sum of the demand quantities

X^* : Vector associated with the best solution found at a certain stage

w_j : Sum of the setup costs of all the candidate elements to augment the current subset S_k.

Step1 (*Initial conditions*)

Read the $s, r, B, c_i, e_i \ \forall i \in \{1,2,\dots,s\}$ and $d_i \ \forall i \in \{1,2,\dots,r\}$.

Set $A = \sum_{i=1}^{r} d_i$, $k = 0$, $ESk = e_1$, $ECk = c_1$, $i = 0$, $S_0 = \varnothing$, $z = \infty$.

$\forall j \in \{s, s-1, \dots, 2\}$ set $w_{j-1} = w_j + e_j$, $ESk = \sum_{j=1}^{s} e_j$, $CSk = \sum_{j=1}^{s} c_j$

Set $i = i+1$ and proceed to next step.

Step 2 (*Branching – feasibility test*)

If $i > s$ then goto step 4

else set $k = k+1, s_k = i$, $S_k = S_{k-1} \cup \{s_k\}$, $ESk = ESk - e_i$, $CSk = CSk - c_i$.

If $ESk > B$ then set $i = i+1$ and repeat step 2

else

if $CSk < A$ then goto step 4

else proceed to next step.

Step 3 (*maximal vector detection, computation of $T(X)$, optimality test*)

$\forall s_j \in S_k$ set $x_{s_j} = 0$ else set $x_{s_j} = 1$ and find $T(X)$.

If $z > T(X)$ then set $z = T(X)$, $X^* = X$. Proceed to next step.

Step 4 (*Backtrack – maximality test and termination test*)

Set $j = s_k$, $ESk = ESk + e_j$, $CSk = CSk + c_j$, $k = k-1$.

If $k < 0$ then write z, X^* and stop

else

if $ESk + w_j < B$ then repeat step 4

else set $i = j+1$ and goto step 2.

4. NUMERICAL EXAMPLE

To get a taste about the useful application of the proposed algorithm a numerical example is next exposed.

Let $G = (V, E)$ be a distribution network as illustrated in Figure 1.

