

ON STRONGLY REGULAR GRAPHS WITH $m_2 = qm_3$ AND $m_3 = qm_2$ FOR $q = \frac{7}{2}, \frac{7}{3}, \frac{7}{4}, \frac{7}{5}, \frac{7}{6}$

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Abstract: We say that a regular graph G of order n and degree $r \geq 1$ (which is not the complete graph) is strongly regular if there exist non-negative integers τ and θ such that $|S_i \cap S_j| = \tau$ for any two adjacent vertices i and j , and $|S_i \cap S_j| = \theta$ for any two distinct non-adjacent vertices i and j , where S_k denotes the neighborhood of the vertex k . Let $\lambda_1 = r$, λ_2 and λ_3 be the distinct eigenvalues of a connected strongly regular graph. Let $m_1 = 1$, m_2 and m_3 denote the multiplicity of r , λ_2 and λ_3 , respectively. We here describe the parameters n , r , τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = \frac{7}{2}, \frac{7}{3}, \frac{7}{4}, \frac{7}{5}, \frac{7}{6}$.

Keywords: Strongly Regular Graph, Conference Graph, Integral Graph.

MSC: 05C50.

1. INTRODUCTION

Let G be a simple graph of order n with vertex set $V(G) = \{1, 2, \dots, n\}$. The spectrum of G consists of the eigenvalues $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ of its $(0,1)$ adjacency matrix A and is denoted by $\sigma(G)$. We say that a regular graph G of order n and degree $r \geq 1$ (which is not the complete graph K_n) is strongly regular if there exist non-negative integers τ and θ such that $|S_i \cap S_j| = \tau$ for any two adjacent vertices i and j , and $|S_i \cap S_j| = \theta$ for any two distinct non-adjacent vertices i and j , where $S_k \subseteq V(G)$ denotes the neighborhood of the vertex k . We know that a regular connected graph G is strongly regular if and only if it has exactly three distinct eigenvalues [1] (see also [2]). Let $\lambda_1 = r$, λ_2 and λ_3 denote the distinct eigenvalues of a connected strongly regular graph G . Let $m_1 = 1$, m_2 and m_3 denote the multiplicity of r , λ_2 and λ_3 . Further, let $\bar{r} = (n-1) - r$, $\bar{\lambda}_2 = -\lambda_3 - 1$ and $\bar{\lambda}_3 = -\lambda_2 - 1$ denote the distinct eigenvalues of the strongly regular graph $\bar{G}$,

where $\overline{G}$ denotes the complement of G . Then $\overline{\tau} = n - 2r - 2 + \theta$ and $\overline{\theta} = n - 2r + \tau$ where $\overline{\tau} = \tau(\overline{G})$ and $\overline{\theta} = \theta(\overline{G})$. Next, according to [1] and [2] we have the following two remarks.

Remark 1. (i) if G is a disconnected strongly regular graph of degree r then $G = mK_{r+1}$, where mH denotes the m -fold union of the graph H and (ii) G is a disconnected strongly regular graph if and only if $\theta = 0$.

Remark 2. (i) a strongly regular graph G of order $n = 4k + 1$ and degree $r = 2k$ with $\tau = k - 1$ and $\theta = k$ is called a conference graph; (ii) a strongly regular graph is a conference graph if and only if $m_2 = m_3$ and (iii) if $m_2 \neq m_3$ then G is an integral graph.

We have recently started to investigate strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, where q is a positive integer [3]. In the same work we have described the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 2, 3, 4$. Besides, (i) we have described in [4] the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 5, 6, 7, 8$; (ii) we have described in [5] the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 9, 10$ and (iii) we have described in [6] the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 11, 12$. In particular, we have recently started to investigate strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, where q is a positive rational number [7]. In the same work we have described the parameters n, r, τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = \frac{3}{2}, \frac{4}{3}, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \frac{6}{5}$. We now proceed to describe the parameters of strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = \frac{7}{2}, \frac{7}{3}, \frac{7}{4}, \frac{7}{5}, \frac{7}{6}$, as follows. First,

Theorem 3 (Lepović [7]). Let G be a connected strongly regular graph of order n and degree r with $m_2 = ap$ and $m_3 = bp$, where $a, b, p \in \mathbb{N}$ so that $(a, b) = 1$ and $a > b$. Then:

$$(1^0) \quad n = (a + b)p + 1;$$

$$(2^0) \quad r = pt;$$

$$(3^0) \quad \tau = \left(pt - \frac{ak^2 + kt}{b} \right) - \frac{(a - b)k + t}{b};$$

$$(4^0) \quad \theta = pt - \frac{ak^2 + kt}{b};$$

$$(5^0) \quad \lambda_2 = k;$$

We say that a connected or disconnected graph G is integral if its spectrum $\sigma(G)$ consists only of integral values.

$$(6^0) \quad \lambda_3 = -\frac{ak+t}{b};$$

$$(7^0) \quad \delta = \frac{(a+b)k+t}{b};$$

$$(8^0) \quad (bp+1)t^2 - b((a+b)p+1)t + a(a+b)k^2 + 2akt = 0,$$

for $k \in \mathbb{N}$ and $t = 1, 2, \dots, a+b-1$, where $\delta = \lambda_2 - \lambda_3$.

Theorem 4 (Lepović [7]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = bp$ and $m_3 = ap$, where $a, b, p \in \mathbb{N}$ so that $(a, b) = 1$ and $a > b$. Then:*

$$(1^0) \quad n = (a+b)p+1;$$

$$(2^0) \quad r = pt;$$

$$(3^0) \quad \tau = \left(pt - \frac{ak^2 - kt}{b}\right) + \frac{(a-b)k - t}{b};$$

$$(4^0) \quad \theta = pt - \frac{ak^2 - kt}{b};$$

$$(5^0) \quad \lambda_2 = \frac{ak - t}{b};$$

$$(6^0) \quad \lambda_3 = -k;$$

$$(7^0) \quad \delta = \frac{(a+b)k - t}{b};$$

$$(8^0) \quad (bp+1)t^2 - b((a+b)p+1)t + a(a+b)k^2 - 2akt = 0,$$

for $k \in \mathbb{N}$ and $t = 1, 2, \dots, a+b-1$, where $\delta = \lambda_2 - \lambda_3$.

2. MAIN RESULTS

Remark 5. *Since $m_2(\overline{G}) = m_3(G)$ and $m_3(\overline{G}) = m_2(G)$ we note that if $m_2(G) = qm_3(G)$ then $m_3(\overline{G}) = qm_2(\overline{G})$.*

Remark 6. *In Theorems 11–15 the complements of strongly regular graphs appear in pairs in (k^0) and $(\overline{k}^0)$ classes, where k denotes the corresponding number of a class.*

Remark 7. *$\overline{\alpha K_\beta}$ is a strongly regular graph of order $n = \alpha\beta$ and degree $r = (\alpha-1)\beta$ with $\tau = (\alpha-2)\beta$ and $\theta = (\alpha-1)\beta$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -\beta$ with $m_2 = \alpha(\beta-1)$ and $m_3 = \alpha-1$.*

In order to demonstrate a method which is applied for describing the parameters n , r , τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = \frac{7}{2}, \frac{7}{3}, \frac{7}{4}, \frac{7}{5}, \frac{7}{6}$, we shall here establish the parameters of strongly regular graphs with $m_2 = (\frac{7}{2})m_3$ and $m_3 = (\frac{7}{2})m_2$. In a similar way, we can establish Theorems 12, 13, 14 and 15.

