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APPROXIMATE REASONING BASED OPTIMIZATION

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Abstract: A new approach to fuzzy optimization is proposed. It is based on application of approximate reasoning categories in order to obtain a more flexible representation of logical aggregation and defuzzification. It allows to design a non-iterative algorithm for fuzzy optimization which surpass the well-known Zimmermann's approach. Bellman-Zadeh's method can be considered as a special case of the approach proposed here. An illustrative example is presented.

Keywords: approximate reasoning, fuzzy optimization, aggregation, defuzzification.

1. INTRODUCTION

Fuzzy set theory was applied to optimization problems and *fuzzy optimization* was introduced [7, 2]. Usually aggregation of constraints AND criteria in these problems is made by using so called "min-aggregator" [9]. Zimmermann has emphasized its shortcomings and has introduced *compensatory AND* as a more appropriate and respective to the *logical AND*.

Bellman and Zadeh [1] have used *mean of maximum* (MOM) - like defuzzification procedure in order to find a crisp solution. In the case when there are more than one maximum of decision membership function any of them is taken as a crisp solution in Bellman-Zadeh's approach. However, information about other possible solutions (these which have degree of membership to decision fuzzy set less than the maximal degree and other solution which have maximal degree of membership, if they exist) is loosed in this case.

Recently, Filev and Yager [4] have introduced a generalized defuzzification method called BADD. It implies MOM and *center of area* (COA) defuzzification methods as a special cases.

Fuzzy optimization can be considered as a two–stage process which includes, Figure 1:

i) aggregation of fuzzy conditions (criteria and constraints) by logical conjunction in order to receive decision fuzzy set;

ii) defuzzification of decision fuzzy set for determination of a crisp solution of the problem.

A more *flexible* aggregation operator and a new defuzzification method are used. It allows to formulate a more realistic optimization problem. Furthermore, it allows to design a non–iterative algorithm and to avoid numerical methods.

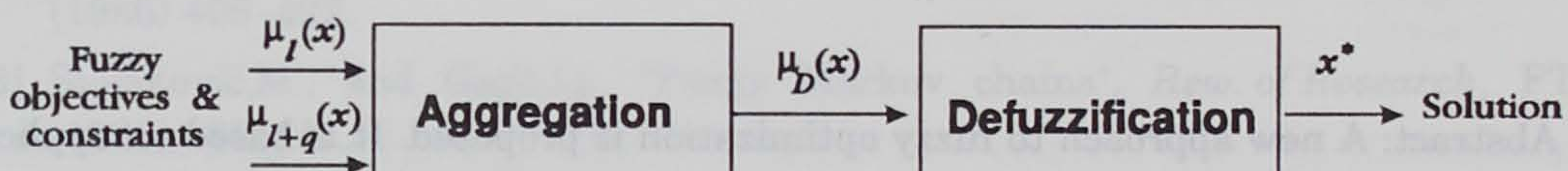

Figure 1: Common approach

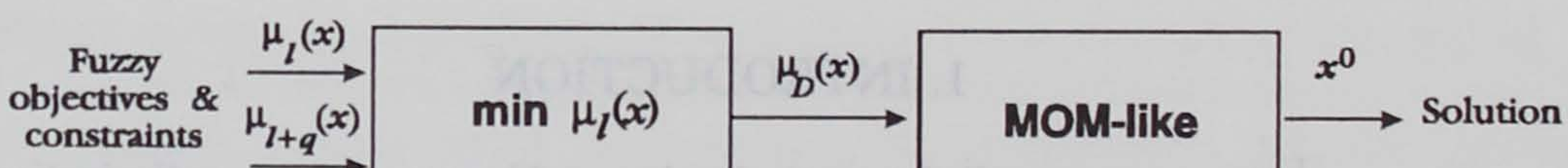

Figure 2: Bellman–Zadeh' s approach

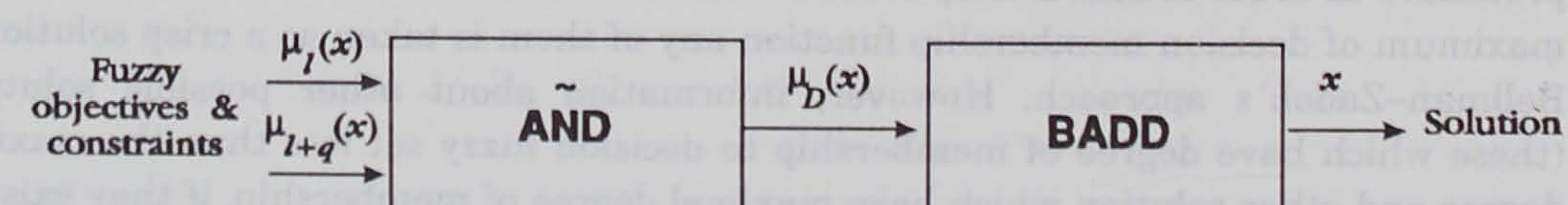

Figure 3: A new *flexible* approach

2. FUZZY OPTIMIZATION PROBLEM FORMULATION

The following description of fuzzy optimization generalizes such problems as fuzzy mathematical programming, decision making in a fuzzy environment, fuzzy dynamical programming, fuzzy multiobjective programing etc. [6, 3]:

The degree of membership to the fuzzy set of the i–th criterion is $\mu_i(x) : x \to [0, 1]$; $x \in R^n$; $i = \{1, \dots, l\}$. The degree of membership to the j–th constraint is $\mu_j(x) : x \to [0, 1]$; $j = \{l, \dots, q\}$. Problem solution has to satisfy both criteria and constraints and has to belong to all fuzzy sets involved. The space of such solutions is again fuzzy set called *decision* with membership function $\mu_D(x) : x \to [0, 1]$. Crisp solution is reached by defuzzification of the decision fuzzy set.

