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ON POWER OF CONTROL CHART FOR THE RATIO OF TWO POISSON DISTRIBUTIONS UNDER MISCLASSIFICATION ERROR

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Abstract: When the control charts for the ratio of two Poisson distributions need to be constructed, a situation may require controlling the ratio rather than a single parameter. Chakraborty and Khurshid [6] constructed Shewhart control chart and Chakraborty and Khurshid [7] studied measurement error effect on control chart for the ratio of two Poisson distributions, respectively. The effects of misclassification on the performance of control charts have been investigated by several authors. Measurement error variability has uncertainty that can arise from several sources. In this paper, we study the effect of the two sources of variability on the power characteristics of control chart under misclassification error for the ratio of two Poisson distributions as studied by Sahai and Khurshid [36]. Probabilities of misclassification of conforming and non-conforming units for grid of values are provided.

Keywords: Misclassification Error, Ratio of Poisson Distribution, Power, Lavin Equation.

MSC: 62P30.

1. INTRODUCTION

A control chart, a popular statistical tool, is widely used to monitor and/or improve a process. For example, on a production assembly/line each item is inspected and classified as conforming or nonconforming to its predefined quality inspection. Due to measurement variation, conforming items can be rejected and nonconforming items can be accepted. This is known as misclassification [2].

It is widely recognized that the measurement error often exists in practice and may considerably affect the performance of control charts in some cases. Measurement error variability has uncertainty which can be from several sources. Misclassification is a particular form of measurement error and misclassification error is generally studied separately from measurement error, although there is clearly much overlap [5]. There is abundant literature on the consequences of measurement error on the actual performance of various control charts (see, e.g. [1, 3, 4, 7, 8, 9, 12, 16, 19, 22, 23, 24, 25, 26, 27, 29, 30, 31, 34, 37, 41, 42, 43, 44, 46, 47, 48]).

Misclassification is a common problem in quality control literature and considerable amount of work has been done by various authors. However, the misclassification error effects on the control chart for the ratio of two Poisson distributions are still not considered. When the control charts for the ratio of two Poisson distributions need to be constructed, a situation may require controlling the ratio rather than a single parameter. Chakraborty and Khurshid [6] constructed Shewhart control chart, and Chakraborty and Khurshid [7] studied measurement error effect on control chart for the ratio of two Poisson distributions, respectively. In a recent article, Yamauchi and Ho [45], compared Shewhart and exponentially weighted moving average cntrol charts for the ratio of two Poisson rates.

The purpose of this paper is to calculate the power of control chart for the ratio of two Poisson distributions as studied by Sahai and Khurshid [36] by considering approximate expression for calculating the probabilities of errors of misclassification due to measurement error.

2. RATIO OF TWO POISSON DISTRIBUTIONS

The problem of making inferences on the ratio of two Poisson parameters from corresponding independent Poisson distributions arises in many scientific investigations and scenarios. It has drawn considerable interest not only in the field of statistics [33, 36] but also in numerous fields like the number of automobile accident deaths on roads before and after a safety training program [40], the number of leukemia event rate per year in a pre and post nuclear accident period [13]. In a breast cancer study [15, 35] two groups of women were compared to ascertain whether those who had been inspected using X-ray fluoroscopy during treatment for tuberculosis had a higher rate of breast cancer than those who had not been inspected using X-ray fluoroscopy. [10] considered the ratio of ion-counting signals in isotope where the distribution of ion-counts follows two independent Poisson distributions. Its application also includes (i) the evaluation of machines break downs over time; (ii) number of defective items from two different suppliers; (iii) the ratio of bacteria growing on two culture plates with different areas.

It is noteworthy that the ratio of two Poisson variables does not follow a Poisson distribution, instead it can be represented by the binomial distribution and can be appreciated in the following way: The production of items from a machine can be viewed as a collection of n independent Bernoulli trials with each unit being either defective a or non-defective b. The probability of selecting a defective items at any particular trial is $a/(a+b)$. Thus, the number of defective items in the sample follows a binomial distribution with parameters n and $a/(a+b)$. Let X and Y be two independent Poisson random variables with parameters a and b, respectively. The conditional distribution of X, given $X+Y=n$, follows a binomial distribution of the form [18, 21]

$$P[X=d|(X+Y=n)] = \binom{n}{d}\left(\frac{a}{a+b}\right)^{d}\left(\frac{b}{a+b}\right)^{n-d} \;;\; d=0,1,2,\ldots,n \quad (1)$$