Proposition 8. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{7}{2})m_3$. Then G belongs to the class $(\bar{2}^0)$ or (3^0) or (4^0) or $(\bar{5}^0)$ or $(\bar{6}^0)$ or (7^0) or $(\bar{8}^0)$ or (9^0) represented in Theorem 11.*

Proof. Let $m_2 = 7p$, $m_3 = 2p$ and $n = 9p + 1$ where $p \in \mathbb{N}$. Let $\lambda_2 = k$ where k is a positive integer. Then according to Theorem 3 we have (i) $\lambda_3 = -\frac{7k+t}{2}$; (ii) $\tau - \theta = -\frac{5k+t}{2}$; (iii) $\delta = \frac{9k+t}{2}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{7k^2+kt}{2}$, where $t = 1, 2, \dots, 8$. In this case we can easily see that Theorem 3 (8^0) is reduced to

$$(2p+1)t^2 - 2(9p+1)t + 63k^2 + 14kt = 0. \quad (1)$$

Case 1. ($t = 1$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+1}{2}$, $\tau - \theta = -\frac{5k+1}{2}$, $\delta = \frac{9k+1}{2}$, $r = p$ and $\theta = p - \frac{7k^2+k}{2}$. Using (1) we find that $16p+1 = 7k(9k+2)$. Replacing k with $4k-1$ we arrive at $p = 63k^2 - 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k-2)^2$ and degree $r = 63k^2 - 28k + 3$ with $\tau = 7k^2 - 12k + 2$ and $\theta = k(7k-2)$.

Case 2. ($t = 2$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+2}{2}$, $\tau - \theta = -\frac{5k+2}{2}$, $\delta = \frac{9k+2}{2}$, $r = 2p$ and $\theta = 2p - \frac{7k^2+2k}{2}$. Using (1) we find that $4p = k(9k+4)$. Replacing k with $2k$ we arrive at $p = k(9k+2)$. So we obtain that G is a strongly regular graph of order $n = (9k+1)^2$ and degree $r = 2k(9k+2)$ with $\tau = 4k^2 - 3k - 1$ and $\theta = 2k(2k+1)$.

Case 3. ($t = 3$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+3}{2}$, $\tau - \theta = -\frac{5k+3}{2}$, $\delta = \frac{9k+3}{2}$, $r = 3p$ and $\theta = 3p - \frac{7k^2+3k}{2}$. Using (1) we find that $12p-1 = 7k(3k+2)$. Replacing k with $6k+1$ we arrive at $p = 63k^2 + 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k+2)^2$ and degree $r = 3(63k^2 + 28k + 3)$ with $\tau = 9k(7k+2)$ and $\theta = (3k+1)(21k+4)$.

Case 4. ($t = 4$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+4}{2}$, $\tau - \theta = -\frac{5k+4}{2}$, $\delta = \frac{9k+4}{2}$, $r = 4p$ and $\theta = 4p - \frac{7k^2+4k}{2}$. Using (1) we find that $40p-8 = 7k(9k+8)$. Replacing k with $20k+4$ we arrive at $p = 630k^2 + 280k + 31$. So we obtain that G is a strongly regular graph of order $n = 70(9k+2)^2$ and degree $r = 4(630k^2 + 280k + 31)$ with $\tau = 2(560k^2 + 235k + 24)$ and $\theta = 20(4k+1)(14k+3)$.

Case 5. ($t = 5$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+5}{2}$, $\tau - \theta = -\frac{5k+5}{2}$, $\delta = \frac{9k+5}{2}$, $r = 5p$ and $\theta = 5p - \frac{7k^2+5k}{2}$. Using (1) we find that $40p-15 = 7k(9k+10)$. Replacing k with $20k-5$ we arrive at $p = 630k^2 - 280k + 31$. So we obtain that G is a strongly regular graph of order $n = 70(9k-2)^2$ and degree $r = 5(630k^2 - 280k + 31)$ with $\tau = 10(5k-1)(35k-9)$ and $\theta = 10(5k-1)(35k-8)$.

Case 6. ($t = 6$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+6}{2}$, $\tau - \theta = -\frac{5k+6}{2}$, $\delta = \frac{9k+6}{2}$, $r = 6p$ and $\theta = 6p - \frac{7k^2+6k}{2}$. Using (1) we find that $12p - 8 = 7k(3k + 4)$. Replacing k with $6k - 2$ we arrive at $p = 63k^2 - 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k - 2)^2$ and degree $r = 6(63k^2 - 28k + 3)$ with $\tau = 3(84k^2 - 39k + 4)$ and $\theta = 2(6k - 1)(21k - 5)$.

Case 7. ($t = 7$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+7}{2}$, $\tau - \theta = -\frac{5k+7}{2}$, $\delta = \frac{9k+7}{2}$, $r = 7p$ and $\theta = 7p - \frac{7k^2+7k}{2}$. Using (1) we find that $4p - 5 = k(9k + 14)$. Replacing k with $2k - 1$ we arrive at $p = k(9k - 2)$. So we obtain that G is a strongly regular graph of order $n = (9k - 1)^2$ and degree $r = 7k(9k - 2)$ with $\tau = 49k^2 - 12k - 1$ and $\theta = 7k(7k - 1)$.

Case 8. ($t = 8$). Using (i)–(v) we find that $\lambda_2 = k$ and $\lambda_3 = -\frac{7k+8}{2}$, $\tau - \theta = -\frac{5k+8}{2}$, $\delta = \frac{9k+8}{2}$, $r = 8p$ and $\theta = 8p - \frac{7k^2+8k}{2}$. Using (1) we find that $16p - 48 = 7k(9k + 16)$. Replacing k with $4k$ we arrive at $p = 63k^2 + 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k + 2)^2$ and degree $r = 8(63k^2 + 28k + 3)$ with $\tau = 2(7k + 2)(32k + 5)$ and $\theta = 8(4k + 1)(14k + 3)$. $\square$

Proposition 9. Let G be a connected strongly regular graph of order n and degree r with $m_3 = (\frac{7}{2})m_2$. Then G belongs to the class (2^0) or (3^0) or (4^0) or (5^0) or (6^0) or (7^0) or (8^0) or (9^0) represented in Theorem 11.

Proof. Let $m_2 = 2p$, $m_3 = 7p$ and $n = 9p + 1$ where $p \in \mathbb{N}$. Let $\lambda_3 = -k$ where k is a positive integer. Then according to Theorem 4 we have (i) $\lambda_2 = \frac{7k-1}{2}$; (ii) $\tau - \theta = \frac{5k-t}{2}$; (iii) $\delta = \frac{9k-t}{2}$; (iv) $r = pt$ and (v) $\theta = pt - \frac{7k^2-kt}{2}$, where $t = 1, 2, \dots, 8$. In this case we can easily see that Theorem 4 (8^0) is reduced to

$$(2p+1)t^2 - 2(9p+1)t + 63k^2 - 14kt = 0. \quad (2)$$

Case 1. ($t = 1$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-1}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-1}{2}$, $\delta = \frac{9k-1}{2}$, $r = p$ and $\theta = p - \frac{7k^2-k}{2}$. Using (2) we find that $16p + 1 = 7k(9k - 2)$. Replacing k with $4k + 1$ we arrive at $p = 63k^2 + 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k + 2)^2$ and degree $r = 63k^2 + 28k + 3$ with $\tau = 7k^2 + 12k + 2$ and $\theta = k(7k + 2)$.

