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OPTIMIZING PRESERVATION STRATEGIES FOR DETERIORATING ITEMS WITH TIME-VARYING HOLDING COST AND STOCK-DEPENDENT DEMAND

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Abstract: Organizations are keen on rethinking and optimizing their existing inventory strategies so as to attain profitability. The phenomenon of deterioration is a common phenomenon while managing any inventory system. However, it could become a major challenge for the business if not dealt carefully. An investment in preservation technology is by far the most influential move towards dealing with deterioration proficiently. Additionally, it is noticed that the demand pattern of many products is reliant on its availability and usability. Thus, considering demand of the product to be "stock-dependent" is a more practical approach. Further, in case of deteriorating items, it is observed that the longer an item stays in the system the higher is its holding cost. Therefore, the model assumes the holding cost to be time varying. Hence, the proposed framework aims to develop an inventory model for deteriorating items with stock-dependent demand and time-varying holding cost under an investment in preservation technology. The objective is to determine the optimal investment in preservation technology and the optimal cycle length so as to minimize the total cost. Numerical example with various special cases have been discussed which signifies the effect of preservation technology investment in controlling the loss due to deterioration. Finally, the effect of key model features on the optimal solution is studied through sensitivity analysis which provides some important managerial implications.

Keywords: Deterioration, Stock-Dependent demand, Preservation technology investment, Time varying holding cost.

MSC: 90B05.

1. INTRODUCTION

The loss of inventory due to deterioration results in the financial loss which ultimately leads to an increase in the total cost significantly. The very first research in this area was carried out by Ghare and Schrader [10] which was then followed by Dave and Patel [8], Wee [37], Chakrabarti and Chaudhari [4], Goyal and Giri [12], Singh et al. [30], Jaggi and Kausar [14], Jaggi et al. [15]. Recently, Mishra [19] developed an inventory model for controllable probabilistic deterioration rate under shortages. Tiwari et al. [32] proposed an inventory model with deteriorating items and limited storage capacity in a supply chain. Further, an inventory model for deteriorating items with variable demand pattern under trade credit was developed by Shaikh et al. [26]. In such a scenario, preservation technology is an efficient tool that aids in pacifying the loss due to deterioration considerably. Although investment in preservation techniques contributes to the total cost component but at the same time it reduces the cost incurred due to deterioration. In this direction, Hsu et al. [13] obtained the optimal inventory policy incorporating preservation technology investment in the model. Later, Dye and Hsieh [9] extended this model (Hsu et al. [13]) by taking preservation technology investment as a function of total cycle time. Singh and Sharma [28] worked on a two-level trade credit financing model with preservation technology. Lately, Zhang et al. [38], Singh et al. [29], Giri et al. [11] and Bardhan et al. [2] have proposed different inventory models with preservation technology investment.

In today's market, the demand of the product is dynamic in nature which is influenced with various factors viz. price, stock-on-hand, time, advertisements etc. The customers are often fascinated by the availability and display of the items. The pioneer work in the area of stock-dependent demand was proposed by Mandal and Phaujdar [17] in which he considered displayed stock level as a promotional tool to boost the demand rate. Several researchers viz. Baker and Urban [1], Datta and Pal [7], Urbanand Baker [35] considered the impact of stock level on the demand rate and ultimately on the total revenue for different inventory systems. Min et al. [18] developed an EOQ model with deteriorating item and stock-dependent demand rate. Tyagi et al. [34] developed an optimal replenishment policy for deteriorating items which are non-instantaneous in nature and the demand is stock-dependent. A two-warehouse inventory model for non-instantaneously deteriorating items with stock-dependent demand under inflation was developed by Palanival et al. [22]. Bhuniya et al. [3] proposed a model for deteriorating

Authors	**Stock-dependent Demand**	**Deterio-ration**	**Preservation Technology**	**Time varying holding cost**
Min et al. (2010)	Yes	Yes	No	No
Dye and Hsieh (2012)	No	Yes	Yes	No
Dash et al. (2014)	No	Yes	No	Yes
Choudhury et al. (2015)	Yes	Yes	No	Yes
Tayal et al. (2015)	No	Yes	No	Yes
Mishra (2016)	Yes	Yes	No	No
Singh et al. (2016)	Yes	Yes	Yes	No
Giri et al. (2017)	No	Yes	Yes	No
Bhunia et al. (2018)	Yes	Yes	No	No
Mishra and Talati (2018)	Yes	Yes	Yes	No
Tripathi et al. (2018)	Yes	No	No	Yes
Bardhan et al. (2019)	Yes	Yes	Yes	No
Present Paper	Yes	Yes	Yes	Yes