One special case of this problem formulation is fuzzy linear programming problem introduced by Zimmermann [8]:

$$C\,x \to \widetilde{\min} \tag{1}$$

$$A\,x \,\tilde{\le}\, b \tag{2}$$

where $\widetilde{\min}$ denotes fuzzy minimization represented by $\mu_i(x)$

$\tilde{\le}$ denotes fuzzy constraints represented by $\mu_j(x)$

A, b, C – parameters; $A \in R^{q \times n}$, $b \in R^q$; $C \in R^n$

x – arguments; $x \in R^n$

According to Bellman–Zadeh' s approach [1], decision fuzzy set is found as:

$$\mu_D(x) = \min_{k=1}^{l+q} \mu_k(x) \tag{3}$$

Crisp solution x^0 is found by a MOM – like defuzzification procedure:

$$x^0_{\max} = \left\{ x \mid \mu_D(x^0) = \max \mu_D(x) \right\} \tag{4}$$

Zimmermann [8] has transformed the problem to the following mathematical programming problem which can be solved by numerical methods:

$$\max \lambda \tag{5}$$

$$\lambda \le \mu_i(x) \qquad i = 1, \dots, l \tag{6}$$

$$\lambda \le \mu_j(x) \qquad j = l, \dots, q \tag{7}$$

$$0 \le \lambda \le 1 \tag{8}$$

3. A NEW APPROACH TO FUZZY OPTIMIZATION

At first stage of fuzzy optimization all constraints have to be combined in appropriate way. The result of this stage is again fuzzy set which has membership function $\mu_D(x)$. It represents the degree of satisfaction of fuzzy objectives ($\mu_i(x)$, $i = 1, \dots, l$) AND fuzzy constraints ($\mu_j(x)$, $j = l, \dots, q$). This logical aggregation is an analogy to the Langrange multipliers and to the penalty functions method in crisp (non–fuzzy) optimization problems. In Belman–Zadeh' s approach [1] $\mu_D(x)$ is reached by "min – aggregator", Figure 2. Zimmermann has discussed shortcomings of this operator and has introduced *compensatory AND* represented by *γ–operator* [9, 8]:

$$\mu_D(x) = (1-\gamma)\prod_{k=1}^{l+q} \mu_k{}^{\delta_k}(x) + \gamma\left\{1 - \prod_{k=1}^{l+q}\left(1-\mu_k{}^{\delta_k}(x)\right)\right\} \tag{9}$$

where γ – compensation coefficient; [0, 1]

δ_k – weights assigned to the different fuzzy sets

This operator closes to the *human AND* [8] and allows to choose the strength of the AND. It can be denoted as "AND", Figure 3. There are another parametric intersection operators. However, they are not better than the *γ–operator* by means of *human AND* [9]. An overview of such operators is given by Klir and Folger [5]:

i) Frank' s operator (β–parameter; $\beta \in [0; \infty)$):

$$\mu_1(x)\tilde{\bigcap}\mu_2(x) = \log_\beta\left[1 + \frac{\left(\beta^{\mu_1}-1\right)\left(\beta^{\mu_2}-1\right)}{\beta-1}\right] \tag{10}$$

ii) Dombi' s operator (β–parameter; $\beta \in [0; \infty)$):

$$\mu_1(x)\tilde{\bigcap}\mu_2(x) = \frac{1}{1-\left[\left(\dfrac{1}{\mu_1(x)}-1\right)^{\beta} + \left(\dfrac{1}{\mu_2(x)}-1\right)^{\beta}\right]^{1-\beta}} \tag{11}$$

iii) Hamacher' s operator (β–parameter; $\beta \in [0; \infty)$):

$$\mu_1(x)\tilde{\bigcap}\mu_2(x) = \frac{\mu_1(x)\mu_2(x)}{\beta-(1-\beta)\left[\mu_1(x)-\mu_2(x)-\mu_1(x)\mu_2(x)\right]} \tag{12}$$

At second stage of fuzzy optimization a crisp solution (x^*) is derived from decision fuzzy set $\mu_D(x)$. In Bellman–Zadeh' s approach MOM – like defuzzification procedure is applied. However, in this method the most acceptable argument (x^0) is taken only. The recently introduced BADD method takes into account all arguments and their membership values instead of MOM method but takes them with various degrees of acceptance instead of COA method. All possible fuzzy solutions are included in determination of crisp solution by BADD. It makes the result more realistic and appropriate to the problem formulation:

$$\bar{x} = \sum_{j=1}^{N} v_j x_j \qquad N = \mathrm{card}(x) \tag{13}$$

$$v_j = \frac{\mu_D{}^{\alpha}(x_j)}{\sum_{i=1}^{N}\mu_D{}^{\alpha}(x_i)} \tag{13a}$$

where α – parameter; $\alpha \in [0; \infty)$

$\bar{x}$ – crisp solution determined by BADD method.