The mean and variance of the above function are

$$\mu = E(X) = \frac{na}{a+b}, \quad (2)$$

and

$$\sigma^2 = Var(X) = \frac{nab}{(a+b)^2}. \quad (3)$$

3. ASSUMPTIONS

Our assumptions can be summarized as follows:
The quality characteristic x is normally distributed with mean μ and standard deviation σ_p; thus

$$f(x)dx = \frac{1}{\sigma_p\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma_p}\right)^2\right]dx. \quad (4)$$

The variable v, denoting measurement error, is also normal with mean x and standard deviation σ_e

$$f(v)dv = \frac{1}{\sigma_e\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{v-x}{\sigma_e}\right)^2\right]dv. \quad (5)$$

The units beyond $x=\mu \pm K\sigma_p$ are defective, and the units within $x=\mu \pm K\sigma_p$ are non-defective.

It is also assumed that the measurements have been taken only to classify the production items into acceptable and rejectable units with certain specifications that can be expressed in terms of mean and standard deviation of the measurable quality characteristics.

Notation
TFD (true fraction defective) is the proportion of defective items when there is no error of misclassification and is denoted by P;
AFD (apparent fraction defective) is the proportion of defective items if error of misclassification is present, and is denoted by π;
$AFD = TFD$, if the misclassification error is zero.

4. EVALUATING PROBABILITIES OF MISCLASSIFICATION

In a production assembly/line each item is inspected and classified as conforming or nonconforming to its predefined quality inspection, which under ideal conditions has no errors. One important way of judging the performance of any classification procedure is to calculate its error (type I and type II) rate or misclassification probabilities. In this background, a type I error occurs when an item that is good is misclassified as a nonconforming, whereas a type II error occurs when a defective item is misclassified as conforming. Let P_1 and P_2 be the type I and type II error probabilities respectively, and take the values between 0 and 1, then following [38], with the above mentioned assumptions (Section 3), P_1 and P_2 can be evaluated as

$$P_1 = \int_{-K\sigma_p}^{K\sigma_p} f(x)dx[1 - \int_{-K\sigma_p}^{K\sigma_p} f(v)dv] \tag{6}$$

and

$$P_2 = \int_{K\sigma_p}^{\infty} f(x)dx \int_{-K\sigma_p}^{K\sigma_p} f(v)dv + \int_{-\infty}^{-K\sigma_p} f(x)dx \int_{-K\sigma_p}^{K\sigma_p} f(v)dv. \tag{7}$$

Singh [38] studied measurement error in acceptance sampling plan and calculated P_1 and P_2 based on the graphic representation of the probabilities of misclassification data for different values of K and $a = \sigma_e/\sigma_p$. Singh [38] approximated expressions for P_1 and P_2 as:

$$P_1 = 2T(h,a) + \{\Phi(k) - \Phi(h)\} \tag{8}$$

and

$$P_2 = 2T(h,a) - \{\Phi(k) - \Phi(h)\} \tag{9}$$

where
$a = \frac{\sigma_e}{\sigma_p}$, $h = \frac{K\sigma_p}{\sqrt{\sigma_e^2+\sigma_p^2}}$, $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{1}{2}v^2\right] dv$ and
$T(h,a) = \frac{1}{\sqrt{2\pi}} \int_0^a \frac{\exp\left[-\frac{1}{2}h^2(1+x^2)\right]}{1+x^2} dx$.

Here, $1.5 \leq K \leq 3$ and $(\sigma_e/\sigma_p) \leq 0.5$ hold good for finding P_1 and P_2. Now if P denotes the incoming true fraction defective of the lots then the expression of AFD, following Lavin [20] is denoted by

$$\pi = P(1 - P_2) + P_1(1 - P). \tag{10}$$

Here π yields a random variable X whose binomial distribution has parameter π instead of P. For published material based on Lavin equation, see [11, 14, 17, 28].