Table 1: Literature summary

items with variable demand dependent on marketing strategy and displayed stock level. Tripathi et al. [33] established the EOQ model under different trade-credits with stock-dependent demand.In the same year, Mishra and Talati [20] proposed a quantity discount model for advertisement and stock-dependent demand with preservation technology in an integrated environment. Additionally, Shaikh et al. [27] investigated the effect of inflation on a two-warehouse inventory model for non-instantaneous deteriorating items with stock-dependent demand. Recently, Lim [16] has written a note on a robust inventory model with stock-dependent demand.

Furthermore, most of the inventory models assume holding cost to be constant, however in realistic situations it is observed that the holding cost may increase with time, especially, in case of deteriorating items. The longer an item stays in the system the higher is its holding cost, for e.g., food grains, fruits, vegetables. In this regard, many researchers viz. Naddor [21], Van der Veen [36], Roy [25], and Pando et al. [23] have explored the effect of time-dependent holding cost on inventory models. Further research in this area was carried out by Dash et al. [6], Tayal et al. [31] with exponential declining demand and deteriorating items for an EOQ and EPQ model respectively. Further, Choudhury et al. [5] developed an inventory model for deteriorating items with stock-dependent demand, time-varying holding cost and shortages, however they didn't take into account any investment in preservation technology. Recently, Rastogi et al. [24] has proposed an inventory model with variable holding cost under credit limit policy and cash discount.

Table 1 briefly presents a quick comparison of the existing and present research

From the above literature review, it is observed that the effect of preservation technology investment in controlling the deterioration of items with stock-dependent demand and time-varying holding cost in an inventory system has not been considered yet. Usually the deterioration rate is considered to be constant, which is not practical if an investment in preservation technology has been done. In view of this, deterioration rate is considered as a decreasing function of the investment in preservation technology in the proposed model. Moreover, the holding cost for deteriorating items does not remain constant over a period of time. It usually increases with time, so time-varying holding cost is a more realistic assumption. Further, the displayed stock plays an important role in increasing the sales. Hence, the present paper attempts to investigate the effect of the above-discussed features in an inventory system dealing with deteriorating items. The objective is to determine the optimal investment in preservation technology and cycle time, so as to minimize the total cost of the system. Numerical examples and sensitivity analysis of the key parameters have been carried out to impart significant model characteristics. Results indicate the significance of preservation technology investment in managing the loss due to deterioration. Some important managerial insights have also been elucidated from the sensitivity analysis.

2. ASSUMPTIONS AND NOTATIONS

2.1. Assumptions

1. The rate of demand is directly proportional to the stock level. i.e.,

$$D(I(t)) = a + bI(t), \quad a > 0,\ 0 < b < 1\,.$$

2. The time horizon is infinite with negligible lead time.
3. The model does not allow shortages.
4. There exists a constant deterioration rate for all the items which can be controlled by preservation technology investment.
5. Preservation technology investment reduces the rate of deterioration gradually. To represent the same, following function is used: $y(\tau) = y_0 e^{-u\tau}$, which satisfies the conditions: $\partial y(\tau)/\partial\tau < 0$, $\partial^2 y(\tau)/\partial\tau^2 > 0$, and $y(0) = y_0$, where u is sensitivity parameter of investment $0 < u < 1$.
6. The holding cost is considered to be dependent on time as $C_h(t) = h + rt$, $0 < r < 1$.

2.2. Notations

Important notations are given below:

Decision Variables

T	Length of cycle
τ	Investment in preservation technology per unit time

Constant Parameters

C_o	Cost of ordering (per order)
C_d	Unit cost due to deterioration
$C_h(t) = h + rt$	Holding cost per unit per unit time at any time t
y_0	Deterioration rate without investment in preservation technology
$y(\tau)$	Deterioration rate with investment in preservation technology

3. MATHEMATICAL MODELING

Consider an inventory system where the inventory level varies with demand and deterioration. The demand rate is taken to be a function of the on-hand inventory level at any time t. The behavior of the inventory with time is presented in Figure 1.

