

PORTFOLIO SELECTION USING R

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Abstract: In this paper, we consider the Markowitz mean-variance model to minimize the risk on two assets and develop the program in R software to improve the performance of the model for two real stocks data with various combinations of the portfolios. We have taken two real stocks data upto 4514 each from yahoo database finance using our R program to show how fast our calculations are.

Keywords: Portfolio selection, Programming.

MSC: 90B85, 90C26.

1. INTRODUCTION

The portfolio selection plays an important role in the financial management and investment decision making. The goal is to maximize the profit and minimize the risk. Theory of portfolio was first proposed by Markowitz [6, 2, 3] and was based on the mean-variance model for the single-period model. From then on, numerous portfolio selection models have been developed by considering the return and risk in the form of the mean-variance model, see [8, 9, 10, 11, 4, 7]. In the financial industries, investors often have to face difficulties to take decisions for portfolio selection. In general, portfolio selection consists of two main processes: asset selection, and portfolio optimization. Several attempts are made to develop a framework for portfolio selection, e.g., [12], see the references therein, but these

have failed to address the important issues such as flexibility, and a managerially oriented decision support.

Many PC-based mean-variance Optimization software packages become available to support portfolio selection [13, 14], see the references therein. The rapid advancement in the Web technologies and the emergence of the e-Business influence the design and implementation of the financial Decision Systems, but they are just like a black box where users input the data and get the result without knowing internal computations. In this paper, we apply Markowitz mean - variance model to develop an optimization model for two real assets, and the R program to find the minimum risk.

The organization of the paper is given as follows: Section 2 presents the optimization for two real assets. The R program is given in Section 3, and computational results are shown in Section 4. Finally, the conclusion is given in Section 5.

2. OPTIMIZATION MODEL FOR TWO REAL ASSETS

Modern portfolio theory states that the investors are risk-opposing. Depending on their own economic conditions, investors have different levels of tolerance to risk. In the Markowitz model, the return on a portfolio is calculated by the expected value of the portfolio return, and the corresponding risk is quantified by the variance of the portfolio return. When an investor wants to construct a portfolio which yields a particular rate of return and simultaneously minimize the portfolio risk, we need to construct the optimization model. We present the return [1] at time 't' of stock as:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}, \quad (1)$$

where P_t, P_{t-1} are the closing prices at time t and $t - 1$, respectively. Let R_A and R_B be the returns on two assets A and B, respectively, and let x_A and x_B be the proportions of the available funds invested in each of the assets. Then, the resulting portfolio is $var(x_A R_A + x_B R_B)$. Thus, Markowitz mean-variance model is presented in the form of mathematical programming as:

$$\text{minimize } x_A^2 var(R_A) + x_B^2 var(R_B) + 2x_A x_B cov(R_A, R_B), \quad (2)$$

subject to the budget constraint

$$x_A + x_B = 1, \quad (3)$$

where

$$x_A \geq 0, \quad (4)$$

$$x_B \geq 0. \quad (5)$$

The objective function (2) represents the risk of the portfolio using variance measure. A risk-averse investor likes to minimize his/her risk.

The constraint (3) is the affection constraint, to assure that the investment does not exceed 100% of the invested capita.

The last two inequalities, (4) and (5), express the positivity of the unknown variables. We differentiate (2) with respect to x_A and we put it equal to zero to get

$$x_A = \frac{\text{var}(R_B) - \text{cov}(R_A, R_B)}{\text{var}(R_A) + \text{var}(R_B) - 2\text{cov}(R_A, R_B)} \quad (6)$$

Thus,

$$x_B = \frac{\text{var}(R_A) - \text{cov}(R_A, R_B)}{\text{var}(R_A) + \text{var}(R_B) - 2\text{cov}(R_A, R_B)}. \quad (7)$$

We observe that if $\text{var}(R_A) > \text{var}(R_B)$, then asset A is riskier than B , and we want to invest more in asset B than in A in order to minimize the risk of the portfolio.

3. THE R PROGRAM

The following R functions have been developed on the basis of definitions given in [1].

