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CONTINUOUS REVIEW INVENTORY MODEL WITH LOST SALE REDUCTION AND ORDERING COST DEPENDENT ON LEAD TIME FOR THE MIXTURES OF DISTRIBUTIONS

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Abstract: We consider a continuous review inventory system for inventory model involving lost sales reduction through capital investment cost function and the reduction of lead time further which reduces the ordering cost. To reduce the lost sales rate, two forms of capital investment cost function, viz. logarithmic and power are employed. Two relationships between ordering cost and lead time, viz. linear and logarithmic are considered. We develop four inventory models by taking different combinations of capital investment cost function and ordering cost lead time relationship. Objective of the study is to reduce the total related cost by simultaneously optimizing the order quantity, safety factor, fraction of the shortages during the stock-out period that will be lost and length of lead time. The lead time demand is assumed to follow a mixture of normal distributions. The optimal solution is derived by developing computer programs using the software MATLAB. We also provide four numerical examples to illustrate the models.

Keywords: Continuous Review, Variable Lead Rime, Lost Sales Reduction, Mixtures of Normal Distribution.

MSC: 90B05.

1. INTRODUCTION

Recently, numerous research studies in the area of inventory management have been undertaken to deliver realistic models for decision makers. Various factors such as lead time, demand, order quantity, safety stock, shortages, etc. are important in the study of inventory management. We can get different inventory models by taking different combinations of the above factors.
Several researchers pay attention to lead time reduction to improve the productivity measure. Liao and Shyu [1] first presented an inventory model considering the lead time as composed of n mutually independent components, each having a different crashing cost for reducing lead time where lead time is a unique variable. Many researchers considered lead time as a controllable variable [2, 3, 4, 5, 6, 7]. Earlier, in economic order quantity (EOQ) and economic production quantity (EPQ) models, ordering cost was assummed as a constant. Practically, ordering cost can be controlled viz. [8, 9, 10, 11]
Researchers in above papers [8, 9, 10, 11] assumed the lead time and ordering cost reductions to act independently. However, this is only one of the possible cases. Usually, the lead time and ordering cost reductions may be related closely. Chen *et* al. [13] analyzed the effect of lead time reduction on continuous review inventory systems with partial backorders. Precisely, they modified Moon and Choi's [4] model to include the cases of the linear and logarithmic relationships between lead time and ordering cost reductions. Ouyang *et* al. [14] and Lin [15] examined the lead time and ordering cost reductions as interdependent in the periodic review inventory model with backorder price discount. Lin [16] considered the lead time and ordering cost reductions work interdependently with backorder price discount in the continuous review inventory model.
Since the demand by the customers vary in the lead time hence, the distribution of demand for each customer can be adequately approximated by a single distribution. But the overall distribution of demand is a mixture. Hence, a single distribution to describe the demand in the lead time cannot be only used as in [2, 3, 5, 8, 13, 14]. Consequently, Wu and Tsai [17] examined the lead time demand with the mixture of normal distributions and the fixed back-order rate. Lee *et* al. [18] developed a mixture inventory model with backorders and lost sales, in which the order quantity, lead time, and reorder point are decision variables. Lee [19] derived an inventory model involving controllable backorder rate and variable lead time demand with the mixture of distributions. Lee *et* al. [20] discussed computational algorithms to evaluate optimal order quantity and optimal lead time under service level constraint, where lead time demand follows the mixture of distributions and backorder rate is assumed to be negative exponential. To adapt more real features of the inventory system, Lin [21] assumed a random number of defective goods in buyer's arriving order lot considering partial lost sales for the mixture of distributions of the controllable lead time demand.
Owing to uncertainty of usual environment, the shortages in an inventory system are obvious. Traditional models of inventory systems consider that during the stock out either all demand is backlogged or completely lost. Practically, it is not

true. Usually during stock out, some of the customers wait for next replenishment, whereas some impatient customers go elsewhere. Taking this into account, various researchers studied different models where a portion of shortages is backlogged while the remaining shortages incur lost sale penalties [3, 4, 5, 17, 22, 23]. Lost-sales cost is recognized as the opportunity cost of lost revenue and the intangible cost associated with the loss of customer goodwill or reliability (Hillier and Lieberman[24]). Hence, the decision maker should concentrate on reducing the shortage cost of lost-sales and the total expected cost through investment.