Case 2. ($t = 2$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-2}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-2}{2}$, $\delta = \frac{9k-2}{2}$, $r = 2p$ and $\theta = 2p - \frac{7k^2-2k}{2}$. Using (2) we find that $4p = k(9k - 4)$. Replacing k with $2k$ we arrive at $p = k(9k - 2)$. So we obtain that G is a strongly regular graph of order $n = (9k - 1)^2$ and degree $r = 2k(9k - 2)$ with $\tau = 4k^2 + 3k - 1$ and $\theta = 2k(2k - 1)$.

Case 3. ($t = 3$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-3}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-3}{2}$, $\delta = \frac{9k-3}{2}$, $r = 3p$ and $\theta = 3p - \frac{7k^2-3k}{2}$. Using (2) we find that $12p - 1 = 7k(3k - 2)$. Replacing k with $6k - 1$ we arrive at $p = 63k^2 - 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k - 2)^2$ and degree $r = 3(63k^2 - 28k + 3)$ with $\tau = 9k(7k - 2)$ and $\theta = (3k - 1)(21k - 4)$.

Case 4. ($t = 4$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-4}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-4}{2}$, $\delta = \frac{9k-4}{2}$, $r = 4p$ and $\theta = 4p - \frac{7k^2-4k}{2}$. Using (2) we find that $40p - 8 = 7k(9k - 8)$. Replacing k with $20k - 4$ we arrive at $p = 630k^2 - 280k + 31$. So we obtain that G is a strongly regular graph of order $n = 70(9k - 2)^2$ and degree $r = 4(630k^2 - 280k + 31)$ with $\tau = 2(560k^2 - 235k + 24)$ and $\theta = 20(4k - 1)(14k - 3)$.

Case 5. ($t = 5$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-5}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-5}{2}$, $\delta = \frac{9k-5}{2}$, $r = 5p$ and $\theta = 5p - \frac{7k^2-5k}{2}$. Using (2) we find that $40p - 15 = 7k(9k - 10)$. Replacing k with $20k + 5$ we arrive at $p = 630k^2 + 280k + 31$. So we obtain that G is a strongly regular graph of order $n = 70(9k + 2)^2$ and degree $r = 5(630k^2 + 280k + 31)$ with $\tau = 10(5k + 1)(35k + 9)$ and $\theta = 10(5k + 1)(35k + 8)$.

Case 6. ($t = 6$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-6}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-6}{2}$, $\delta = \frac{9k-6}{2}$, $r = 6p$ and $\theta = 6p - \frac{7k^2-6k}{2}$. Using (2) we find that $12p - 8 = 7k(3k - 4)$. Replacing k with $6k + 2$ we arrive at $p = 63k^2 + 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k + 2)^2$ and degree $r = 6(63k^2 + 28k + 3)$ with $\tau = 3(84k^2 + 39k + 4)$ and $\theta = 2(6k + 1)(21k + 5)$.

Case 7. ($t = 7$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-7}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-7}{2}$, $\delta = \frac{9k-7}{2}$, $r = 7p$ and $\theta = 7p - \frac{7k^2-7k}{2}$. Using (2) we find that $4p - 5 = k(9k - 14)$. Replacing k with $2k + 1$ we arrive at $p = k(9k + 2)$. So we obtain that G is a strongly regular graph of order $n = (9k + 1)^2$ and degree $r = 7k(9k + 2)$ with $\tau = 49k^2 + 12k - 1$ and $\theta = 7k(7k + 1)$.

Case 8. ($t = 8$). Using (i)–(v) we find that $\lambda_2 = \frac{7k-8}{2}$ and $\lambda_3 = -k$, $\tau - \theta = \frac{5k-8}{2}$, $\delta = \frac{9k-8}{2}$, $r = 8p$ and $\theta = 8p - \frac{7k^2-8k}{2}$. Using (2) we find that $16p - 48 = 7k(9k - 16)$. Replacing k with $4k$ we arrive at $p = 63k^2 - 28k + 3$. So we obtain that G is a strongly regular graph of order $n = 7(9k - 2)^2$ and degree $r = 8(63k^2 - 28k + 3)$ with $\tau = 2(7k - 2)(32k - 5)$ and $\theta = 8(4k - 1)(14k - 3)$. $\square$

Remark 10. We note that $\overline{7K_4}$ is a strongly regular graph with $m_2 = (\frac{7}{2})m_3$. It is obtained from the class Theorem 11 ($\overline{6}^0$) for $k = 0$.

Theorem 11. Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{7}{2})m_3$ or $m_3 = (\frac{7}{2})m_2$. Then G is one of the following strongly regular graphs:

- (1⁰) G is the strongly regular graph $\overline{7K_4}$ of order $n = 28$ and degree $r = 24$ with $\tau = 20$ and $\theta = 24$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -4$ with $m_2 = 21$ and $m_3 = 6$;
- (2⁰) G is a strongly regular graph of order $n = (9k - 1)^2$ and degree $r = 2k(9k - 2)$ with $\tau = 4k^2 + 3k - 1$ and $\theta = 2k(2k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k - 1$ and $\lambda_3 = -2k$ with $m_2 = 2k(9k - 2)$ and $m_3 = 7k(9k - 2)$;
- ($\overline{2}^0$) G is a strongly regular graph of order $n = (9k - 1)^2$ and degree $r = 7k(9k - 2)$ with $\tau = 49k^2 - 12k - 1$ and $\theta = 7k(7k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k - 1$ and $\lambda_3 = -7k$ with $m_2 = 7k(9k - 2)$ and $m_3 = 2k(9k - 2)$;