- `returnsof()`
- `meanof()`
- `varianceof()`
- `covarianceof()`

For our case, as we have linear constraint (3), we develop the following R program to minimize the risk of the portfolios on two assets, the Reliance Industries Ltd., and the Indian Oil Corporation Ltd.

```

1 returnsof<-function(A)
2 {
3   Reliance=A
4   new_date<- as.Date(Reliance$Date, "%d/%m/%Y")
5   year=strftime(new_date, "%Y")
6   year= as.integer(year)
7   month=strftime(new_date, "%m")
8   month= as.integer(month)
9   da=Reliance[,5]
10  ymc=data.frame(month, year, da)
11  k=1
12  p <- c()
13  r<-c()
14  p[k]=ymc[1,1]

```

```

15 r[k]=ymc[1,1]
16 q=p[k]
17 len=length(ymc[,1])
18 for(i in 1:len)
19 {
20   if (q==ymc[i,1])
21   {
22     p[k]=ymc[i,1]
23     r[k]=ymc[i,3]
24     k=k+1
25     q=q+1
26   }
27   else if (q>=13)
28   {
29     q=1
30     if (q==ymc[i,1])
31     {
32       p[k]=ymc[i,1]
33       r[k]=ymc[i,3]
34       k=k+1
35       q=q+1
36     }
37   }
38 }
39 R<-c()
40 return<-data.frame(p,r)
41 len=length(p)
42 for(i in 1:len-1)
43 {
44   R[i]=(return[i+1,2]-return[i,2])/return[i,2]
45 }
46 hist(R,main="Returns on Stocks",xlab="Returns on Stocks",
47      ylab="Frequency")
48 return(R)
49 }
50 A=file.choose()
51 A=read.csv(A)
52 B=file.choose()
53 B=read.csv(B)
54 returnsof(A)
55 returnsof(B)
56 #Compute Mean of the Reliance Industries Ltd. and
57 # the Indian Oil Corporation ltd.
58 meanof<-function(A)
59 {
60   rs=returnsof(A)
61   sum=0
62   n=length(rs)
63   for(i in 1:n)
64   {
65     sum=sum+rs[i]
66   }
67   meanofrestock=sum/n
68   return(meanofrestock)
69 }
70 meanof(A)

```

```

71 meanof(B)
72 #To compute Variance of the Reliance Industries Ltd.
73 # and the Indian Oil Corporation ltd.
74 varianceof<-function(A)
75 {
76 rs=returnsof(A)
77 n=length(rs)
78 mean=meanof(A)
79 dis=0
80 for(i in 1:n)
81 {
82 dis=dis+(rs[i]-mean)^2
83 }
84 variance=dis/(n-1)
85 return(variance)
86 }
87 varianceof(A)
88 varianceof(B)
89 #To compute covariance of the Reliance Industries Ltd.
90 # and the Indian Oil Corporation ltd.
91 covarianceof=function(A, B)
92 {
93 rsa=returnsof(A)
94 n=length(rsa)
95 meana=meanof(A)
96 rsb=returnsof(B)
97 n=length(rsb)
98 meanb=meanof(B)
99 dis=0
100 for(i in 1:n)
101 {
102 dis=dis+(rsa[i]-meana)*(rsb[i]-meanb)
103 }
104 covariance=dis/(n-1)
105 return(covariance)
106 }
107 covarianceof(A,B)
108 #Compute minimum risk value of the Reliance Industries Ltd.
109 and the Indian Oil Corporation ltd.
110 minriskof<-function(A,B)
111 {
112 xa=(varianceof(B)-covarianceof(A,B))/(varianceof(A)+varianceof(B)
113 -2*covarianceof(A,B))
114 xb=1-xa
115 ma=meanof(A)
116 mb=meanof(B)
117 exp=xa*ma+xb*mb
118 varia_portfo=xa^2*varianceof(A)+xb^2*varianceof(B)
119 +2*xa*xb*covarianceof(A,B)
120 risk=sqrt(varia_portfo)
121 print("Optimum Proportion")
122 print("xa")
123 print(xa*100)
124 print("xb")
125 print(xb*100)
126 print("Expected Returns ")