Ouyang and Chang [25] considered capital investment to reduce the lost-sales rate in continuous review inventory model. Annadurai and Uthayakumar [26] developed (T, R, L) an inventory model with controllable lead time and analyzed the effects of increasing two different types of investments to reduce the lost-sales rate, in which the review period, lead time, and lost-sales rate are considered as decision variables. Lin [21] examined the effects of the increase in investment to minimize the lost sales rate when the order quantity, reorder point, lost sales rate, and lead time are taken as decision variables. Soni and Patel [27] developed a continuous review inventory system for the inventory model involving fuzzy random demand, variable lead-time with lost sales by considering two forms of capital investment cost function viz. logarithmic and power for reducing the lost-sales rate.

This paper considers a continuous review inventory system for inventory models comprising lost sales reduction through capital investment cost function and the reduction of lead time, which thereby decreases the ordering cost. Further, it is assumed that the lead time demand follows a mixture of normal distributions to minimize the total related cost by optimizing the order quantity, safety factor, length of lead time, and fraction of the shortages during the stock-out period that will be lost simultaneously.

The rest of the paper is organized as follows. In section 2, notations and assumptions are given, used to develop the proposed models. Section 3 deals with development of our mathematical models. To demonstrate the effectiveness of the solution methodology, some numerical examples are provided in section 4. Finally, conclusion is covered in section 5.

2. NOTATION AND ASSUMPTIONS

To develop the proposed models, the following notation and assumptions are employed.

a) Notation:

b) Assumptions:

1. Inventory is continuously reviewed and replenishments are made whenever the inventory level falls to the reorder point r.
2. The lead time L consists of n mutually independent components. The ith component has a minimum duration a_i, a normal duration b_i, and a crashing cost c_i per unit time. Furthermore, these c_i are assumed to be arranged such that $c_1 \leq c_2 \leq \cdots \leq c_n$.

Decision Variables	
Q	order quantity
α	fraction of the shortages during the stock-out period that will be lost
L	length of lead time (in weeks)
k	safety factor
Parameter	
D	average demand per year
A	ordering cost per order ($ /order)
π	stock out cost per unit short ($ / unit)
π_0	gross marginal profit (i.e. cost of lost demand) per unit ($ / unit)
h	holding cost per unit per year ($ / unit / year)
r	reorder point
α_0	original fraction of the shortage that will be lost
$I(\alpha)$	investment required to reduce the lost sales fraction from α_0 to α ($ / year)
θ	fractional opportunity cost of capital per unit time (per $ / year)
p	the weight of the component normal distributions, $0 \leqslant p \leqslant 1$
X	lead time demand with mixture of normal distribution

3. The lead time is deterministic and the lead time demand X follows the mixture of normal distributions with the probability density function (p.d.f.) given by

$$f(x) = p\frac{1}{\sqrt{2\pi}\sigma\sqrt{L}}e^{-\frac{1}{2}\left(\frac{x-\mu_1 L}{\sigma\sqrt{L}}\right)^2} + (1-p)\,\frac{1}{\sqrt{2\pi}\sigma\sqrt{L}}e^{-\frac{1}{2}\left(\frac{x-\mu_2 L}{\sigma\sqrt{L}}\right)^2}$$

where $\mu_1 - \mu_2 = \eta\sigma/\sqrt{L}$ or $\mu_1 L - \mu_2 L = \eta\sigma\sqrt{L}, \eta > 0, x \in R, 0 \leq p \leq 1, \sigma > 0$. The mixture of normal distribution is unimodal for all p if $(\mu_1 - \mu_2)^2 < 27\sigma^2/8L \left(\text{or}\eta < \sqrt{27/8}\right)$.

4. The reorder point r = expected demand during lead time + safety stock (SS), and SS = k × (standard deviation of lead time demand), that is $r = \mu_* L + k\sigma_*\sqrt{L}$ where $\mu_* = p\mu_1 + (1-p)\,\mu_2, \sigma_* = \sqrt{1 + p\,(1-p)\,\eta^2}\sigma$ $\mu_1 = \mu_* + (1-p)\,\eta\sigma/\sqrt{L}, \mu_2 = \mu_* - p\eta\sigma/\sqrt{L}$ and k is the safety factor.
5. The components of the lead time are crashed one at a time, starting with the component of the least c_i and so on.
6. If we let L_i be the length of lead time with components 1, 2, . . . i crashed to their minimum duration, then $L_{\min} = \sum\limits_{i=1}^{n} a_i \leq L \leq \sum\limits_{i=1}^{n} b_i = L_{\max}, L_i = L_{\max} - \sum\limits_{j=1}^{i} (b_j - a_j)$ and the lead time crashing cost per cycle $R(L)$ for a given $L \in [L_i, L_{i-1}]$ is given by $R\,(L) = c_i\,(L_{i-1} - L) + \sum\limits_{j=1}^{i-1} c_j\,(b_j - a_j)$.
7. The lost sales fraction α can be reduced by capital investment $I(\alpha)$.
8. Ordering cost is a function of lead time.