- (3⁰) G is a strongly regular graph of order $n = (9k+1)^2$ and degree $r = 2k(9k+2)$ with $\tau = 4k^2 - 3k - 1$ and $\theta = 2k(2k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k$ and $\lambda_3 = -(7k+1)$ with $m_2 = 7k(9k+2)$ and $m_3 = 2k(9k+2)$;
- (3⁰) G is a strongly regular graph of order $n = (9k+1)^2$ and degree $r = 7k(9k+2)$ with $\tau = 49k^2 + 12k - 1$ and $\theta = 7k(7k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k$ and $\lambda_3 = -(2k+1)$ with $m_2 = 2k(9k+2)$ and $m_3 = 7k(9k+2)$;
- (4⁰) G is a strongly regular graph of order $n = 7(9k-2)^2$ and degree $r = 63k^2 - 28k+3$ with $\tau = 7k^2 - 12k+2$ and $\theta = k(7k-2)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 4k - 1$ and $\lambda_3 = -(14k - 3)$ with $m_2 = 7(63k^2 - 28k + 3)$ and $m_3 = 2(63k^2 - 28k + 3)$;
- (4⁰) G is a strongly regular graph of order $n = 7(9k-2)^2$ and degree $r = 8(63k^2 - 28k+3)$ with $\tau = 2(7k-2)(32k-5)$ and $\theta = 8(4k-1)(14k-3)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 14k - 4$ and $\lambda_3 = -4k$ with $m_2 = 2(63k^2 - 28k + 3)$ and $m_3 = 7(63k^2 - 28k + 3)$;
- (5⁰) G is a strongly regular graph of order $n = 7(9k-2)^2$ and degree $r = 3(63k^2 - 28k+3)$ with $\tau = 9k(7k-2)$ and $\theta = (3k-1)(21k-4)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 21k-5$ and $\lambda_3 = -(6k-1)$ with $m_2 = 2(63k^2 - 28k + 3)$ and $m_3 = 7(63k^2 - 28k + 3)$;
- (5⁰) G is a strongly regular graph of order $n = 7(9k-2)^2$ and degree $r = 6(63k^2 - 28k+3)$ with $\tau = 3(84k^2 - 39k+4)$ and $\theta = 2(6k-1)(21k-5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 6k-2$ and $\lambda_3 = -(21k-4)$ with $m_2 = 7(63k^2 - 28k + 3)$ and $m_3 = 2(63k^2 - 28k + 3)$;
- (6⁰) G is a strongly regular graph of order $n = 7(9k+2)^2$ and degree $r = 63k^2 + 28k+3$ with $\tau = 7k^2 + 12k+2$ and $\theta = k(7k+2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 14k+3$ and $\lambda_3 = -(4k+1)$ with $m_2 = 2(63k^2 + 28k + 3)$ and $m_3 = 7(63k^2 + 28k + 3)$;
- (6⁰) G is a strongly regular graph of order $n = 7(9k+2)^2$ and degree $r = 8(63k^2 + 28k+3)$ with $\tau = 2(7k+2)(32k+5)$ and $\theta = 8(4k+1)(14k+3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 4k$ and $\lambda_3 = -(14k+4)$ with $m_2 = 7(63k^2 + 28k + 3)$ and $m_3 = 2(63k^2 + 28k + 3)$;
- (7⁰) G is a strongly regular graph of order $n = 7(9k+2)^2$ and degree $r = 3(63k^2 + 28k+3)$ with $\tau = 9k(7k+2)$ and $\theta = (3k+1)(21k+4)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 6k+1$ and $\lambda_3 = -(21k+5)$ with $m_2 = 7(63k^2 + 28k + 3)$ and $m_3 = 2(63k^2 + 28k + 3)$;
- (7⁰) G is a strongly regular graph of order $n = 7(9k+2)^2$ and degree $r = 6(63k^2 + 28k+3)$ with $\tau = 3(84k^2 + 39k+4)$ and $\theta = 2(6k+1)(21k+5)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 21k+4$ and $\lambda_3 = -(6k+2)$ with $m_2 = 2(63k^2 + 28k + 3)$ and $m_3 = 7(63k^2 + 28k + 3)$;

- (8^0) G is a strongly regular graph of order $n = 70(9k - 2)^2$ and degree $r = 4(630k^2 - 280k + 31)$ with $\tau = 2(560k^2 - 235k + 24)$ and $\theta = 20(4k - 1)(14k - 3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 70k - 16$ and $\lambda_3 = -(20k - 4)$ with $m_2 = 2(630k^2 - 280k + 31)$ and $m_3 = 7(630k^2 - 280k + 31)$;
- ($\bar{8}^0$) G is a strongly regular graph of order $n = 70(9k - 2)^2$ and degree $r = 5(630k^2 - 280k + 31)$ with $\tau = 10(5k - 1)(35k - 9)$ and $\theta = 10(5k - 1)(35k - 8)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 20k - 5$ and $\lambda_3 = -(70k - 15)$ with $m_2 = 7(630k^2 - 280k + 31)$ and $m_3 = 2(630k^2 - 280k + 31)$;
- (9^0) G is a strongly regular graph of order $n = 70(9k + 2)^2$ and degree $r = 4(630k^2 + 280k + 31)$ with $\tau = 2(560k^2 + 235k + 24)$ and $\theta = 20(4k + 1)(14k + 3)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 20k + 4$ and $\lambda_3 = -(70k + 16)$ with $m_2 = 7(630k^2 + 280k + 31)$ and $m_3 = 2(630k^2 + 280k + 31)$;
- ($\bar{9}^0$) G is a strongly regular graph of order $n = 70(9k + 2)^2$ and degree $r = 5(630k^2 + 280k + 31)$ with $\tau = 10(5k + 1)(35k + 9)$ and $\theta = 10(5k + 1)(35k + 8)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 70k + 15$ and $\lambda_3 = -(20k + 5)$ with $m_2 = 2(630k^2 + 280k + 31)$ and $m_3 = 7(630k^2 + 280k + 31)$.

Proof. First, according to Remark 7 we have $2\alpha(\beta - 1) = 7(\alpha - 1)$, from which we find that $\alpha = 7$, $\beta = 4$. In view of this we obtain the strongly regular graph represented in Theorem 11 (1^0). Next, according to Proposition 8 it turns out that G belongs to the class ($\bar{2}^0$) or (3^0) or (4^0) or ($\bar{5}^0$) or ($\bar{6}^0$) or (7^0) or ($\bar{8}^0$) or (9^0) if $m_2 = (\frac{7}{2})m_3$. According to Proposition 9 it turns out that G belongs to the class (2^0) or ($\bar{3}^0$) or ($\bar{4}^0$) or (5^0) or (6^0) or ($\bar{7}^0$) or (8^0) or ($\bar{9}^0$) if $m_3 = (\frac{7}{2})m_2$. $\square$

Theorem 12. Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{7}{3})m_3$ or $m_3 = (\frac{7}{3})m_2$. Then G is one of the following strongly regular graphs:

- (1^0) G is the strongly regular graph $\overline{7K_3}$ of order $n = 21$ and degree $r = 18$ with $\tau = 15$ and $\theta = 18$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -3$ with $m_2 = 14$ and $m_3 = 6$;
- (2^0) G is a strongly regular graph of order $n = (10k - 1)^2$ and degree $r = 6k(5k - 1)$ with $\tau = 9k^2 + k - 1$ and $\theta = 3k(3k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k - 1$ and $\lambda_3 = -3k$ with $m_2 = 6k(5k - 1)$ and $m_3 = 14k(5k - 1)$;
- ($\bar{2}^0$) G is a strongly regular graph of order $n = (10k - 1)^2$ and degree $r = 14k(5k - 1)$ with $\tau = 49k^2 - 11k - 1$ and $\theta = 7k(7k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k - 1$ and $\lambda_3 = -7k$ with $m_2 = 14k(5k - 1)$ and $m_3 = 6k(5k - 1)$;
- (3^0) G is a strongly regular graph of order $n = (10k + 1)^2$ and degree $r = 6k(5k + 1)$ with $\tau = 9k^2 - k - 1$ and $\theta = 3k(3k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k$ and $\lambda_3 = -(7k + 1)$ with $m_2 = 14k(5k + 1)$ and $m_3 = 6k(5k + 1)$;