5. POWER OF CONTROL CHART FOR RATIO OF TWO POISSON DISTRIBUTIONS UNDER MISCLASSIFICATION

Kanazuka [19] has shown that the power of detecting the change of process parameter for the control chart can be found

$$P_d = P\{X \geq UCL\} + P\{X \leq LCL\}$$

where UCL and LCL are upper and lower control limits respectively. Hence, following Equation 1, we have

$$P_d = \left[1 - \sum_{d=0}^{UCL-1} \binom{n}{d} \left(\frac{a}{a+b}\right)^d \left(\frac{b}{a+b}\right)^{n-d}\right] + \left[\sum_{d=0}^{LCL} \binom{n}{d} \left(\frac{a}{a+b}\right)^d \left(\frac{b}{a+b}\right)^{n-d}\right]. \quad (11)$$

Let $P = \left(\frac{a}{a+b}\right)$ denotes the incoming true function defective of the lots, then from Equation 10, the apparent fraction defective is given by

$$\pi = \left(\frac{a}{a+b}\right)(1 - P_2) + P_1\left(1 - \frac{a}{a+b}\right). \quad (12)$$

Thus under misclassification, the control limits (UCL and LCL) are $\pi \pm K\sqrt{\frac{\pi(1-\pi)}{n}}$ and center line (CL) is π. Hence, the power of the control chart under misclassification is

$$P_d = \left[1 - \sum_{d_e=0}^{UCL-1} \binom{n}{d_e} \pi^{d_e}(1-\pi)^{n-d_e}\right] + \left[\sum_{d_e=0}^{LCL} \binom{n}{d_e} \pi^{d_e}(1-\pi)^{n-d_e}\right], \quad (13)$$

where d_e is the number of apparent fraction defectives observed by the inspectors.

For our calculations here, we have kept $\left(\frac{a}{a+b}\right) = p = \bar{p} = 0.2$, the overall sample proportion of defective fixed and the values of n being changed in different situations to see the effect of the size of the sample on the power of control chart.

6. CALCULATIONS AND CONCLUSIONS

To obtain the power of control chart (P_d) and operating characteristic (OC) curve ($P_e(\pi)$) under misclassification error, first we have to find $\pi = P(1 - P_2) + P_1(1-P)$ based on the approximate expressions for P_1 and P_2 (Equations 8 and 9).

Tables 1, 2, and 3 give the values of $h = \frac{K}{\sqrt{a^2+1}}$ for different combinations of $a = \sigma_e/\sigma_p$ and $T(h, a)$. Here we have used Monte Carlo simulation to find $T(h, a)$. True values of fraction defective P can be obtained from the normal probability table for different values of K. The values of P_1 and P_2 for different combinations of $T(h, a)$ and $\Phi(h)$ for fixed K have been tabulated in Tables 1, 2, and 3. It has been observed from these tables that for fixed K, the values of P_1 and P_2 show a decreasing trend if the measurement error $a = \sigma_e/\sigma_p$ decreases. On the other hand, we also observe that for fixed $a = \sigma_e/\sigma_p$ the values of P_1 is greater than P_2 and when $h \cong K$, then $P_1 = P_2$.

The relationship between apparent fraction defective (AFD) and true fraction defective (TFD) is shown in Table 4. It is observed that for fixed K and $a = \sigma_e/\sigma_p$ as the values of the true fraction defective (P) increase, the values of π i.e., apparent (observed) fraction defective also increase and also for fixed P, as the values of measurement error $a = \sigma_e/\sigma_p$ increase, there is considerable increase in the values of π.

Table 5 depicts the effect of K on probabilities of misclassification of conforming units (P_1) and non-conforming units (P_2). For fixed $a = \sigma_e/\sigma_p$, if we increase the values of K, there is a decreasing trend for P_1 but for fixed K, the values of P_1 increase as $a = \sigma_e/\sigma_p$ is increased.

Tables 6 and 7 offer us the idea how the values of AFD (π) influence the control limits for fraction defective charts. It has been observed from the tables that for fixed K, the values of both UCL and LCL increase as there is an increase in the values of $a = \sigma_e/\sigma_p$. For fixed $a = \sigma_e/\sigma_p$, the difference between UCL and LCL increases as we go on increasing K when the corresponding values of π decrease (which depends on P_1, P_2 and P). It is observed that the range of UCL and LCL is less when the size of the sample is increased.