3. MODEL FORMULATION

In this study we extend the model of Wu and Tsai [17] for variable lead time demand with the mixture of normal distributions. In that model the total expected annual cost (EAC) is given by

$$\begin{aligned} EAC\left(Q,L\right) &= A\tfrac{D}{Q} + h\tfrac{Q}{2} + h\sigma\sqrt{L}pr_1\Phi\left(\tfrac{\mu_*\sqrt{L}}{\sigma} + (1-p)\eta\right) - h\sigma\sqrt{L}p\phi \\ &\times\left(\tfrac{\mu_*\sqrt{L}}{\sigma} + (1-p)\eta\right) + h\sigma\sqrt{L}\left(1-p\right)r_2\Phi\left(\tfrac{\mu_*\sqrt{L}}{\sigma} - p\eta\right) - h\sigma\sqrt{L}\phi \\ &\times\left(\tfrac{\mu_*\sqrt{L}}{\sigma} - p\eta\right) + \left[h\left(1-\beta\right) + \tfrac{D}{Q}\left(\pi + \pi_0\left(1-\beta\right)\right)\right]B\left(r\right) + \tfrac{D}{Q}R\left(L\right) \end{aligned} \tag{1}$$

when the demand of the lead time X follows the mixture of normal p.d.f. $f(x)$ and the reorder point is $r = \mu_* L + k\sigma_*\sqrt{L}$, where μ_*, σ_* and k are defined as above. Then the expected shortage at the end of the cycle is given by $B\left(r\right) = E[X-r]^+ = \int_r^\infty \left(x-r\right)dF_*\left(x\right) = \sigma\sqrt{L}\psi\left(r_1, r_2, p\right)$, where $\psi\left(r_1, r_2, p\right) = p\left\{\phi\left(r_1\right) - r_1\left[1-\Phi\left(r_1\right)\right]\right\} + \left(1-p\right)\left\{\phi\left(r_2\right) - r_2\left[1-\Phi\left(r_2\right)\right]\right\}$, ϕ and Φ denote the standard normal p.d.f and cumulative distribution function (c.d.f.), respectively. Here $r_1 = k\sqrt{1+\eta^2 p\left(1-p\right)} - \left(1-p\right)\eta$ and $r_2 = k\sqrt{1+\eta^2 p\left(1-p\right)} + \eta p$. Hence, $\psi(r_1, r_2, p)$ can be written as

$$\begin{aligned} &\psi\left(k,p\right) = p\phi\left(kF_*\left(p\right) - \left(1-p\right)\eta\right) - p\left\{kF_*\left(p\right) - \left(1-p\right)\eta\right\}\left(1 - \Phi kF_*\left(p\right)\right. \\ &\left. - \Phi\left(1-p\right)\eta\right) + \left(1-p\right)\phi\left(kF_*\left(p\right) + p\eta\right) - \left(1-p\right)\left\{kF_*\left(p\right) + p\eta\right\} \\ &\times\left(1 - \Phi\left(kF_*\left(p\right) + p\eta\right)\right) \end{aligned} \tag{2}$$

where

$$F_*\left(p\right) = \sqrt{1+\eta^2 p\left(1-p\right)} \tag{3}$$

so the safety factor k can be treated as a decision variable. Thus equation 1 reduces to

$$\begin{aligned} &EAC\left(Q,k,L\right) = A\tfrac{D}{Q} + h\tfrac{Q}{2} + h\sigma\sqrt{L}\left\{pr_1\Phi\left(\tfrac{\mu_*\sqrt{L}}{\sigma} + \left(1-p\right)\eta\right) - p\phi\tfrac{\mu_*\sqrt{L}}{\sigma}\right. \\ &\left. + p\phi(1-p)\eta\right\} + h\sigma\sqrt{L}\left\{\left(1-p\right)\left[r_2\Phi\left(\tfrac{\mu_*\sqrt{L}}{\sigma} - p\eta\right) - \phi\left(\tfrac{\mu_*\sqrt{L}}{\sigma} - p\eta\right)\right]\right\} \\ &+ \left[h\left(1-\beta\right) + \tfrac{D}{Q}\left(\pi + \pi_0\left(1-\beta\right)\right)\right]\sigma\sqrt{L}\psi\left(k,p\right) + \tfrac{D}{Q}R\left(L\right) \end{aligned} \tag{4}$$