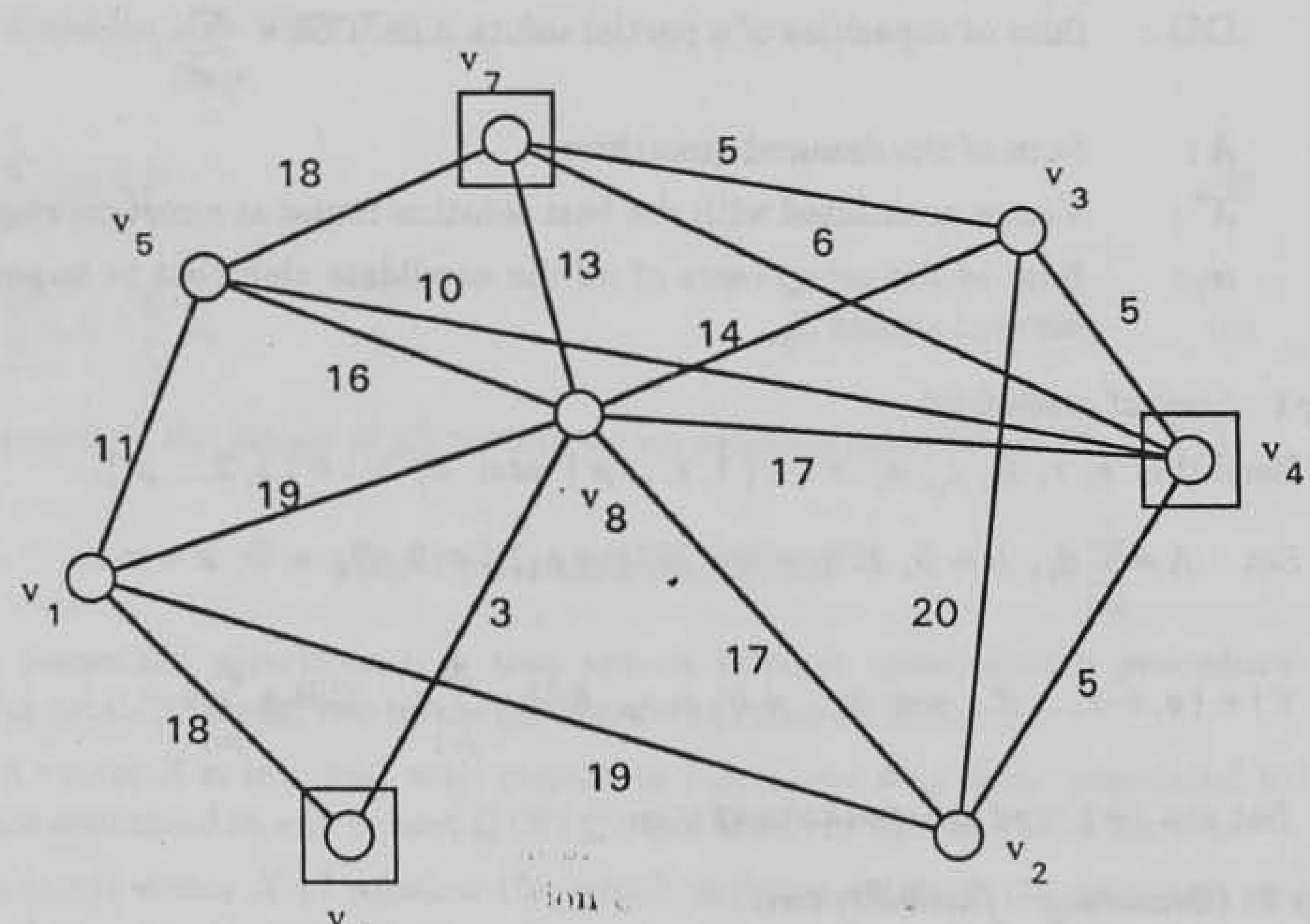

Figure 1.

The following data concerning the capacity, setup cost and demand quantities have been generated randomly

$$\mathcal{C} = [\,284\ 230\ 215\ 214\ 171\ 54\ 47\ 24\,], \qquad c_i \in [\,10, 300\,]$$

$$\mathcal{E} = [\,134\ 121\ 117\ 117\ 104\ 58\ 54\ 39\,], \qquad e_i \in [\,30, 150\,]$$

$$\mathcal{D} = [\,25\ 22\ 46\ 35\ 24\ 28\ 11\ 10\,], \qquad d_i \in [\,10, 50\,],\ A = \sum_{i=1}^{8} d_i = 201$$

We set $B = \dfrac{\sum_{i=1}^{8} e_i}{3} = 248$.

Without loss of generality we assume that the transportation cost of a product unit per distance unit is one, and that $S = D = V$. Therefore the transportation cost from any $v_i \in S$ to any $v_j \in D$ is the value of the shortest path between these nodes. The obtained results using the above data is shown in Table 1.

Table 1.

Serial Number	Maximal Vectors	Distribution cost	Setup Cost	Capacity	DV %	
1	5, 7, 8	1593	197	242	64	
2	5, 6, 8	1785	201	249	72	
3	5, 6, 7	1315	216	272	53	
4	4, 7, 8	1371	210	285	55	
5	4, 6, 8	1096	214	292	44	
6	4, 6, 7	1087	229	315	43	optimal solution
7	4, 5	1373	221	385	55	
8	3, 7, 8	1671	210	286	67	
9	3, 6, 8	1260	214	293	51	
10	3, 6, 7	1307	229	316	52	
11	3, 5	1341	221	386	54	
12	3, 4	1546	234	429	62	
13	3, 7, 8	1612	214	301	65	
14	2, 6, 8	1566	218	308	63	
15	2, 6, 7	1304	233	321	52	
16	2, 5	1723	225	401	69	
17	2, 4	1741	238	444	70	
18	2, 3	1681	238	445	67	
19	1, 7, 8	2151	227	355	86	
20	1, 6, 8	2486	231	362	100	
21	1, 6, 7	1779	246	385	71	
22	1, 5	2210	238	455	88	

The squared nodes of Figure 1 indicates the optimal locations of supply centers.

The distribution of the products which corresponds to the optimal solution (serial number 6) is performed according to Table 2.

Table 2.

Supply Node	Demand Node	Distributed quantity
4	3	10
4	2	22
7	3	36
4	1	9
4	5	24
6	1	16
6	8	10

The last column of Table 1 expresses the percentage deviation between the corresponding to each line distribution cost and the worst obtained one (serial number 20). As we can observe the distribution cost of the optimal solution is less than the half cost of the worst solution, which might be an unplanned selected solution.

Another significant remark is that the total supply capacity of the optimal solution is smaller than the worst one, and that the solution which corresponds to the minimum capacity (serial number 1) is 36% far from the worst solution and 21% far from the optimal one.

5. CONCLUSIONS

The computation of $T(X)$ and the corresponding supply distribution scheme of step 3 was accomplished by routine H03ABF available in NAG Library. The proposed algorithm is an exponential [7] time algorithm due to the procedure which generates the maximal vectors. However, in practical situation the number of the involved variables and the relation between the parameters of inequality (2) permits the application of the stated algorithm successfully.

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