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OPTIMIZATION OF AIR DISTRIBUTION IN MINE VENTILATION NETWORKS

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Abstract:* This paper deals with application of the Sequential Unconstrained Minimization Technique (SUMT) for solution of the problems of partially controlled air distribution in mine ventilation networks. All papers in this area in Yugoslav mining literature [8], [14], [13], and practice were based, particularly when use of contemporary computers are in question, on the well known Hardy Cross Iterative Method. Having in mind that the problems is related to nonlinear programming, the SUMT method can be used to find optimal solution.

Keywords: Mine ventilation, SUMT method, air distribution, optimization.

1. INTRODUCTION

The most common problem of solving mine ventilation systems refers to definition of partially controlled air distribution in the network, i.e. determination of the values of air quantities through network branches based on network configuration, individual predetermined flows, branch resistances and fans as depression sources in the network.

Mine ventilation network are fully defined by three equations: Atkinson's equation, Kirchhoff's current law equation and Kirchhoff's voltage law equation. Analyses of ventilation system in our mining literature, and in practice, are related to determination of natural air distribution, using the Hardy Cross Method [3], [12].

Ventilation networks, mathematically defined by above mentioned equations, reduce the problem of partially controlled distribution to nonlinear programming solutions. The problem defined in such a manner may be solved, in addition to the Hardy Cross Method, by the use of optimization gradient methods with high rate of convergence.

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The paper outlines the SUMT algorithm applied for the solution of partially controlled air distribution in mine ventilation networks as restricted optimizations.

2. MATHEMATICAL MODEL

The mathematical model for determination of partially controlled air distribution in mine ventilation networks, based on Telegan's Theorem [16], [11], defining the criteria function and Kirchhoff s current and voltage laws as principal constraints, reduced the problem to nonlinear programming.

Optimization of air flow distribution through the ventilation network in which some flows are predetermined is made by use of Telegan' s Theorem on total engaged power for achievement of the criteria function to be minimized, i.e.

$$F(h,v) = \sum_{n_v \in N_v} h_{n_v} v_{n_v} \rightarrow \min \qquad (1)$$

where: n_v - number of a branch with fan, N_v - set of branches with fans, h_{n_v} - fan pressure in branch n_v, v_{n_v} - air flow in fan branch n_v.

Main and auxiliary (booster) fans pressures are a function of air flow through fan branches. The functions are usually expressed in polynomial form.

Selection of ventilation network polygon or fundamental meshes is made so that the branches with fixed air flow represent independent branches with respect to the spanning tree. If the total number of fundamental meshes is designated as m, and the number of fixed flows as n_f, then the number of independent network branches whose flows are determined by power minimization equals $m - n_f$.

Definition of air flow through all branches is made, upon numbering of branches with fixed flows from 1 to n_f, unknown flows through independent branches from $n_f + 1$ to m, and network tree branches from m + 1 to n_0, as follows:

$$v_j = \sum_{i=1}^{n_f} b_{ij} v_i + \sum_{i=n_f+1}^{m} b_{ij} v_i, \qquad \text{for } j = 1, 2, \ldots, n_0 \qquad (2)$$

where: b_{ij} - elements of fundamental mesh matrix $(1, 0, -1)$, n_0 - total number of network branches.

Relation (2) is a form of Kirchhoff s current law where the first part represents the sum of known accepted flows, and the second one the sum of variable ones to be determined.

Air ways through the ventilation network may be defined as an oriented chain of branches from the incoming network node towards the outgoing one. A path matrix is used for description of air paths through networks designated as:

$$L = [l_{ij}]_{n_l \times n_0},$$

where: $l_{ij} = 1$, when path i contains branch j, $l_{ij} = 0$, when path i does not contain branch j, and n_l - number of paths.

Then the Kirchhoff s voltage law may be expressed as follows:

$$\sum_{n_v \in N_v} l_{in_v} h_{n_v} - \sum_{j=1}^{n_0} l_{ij} R_j v_j^2 \geq 0 \tag{3}$$

where: h_{n_v} – fan pressure in branch n_v, n_v – number of a branch with fan, N_v – the set of branches containing fans.

If relation (3) left hand side is larger than zero, it is necessary to install a flow control along the path in order to meet the Kirchhoff s voltage law.