Moreover, the capital investment $I(\alpha)$ will be made to prevent the possible lost sales demand caused by stock-out. Here investment $I(\alpha)$ is the one time investment cost whose benefits will envisage indefinitely into the future, thus the annual cost of such an investment is $\theta I(\alpha)$, where θ is fractional annual opportunity cost of capital, i.e., one can consider the lost sales rate as a decision variable and try to minimize the sum of the capital investment cost of reducing lost sales rate and the inventory related costs by optimizing over $Q,\ k,\ \alpha,\ L$ constrained $0 < \alpha \leqslant 1$, where $0 < \alpha \leqslant \alpha_0$, where α_0 is the original fraction of the shortage that will be lost. The lost sales fraction α can be reduced by capital investment $I(\alpha)$. More precisely, according to our new model, the objective of our problem is to minimize the following total expected annual cost, composed of the investment opportunity cost for lost sales reduction, ordering cost, holding cost, stock out cost, and lead time crashing cost. In mathematical notation, the problem is given by

$$\begin{aligned} EAC\left(Q,k,\alpha,L\right) = {} & \theta I\left(\alpha\right) + A\left(L\right)\tfrac{D}{Q} + h\tfrac{Q}{2} + h\sigma\sqrt{L}p\left(\tfrac{\mu_*\sqrt{L}}{\sigma}\right)\left(r_1\Phi-\phi\right) \\ & + h\sigma\sqrt{L}p(1-p)\eta\left(r_1\Phi-\phi\right) + h\sigma\sqrt{L}\left(1-p\right) \\ & \times\left(\tfrac{\mu_*\sqrt{L}}{\sigma}\right)\left(r_2\Phi-\phi\right) - h\sigma\sqrt{L}\left(1-p\right)p\eta\left(r_2\Phi-\phi\right) \\ & + \left[h\alpha + \tfrac{D}{Q}\left(\pi+\pi_0\alpha\right)\right]\sigma\sqrt{L}\psi\left(k,p\right) + \tfrac{D}{Q}R\left(L\right) \end{aligned} \tag{5}$$

In this study we consider two possibilities. Namely, capital investment function is logarithmic, and power function of lost sales fraction, α. More precisely, in the first subsection capital investment is logarithmic function of lost sales fraction α, and in the second subsection capital investment is power function of lost sales fraction α. In each subsection, we consider the situation where shortening lead time accompanies the decrease of ordering cost. Specifically, we consider the cases that the relationship between ordering cost and lead time is linear and logarithmic.

3.1. Capital investment is logarithmic function of lost sales fraction

In this subsection capital investment function $I(\alpha)$ follows a logarithmic function given by

$$I\left(\alpha\right) = v\ln\left(\frac{\alpha_0}{\alpha}\right) \text{ for } 0<\alpha\le\alpha_0 \text{ and } \alpha=1-\beta \tag{6}$$

Also, we consider two situations when relationship between ordering cost and lead time is linear and logarithmic.

3.1.1. Ordering cost is a linear function of lead time

Here lead time and ordering cost reductions are related by the following relationship:

$$\frac{L_0-L}{L_0} = \varepsilon\left(\frac{A_0-A}{A_0}\right) \tag{7}$$

where $\varepsilon(>0)$ is a constant scaling parameter, used to describe the linear relationship between percentages of reductions in lead time and ordering cost. By considering relationship (7), the ordering cost A can be written as a linear function of L, i.e,

$$A\left(L\right) = a + bL \tag{8}$$

where $a = \left(1-\frac{1}{\varepsilon}\right)A_0$ and $b = \frac{A_0}{\varepsilon L_0}$. Now, the total expected annual cost is given by