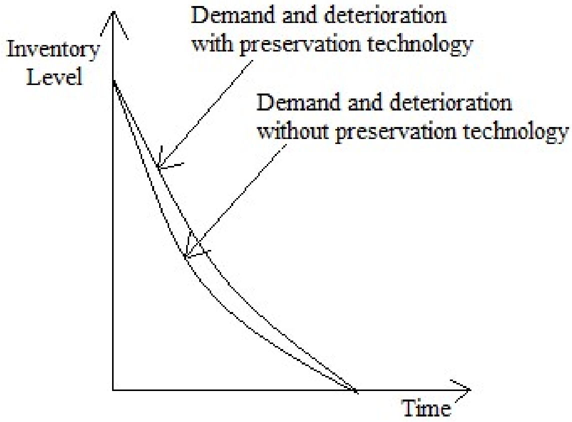

Figure 1: Graph of the inventory depletion with and without preservation technology

The equation governing the inventory level is given as:

$$\frac{dI(t)}{dt} + y(\tau)I(t) = -(a + bI(t)), \quad 0 \leq t \leq T\,. \tag{1}$$

The solution of equation (1) using the boundary condition $I(T) = 0$, is given as:

$$I(t) = a\left[(T-t) + \frac{(T-t)^2(y(\tau)+b)}{2}\right] \tag{2}$$

and the initial inventory level is:

$$Q = I(0) = a\left[T + \frac{T^2(y(\tau)+b)}{2}\right] \tag{3}$$

The total cost of the inventory system for one cycle comprises of the following components:

- Cost of ordering $= C_o$ (4)
- Preservation technology investment $= \tau T$ (5)
- Holding cost $= \displaystyle\int_0^T (h + rt)I(t)dt$

$$= \left[\frac{ahT^2}{2} + \frac{arT^3}{6} + \frac{a(y(\tau)+b)}{2}\left(\frac{hT^3}{3} + \frac{rT^4}{12}\right)\right] \tag{6}$$

- Deterioration cost $= C_d\left(Q - \int_0^T D(I(t))dt\right)$

$$= C_d\left[\frac{T^2y(\tau)(3a-abT) - ab^2T^3}{6}\right] \tag{7}$$

Using equations (4)-(7), we get the total cost of the system per unit time as:

$$TC(T,\tau) = \frac{C_o}{T} + \frac{1}{T}\left[\frac{ahT^2}{2} + \frac{arT^3}{6} + \frac{a(y(\tau)+b)}{2}\left(\frac{hT^3}{3} + \frac{rT^4}{12}\right)\right] + \frac{C_d}{T}\left[\frac{T^2y(\tau)(3a-abT) - ab^2T^3}{6}\right] + \frac{\tau T}{T} \tag{8}$$

3.1. Optimal solution

The main objective of the present study is to minimize the total cost of the model by jointly optimizing the cycle time (T) and the total investment in preservation technology (τ). To establish optimality, the necessary and sufficient conditions are given below:

Necessary conditions:

$$\frac{\partial TC}{\partial T} = 0, \quad \frac{\partial TC}{\partial \tau} = 0 \tag{9}$$

$$\frac{\partial TC}{\partial T} = \frac{ah}{2} + \frac{arT}{3} + \frac{ahT(e^{-u\tau}+b)}{3} + \frac{arT^2(e^{-u\tau}+b)}{8} - \frac{C_o}{T^2} + C_d\left(\frac{3ay_0e^{-u\tau} - 2Taby_0e^{-u\tau} - 2Tab^2}{6}\right) = 0 \tag{10}$$

$$\frac{\partial TC}{\partial \tau} = 1 - \frac{uahT^2y_0e^{-u\tau}}{6} - \frac{uarT^3y_0e^{-u\tau}}{24} - C_duTy_0e^{-u\tau}\left(\frac{3a-abT}{6}\right) = 0 \tag{11}$$

Now, solving equation (10) and equation (11) simultaneously, we get the optimal values of T and τ as T^* and τ^*. After substituting these values in equation (8), we get optimal total cost of the system per unit time, and the optimal value of order quantity (Q^*) is found using equation (3).

