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OPTIMAL LOCATION OF PRODUCTION CENTERS, INTERCONNECTED MATERIALS AND PRODUCTS ALLOCATION

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Abstract: This paper deals with a complex problem, consisting of the selection of production centers, the allocation of materials and products as well as the consideration of the production levels.

Production centers may be located at different sites, i.e. nodes of a graph. Multiple components or raw materials are transported to the production centers and a variety of final products may be manufactured and distributed from them. Each product has specific unit requirements of component materials.

There are multiple supply centers for each type of material and demand centers for each product, i.e. nodes of the graph, with capacity or demand limits. All distances between nodes of the graph i.e. supply, demand and production centers, are known. Transportation costs may be nonlinear functions of distance. Other variables like costs or profits are considered. The optimisation criterion may be the cost required for demand satisfaction or the profit. The problem is formulated as a linear programming model.

Keywords: Production centers, production level, transportation cost.

1. DESCRIPTION OF THE PROBLEM

The problem can be described as follows : There are $|P|$ types of products which can be manufactured from $|M|$ types of components or materials. Each unit of product $p \in P$ requires a specific quantity $a_{m,p}$ of each material $m \in M$.

A transportation network connects nodes or centers, i.e. cities. A node may be a supply, demand or production center for one or more materials and products. There are $\boldsymbol{I_m} = \mathrm{card}I_m$ fixed supply centers for each material m, $\boldsymbol{J_p} = \mathrm{card}J_p$ fixed demand centers for each product p and $|N|$ candidate production centers.

A supply center $i(m) \in I_m$ has an upper capacity limit $A_{i(m)}$ and a demand center $j(p) \in J_p$ has an upper demand limit $D_{j(p)}$. Production centers may be existing plants or firms which can undertake the manufacture of some products. Therefore there may be limits $B_{n,p}$ or $B_{n,m}$ to the capacity of center $n \in N$ to manufacture product p or to handle material m.

Manufacturing processes may impose additional constraints, such as quantitative relations between products, i.e. there may be strict or upper bound proportions of quantities of some or all products manufactured at center n.

All distances $S_{i(m),p}$ and $S_{n,j(p)}$ between production centers and supply or demand centers are known. It is assumed that these are the "shortest" or least cost distances, depending on the items to be transported and the available transportation facilities.

Total unit costs $C_{i(m),p}$ for materials and $C_{n,j(p)}$ for products, include the unit transportation costs per total distance $K_{i(m),p}$ and $K_{n,j(p)}$, as well as any other variable costs such as procurement costs, handling or manufacturing costs, distribution and retail costs, e.t.c.

Transportation costs may be nonlinear functions of distance. In general, transportation costs per unit of item t and total distance $S_{i,j}$ must be calculated for each t and $S_{i,j}$ using the appropriate functions.

There is an evidence that in some practical situations the cost factors $K^0_{(t)i,j}$ per unit of item and unit of distance can be expressed as a decreasing function of $S_{i,j}$, i.e. $K^0_{(t)i,j} = C_{1(t)}\, S_{i,j}^{a(t)} + C_{2(t)}$, $a(t) < 0$. The unit cost per total distance $S_{i,j}$ would then be $K_{(t)i,j} = C_{1(t)}\, S_{i,j}^{1+a(t)} + C_{2(t)}\, S_{i,j}$. Usually the cost factors of all items have the same exponent $a(t) = a$ and the same ratio of coefficients $C_{1(t)} / C_{2(t)} = C$. In that case, it is practical to transform all "real" distances $S_{i,j}$ to "equivalent" distances $S^*_{i,j}$, $S^*_{i,j} = 0.5\,(CS^a_{i,j} + 1)\,S_{i,j}$. Subsequently, the "equivalent" unit cost factors are constants, $K^*_{(t)} = 2C_{2(t)}$ and the unit costs per distance $S_{i,j}$ are linear functions of $S^*_{i,j}$, $K_{(t)i,j} = K^*_{(t)}\, S^*_{i,j}$.

The optimisation criterion may be the cost of the satisfaction of demands. In that case a uniform fictitious "price" must be attributed to every product. The exact value of this price is not important, as long as it is estimated to exceed the most unfavourable combination of unit costs associated with any product and its components. Alternatively, a sufficiently high "price" can be assigned to all component materials and products. If real profit is the criterion, then real prices $g_{j(p)}$ must be established for every product and demand center. This, of course, may lead to reductions or differentiations of production levels, resource utilisation, demand satisfaction, location of production centers etc.