$$\begin{aligned} EAC\left(Q,k,\alpha,L\right) = {} & \theta v\ln\left(\tfrac{\alpha_0}{\alpha}\right) + A\left(L\right)\tfrac{D}{Q} + h\tfrac{Q}{2} + h\sigma\sqrt{L}p\left(\tfrac{\mu_*\sqrt{L}}{\sigma}\right)\left(r_1\Phi-\phi\right) \\ & + h\sigma\sqrt{L}p(1-p)\eta\left(r_1\Phi-\phi\right) + h\sigma\sqrt{L}\left(1-p\right) \\ & \times\left(\tfrac{\mu_*\sqrt{L}}{\sigma}\right)\left(r_2\Phi-\phi\right) - h\sigma\sqrt{L}\left(1-p\right)r_2\Phi p\eta\left(r_2\Phi-\phi\right) \\ & + \left[h\alpha + \tfrac{D}{Q}\left(\pi+\pi_0\alpha\right)\right]\sigma\sqrt{L}\psi\left(k,p\right) + \tfrac{D}{Q}R\left(L\right) \end{aligned} \tag{9}$$

In order to solve this nonlinear programming problem, first ignore the constraint $0 < \alpha \leq \alpha_0$ and take the first order partial derivatives of $EAC\,(Q, k, \alpha, L)$ with respect to Q, k, α and $L \in [L_i, L_{i-1}]$, respectively. One obtains

$$\frac{\partial EAC}{\partial Q} = -\frac{A(L)D}{Q^2} + \frac{h}{2} - \frac{D}{Q^2}\left[R(L) + (\pi + \pi_0\alpha)\sigma\sqrt{L}\psi(k, p)\right] \tag{10}$$

$$\begin{aligned}\tfrac{\partial EAC}{\partial k} &= h\sigma\sqrt{L}F_*\,(p)\left\{p\Phi\left(\tfrac{\mu_*\sqrt{L}}{\sigma} + (1-p)\,\eta\right) + (1-p)\,\Phi\left(\tfrac{\mu_*\sqrt{L}}{\sigma} - p\eta\right)\right\} \\ &+ \sigma\sqrt{L}F_*\,(p)\left[h\alpha + \tfrac{D}{Q}\,(\pi + \pi_0\alpha)\right][G_*\,(k,p) - 1]\end{aligned} \tag{11}$$

where

$$G_*\,(k,p) = p\Phi\,(kF_*\,(p) - (1-p)\,\eta) + (1-p)\,\Phi\,(kF_*\,(p) + p\eta) \tag{12}$$

$$\frac{\partial EAC}{\partial \alpha} = -\frac{\theta v}{\alpha} + \left[h + \frac{D}{Q}\pi_0\right]\sigma\sqrt{L}\psi\,(k,p) \tag{13}$$

$$\begin{aligned}\tfrac{\partial EAC}{\partial L} &= \tfrac{bD}{Q} + \tfrac{h\sigma}{2\sqrt{L}}p\left[r_1\Phi\left(\tfrac{\mu_*\sqrt{L}}{\sigma} + (1-p)\,\eta\right) - \phi\left(\tfrac{\mu_*\sqrt{L}}{\sigma} + (1-p)\,\eta\right)\right] \\ &+ \tfrac{h\sigma}{2\sqrt{L}}\,(1-p)\left[r_2\Phi\left(\tfrac{\mu_*\sqrt{L}}{\sigma} - p\eta\right) - \phi\left(\tfrac{\mu_*\sqrt{L}}{\sigma} - p\eta\right)\right] \\ &+ \tfrac{h\mu_*}{2}\left\{p\left(r_1 + \tfrac{\mu_*\sqrt{L}}{\sigma} + (1-p)\,\eta\right)\phi\left(\tfrac{\mu_*\sqrt{L}}{\sigma} + (1-p)\,\eta\right)\right\} \\ &+ \tfrac{h\mu_*}{2}\left\{(1-p)\left(r_2 + \tfrac{\mu_*\sqrt{L}}{\sigma} - p\eta\right)\phi\left(\tfrac{\mu_*\sqrt{L}}{\sigma} - p\eta\right)\right\} \\ &+ \left[h\alpha + \tfrac{D}{Q}\,(\pi + \pi_0\alpha)\right]\tfrac{\sigma\psi(k,p)}{2\sqrt{L}} - \tfrac{D}{Q}c_i\end{aligned} \tag{14}$$