Sufficiency conditions:

$$\frac{\partial^2 TC}{\partial T^2} > 0 \tag{12}$$

and

$$\left(\left(\frac{\partial^2 TC}{\partial T^2}.\frac{\partial^2 TC}{\partial \tau^2}\right) - \left(\frac{\partial^2 TC}{\partial T\partial \tau}.\frac{\partial^2 TC}{\partial \tau\partial T}\right)\right) > 0. \tag{13}$$

All the second order derivatives are calculated in Appendix A.

Since all the second order derivatives are extremely non-linear in nature, therefore the optimality has been established graphically (Figure 2).

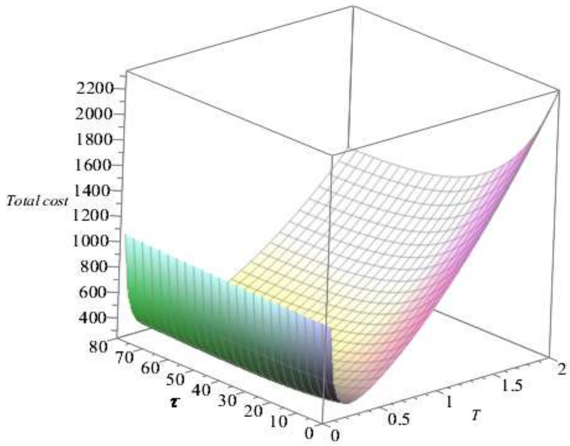

Figure 2: Graphical representation of the total cost versus the investment in preservation technology and cycle time

4. NUMERICAL ANALYSIS

4.1. Illustrative example

The proposed study has been illustrated with a numerical example by using the following data in appropriate units:

$C_o = 40$/order, $C_d = 50$/year, $y_0 = 0.09$, $h = 0.7$/unit/year, $r = 5$, $u = 0.05$, $a = 260$, $b = 0.1$.

The total cost of this model with preservation technology investment, yields the total cost $TC = \$230.390$, the cycle time $T = 0.388$ year and investment in preservation technology $\tau = \$\,48.420$.

4.2. Special Cases

The proposed model has been investigated under various special conditions. Following inferences are made on the basis of comparative results summarized in Table 2.

Case 1. Demand is constant

Taking $b = 0 \Rightarrow D = a$, i.e., constant demand. In this case, the order quantity and cycle length decreases while the total cost of the system increases due to increase in deterioration cost. Since, stock-dependent demand influence the demand rate positively, as one is able to increase the sales and hence the order size is larger in the proposed model as compared to that of with constant demand.

Case 2. Without preservation technology investment
If the system does not consider the investment in preservation technology ($\tau = 0$), the total cost increases significantly. Since, deterioration rate cannot be controlled in this case, it results in an increase of deteriorating units which eventually increases the total cost. In this case, it would be better to place small orders more frequently. This observation implies that the preservation technology influences the total cost of the system positively.

Case 3. Demand is constant without investment in preservation technology
$b = 0$ and $\tau = 0$, i.e., the rate of demand is constant and the value of deterioration rate cannot be controlled since there is no investment in preservation technology. Here the cost of the system further increases as compared to the above two cases since there is a dual impact of constant demand and constant deterioration rate throughout the cycle.

Case 4. Constant holding cost without investment in preservation technology
$\tau = 0$ and $r = 0$, i.e., the holding cost is constant and value of the deterioration parameter will be constant and equal to initial deterioration rate throughout the cycle time. In this case, as the holding cost is not increasing with the time, one is able to minimize the total cost. However with no investment in preservation technology, the loss due to deterioration increases and hence total cost increases.

Case 5. Demand and holding cost are constant without preservation technology investment
This case represents the simple case of EOQ model with deterioration, in which the total cost of the system is higher than in proposed model. Hence, it shows that the parameters incorporated in the model, i.e., investment in preservation technology, time-varying holding cost as well as stock-dependent demand rate, influence the inventory system in a positive way, since the total cost is minimum in the proposed model as compared to all the above discussed cases.