2. MODEL FORMULATION

The problem will be formulated as a L.P. model, using the previous notation. If $x_{i(m),n}$ is the quantity of material m supplied to production center n from source $i(m)$ and $x_{n,j(p)}$ is the quantity of final product p distributed from n to demand center $j(p)$ then the problem may be stated as:

Maximize :

$$Z = \sum_{n\in N}\sum_{p\in P}\sum_{j(p)\in J_p}\left(g_{j(p)} - C_{n,j(p)}\right)x_{n,j(p)} - \sum_{n\in N}\sum_{m\in M}\sum_{i(m)\in I_m} C_{i(m),n}\, x_{i(m),n} \quad (1)$$

subject to:

$$\sum_{n\in N} x_{i(m),n} \le A_{i(m)} \qquad \text{for all } i(m)\in I_m,\, m\in M \quad (2)$$

$$\sum_{n\in N} x_{n,j(p)} \le D_{j(p)} \qquad \text{for all } j(p)\in J_p,\, p\in P \quad (3)$$

$$\sum_{i(m)\in I_m} x_{i(m),n} - \sum_{p\in P} a_{m,p} \sum_{j(p)\in J_p} x_{n,j(p)} = 0 \qquad \text{for all } m\in M, n\in N \quad (4)$$

Constraints (2) and (3) state that capacity limits of supply centers or demand limits of demand centers must not be exceeded.

Constraints (4) are the balance equations stating that the outflow of all products from each production center must be compensate for the inflow of each material, which is the exact quantity required for production.

Additional constraints may be formulated according to the actual conditions in each case, i.e. capacity limits of production centers may be stated as :

$$\sum_{j(p)\in J_p} x_{n,j(p)} \le B_{n,p} \qquad \forall p\in P, \forall n\in N \quad (5)$$

if there is an upper bound on the production of p at center n, or:

$$\sum_{i(m)\in I_m} x_{i(m),n} \le B_{n,m} \qquad \forall m\in M, \forall n\in N \quad (6)$$

if there is an upper bound on the quantity of m which can be handled at n.

Quantitative relations between products at each production center may be equalities or upper bound inequalities, such as:

$$\sum_{j(p^*)\in J_{p^*}} b_{n,p^*}\cdot x_{n,j(p^*)} \le \sum_{j(p^{**})\in J_{p^{**}}} b_{n,p^{**}}\, x_{n,j(p^{**})}, \quad (7)$$

where p^* and p^{**} are elements of disjoint subsets of P and b_{n,p^*}, $b_{n,p^{**}}$ are appropriate coefficients.

3. NUMERICAL APPLICATION

We shall demonstrate the problem by the following example. A ten–nod|e transportation network comprises sources of three types of materials ($M = 3$, $I_1 = |I_1| = 3, I_2 = |I_2| = 2, I_3 = |I_3| = 3$) and demand centers for two products $|P| = 2, J_1 = |J_1| = 2, J_2 = |J_2| = 2$ where $|X|$ denotes cardinal number of the set X. All nodes are considered as candidate sites for production centers ($|N| = 10$).

Table 1 gives the values of the specific quantities $a_{m,p}$ of each material for each product. Table 2 gives the values of transportation cost coefficients $C_{1(t)}$, $C_{2(t)}$ and $K^*_{(t)}$.

Table 1. Unit Requirements $a_{m,p}$

		Materials :		
	p \ m	1	2	3
Products	1	5	4	8
	2	8	2	2

Table 2. Transportation cost coefficients

		$C_{1(t)}$	$C_{2(t)}$	$K^*_{(t)}=2C_{2(t)}$
m	1	1.5	1	2
	2	2.25	1.5	3
	3	3	2	4
p	1	4.5	3	6
	2	4.5	3	6

Table 3. Transportation distances and capacity / demand limits

i \ j (NODES)	DISTANCES BETWEEN NODES										CAPACITY / DEMAND $A_{i(m)}$ MATERIALS (m)			LIMITS $D_{j(p)}$ PRODUCTS (p)	
	1	2	3	4	5	6	7	8	9	10	1	2	3	1	2
1		100.7	50.5	127.2	67.5	63.3	109.8	90.5	56.2	96.5	2000			600	
2	114		118	143.5	62.5	41.7	72.8	159.8	88	125.5			2000		
3	55	135		87.2	60.7	81.3	151.7	66	31	45.7		2000			
4	146	167	98		78.8	99.8	105	118	72	43.5	10000		5000	400	
5	75	69	67	88		27.3	100.7	98	28	67					
6	70	45	91	113	28		77	118	50.5	88					500
7	125	81	177	119	114	86		166.2	122.2	159		5000			300
8	102	187	73	135	111	135	224		73.8	87.3			2000		
9	64	99	32	80	29	55	141	82		47					
10	109	144	54	47	74	99	186	97	51		1000				

equivalent distances

real distances

For the assumed values of $C_{1(t)} / C_{2(t)}$ and $a(t)$ the function $S^*_{i,j}(S_{i,j})$ is increasing and $S^*_{i,j} < S_{i,j}$ for $S_{i,j} > 17$.