The parameter α expresses the degree of preference of x^0, where x^0 is found by (4). It has been proved [4] that for $\alpha \to \infty$ BADD approaches to the MOM method and for $\alpha = 1$ it is equal to the COA method. Decreasing of the entropy with increasing of the value of the parameter α has been proved, too. It means that for a large value of α arguments x for which value of $\mu_D(x)$ is large are taken with higher priority and arguments for which value of $\mu_D(x)$ is small are ignored. For $\alpha = 1$ all arguments are taken with equal priority weighted by their membership functions value $\mu_D(x)$. Defuzzification by BADD method allows to adjust the degree of neglecting of information by learning parameter α, e.g. by a neural network.

If a parametric conjunction operator (Frank' s, Dombi' s or Hamacher' s) is used then Bellman–Zadeh' s approach [1] can be considered as a special case of the approach proposed here for $\alpha \to \infty$ and $\beta \to \infty$. It follows from BADD method and from *flexible* conjunction operators definitions [4,5].

4. A NON–ITERATIVE ALGORITHM FOR FUZZY OPTIMIZATION

The proposed concept can be treated as an extension of the well–known Bellman–Zadeh' s approach for more *flexible* problem formulation. It is an effective alternative to the widely used Zimmermann' s approach. The main advantage of this concept is that it avoids numerical methods and gives the global extremum (13).

The following simple algorithm for non–iterative solving of fuzzy optimization problem is proposed:

1. Determination of parameters α, γ, and δ_k. Parameter α is a measure of belief in the possible alternatives contained in a fuzzy solution $\mu_D(x)$. Parameter γ represents the degree of compensation between "strong AND" and its alternative. Weights δ_k, $k = 1, \dots, l+q$ express the importance of every objective and constraint.

   Two approaches for parameters determination are possible:

   i) by subjective estimation;

   ii) by learning procedure (e.g. by neural network).

2. Fuzzy solution determination by (9).

   Another operator (10)–(12) can also be applied.

3. Crisp solution determination by (13).

It should be mentioned that the result depends on the discretization of the arguments.

EXAMPLE

Let us consider the following fuzzy mathematical programming problem:

$$J = 20x_1 + 25x_2 \to \tilde{\max}$$

$$6x_1 + 10x_2 \tilde{\le} 660$$

$$0.5x_1 + 0.3x_2 \tilde{\le} 47$$

$$\mu_1 = \begin{cases} 1 & ; \; J > 2200 \\ \dfrac{20x_1 + 25x_2 - 2000}{200} & ; \; 2000 \le J \le 2200 \\ 0 & ; \; J < 2000 \end{cases}$$

$$\mu_2 = \begin{cases} 1 & ; \; 6x_1 + 10x_2 < 660 \\ \dfrac{1 - 6x_1 + 10x_2 - 660}{40} & ; \; 660 \le 6x_1 + 10x_2 \le 700 \\ 0 & ; \; 6x_1 + 10x_2 > 700 \end{cases}$$

$$\mu_3 = \begin{cases} 1 & ; \; 0.5x_1 + 0.3x_2 < 47 \\ 1 - \dfrac{0.5x_1 + 0.3x_2}{3} & ; \; 47 \le 0.5x_1 + 0.3x_2 \le 50 \\ 0 & ; \; 0.5x_1 + 0.3x_2 > 50 \end{cases}$$

The following discrete values of arguments are considered:

$$x_1 = \{20; 50; 70; 80; 90; 100\}$$

$$x_2 = \{5; 10; 15; 20\}$$

$$N_1 = 6; \; N_2 = 4; \; N = N_1 N_2 = 24$$

Fuzzy solution for $\gamma = 0.7$ and $\delta_1 = \delta_2 = \delta_3 = 1$ is:

$$\mu_D(x) = 0.3\, \mu_1(x)\, \mu_2(x)\, \mu_3(x) + 0.7\, \{ 1 - (1 - \mu_1(x))\, (1 - \mu_2(x))\, (1 - \mu_3(x)) \}$$

Crisp solution is defined for $\alpha = 10$:

$$\bar{x} = \sum_{j=1}^{N} \frac{\mu_D{}^{10}(x_j)}{\sum_{i=1}^{N} \mu_D{}^{10}(x_i)} x_j$$

$$\bar{x} = \begin{vmatrix} 87.1 \\ 15.2 \end{vmatrix}$$

5. CONCLUSION

Fuzzy optimization is presented as a two stage process. Using approximate reasoning categories a more *flexible* and realistic problem is formulated. A non-iterative algorithm is designed. Our efforts in the future will be concentrated to the application of the proposed algorithm to various fuzzy optimization problems in biotechnology.

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