- ($\bar{3}^0$) G is a strongly regular graph of order $n = (10k+1)^2$ and degree $r = 14k(5k+1)$ with $\tau = 49k^2 + 11k - 1$ and $\theta = 7k(7k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k$ and $\lambda_3 = -(3k+1)$ with $m_2 = 6k(5k+1)$ and $m_3 = 14k(5k+1)$;
- (4^0) G is a strongly regular graph of order $n = 21(10k-1)^2$ and degree $r = 210k^2 - 42k + 2$ with $\tau = 21k^2 - 15k + 1$ and $\theta = 3k(7k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 9k-1$ and $\lambda_3 = -(21k-2)$ with $m_2 = 7(210k^2 - 42k + 2)$ and $m_3 = 3(210k^2 - 42k + 2)$;
- ($\bar{4}^0$) G is a strongly regular graph of order $n = 21(10k-1)^2$ and degree $r = 9(210k^2 - 42k + 2)$ with $\tau = 3(567k^2 - 113k + 5)$ and $\theta = 9(9k-1)(21k-2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 21k-3$ and $\lambda_3 = -9k$ with $m_2 = 3(210k^2 - 42k + 2)$ and $m_3 = 7(210k^2 - 42k + 2)$;
- (5^0) G is a strongly regular graph of order $n = 21(10k-1)^2$ and degree $r = 2(210k^2 - 42k + 2)$ with $\tau = 84k^2 - 4k - 1$ and $\theta = (6k-1)(14k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 28k-3$ and $\lambda_3 = -(12k-1)$ with $m_2 = 3(210k^2 - 42k + 2)$ and $m_3 = 7(210k^2 - 42k + 2)$;
- ($\bar{5}^0$) G is a strongly regular graph of order $n = 21(10k-1)^2$ and degree $r = 8(210k^2 - 42k + 2)$ with $\tau = 4(336k^2 - 68k + 3)$ and $\theta = 4(12k-1)(28k-3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k-2$ and $\lambda_3 = -(28k-2)$ with $m_2 = 7(210k^2 - 42k + 2)$ and $m_3 = 3(210k^2 - 42k + 2)$;
- (6^0) G is a strongly regular graph of order $n = 21(10k-1)^2$ and degree $r = 5(210k^2 - 42k + 2)$ with $\tau = 525k^2 - 115k + 5$ and $\theta = (15k-1)(35k-4)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 15k-2$ and $\lambda_3 = -(35k-3)$ with $m_2 = 7(210k^2 - 42k + 2)$ and $m_3 = 3(210k^2 - 42k + 2)$;
- ($\bar{6}^0$) G is a strongly regular graph of order $n = 21(10k-1)^2$ and degree $r = 5(210k^2 - 42k + 2)$ with $\tau = 525k^2 - 95k + 3$ and $\theta = (15k-2)(35k-3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 35k-4$ and $\lambda_3 = -(15k-1)$ with $m_2 = 3(210k^2 - 42k + 2)$ and $m_3 = 7(210k^2 - 42k + 2)$;
- (7^0) G is a strongly regular graph of order $n = 21(10k+1)^2$ and degree $r = 210k^2 + 42k + 2$ with $\tau = 21k^2 + 15k + 1$ and $\theta = 3k(7k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 21k+2$ and $\lambda_3 = -(9k+1)$ with $m_2 = 3(210k^2 + 42k + 2)$ and $m_3 = 7(210k^2 + 42k + 2)$;
- ($\bar{7}^0$) G is a strongly regular graph of order $n = 21(10k+1)^2$ and degree $r = 9(210k^2 + 42k + 2)$ with $\tau = 3(567k^2 + 113k + 5)$ and $\theta = 9(9k+1)(21k+2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 9k$ and $\lambda_3 = -(21k+3)$ with $m_2 = 7(210k^2 + 42k + 2)$ and $m_3 = 3(210k^2 + 42k + 2)$;
- (8^0) G is a strongly regular graph of order $n = 21(10k+1)^2$ and degree $r = 2(210k^2 + 42k + 2)$ with $\tau = 84k^2 + 4k - 1$ and $\theta = (6k+1)(14k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k+1$ and $\lambda_3 = -(28k+3)$ with $m_2 = 7(210k^2 + 42k + 2)$ and $m_3 = 3(210k^2 + 42k + 2)$;

- ($\bar{8}^0$) G is a strongly regular graph of order $n = 21(10k + 1)^2$ and degree $r = 8(210k^2 + 42k + 2)$ with $\tau = 4(336k^2 + 68k + 3)$ and $\theta = 4(12k + 1)(28k + 3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 28k + 2$ and $\lambda_3 = -(12k + 2)$ with $m_2 = 3(210k^2 + 42k + 2)$ and $m_3 = 7(210k^2 + 42k + 2)$;
- (9^0) G is a strongly regular graph of order $n = 21(10k + 1)^2$ and degree $r = 5(210k^2 + 42k + 2)$ with $\tau = 525k^2 + 95k + 3$ and $\theta = (15k + 2)(35k + 3)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 15k + 1$ and $\lambda_3 = -(35k + 4)$ with $m_2 = 7(210k^2 + 42k + 2)$ and $m_3 = 3(210k^2 + 42k + 2)$;
- ($\bar{9}^0$) G is a strongly regular graph of order $n = 21(10k + 1)^2$ and degree $r = 5(210k^2 + 42k + 2)$ with $\tau = 525k^2 + 115k + 5$ and $\theta = (15k + 1)(35k + 4)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 35k + 3$ and $\lambda_3 = -(15k + 2)$ with $m_2 = 3(210k^2 + 42k + 2)$ and $m_3 = 7(210k^2 + 42k + 2)$.

Theorem 13. Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{7}{4})m_3$ or $m_3 = (\frac{7}{4})m_2$. Then G is one of the following strongly regular graphs:

- (1^0) G is a strongly regular graph of order $n = (11k - 1)^2$ and degree $r = 4k(11k - 2)$ with $\tau = 16k^2 - k - 1$ and $\theta = 4k(4k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k - 1$ and $\lambda_3 = -4k$ with $m_2 = 4k(11k - 2)$ and $m_3 = 7k(11k - 2)$;
- ($\bar{1}^0$) G is a strongly regular graph of order $n = (11k - 1)^2$ and degree $r = 7k(11k - 2)$ with $\tau = 49k^2 - 10k - 1$ and $\theta = 7k(7k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 4k - 1$ and $\lambda_3 = -7k$ with $m_2 = 7k(11k - 2)$ and $m_3 = 4k(11k - 2)$;
- (2^0) G is a strongly regular graph of order $n = (11k + 1)^2$ and degree $r = 4k(11k + 2)$ with $\tau = 16k^2 + k - 1$ and $\theta = 4k(4k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 4k$ and $\lambda_3 = -(7k + 1)$ with $m_2 = 7k(11k + 2)$ and $m_3 = 4k(11k + 2)$;
- ($\bar{2}^0$) G is a strongly regular graph of order $n = (11k + 1)^2$ and degree $r = 7k(11k + 2)$ with $\tau = 49k^2 + 10k - 1$ and $\theta = 7k(7k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k$ and $\lambda_3 = -(4k + 1)$ with $m_2 = 4k(11k + 2)$ and $m_3 = 7k(11k + 2)$;
- (3^0) G is a strongly regular graph of order $n = 14(11k - 2)^2$ and degree $r = 2(154k^2 - 56k + 5)$ with $\tau = k(56k - 13)$ and $\theta = 2(4k - 1)(7k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 21k - 4$ and $\lambda_3 = -(12k - 2)$ with $m_2 = 4(154k^2 - 56k + 5)$ and $m_3 = 7(154k^2 - 56k + 5)$;
- ($\bar{3}^0$) G is a strongly regular graph of order $n = 14(11k - 2)^2$ and degree $r = 9(154k^2 - 56k + 5)$ with $\tau = 18(7k - 1)(9k - 2)$ and $\theta = 9(6k - 1)(21k - 4)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k - 3$ and $\lambda_3 = -(21k - 3)$ with $m_2 = 7(154k^2 - 56k + 5)$ and $m_3 = 4(154k^2 - 56k + 5)$;