By examining the second order sufficient conditions, it can be shown that $EAC(Q,\ k,\ \alpha,\ L)$ is not a convex function of $(Q,\ k,\ \alpha,\ L)$. However, for fixed $(Q,\ k,\ \alpha)$, $EAC(Q,\ k,\ \alpha,\ L)$ is concave in $L \in [L_i, L_{i-1}]$, since

$$\frac{\partial^2 EAC}{\partial L^2} < 0 \tag{15}$$

If $\frac{\mu_*\sqrt{L}}{\sigma} - \eta p > \sqrt{2}$

Therefore, for fixed $Q,\ k\ and\ \alpha$, the minimum total EAC will occur at the end points of the interval $[L_i, L_{i-1}]$. On the other hand, for a given value of $L \in [L_i, L_{i-1}]$ by setting (10), (11), and (13) equal to zero, one gets

$$Q = \left[\frac{2D}{h}\left\{A\,(L) + R\,(L) + (\pi + \pi_0\alpha)\,\sigma\sqrt{L}\psi\,(k,p)\right\}\right]^{1/2} \tag{16}$$

$$1 - G_*\,(k,p) = \frac{h\left\{p\Phi\left(\frac{\mu_*\sqrt{L}}{\sigma} + (1-p)\,\eta\right) + (1-p)\,\Phi\left(\frac{\mu_*\sqrt{L}}{\sigma} - p\eta\right)\right\}}{h\alpha + \frac{D}{Q}\,(\pi + \pi_0\alpha)} \tag{17}$$

where $G_*\,(k,p)$ is given by equation (12)

$$\alpha = \frac{\theta v}{\left[h + \frac{D}{Q}\pi_0\right]\sigma\sqrt{L}\psi\,(k,p)} \tag{18}$$

3.1.2. Ordering cost is a logarithmic function of lead time

Here the lead time and ordering cost reductions are related by the following relationship

$$\frac{A_0 - A}{A_0} = \tau \ln\left(\frac{L}{L_0}\right) \tag{19}$$

where $\tau\,(< 0)$ is a constant scaling parameter used to describe the logarithmic relationship between percentages of reductions in lead time and ordering cost. The ordering cost A can be written as

$$A\left(L\right) = d + e \ln L \tag{20}$$

where $d = A_0 + \tau A_0 \ln L_0,$ and $e = -\tau A_0 > 0$. As discussed in section 3.2.1, the same methodology is being used in this section and one can get the following results:

$$Q = \left[\frac{2D}{h}\left\{\left(d + e\ln L\right) + R\left(L\right) + \left(\pi + \pi_0\alpha\right)\sigma\sqrt{L}\psi\left(k,p\right)\right\}\right]^{1/2} \tag{21}$$

$$1 - G_*\left(k,p\right) = \frac{h\left\{p\Phi\left(\frac{\mu_*\sqrt{L}}{\sigma} + (1-p)\,\eta\right) + (1-p)\,\Phi\left(\frac{\mu_*\sqrt{L}}{\sigma} - p\eta\right)\right\}}{h\alpha + \frac{D}{Q}\left(\pi + \pi_0\alpha\right)} \tag{22}$$

where $G_*\left(k,p\right)$ is given by equation (12)

$$\alpha = \frac{\theta v}{\left[h + \frac{D}{Q}\pi_0\right]\sigma\sqrt{L}\psi\left(k,p\right)} \tag{23}$$

3.2. Capital investment is power function of lost sales fraction

In this subsection capital investment function $I(\alpha)$ follows a power function given by

$$I\left(\alpha\right) = l\left(\alpha^{-m} - \alpha_0^{-m}\right) \tag{24}$$

Also, we consider two situations, when relationship between ordering cost and lead time is linear, and logarithmic.

3.2.1. Ordering cost is a linear function of lead time

Here relationship between lead time and ordering cost reductions is expressed by (8). Like wise the section 3.2.1, the present section has also followed the same methodology by which one can get the following results:

$$Q = \left[\frac{2D}{h}\left\{A\left(L\right) + R\left(L\right) + \left(\pi + \pi_0\alpha\right)\sigma\sqrt{L}\psi\left(k,p\right)\right\}\right]^{1/2} \tag{25}$$

$$1 - G_*\left(k,p\right) = \frac{h\left\{p\Phi\left(\frac{\mu_*\sqrt{L}}{\sigma} + \left(1-p\right)\eta\right) + \left(1-p\right)\Phi\left(\frac{\mu_*\sqrt{L}}{\sigma} - p\eta\right)\right\}}{h\alpha + \frac{D}{Q}\left(\pi + \pi_0\alpha\right)} \tag{26}$$

where $G_*\left(k,p\right)$ is given by equation (12)

$$\alpha = \left[\frac{\theta lm}{\left(h + \frac{D}{Q}\pi_0\right)\sigma\sqrt{L}\psi\left(k,p\right)}\right]^{\frac{1}{m+1}} \tag{27}$$