Cases	T^*	τ^*	Q^*	TC^*
1	0.362	47.304	94.518	239.082
2	0.232	0	61.710	336.798
3	0.226	0	59.619	341.025
4	0.25	0	66.729	324.077
5	0.243	0	63.874	329.038
Present Model	**0.388**	**48.420**	**103.187**	**230.390**

Table 2: Special cases

4.3. *Interaction between initial deterioration rate and the effectiveness of preservation technology investment*

y_0	u	**0.03**	**0.04**	**0.05**	**0.06**
0.03	T	0.367	0.38	0.388	0.394
	τ	27.162	26.955	26.448	25.31
	Q	97.445	101.014	103.187	104.647
	TC	221.979	214.398	208.417	203.692
0.05	T	0.367	0.38	0.388	0.394
	τ	42.19	39.725	36.664	33.824
	Q	97.445	101.014	103.187	104.647
	TC	239.007	227.169	218.634	212.205
0.07	T	0.367	0.38	0.388	0.394
	τ	53.406	48.137	43.394	39.432
	Q	97.445	101.014	103.187	104.647
	TC	250.223	235.581	225.363	217.813
0.09	T	0.367	0.38	0.388	0.394
	τ	61.783	54.42	48.42	43.621
	Q	97.445	101.014	103.187	104.647
	TC	258.6	241.864	230.39	222.002

Table 3: Interaction between y_0 and u

From Table 3, with an increase in the initial rate of deterioration y_0, the total cost of the system rises, since the number of deteriorated units increases. However, if the effectiveness of the preservation technology investment, u, increases then the total cost decreases. With more effective and better preservation technology techniques, one can control the deterioration rate, which results in less damage and a lesser deterioration cost.

Further, in case of higher initial deterioration rate, ($y_0 = 0$), the total cost can be controlled by increasing the value of the parameter u. With the increase in the value of u, the order size Q increases and the total cost TC decreases. Since a higher value of u implies more effective preservation technology techniques, thus deterioration of items can be controlled which results in a slight increase in order quantity. Moreover, better preservation technology techniques also helps in bringing down the corresponding investment.

Hence, it is recommended for the organizations to adopt effective preservation technology techniques so as to minimize the financial damage caused due to deteriorating items, and at the same time it aids in controlling the investment in preservation technology.

4.4. *Sensitivity analysis*

The impact of the key parameters of our model on the optimal policy are studied in this section.

h	T	Q	τ	Total cost
0.3	0.427	113.659	50.275	208.901
0.5	0.406	108.116	49.317	219.903
0.7	0.388	103.187	48.420	230.390
0.9	0.372	98.782	47.581	240.416
1.1	0.357	94.826	46.794	250.031
a				
220	0.412	92.735	46.245	217.584
240	0.399	98.047	47.380	224.155
260	0.388	103.187	48.420	230.390
280	0.378	108.172	49.381	236.332
300	0.369	113.018	50.272	242.015
r				
3	0.449	119.629	51.278	215.219
4	0.414	110.056	49.667	223.351
5	0.388	103.187	48.420	230.390
6	0.369	97.887	47.399	236.664
7	0.353	93.607	46.531	242.365
u				
0.01	0.271	72.145	75.614	329.093
0.03	0.367	97.445	61.783	258.600
0.05	0.388	103.187	48.420	230.390
0.07	0.398	105.695	39.732	215.580
0.09	0.403	107.099	33.840	206.347

Table 4: Impact of various parameters on the optimal policy

Observations:

- In Table 4, with an increase in the holding cost components (h and r), the order size (Q), cycle length (T), and the preservation technology investment (τ) decrease, however the total cost increases.
- When the demand parameter (a) increases, the order size (Q) increases and the cycle length (T) decreases. The preservation technology investment (τ) and the total cost of the system decrease.
- When the effectiveness parameter (u) increases, the total cost and the investment in preservation technology (τ) decrease, where as the cycle time (T) and the order size (Q) increase.

Managerial insights:

- With increasing holding cost, it would be preferable to order less quantity for a lesser time so as to maintain the usability of the product. Moreover, as the holding cost increases, it is suggested to invest less amount in the preservation technology.