- (4⁰) G is a strongly regular graph of order $n = 14(11k + 2)^2$ and degree $r = 2(154k^2 + 56k + 5)$ with $\tau = k(56k + 13)$ and $\theta = 2(4k + 1)(7k + 1)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 12k + 2$ and $\lambda_3 = -(21k + 4)$ with $m_2 = 7(154k^2 + 56k + 5)$ and $m_3 = 4(154k^2 + 56k + 5)$;
- ($\bar{4}$ ⁰) G is a strongly regular graph of order $n = 14(11k + 2)^2$ and degree $r = 9(154k^2 + 56k + 5)$ with $\tau = 18(7k + 1)(9k + 2)$ and $\theta = 9(6k + 1)(21k + 4)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 21k + 3$ and $\lambda_3 = -(12k + 3)$ with $m_2 = 4(154k^2 + 56k + 5)$ and $m_3 = 7(154k^2 + 56k + 5)$;
- (5⁰) G is a strongly regular graph of order $n = 42(11k - 4)^2$ and degree $r = 3(462k^2 - 336k + 61)$ with $\tau = 18(3k - 1)(7k - 3)$ and $\theta = 6(3k - 1)(21k - 8)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 24k - 9$ and $\lambda_3 = -(42k - 15)$ with $m_2 = 7(462k^2 - 336k + 61)$ and $m_3 = 4(462k^2 - 336k + 61)$;
- ($\bar{5}$ ⁰) G is a strongly regular graph of order $n = 42(11k - 4)^2$ and degree $r = 8(462k^2 - 336k + 61)$ with $\tau = 2(1344k^2 - 975k + 176)$ and $\theta = 24(8k - 3)(14k - 5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 42k - 16$ and $\lambda_3 = -(24k - 8)$ with $m_2 = 4(462k^2 - 336k + 61)$ and $m_3 = 7(462k^2 - 336k + 61)$;
- (6⁰) G is a strongly regular graph of order $n = 42(11k + 4)^2$ and degree $r = 3(462k^2 + 336k + 61)$ with $\tau = 18(3k + 1)(7k + 3)$ and $\theta = 6(3k + 1)(21k + 8)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 42k + 15$ and $\lambda_3 = -(24k + 9)$ with $m_2 = 4(462k^2 + 336k + 61)$ and $m_3 = 7(462k^2 + 336k + 61)$;
- ($\bar{6}$ ⁰) G is a strongly regular graph of order $n = 42(11k + 4)^2$ and degree $r = 8(462k^2 + 336k + 61)$ with $\tau = 2(1344k^2 + 975k + 176)$ and $\theta = 24(8k + 3)(14k + 5)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 24k + 8$ and $\lambda_3 = -(42k + 16)$ with $m_2 = 7(462k^2 + 336k + 61)$ and $m_3 = 4(462k^2 + 336k + 61)$;
- (7⁰) G is a strongly regular graph of order $n = 70(11k - 5)^2$ and degree $r = 770k^2 - 700k + 159$ with $\tau = 2(35k^2 - 25k + 4)$ and $\theta = 5(2k - 1)(7k - 3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 35k - 16$ and $\lambda_3 = -(20k - 9)$ with $m_2 = 4(770k^2 - 700k + 159)$ and $m_3 = 7(770k^2 - 700k + 159)$;
- ($\bar{7}$ ⁰) G is a strongly regular graph of order $n = 70(11k - 5)^2$ and degree $r = 10(770k^2 - 700k + 159)$ with $\tau = 5(1400k^2 - 1273k + 289)$ and $\theta = 10(20k - 9)(35k - 16)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 20k - 10$ and $\lambda_3 = -(35k - 15)$ with $m_2 = 7(770k^2 - 700k + 159)$ and $m_3 = 4(770k^2 - 700k + 159)$;
- (8⁰) G is a strongly regular graph of order $n = 70(11k + 5)^2$ and degree $r = 770k^2 + 700k + 159$ with $\tau = 2(35k^2 + 25k + 4)$ and $\theta = 5(2k + 1)(7k + 3)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 20k + 9$ and $\lambda_3 = -(35k + 16)$ with $m_2 = 7(770k^2 + 700k + 159)$ and $m_3 = 4(770k^2 + 700k + 159)$;
- ($\bar{8}$ ⁰) G is a strongly regular graph of order $n = 70(11k + 5)^2$ and degree $r = 10(770k^2 + 700k + 159)$ with $\tau = 5(1400k^2 + 1273k + 289)$ and $\theta = 10(20k +$

- 9)($35k + 16$), where $k \geq 0$. Its eigenvalues are $\lambda_2 = 35k + 15$ and $\lambda_3 = -(20k + 10)$ with $m_2 = 4(770k^2 + 700k + 159)$ and $m_3 = 7(770k^2 + 700k + 159)$;
- (9^0) G is a strongly regular graph of order $n = 210(11k - 1)^2$ and degree $r = 5(2310k^2 - 420k + 19)$ with $\tau = 10(525k^2 - 93k + 4)$ and $\theta = 15(10k - 1)(35k - 3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 105k - 10$ and $\lambda_3 = -(60k - 5)$ with $m_2 = 4(2310k^2 - 420k + 19)$ and $m_3 = 7(2310k^2 - 420k + 19)$;
- ($\bar{9}^0$) G is a strongly regular graph of order $n = 210(11k - 1)^2$ and degree $r = 6(2310k^2 - 420k + 19)$ with $\tau = 9(840k^2 - 155k + 7)$ and $\theta = 30(12k - 1)(21k - 2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 60k - 6$ and $\lambda_3 = -(105k - 9)$ with $m_2 = 7(2310k^2 - 420k + 19)$ and $m_3 = 4(2310k^2 - 420k + 19)$;
- (10^0) G is a strongly regular graph of order $n = 210(11k + 1)^2$ and degree $r = 5(2310k^2 + 420k + 19)$ with $\tau = 10(525k^2 + 93k + 4)$ and $\theta = 15(10k + 1)(35k + 3)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 60k + 5$ and $\lambda_3 = -(105k + 10)$ with $m_2 = 7(2310k^2 + 420k + 19)$ and $m_3 = 4(2310k^2 + 420k + 19)$;
- ($\bar{10}^0$) G is a strongly regular graph of order $n = 210(11k + 1)^2$ and degree $r = 6(2310k^2 + 420k + 19)$ with $\tau = 9(840k^2 + 155k + 7)$ and $\theta = 30(12k + 1)(21k + 2)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 105k + 9$ and $\lambda_3 = -(60k + 6)$ with $m_2 = 4(2310k^2 + 420k + 19)$ and $m_3 = 7(2310k^2 + 420k + 19)$.