```

```

127 print(exp)
128 print("Minimum Risk")
129 print(risk)
130 }
131 # Compute risk under several portfolios combinations
132 minriskof(A,B)
133 standof<-function(A,B)
134 {
135 k=1
136 xa=0
137 xb=0
138 varia_portfo=0
139 risk=0
140 expected=0
141 for(i in seq(from=.0, to=1, by=.1))
142 {
143 xb[k]=i
144 xa[k]=1-xb[k]
145 expected[k]=xa[k]*meanof(A)+xb[k]*meanof(B)
146 varia_portfo[k]=xa[k]^2*varianceof(A)+xb[k]^2*varianceof(B)
147 +2*xa[k]*xb[k]*covarianceof(A,B)
148 risk[k]=sqrt(varia_portfo[k])
149 k=k+1
150 }
151 retris=data.frame(xa,xb,expected,risk)
152 plot(retris[,4],retris[,3],pch=19,col="blue",type="o",
153       xlab="Risk (standard deviation)",ylab="Expected return")
154 #print(retris)
155 return(retris)
156 }
157 standof(A,B)

```

4. EXPERIMENTAL RESULTS

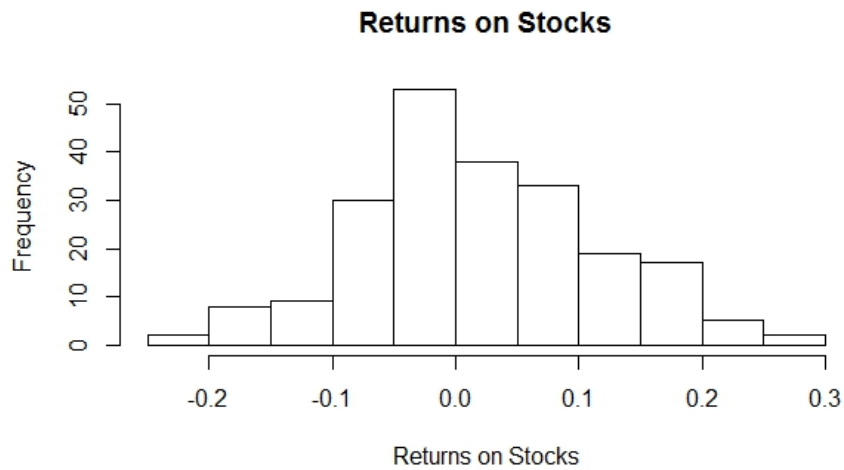
In this part, we tested our approach, and used real financial Data from [5], which included two real stocks data, that is, the Reliance Industries Ltd. and the Indian Oil Corporation Ltd. This website provides both the closing and the adjusted prices. In this paper, the closing prices are used. The price at the end of the day's trading on the day of trading is the closing price. Our study covers the period from May, 2000 to May, 2018 in monthly. After executing the above R program, 4514 data are filtered by taking the closing price of the beginning of each month. The R program computes the returns on stocks using (1). We see the benefit of diversification, which has been calculated using (5) and (6). The combination of 67.43% Reliance Industries Ltd. and 32.57% Indian Oil Corporation Ltd. gives less risk than the individual risk. The corresponding expected return of this portfolio is 0.01867138 with minimum risk 0.08512102.

We have deleted the negative returns on stocks from the portfolio to avoid the risk of the portfolio and to get the chance of profitability. The variance-covariance matrix of the returns on two assets is presented in Table 1. The expected return is maximized for a given level of variance of return, and variance of return is minimized for a given level of expected return. Such a preference ordering

Table 1: **Variance-Covariance Matrix**

	IOC	RELIANCE
IOC	0.01591337	
RELIANCE	0.003059054	0.009267683

system that would apply Investing in the real stock data always bears some risk. The portfolio with maximum expected return does not always give the minimum variance. The expected returns–variance rule [6] has been studied theoretically to handle such situation. The risk may be larger or small. It depends on the variance of the returns on stocks. From Figure 1, we can see the histograms of returns of stock of the Reliance Industries Ltd.

Figure 1: **Returns on stocks of the Reliance Industries Ltd.**

See Figure 2 for the histogram of the calculated returns on stocks of the Indian Oil Corporation Ltd.