It is assumed, for simplicity, that there are no upper bounds on production capacity or any other production constraints, such as quantitative relations between products. Also, total costs include only transportation costs. Therefore total unit costs can be computed as $C_{(t)i,j} = K^*_{(t)}\, S^*_{i,j}$. Table 4 gives the total unit costs $C_{i(m),n}$, for materials and $C_{n,j(p)}$, for products.

The L.P. model is formulated by application of the objective function (1), the upper bound constraints (2) and (3) and the balance equations (4). There are

$\sum_{m \in M} i(m) + \sum_{p \in P} j(p) = 8 + 4 = 12$ inequalities, $|M| \times |N|$ balance equalities and $|N|\left(\sum_{m \in M} i(m) + \sum_{p \in P} j(p)\right) = 120$ primal variables.

Table 4. Total unit costs $C_{i(m),n}$, $C_{n,j(p)}$

		CANDIDATE PRODUCTION NODES										
		n	1	2	3	4	5	6	7	8	9	10
SUPPLY NODES $i(m)$	MATERIAL 1	1(1)	0	201	101	254	135	127	220	181	116	193
		4(1)	254	287	174	0	158	200	339	236	144	87
		10(1)	193	251	99	87	134	176	318	173	94	0
	MATERIAL 2	3(2)	151	354	0	262	182	244	455	198	93	149
		7(2)	329	218	455	509	302	231	0	564	367	477
	MATERIAL 3	2(3)	402	0	472	574	251	167	291	639	352	502
		4(3)	509	574	349	0	315	399	678	472	288	174
		8(3)	362	639	264	472	392	472	752	0	295	345
DEMAND NODES $j(p)$	PROD.1	1(1)	0	604	303	763	406	380	659	543	349	579
		4(1)	763	861	523	0	473	599	1017	708	432	261
	PROD.2	6(2)	380	250	488	599	164	0	462	708	303	528
		7(2)	659	437	910	1017	604	462	0	1129	733	954

Table 5 gives the results from solving three versions of the example. In version 1 a high "price" $g_{i(p)} = 20{,}000$ exceeding the most unfavourable cost combinations, is attached to every product and demand center. Demand is satisfied up to the limit of available resources, which is established by the total of 8,000 units available at the supply centers of material 3.

Table 5. Numerical applications

			Supply of demand		Example 1					Example 2			Example 3	
						Production Nodes					Prod. Nodes			
			Nodes	Limits	$\sum_{n\in N} x$	1	2	4	8	$\sum_{n\in N} x$	2	4	$\sum_{n\in N} x$	4
MATERIALS	m=1	$l_1=3$	1	2000	2000	1333	625		42	625	625			
			2	10000	2400			7400		6500		6500	3600	3600
			10	1000	1000				1000					
			(Σ)	13000	10400	1333	625	7400	1042	7125	625	6500	3600	3600
	m=2	$l_2=2$	3	2000	2000			1167	833	2000		2000	2000	2000
			7	5000	2800	333	500	1967		1500	500	1000		
			(Σ)	7000	4800	333	500	3134	833	3500	500	3000	2000	2000
	m=3	$l_3=3$	2	1000	1000		1000			1000	1000			
			4	5000	5000			5000		5000		5000	3600	3600
			8	2000	2000	333			1667					
			(Σ)	8000	8000	333	1000	5000	1667	6000	1000	5000	3600	3600
PRODUCTS	p=1	$J_1=2$	1	600	400		125	67	208	225	125	100		
			4	400	400			400		400		400	400	400
			(Σ)	1000	800		125	467	208	625	125	500	400	400
	p=2	$J_2=2$	6	500	500			500		500		500	200	200
			7	300	300	167		133						
			(Σ)	800	800	167		633		500		500	200	200
Objective function Z (profit)					29,098,375					1,156,075			456,200	
Total costs $g_{j(p)}$					2,901,625					1,718,925			643,800	
Average Unit cost					1,814					1,528			1,073	

In versions 2 and 3 the objective is profit maximisation. In version 2 prices are set at 3,000 for product 1 and 2,000 for product 2. In version 3 lower prices of 2,000 and 1,500 are set for products 1 and 2 respectively.

As shown in Table 5, four production centers are selected in version 1, producing a maximum total of 800 units of product 1 and 800 units of product 2. In versions 2 and 3, fewer centers are selected, total production levels are decreased but the average unit cost is substantially reduced. There is computational experience from more examples, not included in this paper, with additional production constraints. The model can also be applied to multiperiod planning when quantitative constraints between periods are imposed.

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