Theorem 14. Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{7}{5})m_3$ or $m_3 = (\frac{7}{5})m_2$. Then G is one of the following strongly regular graphs:

- (1^0) G is a strongly regular graph of order $n = (12k - 1)^2$ and degree $r = 10k(6k - 1)$ with $\tau = 25k^2 - 3k - 1$ and $\theta = 5k(5k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k - 1$ and $\lambda_3 = -5k$ with $m_2 = 10k(6k - 1)$ and $m_3 = 14k(6k - 1)$;
- ($\bar{1}^0$) G is a strongly regular graph of order $n = (12k - 1)^2$ and degree $r = 14k(6k - 1)$ with $\tau = 49k^2 - 9k - 1$ and $\theta = 7k(7k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k - 1$ and $\lambda_3 = -7k$ with $m_2 = 14k(6k - 1)$ and $m_3 = 10k(6k - 1)$;
- (2^0) G is a strongly regular graph of order $n = (12k + 1)^2$ and degree $r = 10k(6k + 1)$ with $\tau = 25k^2 + 3k - 1$ and $\theta = 5k(5k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k$ and $\lambda_3 = -(7k + 1)$ with $m_2 = 14k(6k + 1)$ and $m_3 = 10k(6k + 1)$;
- ($\bar{2}^0$) G is a strongly regular graph of order $n = (12k + 1)^2$ and degree $r = 14k(6k + 1)$ with $\tau = 49k^2 + 9k - 1$ and $\theta = 7k(7k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k$ and $\lambda_3 = -(5k + 1)$ with $m_2 = 10k(6k + 1)$ and $m_3 = 14k(6k + 1)$;
- (3^0) G is a strongly regular graph of order $n = 385(12k - 5)^2$ and degree $r = 4620k^2 - 3850k + 802$ with $\tau = 385k^2 - 341k + 75$ and $\theta = 11(5k - 2)(7k - 3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 55k - 23$ and $\lambda_3 = -(77k - 32)$ with $m_2 = 7(4620k^2 - 3850k + 802)$ and $m_3 = 5(4620k^2 - 3850k + 802)$;

- ($\bar{3}^0$) G is a strongly regular graph of order $n = 385(12k - 5)^2$ and degree $r = 11(4620k^2 - 3850k + 802)$ with $\tau = 11(4235k^2 - 3529k + 735)$ and $\theta = 11(55k - 23)(77k - 32)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 77k - 33$ and $\lambda_3 = -(55k - 22)$ with $m_2 = 5(4620k^2 - 3850k + 802)$ and $m_3 = 7(4620k^2 - 3850k + 802)$;
- (4^0) G is a strongly regular graph of order $n = 385(12k + 5)^2$ and degree $r = 4620k^2 + 3850k + 802$ with $\tau = 385k^2 + 341k + 75$ and $\theta = 11(5k + 2)(7k + 3)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 77k + 32$ and $\lambda_3 = -(55k + 23)$ with $m_2 = 5(4620k^2 + 3850k + 802)$ and $m_3 = 7(4620k^2 + 3850k + 802)$;
- ($\bar{4}^0$) G is a strongly regular graph of order $n = 385(12k + 5)^2$ and degree $r = 11(4620k^2 + 3850k + 802)$ with $\tau = 11(4235k^2 + 3529k + 735)$ and $\theta = 11(55k + 23)(77k + 32)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 55k + 22$ and $\lambda_3 = -(77k + 33)$ with $m_2 = 7(4620k^2 + 3850k + 802)$ and $m_3 = 5(4620k^2 + 3850k + 802)$.

Theorem 15. Let G be a connected strongly regular graph of order n and degree r with $m_2 = (\frac{7}{6})m_3$ or $m_3 = (\frac{7}{6})m_2$. Then G is one of the following strongly regular graphs:

- (1^0) G is the strongly regular graph $\overline{7K_2}$ of order $n = 14$ and degree $r = 12$ with $\tau = 10$ and $\theta = 12$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -2$ with $m_2 = 7$ and $m_3 = 6$;
- (2^0) G is a strongly regular graph of order $n = (13k - 1)^2$ and degree $r = 6k(13k - 2)$ with $\tau = 36k^2 - 5k - 1$ and $\theta = 6k(6k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k - 1$ and $\lambda_3 = -6k$ with $m_2 = 6k(13k - 2)$ and $m_3 = 7k(13k - 2)$;
- ($\bar{2}^0$) G is a strongly regular graph of order $n = (13k - 1)^2$ and degree $r = 7k(13k - 2)$ with $\tau = 49k^2 - 8k - 1$ and $\theta = 7k(7k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 6k - 1$ and $\lambda_3 = -7k$ with $m_2 = 7k(13k - 2)$ and $m_3 = 6k(13k - 2)$;
- (3^0) G is a strongly regular graph of order $n = (13k + 1)^2$ and degree $r = 6k(13k + 2)$ with $\tau = 36k^2 + 5k - 1$ and $\theta = 6k(6k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 6k$ and $\lambda_3 = -(7k + 1)$ with $m_2 = 7k(13k + 2)$ and $m_3 = 6k(13k + 2)$;
- ($\bar{3}^0$) G is a strongly regular graph of order $n = (13k + 1)^2$ and degree $r = 7k(13k + 2)$ with $\tau = 49k^2 + 8k - 1$ and $\theta = 7k(7k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k$ and $\lambda_3 = -(6k + 1)$ with $m_2 = 6k(13k + 2)$ and $m_3 = 7k(13k + 2)$;
- (4^0) G is a strongly regular graph of order $n = 14(13k - 1)^2$ and degree $r = 182k^2 - 28k + 1$ with $\tau = 2k(7k - 2)$ and $\theta = 2k(7k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k - 1$ and $\lambda_3 = -(14k - 1)$ with $m_2 = 7(182k^2 - 28k + 1)$ and $m_3 = 6(182k^2 - 28k + 1)$;