In order to prevent the change of air flow direction in branches which would disturb the defined path, protective conditions are introduced:

$$v_j \geq 0, \qquad \text{for } j = m, m+1, \dots, n_0 \tag{4}$$

In line with above described relations (1) through (4) solving of partially controlled air distribution in mine ventilation networks as a nonlinear programming problem may be formulated as follows:

Criterion function:

$$F(h,v) = \sum_{n_v \in N_v} h_{n_v} \left(\sum_{k=1}^{n_f} b_{kn_v} v_k + \sum_{k=n_f+1}^{m} b_{kn_v} v_k \right) \rightarrow \min \tag{5}$$

With constraints:

$$C_i(h,v) = \sum_{n_v \in N_v} l_{in_v} h_{n_v} - \sum_{j=1}^{n_0} l_{ij} R_j \left(\sum_{k=1}^{n_f} b_{kj} v_k + \sum_{k=n_f+1}^{m} b_{kj} v_k \right)^2 \geq 0,$$

$$\text{for } i = 1, \dots, n_l \tag{6}$$

$$C_i(h,v) = h_i \geq 0, \qquad \text{for } i = 1, 2, \dots, nf \tag{7}$$

$$C_i(h,v) = v_i \geq 0, \qquad \text{for } i = nf+1, \dots, m \tag{8}$$

$$C_i(h,v) = \sum_{k=1}^{n_f} b_{ki} v_k' + \sum_{k=n_f+1}^{m} b_{ki} v_k \geq 0, \qquad \text{for } i = m+1, \dots, no \tag{9}$$

Solution of the nonlinear programming model with constraints, as defined in relations (5) through (8), may be performed by use of SUMT.

3. SEQUENTIAL UNCONSTRAINED MINIMIZATION TECHNIQUE (SUMT)

The basic idea underlying this method is to transform the task [1], [7]:

minimize the function $z = f(v)$, with constraints $c_i(v) > 0$,

into the task: minimize the function without constraints.

To achieve this, a new function Z is introduced, defined as:

$$Z = f(v) + P(v), \tag{10}$$

where $P(v)$ is a certain correction of function f, enabling to find the minimized solutions "within" the constraints.

It should be emphasized that function P was not uniformly defined. The suitable form is:

$$P(v) = r\sum_{i=1}^{n} \frac{1}{c_i(v)}, \tag{11}$$

where c_i is the i^{th} constraint, n is the number of the constraints, while r is a positive number to be determined.

SUMT was initially reported by C.W. Carrol in 1961 [2], while his idea was elaborated for practical application by A.V. Fiacco and G.P. McCormick [4], [5].

For the predetermined (given) function $f(v)$ with constraints $c_j(v) > 0$, it is necessary to select a positive number $r = r_0$ in order to form the function $\varphi(v,r_0)$, which is minimized without constraints by the DFP (Davidon–Flecher–Powell) method. This method is outlined in references [6], [7]. To reach the minimum of the function $\varphi(v,r_0)$, the value of r must be decreased. This may be achieved by introduction of $r_1 = r_0 / c$, where $c > 1$ is a constant. Then function $\varphi(v,r_1)$ is minimized again by use of the DFP method. In this way the iterative procedure is developed so that in the k–th step function $\varphi(v,r_k)$ is minimized, whose minimum is the point v_k^*.

It is rational to start from the assumption that point v is located near the minimum of the function:

$$\varphi(v,r) = f(v) + r\sum_{i=1}^{n} \frac{1}{c_i(v)} = f(v) + P(v), \tag{12}$$

and the gradient of function $\varphi(v,r)$ has negligible value. Since the function gradient:

$$\nabla\varphi = \nabla f + \nabla P \tag{13}$$

is negligible, we obtain:

$$r = \frac{-\nabla f^T(v)\nabla P(v)}{\nabla P^T(v)\nabla P(v)} \tag{14}$$

for the norm square.

The SUMT algorithm is shown on Figure 1.

4. NUMERICAL EXAMPLE

Solution of the problem of partially controlled air distribution by use of SUMT will be presented on an actual mine ventilation network example, defined by a closed canonic ventilation diagram shown on Figure 2.

On this diagram volumetric air flows are given in branches 1 (V_1=2.60 m^3/s) and 2 (V_2=2.70 m^3/s), while a fan of type N–AVV–K–80/40–8, "Klima", Celje, is installed in branch 9.