Tables 6 and 7 show the different values of power of control chart (P_d) for the corresponding values of π. Here we observe how power curve (P_d) changes for different values of n, K, $a = \sigma_e/\sigma_p$, UCL and LCL. From Table 6 it is observed that values of P_d go on decreasing as we increase K ($K = 1.5$ to $K = 3$) for fixed $a = \sigma_e/\sigma_p$ and $P_1 = P_2$. Also, no change in the values of P_d being observed if there is marginal increase in the values of $a = \sigma_e/\sigma_p$ for fixed n and fixed K. But if we increase the size of the sample (Column 2 of Table 7) for fixed K and $P_1 = P_2$, there is a change in the values of P_d. The values of the power (P_d) is less if the size of the sample is larger for fixed $a = \sigma_e/\sigma_p$. It is also understood from Table 7, that the values of the power (P_d) is higher, if n increased along with the value of $a = \sigma_e/\sigma_p$.

REFERENCES

[1] Abraham, B., "Control charts and measurement error", *ASQC Technical Conference Transactions*, 33 (1977) 370–374.

[2] Anderson, M. A. , Greenberg, B. S., and Stokes, S. L., "Acceptance sampling with rectification when inspection errors are present", *Journal of Quality Technology*, 33 (4) (2001) 493–505.

[3] Balamurali, S., and Kalyanasundaram, M., "An investigation of the effects of misclassification errors on the analysis of means", *Tamsui Oxford Journal of Information and Mathematical Sciences*, 27 (2) (2011) 117–136.

[4] Bennett, C. A., "Effect of measurement error on chemical process control", *Industrial Quality Control*, 10 (4) (1954) 17–20.

[5] Buzas, J. S. , Stefanski, L. A. , and Tosteson, T. D., *Handbook of Epidemology*, chapter Measurement error, Springer, New York, (2005) 729–765.

[6] Chakraborty, A. B., and Khurshid, A., "Control Charts for binomial when the underlying distribution is ratio of two Poisson means", *Revista Investigacion Operacional*, 32 (3) (2011) 258–264.

[7] Chakraborty, A. B., and Khurshid, A., "Measurement error effect on the power of control chart for the ratio of two Poisson distributions", *Economic Quality Control*, 28 (1) (2013) 15–21.

[8] Chakraborty, A. B., and Khurshid, A., "Measurement error effect on the power of control chart for zero-truncated Poisson distribution", *International Journal for Quality Research*, 7 (3) (2013) 411–419.

[9] Chang, T.C., and Gan, F. F., "Monitoring linearity of measurement gauges", *Journal of Statistical Computation and Simulation*, 76 (10) (2006) 889–911.

[10] Coath, C. D. , Steele, R. C. J., and Lunnon, W. F., "Statistical bias in isotope ratios", *Journal of Analytical Atomic Spectrometry*, 28 (1) (2013) 52–58.

[11] Collins, R. D., and Case, K. E., "The distribution of observed defectives in attribute acceptance sampling plans under inspection errors", *AIIE Transactions*, 8 (3) (1976) 375–378.

[12] Costa, A. F. B., and Castagliola, P., "Effect of measurement error and autocorrelation on the $\bar{X}$ chart", *Journal of Applied Statistics*, 38 (4) (2011) 661–673.

[13] Fleiss, J. L. , Levin, B. , and Paik, M. C., *Statistical Methods for Rates and Proportions.* John Wiley & Sons, New Jersey, 2003.

[14] Govindaraju, K., and Jones, G., *Frontiers in Statistical Quality Control*, chapter Fractional acceptance numbers for lot quality assurance, Springer, Switzerland, (2015) 271–278.

[15] Graham, P. L. , Mengersen, K. , and Morton, A. P., "Confidence limits for the ratio of two rates based on likelihood scores: non-iterative method", *Statistics in Medicine*, 22 (12) (2003) 2071–2083.

[16] Huwang, L., and Hung, Y., "Effect of measurement error on monitoring multivariate process variability", *Statistica Sinica*, 17 (2) (2007) 749–760.

[17] Johnson, N. L. , Kotz, S. , and Kemp, A. W., *Inspection Errors for Attributes in Quality Control*, Chapman and Hall, New York, 1991.

[18] Johnson, N. L. , Kotz, S. , and Kemp, A. W., *Univariate Discrete Distributions*, 3rd edition, Wiley-Interscience, Hoboken, New Jersey, 2005.

[19] Kanazuka, T., "The effect of measurement error on the power of $\bar{X}$ – R charts", *Journal of Quality Technology*, 18 (2) (1986) 91–95.