- ($\bar{4}^0$) G is a strongly regular graph of order $n = 14(13k - 1)^2$ and degree $r = 12(182k^2 - 28k + 1)$ with $\tau = 2(1008k^2 - 155k + 5)$ and $\theta = 12(12k - 1)(14k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 14k - 2$ and $\lambda_3 = -12k$ with $m_2 = 6(182k^2 - 28k + 1)$ and $m_3 = 7(182k^2 - 28k + 1)$;
- (5^0) G is a strongly regular graph of order $n = 14(13k + 1)^2$ and degree $r = 182k^2 + 28k + 1$ with $\tau = 2k(7k + 2)$ and $\theta = 2k(7k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 14k + 1$ and $\lambda_3 = -(12k + 1)$ with $m_2 = 6(182k^2 + 28k + 1)$ and $m_3 = 7(182k^2 + 28k + 1)$;
- ($\bar{5}^0$) G is a strongly regular graph of order $n = 14(13k + 1)^2$ and degree $r = 12(182k^2 + 28k + 1)$ with $\tau = 2(1008k^2 + 155k + 5)$ and $\theta = 12(12k + 1)(14k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k$ and $\lambda_3 = -(14k + 2)$ with $m_2 = 7(182k^2 + 28k + 1)$ and $m_3 = 6(182k^2 + 28k + 1)$;
- (6^0) G is a strongly regular graph of order $n = 35(13k - 4)^2$ and degree $r = 3(455k^2 - 280k + 43)$ with $\tau = 315k^2 - 190k + 28$ and $\theta = 15(3k - 1)(7k - 2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 35k - 11$ and $\lambda_3 = -(30k - 9)$ with $m_2 = 6(455k^2 - 280k + 43)$ and $m_3 = 7(455k^2 - 280k + 43)$;
- ($\bar{6}^0$) G is a strongly regular graph of order $n = 35(13k - 4)^2$ and degree $r = 10(455k^2 - 280k + 43)$ with $\tau = 5(7k - 2)(100k - 33)$ and $\theta = 10(10k - 3)(35k - 11)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 30k - 10$ and $\lambda_3 = -(35k - 10)$ with $m_2 = 7(455k^2 - 280k + 43)$ and $m_3 = 6(455k^2 - 280k + 43)$;
- (7^0) G is a strongly regular graph of order $n = 35(13k + 4)^2$ and degree $r = 3(455k^2 + 280k + 43)$ with $\tau = 315k^2 + 190k + 28$ and $\theta = 15(3k + 1)(7k + 2)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 30k + 9$ and $\lambda_3 = -(35k + 11)$ with $m_2 = 7(455k^2 + 280k + 43)$ and $m_3 = 6(455k^2 + 280k + 43)$;
- ($\bar{7}^0$) G is a strongly regular graph of order $n = 35(13k + 4)^2$ and degree $r = 10(455k^2 + 280k + 43)$ with $\tau = 5(7k + 2)(100k + 33)$ and $\theta = 10(10k + 3)(35k + 11)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 35k + 10$ and $\lambda_3 = -(30k + 10)$ with $m_2 = 6(455k^2 + 280k + 43)$ and $m_3 = 7(455k^2 + 280k + 43)$;
- (8^0) G is a strongly regular graph of order $n = 42(13k - 3)^2$ and degree $r = 4(546k^2 - 252k + 29)$ with $\tau = 2(336k^2 - 153k + 17)$ and $\theta = 12(4k - 1)(14k - 3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 42k - 10$ and $\lambda_3 = -(36k - 8)$ with $m_2 = 6(546k^2 - 252k + 29)$ and $m_3 = 7(546k^2 - 252k + 29)$;
- ($\bar{8}^0$) G is a strongly regular graph of order $n = 42(13k - 3)^2$ and degree $r = 9(546k^2 - 252k + 29)$ with $\tau = 6(567k^2 - 262k + 30)$ and $\theta = 18(9k - 2)(21k - 5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 36k - 9$ and $\lambda_3 = -(42k - 9)$ with $m_2 = 7(546k^2 - 252k + 29)$ and $m_3 = 6(546k^2 - 252k + 29)$;
- (9^0) G is a strongly regular graph of order $n = 42(13k + 3)^2$ and degree $r = 4(546k^2 + 252k + 29)$ with $\tau = 2(336k^2 + 153k + 17)$ and $\theta = 12(4k + 1)(14k + 3)$,

where $k \geq 0$. Its eigenvalues are $\lambda_2 = 36k + 8$ and $\lambda_3 = -(42k + 10)$ with $m_2 = 7(546k^2 + 252k + 29)$ and $m_3 = 6(546k^2 + 252k + 29)$;

- ($\bar{9}^0$) G is a strongly regular graph of order $n = 42(13k + 3)^2$ and degree $r = 9(546k^2 + 252k + 29)$ with $\tau = 6(567k^2 + 262k + 30)$ and $\theta = 18(9k + 2)(21k + 5)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 42k + 9$ and $\lambda_3 = -(36k + 9)$ with $m_2 = 6(546k^2 + 252k + 29)$ and $m_3 = 7(546k^2 + 252k + 29)$;
- (10^0) G is a strongly regular graph of order $n = 105(13k - 1)^2$ and degree $r = 5(1365k^2 - 210k + 8)$ with $\tau = 5(525k^2 - 82k + 3)$ and $\theta = 5(15k - 1)(35k - 3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 60k - 5$ and $\lambda_3 = -(70k - 5)$ with $m_2 = 7(1365k^2 - 210k + 8)$ and $m_3 = 6(1365k^2 - 210k + 8)$;
- ($\bar{10}^0$) G is a strongly regular graph of order $n = 105(13k - 1)^2$ and degree $r = 8(1365k^2 - 210k + 8)$ with $\tau = 2(3360k^2 - 515k + 19)$ and $\theta = 40(12k - 1)(14k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 70k - 6$ and $\lambda_3 = -(60k - 4)$ with $m_2 = 6(1365k^2 - 210k + 8)$ and $m_3 = 7(1365k^2 - 210k + 8)$;
- (11^0) G is a strongly regular graph of order $n = 105(13k + 1)^2$ and degree $r = 5(1365k^2 + 210k + 8)$ with $\tau = 5(525k^2 + 82k + 3)$ and $\theta = 5(15k + 1)(35k + 3)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 70k + 5$ and $\lambda_3 = -(60k + 5)$ with $m_2 = 6(1365k^2 + 210k + 8)$ and $m_3 = 7(1365k^2 + 210k + 8)$;
- ($\bar{11}^0$) G is a strongly regular graph of order $n = 105(13k + 1)^2$ and degree $r = 8(1365k^2 + 210k + 8)$ with $\tau = 2(3360k^2 + 515k + 19)$ and $\theta = 40(12k + 1)(14k + 1)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 60k + 4$ and $\lambda_3 = -(70k + 6)$ with $m_2 = 7(1365k^2 + 210k + 8)$ and $m_3 = 6(1365k^2 + 210k + 8)$;
- (12^0) G is a strongly regular graph of order $n = 231(13k - 2)^2$ and degree $r = 2(3003k^2 - 924k + 71)$ with $\tau = 924k^2 - 275k + 20$ and $\theta = 22(6k - 1)(7k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 77k - 12$ and $\lambda_3 = -(66k - 10)$ with $m_2 = 6(3003k^2 - 924k + 71)$ and $m_3 = 7(3003k^2 - 924k + 71)$;
- ($\bar{12}^0$) G is a strongly regular graph of order $n = 231(13k - 2)^2$ and degree $r = 11(3003k^2 - 924k + 71)$ with $\tau = 11(2541k^2 - 782k + 60)$ and $\theta = 11(33k - 5)(77k - 12)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 66k - 11$ and $\lambda_3 = -(77k - 11)$ with $m_2 = 7(3003k^2 - 924k + 71)$ and $m_3 = 6(3003k^2 - 924k + 71)$;
- (13^0) G is a strongly regular graph of order $n = 231(13k + 2)^2$ and degree $r = 2(3003k^2 + 924k + 71)$ with $\tau = 924k^2 + 275k + 20$ and $\theta = 22(6k + 1)(7k + 1)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 66k + 10$ and $\lambda_3 = -(77k + 12)$ with $m_2 = 7(3003k^2 + 924k + 71)$ and $m_3 = 6(3003k^2 + 924k + 71)$;
- ($\bar{13}^0$) G is a strongly regular graph of order $n = 231(13k + 2)^2$ and degree $r = 11(3003k^2 + 924k + 71)$ with $\tau = 11(2541k^2 + 782k + 60)$ and $\theta = 11(33k + 5)(77k + 12)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 77k + 11$ and $\lambda_3 = -(66k + 11)$ with $m_2 = 6(3003k^2 + 924k + 71)$ and $m_3 = 7(3003k^2 + 924k + 71)$.

3. CONCLUDING REMARKS

Using Theorems 3 and 4, it is possible to describe the parameters n , r , τ and θ , for any connected strongly regular graph by using only one parameter k . In the forthcoming paper we shall describe the parameters n , r , τ and θ , for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = \frac{8}{3}, \frac{8}{5}, \frac{8}{7}$.

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The all results presented in this work are verified by using a computer program `srgpar.exe`, which has been written by the author in the programming language `Borland C++ Builder 5.5`.