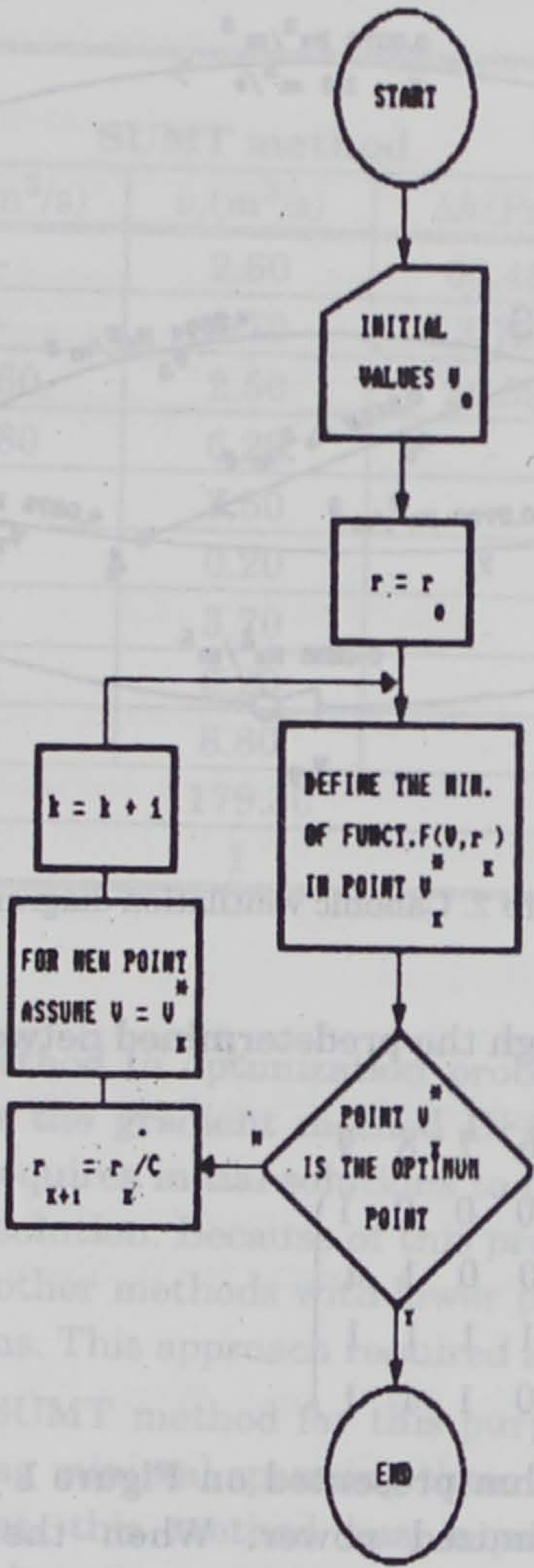

Figure 1. SUMT algorithm

Fan characteristic curve is approximated by the following polynomial:

$$h_{n_v} = a + b v_{n_v} + c v_{n_v}^2 + \dots \qquad (15)$$

In accordance with the assumed network tree, and taking into account 4 meshes in the network ($m = n_0 - n_n + 1 = 9 - 6 + 1 = 4$), where n_n - total number of nodes, fundamental mesh matrix has the following form:

$$\boldsymbol{B} = \begin{array}{c c} & \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \end{array}$$