- In case of increasing demand the retailer should increase the order size, but should place the orders more frequently. Since the ordered quantity is large, the cost incurred due to deterioration would also be high, which results in higher total cost. Thus, to reduce this cost it would be feasible to hold the inventory for lesser time. The preservation technology cost increases, as the elevated demand leads to a bigger order size, and consequently requires more investment in preservation technology.
- Retailer should employ better and good quality techniques for preservation technology. Since, more effective the technique, lesser is the investment in preservation technology, which helps in minimizing the total cost. Here it is recommended to order a bigger lot for a longer span of time.

5. CONCLUSION AND FUTURE RESEARCH DIRECTIONS

This paper develops an inventory model for the items which are deteriorating in nature taking into account investment in preservation technology. The demand of the product is dependent on on-hand stock level, and the holding cost is assumed to be increasing with time. The main aim of the study is to jointly optimize the cycle length and investment in preservation technology so as to minimize the total cost of the system. The model is analyzed under some special conditions, which indicates that stock-dependent demand rate has a positive impact on customer preference. It helps to increase the sales and at the same time to reduce the total cost and investment in preservation technology. Although, the time-varying holding cost and investment in preservation technology contribute to the cost components, but the net effect of these parameters with stock-dependent demand is noteworthy in reducing the total cost of the system. Further, sensitivity analysis has been performed on key model parameters which gives some important managerial insights. In case of higher holding cost, small orders should be placed more frequently. Whereas in case of increasing demand, large orders should be placed at short intervals of time. The study suggests the organizations to adopt effective and better preservation technology techniques, which aids in controlling the deterioration rate and thus helps in reducing the total cost of the system.

For future research, the model can be extended for multi-item inventory system with different demand patterns viz., freshness-dependent demand, price sensitive demand, advertisement dependent demand. Another punctual research direction would be to study the impact of trade credit financing and inflation on the existing model. The extension of the present model under the effect of carbon-emissions for sustainable environment would be a significant contribution to the current literature.

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Appendix A1.

$$\frac{\partial TC}{\partial T} = \frac{ah}{2} + \frac{arT}{3} + \frac{ahT(e^{-u\tau}+b)}{3} + \frac{arT^2(e^{-u\tau}+b)}{8} - \frac{C_o}{T^2} + C_d\left(\frac{3ay_0e^{-u\tau} - 2Taby_0e^{-u\tau} - 2Tab^2}{6}\right) \tag{A.1}$$

$$\frac{\partial TC}{\partial \tau} = 1 - \frac{uahT^2y_0e^{-u\tau}}{6} - \frac{uarT^3y_0e^{-u\tau}}{24} - C_duTy_0e^{-u\tau}\left(\frac{3a-abT}{6}\right) \tag{A.2}$$

$$\frac{\partial^2 TC}{\partial T^2} = \frac{ar}{3} + \frac{ah(y_0e^{-u\tau}+b)}{3} + \frac{arT(y_0e^{-u\tau}+b)}{4} + \frac{2C_o}{T^3} - C_d\left(\frac{aby_0e^{-u\tau}+ab^2}{3}\right) \tag{A.3}$$

$$\frac{\partial^2 TC}{\partial \tau^2} = \frac{u^2ahT^2y_0e^{-u\tau}}{6} + \frac{u^2arT^3y_0e^{-u\tau}}{24} + C_du^2Ty_0e^{-u\tau}\left(\frac{3a-abT}{6}\right) \tag{A.4}$$

$$\frac{\partial^2 TC}{\partial \tau \partial T} = -\frac{uahTy_0e^{-u\tau}}{3} - \frac{uarT^2y_0e^{-u\tau}}{8} - C_duy_0e^{-u\tau}\left(\frac{3a-abT}{6}\right) + \frac{abC_duTy_0e^{-u\tau}}{6} \tag{A.5}$$

$$\frac{\partial^2 TC}{\partial T \partial \tau} = \frac{-uahy_0Te^{-u\tau}}{3} - \frac{uary_0T^2e^{-u\tau}}{8} - uC_d\left(\frac{3ay_0e^{-u\tau} - 2Taby_0e^{-u\tau}}{6}\right) \tag{A.6}$$