3.2.2. Ordering cost is a logarithmic function of lead time

Here relationship between lead time and ordering cost reductions is expressed by (20). Acknowledging the section 3.2.1, in the present section the same method is applied, by which one can reach to the following results:

$$Q = \left[\frac{2D}{h}\left\{\left(d + e\ln L\right) + R\left(L\right) + \left(\pi + \pi_0\alpha\right)\sigma\sqrt{L}\psi\left(k,p\right)\right\}\right]^{1/2} \tag{28}$$

$$1 - G_*\left(k,p\right) = \frac{h\left\{p\Phi\left(\frac{\mu_*\sqrt{L}}{\sigma} + \left(1-p\right)\eta\right) + \left(1-p\right)\Phi\left(\frac{\mu_*\sqrt{L}}{\sigma} - p\eta\right)\right\}}{h\alpha + \frac{D}{Q}\left(\pi + \pi_0\alpha\right)} \tag{29}$$

where $G_*\left(k,p\right)$ is given by equation (12)

$$\alpha = \left[\frac{\theta lm}{\left(h + \frac{D}{Q}\pi_0\right)\sigma\sqrt{L}\psi\left(k,p\right)}\right]^{\frac{1}{m+1}} \tag{30}$$

4. NUMERICAL EXAMPLES

Example 1. In order to illustrate the above solution procedure and the effects of investing in lost sales rate reduction, let us consider an inventory system with the following data: $D = 600$ units/year, $A_0 = \$200$, $h = \$20$, $\pi = \$50$, $\pi_0 = \$150$, $\sigma = 7$ units/week, and the lead time has three components with data shown in Table1. In this example the capital investment is logarithmic function of lost sales

Table 1: Lead time data.

Lead time component i	Normal duration b_i (days)	Minimum duration a_i (days)	Unit crashing cost c_i (\$/day)
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

fraction and the relationship between lead time and ordering cost is linear. We solve the problem when $p = 0$, 0.5, 1, $\eta = 0.6$, $\mu = 11$, $\alpha_0 = 0.2$, $\theta = 0.005$, v = 1/0.001, $\varepsilon = 6$. A summary of these optimal results, obtained at $L = 4$ is presented in $\left(Q^*, k^*, \alpha^*, L^*\right)$ and $EAC\left(Q^*, k^*, \alpha^*, L^*\right)$ of Table 2.

Table 2: Summary of solutions procedure of example 1 when $\theta = 0.005$ (L^* in weeks)

p	L^*	$A(L)$	k^*	r	Q^*	α^*	$EAC(.)$
0	4	183.33	1.4383	64.14	117.54	0.0135	2765.92
0.5	4	183.33	1.4373	65.01	117.80	0.0130	2788.14
1	4	183.33	1.4383	64.14	117.54	0.0135	2766.02

Now we examine the effect on the values of $Q^*,\ k^*,\ \alpha^*,\ L\ and\ EAC(.)$ by taking different values of θ and considering only optimum values, obtained at L = 4. The results are shown in Table 3.

Example 2: Using the same data proposed in example 1, except the relationship between lead-time and ordering cost. In this example the relationship between lead-time and ordering cost is logarithmic with $\tau = -0.8$. The results of the solution procedure are summarized in Table 3.

Table 3: Summary of solutions procedure of example 1 and 2 for different values of θ (L^* in weeks)

			Relationship between lead time and ordering cost is linear					Relationship between lead time and ordering cost is logarithmic				
θ	p	L^*	$A(L)$	k^*	Q^*	α^*	$EAC(.)$	$A(L)$	k^*	Q^*	α^*	$EAC(.)$
	0.00	4	183.33	1.4383	117.54	0.0135	2765.92	188.91	1.4315	119.05	0.0135	2794.20
0.005	0.50	4	183.33	1.4373	117.80	0.0130	2788.14	188.91	1.4305	119.31	0.0129	2816.36
	1.00	4	183.33	1.4383	117.54	0.0135	2766.02	188.91	1.4315	119.05	0.0135	2794.30
	0.00	4	183.33	1.4613	117.48	0.0284	2777.34	188.91	1.4545	118.99	0.0283	2805.64
0.010	0.50	4	183.33	1.4593	117.74	0.0272	2799.76	188.91	1.4525	119.25	0.0271	2828.00
	1.00	4	183.33	1.4613	117.48	0.0284	2777.44	188.91	1.4545	118.99	0.0283	2805.74
	0.00	4	183.33	1.4855	117.42	0.0450	2785.88	188.91	1.4787	118.93	0.0448	2814.20
0.015	0.50	4	183.33	1.4825	117.68	0.0430	2808.52	188.91	1.4757	119.19	0.0429	2836.77
	1.00	4	183.33	1.4855	117.42	0.0450	2785.98	188.91	1.4787	118.93	0.0448	2814.30