[20] Lavin, M., "Inspection efficiency and sampling inspection plans", *Journal of the American Statistical Association*, 41 (236) (1946) 432–438.

[21] Lehmann, E. L., and Romano, J. P., *Testing Statistical Hypotheses*,3rd edition, Springer-Verlag, New York, 2005.

[22] Linna, K. W., and Woodall, W. H., "Effect of measurement error on Shewhart control charts", *Journal of Quality Technology*, 33 (2) (2001) 213–222.

[23] Linna, K. W., Woodall, W. H. , and Busby, K. L., "The performances of multivariate effect control charts in presence of measurement error", *Journal of Quality Technology*, 33 (3) (2001) 349–355.

[24] Linna, K.W., "Control chart performance under linear covariate measurement processes", PhD thesis, University of Alabama, USA, 1991.

[25] Maravelakis, P. E. , Panaretos, J. , and Psarakis, S., "EWMA chart and measurement error", *Journal of Applied Statistics*, 31 (4) (2004) 445–455.

[26] Maravelakis, P.E., "Measurement error effect on the cusum control chart", *Journal of Applied Statistics*, 39 (2) (2012) 323–336.

[27] Mittag, H.-J., *"Measurement error effect on control chart performance"*, in: ASQC Technical Conference Transactions, American Society for Quality Control, Milwaukee, WI, 1995, 66–73.

[28] Mittag, H.-J. and Rinne, D., *Statistical Methods of Quality Assurance.* Chapman and Hall/CRC, New York, 1993.

[29] Mittag, H.-J., and Stemann, D., "Gauge inspection effect on the performance of the $\bar{X}$ – S control chart", *Journal of Applied Statistics*, 25 (1998) 307–317.

[30] Mizuno, S., "Problems on measurement errors in process control", *Bulletin of the Interna-*

tional Statistical Institute, 38 (1961) 405–415.

[31] Moameni, M. , Saghaei, A. , and Salanghooch, M. G., “The effect of measurement error on $\bar{X}$ – R fuzzy control charts”, *ETSAR- Engineering, Technology and Applied Science Research*, 2 (1) (2012) 173–176.

[32] Owen, D. B., “Tables for computing bivariate normal probabilities”, *Annals of Mathematical Statistics*, 27 (4) (1956) 1075–1090.

[33] Price, R. M., and Bonett, D. G., “Estimating the ratio of two Poisson rates”, *Computational Statistics & Data Analysis*, 34 (3) (2000) 345–356.

[34] Rahim, M. A., “Economic model of charts under non-normality and measurement error”, *Computers and Operations Research*, 12 (3) (1985) 291–299.

[35] Rothman, K. J., and Greenland, S., *Modern Epidemiology*, 2nd edition, Lippincott-Raven, Philadelphia, 1998.

[36] Sahai, H., and Khurshid, A., “Confidence intervals for the ratio of two Poisson means”, *The Mathematical Scientist*, 18 (1) (1993) 43–50.

[37] Shore, H., “Determining measurement error requirements to satisfy statistical process control performance requirements”, *IIE Transactions*, 36 (9) (2004) 881–890.

[38] Singh, H. R., “Measurement error in acceptance sampling for attributes”, *Indian Society of Quality Control Bulletin*, x (1964) 29–36.

[39] Smirnov, N. V., and Bolsev, L. N., *Tables for evaluating function of a two dimensional normal distribution*, Izdat.Akad.Nauk.USSR, Moscow, 1962.

[40] Stapleton, J. H., *Linear Statistical Models*, John Wiley, New York, 1995.

[41] Stemann, D., and Weihs, C., “The EWMA-X-S control chart and its performance in the case of precise and imprecise data”, *Statistical Papers*, 42 (2) (2011) 207–223.

[42] Tricker, A., Coates, E., and Okell, E., “The effect on the R chart of precision of measurement”, *Journal of Quality Technology*, 30 (3) (1998) 232–239.

[43] Walden, C. T., *An analysis of variable control charts in the presence of measurement error*, Unpublished Master’s Thesis, Department of Industrial Engineering, Mississippi State University, USA, 1990.

[44] Xiaohong, L., and Zhaojun, W., “The CUSUM control chart for the autocorrelated data with measurement error”, *Chinese Journal of Applied Probability*, 25 (5) (2009) 461–474.