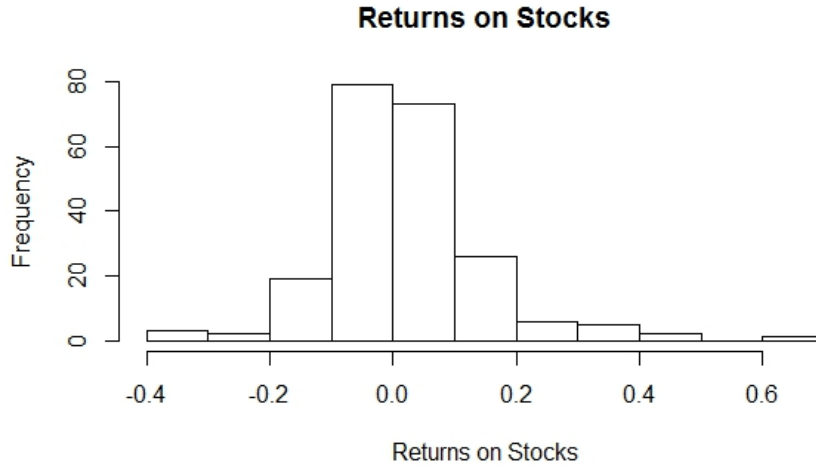


Figure 2: Returns on stocks of the Indian Oil Corporation Ltd.

The investor may not invest his/her funds in the computed combination of the two stocks because some people can tolerate more risk than the minimum risk. Any other combination of the two risks, under the given constraint (2), will give risk higher than the minimum risk. In Table 2, we present output of the R program.

Table 2: Expected return and Risk on the portfolio for values x_A and x_B

No.	x_a	x_b	Expected	Risk
1	1.0	0.0	0.01812424	0.09626881
2	0.9	0.1	0.01829223	0.09064539
3	0.8	0.2	0.01846023	0.08687203
4	0.7	0.3	0.01862822	0.08519490
5	0.6	0.4	0.01879622	0.08573710
6	0.5	0.5	0.01896422	0.08845784
7	0.4	0.6	0.01913221	0.09316645
8	0.3	0.7	0.01930021	0.09958134
9	0.2	0.8	0.01946821	0.10739720
10	0.1	0.9	0.01963620	0.11633200
11	0.0	1.0	0.01980420	0.12614819

We have taken different values of x_A and x_B to get the expected return and risk on the portfolios. We have shown the portfolio possibilities curve in Figure 3, which is drawn using Table 2. We observe that the efficient portfolio points are located on the top part of the graph between the minimum risk portfolio point and the maximum return portfolio point, which is called the efficient frontier. From Figure

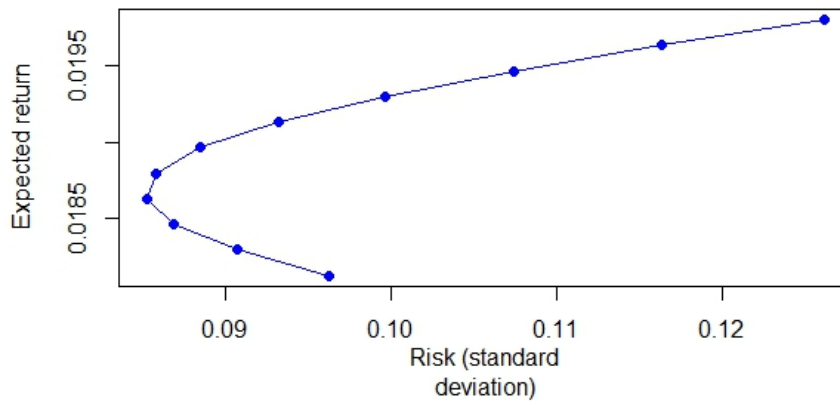


Figure 3: **Portfolio possibilities curve for two real stocks data**

3, we observe that the efficient portfolios should provide higher expected return for the same level of risk or lower risk for the same level of expected return, and the individual securities can be diversified away. Even though equal allocation is also not the optimum solution. But, diversification reduces the risk when investing in securities that are uncorrelated or in securities for which the average covariance is small.

5. CONCLUSIONS

We have minimized the risk and maximized the expected returns for portfolio selection for stocks data of two companies the Reliance Industries Ltd. and the Indian Oil Corporation Ltd., respectively. The developed R program for Markowitz's mean-variance model is used to find the expected return and risk under several combinations of portfolios. The task to compute the statistical measures becomes easy due to our R program. We have not used any predefined functions to calculate the statistical measures. The problem becomes more complicated as the number of stocks increases. This could be solved through quadratic programming, which is our possible task for the future research.

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