Example 3: In this example the capital investment is a power function of lost sales fraction and the relationship between lead time and ordering cost is linear. Using the same data as in example 1, we solve this problem for different 4 combinations of the values of l and m, respectively as (0.15, 0.6), (0.15, 0.8), (0.05, 0.6), (0.05, 0.8). The results of the solution procedure are summarized in Table 4.

Table 4: Summary of solutions procedure of example 3 and 4 for different values of p (L^* in weeks)

			Relationship between lead time and ordering cost is linear					Relationship between lead time and ordering cost is logarithmic				
l	m	L^*	$A(L)$	k^*	Q^*	α^*	$EAC(.)$	$A(L)$	k^*	Q^*	α^*	$EAC(.)$
p = 0								p = 0				
	0.6	4	183.33	1.4167	117.60	0.0002	2747.52	188.91	1.4099	119.11	0.0002	2775.79
0.15	0.8	4	183.33	1.4174	117.60	0.0006	2747.83	188.91	1.4105	119.10	0.0006	2776.10
	0.6	4	183.33	1.4165	117.60	0.0001	2747.42	188.91	1.4097	119.11	0.0001	2775.69
0.05	0.8	4	183.33	1.4169	117.60	0.0003	2747.60	188.91	1.4101	119.11	0.0003	2775.87
p = 0.5								p = 0.5				
	0.6	4	183.33	1.4165	117.86	0.0002	2769.54	188.91	1.4097	119.37	0.0002	2797.75
0.15	0.8	4	183.33	1.4171	117.86	0.0006	2769.86	188.91	1.4103	119.37	0.0006	2798.07
	0.6	4	183.33	1.4163	117.86	0.0001	2769.44	188.91	1.4095	119.37	0.0001	2797.65
0.05	0.8	4	183.33	1.4167	117.86	0.0003	2769.62	188.91	1.4099	119.37	0.0003	2797.83
p = 1								p = 1				
	0.6	4	183.33	1.4167	117.60	0.0002	2747.62	188.91	1.4099	119.11	0.0002	2775.89
0.15	0.8	4	183.33	1.4174	117.60	0.0006	2747.94	188.91	1.4105	119.10	0.0006	2776.20
	0.6	4	183.33	1.4165	117.60	0.0001	2747.52	188.91	1.4097	119.11	0.0001	2775.79
0.05	0.8	4	183.33	1.4169	117.60	0.0003	2747.70	188.91	1.4101	119.11	0.0003	2775.97

Example 4: In this example the Capital investment is a power function of lost sales fraction and the relationship between lead time and ordering cost is logarithmic. Using the same data as in example 1, we solve this problem for different

4 combinations of the values of l and m, respectively as (0.15, 0.6), (0.15, 0.8), (0.05, 0.6), (0.05, 0.8). The results of the solution procedure are summarized in Table 4.

One can observe a significant reduction through capital investment function in original fraction of the shortages that will be lost. The original fraction of the shortages that will be lost reduces from 20% to the range of 1.29% - 8.45% when capital investment is logarithmic function of lost sales fraction, and to the range of .01% - 0.35% when capital investment is a power function of lost sales fraction. Hence, through capital investment seller can improve his goodwill, which is one of the most essential factor that motivates the customersâ€™ intention of backorder, so helping the retailer to gain competitive edge in business and drive customer satisfaction. This results in lock-in the prevailing customers and attract the new customers as well.

5. CONCLUSIONS

The objective of this study is to investigate the inventory system with lost sales reduction through capital investment and the reduction of lead time that accompanies a decrease of ordering cost. We seek to minimize the total expected annual cost by simultaneously optimizing the order quantity Q, safety factor k, fraction of the shortages that will be lost during the stock-out period α, and length of lead time L, under the assumption that the lead time demand follows mixture of normal distributions. Results of numerical examples show that significant reduction in lost sales can be achieved through capital investment.

A future research scope of this study can include different general types of investment functions and related marginal cost behavior. Also, the present study can be extended by considering the distribution free case where only mean and standard deviations of lead time demands are known and finite.

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