[45] Yamauchi, T., and Lee Ho, L., “Control charts for monitoring the ratio of two poisson rates”, *Quality and Reliability Engineering International*, 36 (1) (2019) 1–17.

[46] Yang, S. F., “The effects of imprecise measurement on the economic asymmetric and S control charts”, *The Asian Journal on Quality*, 3 (2) (2002) 46–55.

[47] Yang, S. F., Han-Wei, H., and Rahim, M. A., “Effect of measurement error on controlling two dependent process steps”, *Economic Quality Control*, 22 (1) (2007) 127–139.

[48] Yang, S. F., and Yang, C. M., “Effects of imprecise measurement on the two dependent processes control for the autocorrelated observations”, *The International Journal of Advanced Manufacturing Technology*, 26 (5) (2011) 623–630.

When $K = 1.5$ and $\Phi(K) = 0.9332$					
$a = \sigma_e/\sigma_p$	$h = \frac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.50	1.34	0.07039360	0.9099	0.16408720	0.11748720
0.40	1.39	0.05503907	0.9177	0.12557814	0.09457814
0.30	1.44	0.04001047	0.9251	0.08812094	0.07192094
0.25	1.46	0.03294319	0.9279	0.07118638	0.06058638
0.20	1.47	0.02635462	0.9292	0.05670924	0.04870924
0.15	1.48	0.01970635	0.9306	0.04201270	0.03681270
0.10	1.49	0.01305494	0.9319	0.02740988	0.02480988
0.05	1.50	0.00646451	0.9332	0.01292902	0.01292902

Table 1: Values of $T(h,a) = \frac{1}{\sqrt{2\pi}} \int_0^a \frac{\exp[-\frac{1}{2}h^2(1+x^2)]}{1+x^2} dx$, $\Phi(h)$, $P_1 = 2T(h,a) + \{\Phi(k) - \Phi(h)\}$ and $P_2 = 2T(h,a) - \{\Phi(k) - \Phi(h)\}$.

The function $T(h,a)$ has been tabulated by [32, 39]. Interested readers may obtain a simple QBASIC program from the first author.

When $K = 1.75$ and $\Phi(K) = 0.9599$					
$a = \sigma_e/\sigma_p$	$h = \frac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.50	1.57	0.04915861	0.9418	0.11641722	0.08021722
0.40	1.63	0.03763664	0.9484	0.08677328	0.06377328
0.30	1.68	0.02722368	0.9535	0.06084736	0.04804736
0.25	1.70	0.02237561	0.9554	0.04925122	0.04025122
0.20	1.72	0.01759671	0.9573	0.03779342	0.03259342
0.15	1.73	0.01315372	0.9582	0.02800744	0.02460744
0.10	1.74	0.008706727	0.9591	0.018213454	0.016613454
0.05	1.75	0.004304809	0.9599	0.008609618	0.008609618

Table 2: Values of $T(h,a) = \frac{1}{\sqrt{2\pi}} \int_0^a \frac{\exp[-\frac{1}{2}h^2(1+x^2)]}{1+x^2} dx$, $\Phi(h)$, $P_1 = 2T(h,a) + \{\Phi(k) - \Phi(h)\}$ and $P_2 = 2T(h,a) - \{\Phi(k) - \Phi(h)\}$.

When $K = 2.0$ and $\Phi(K) = 0.9772$					
$a = \sigma_e/\sigma_p$	$h = \frac{K}{\sqrt{a^2+1}}$	$T(h,a)$	$\Phi(h)$	P_1	P_2
0.50	1.79	0.03308721	0.9633	0.08007442	0.05227442
0.40	1.86	0.02471443	0.9686	0.05802886	0.04082886
0.30	1.92	0.01745997	0.9726	0.03951994	0.03031994
0.25	1.94	0.01433163	0.9738	0.03206326	0.02526326
0.20	1.96	0.01125022	0.9750	0.02470044	0.02030044
0.15	1.98	0.008244447	0.9761	0.017588894	0.015388894
0.10	1.99	0.00545368	0.9767	0.0114073	0.01040736
0.05	2.00	0.002692772	0.9772	0.005385544	0.005385544

Table 3: Values of $T(h,a) = \frac{1}{\sqrt{2\pi}} \int_0^a \frac{\exp[-\frac{1}{2}h^2(1+x^2)]}{1+x^2} dx$, $\Phi(h)$, $P_1 = 2T(h,a) + \{\Phi(k) - \Phi(h)\}$ and $P_2 = 2T(h,a) - \{\Phi(k) - \Phi(h)\}$.

P	π		
	$a = \sigma_e/\sigma_p = 0.05$	$a = \sigma_e/\sigma_p = 0.10$	$a = \sigma_e/\sigma_p = 0.15$
0	0.005385	0.011407	0.017589
0.01	0.015280	0.021119	0.027259
0.02	0.025170	0.030971	0.036929
0.03	0.035116	0.040753	0.046600
0.04	0.044955	0.050535	0.056270
0.05	0.054846	0.060317	0.065940

Table 4: Relationship between $TFD(= P)$ and $AFD(= \pi)$ for different values of $a = \sigma_e/\sigma_p$ when $K = 2.0$.

K	$a = 0.10$		$a = 0.15$		$a = 0.20$	
	P_1	P_2	P_1	P_2	P_1	P_2
1.50	0.027409880	0.024809880	0.042012700	0.036812700	0.056709240	0.048709240
1.75	0.018213454	0.016613454	0.028007440	0.024607440	0.037793420	0.032593420
2.00	0.011407360	0.010407360	0.017588894	0.015388894	0.024700440	0.020300440
2.25	0.006716798	0.006116798	0.010404278	0.009004278	0.015270486	0.011870486
2.50	0.003745750	0.003345750	0.005762864	0.004962864	0.008432314	0.006632314
2.75	0.001940131	0.001740131	0.003160336	0.002560336	0.004424558	0.003424558
3.00	0.000996947	0.000796947	0.001597271	0.001197271	0.002277630	0.001677630

Table 5: Probabilities of misclassification of conforming units (P_1) and nonconforming units (P_2) for different values of K and $a = \sigma_e/\sigma_p$.

	P_d		
π	$K = 1.5,\ P_1 = P_2 = 0.013,$ $CL = 3.1,\ UCL = 5,\ LCL = 1$	$K = 2,\ P_1 = P_2 = 0.005,$ $CL = 3,\ UCL = 5,\ LCL = 0$	$K = 3,\ P_1 = P_2 = 0.00044,$ $CL = 3.005,\ UCL = 7,\ LCL = 0$
0.01	0.9904	0.8601	0.8601
0.02	0.9674	0.7386	0.7386
0.04	0.8811	0.5421	0.5421
0.05	0.8296	0.4634	0.4633
0.07	0.7196	0.3370	0.3367
0.09	0.6117	0.2443	0.2432
0.10	0.5617	0.2081	0.2062
0.15	0.3354	0.1042	0.0910
0.20	0.3313	0.0963	0.0533
0.25	0.3937	0.1618	0.0700
0.35	0.6623	0.4373	0.2468
0.45	0.8813	0.7393	0.5479
0.50	0.9413	0.8491	0.6964
0.65	0.9972	0.9876	0.9578
0.75	0.9999	0.9992	0.9958

Table 6: Power of control chart for the ratio of two Poisson under misclassification due to measurement error ($a = \sigma_e/\sigma_p = 0.05,\ p = \bar{p} = 0.2,\ n = 15$).

	P_d		
π	$K = 1.5,\ P_1 = P_2 = 0.013,$ $CL = 4.16,\ UCL = 7,\ LCL = 1$	$K = 1.5,\ P_1 = 0.164,\ P_2 = 0.1175,$ $CL = 6.16,\ UCL = 9,\ LCL = 3$	$K = 3,\ P_1 = 0.010,\ P_2 = 0.005,$ $CL = 4,\ UCL = 9,\ LCL = 0$
0.01	0.9831	1.0000	0.8179
0.02	0.9401	0.9994	0.6676
0.04	0.8103	0.9926	0.4420
0.05	0.7358	0.9841	0.3585
0.10	0.3941	0.8671	0.1217
0.20	0.1559	0.4214	0.0215
0.25	0.2385	0.2661	0.0441
0.35	0.5855	0.2820	0.2378
0.40	0.7505	0.4204	0.4044
0.45	0.8702	0.5906	0.5857
0.50	0.9432	0.7496	0.7483
0.65	0.9887	0.9804	0.9804

Table 7: Power of control chart for the ratio of two Poisson under misclassification due to measurement error ($a = \sigma_e/\sigma_p = 0.05,\ p = \bar{p} = 0.2,\ n = 20$).