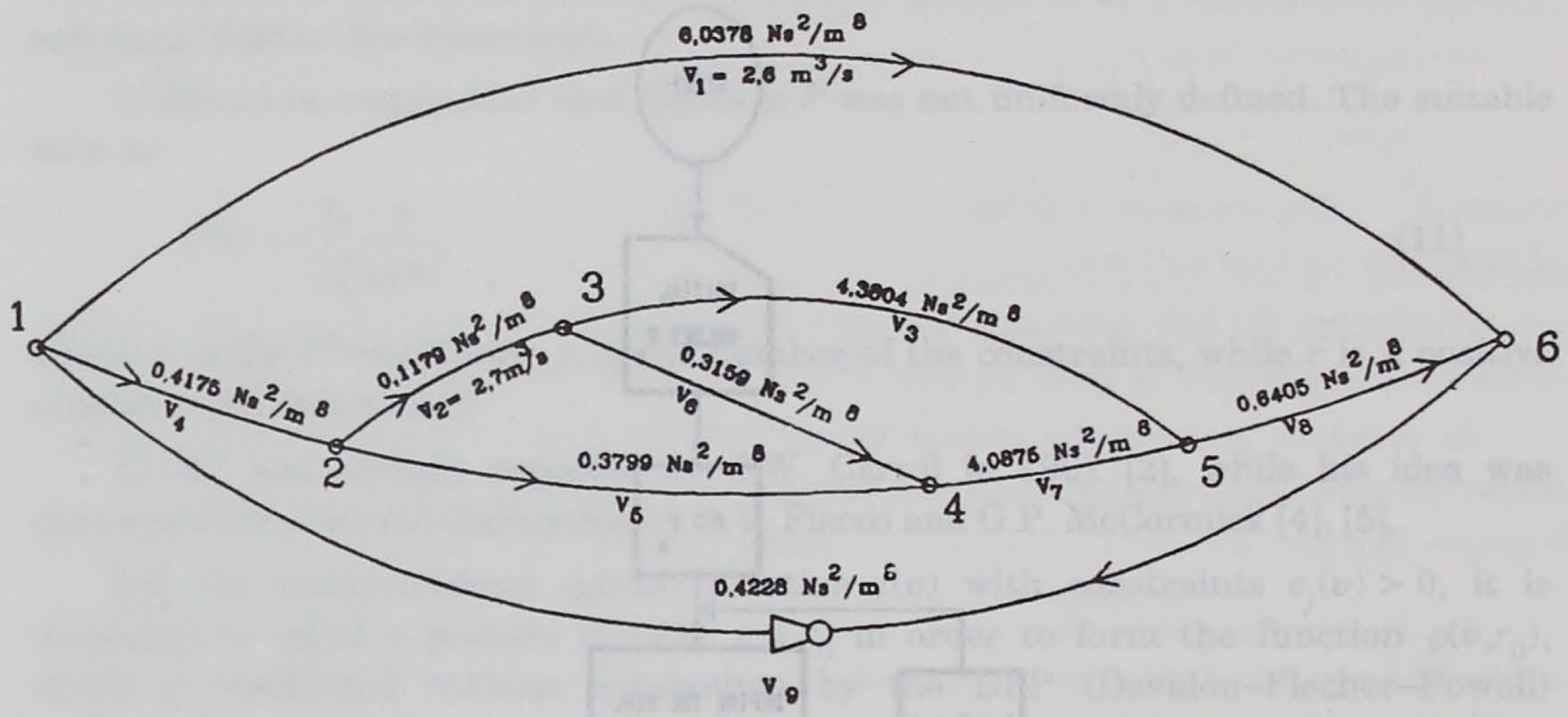

Figure 2. Canonic ventilation diagram

The air path matrix through the predetermined network is defined as follows:

$$
L = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c} \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\
\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \end{array}
$$

Application of the algorithm presented on Figure 1 yields air flows through the ventilation network at minimized power. When the volumetric air flows are determined in this way, flow regulators are defined by the critical path method, because the problems is now transformed into linear programming.

Comparison of the numerical results of SUMT and Hardy Cross methods is given for the optimal air distribution in mine ventilation network, see Figure 2 and Table 1.

It is obvious from this results that differences of air quantities for most branches in network are negligible. Most differences appears in branches 3, 6 and 7. In the case of 6-th branch, Hardy Cross method gave opposite air flow direction (-0.386 m^3/s), what disturbed given path of the air flow in network. SUMT method produced result in one iterative step, while Hardy Cross required five steps. Small number of iterative steps in a case of Hardy Cross method was consequence of good choice of initial solutions. Number of unknown air quantities, as well as range of constraints, which are defined by the configuration of ventilation network, influenced both the convergence rate and the minimal value of criteria function (engaged power). This is why used constraints and initial solutions produced final solutions with higher engaged power (1179.30W) than in the case of Hardy Cross.

Table 1.

Branch	R_i	**SUMT method**			**Hardy Cross method**	
	(Ns^2/m^8)	$v_{0i}(m^3/s)$	$v_i(m^3/s)$	$\Delta h(Pa)$	$v_i(m^3/s)$	$\Delta h(Pa)$
1	6.0378	-	2.60	60.46	2.600	47.619
2	0.1179	-	2.70	3.78	2.700	4.063
3	4.3804	2.60	2.50	28.59	3.086	-
4	0.4175	2.80	6.20	-	6.282	-
5	0.3799	-	3.50	-	3.582	-
6	0.3159	-	0.20	-	−0.386	-
7	4.0875	-	3.70	-	3.196	-
8	0.6405	-	6.20	-	6.282	-
9	0.4228	-	8.80	-	8.882	-
P(W)		1179.30			1081.80	
No. of iterations		1			5	

Application of SUMT method to optimization problems of the mine ventilation networks, which incorporates the gradient method DFP in its algorithm, has some disadvantages. This method requires initial solutions to be very close to final solution, or it will not give an optimal solution. Because of this problem, we suggest that initial solutions should be found by other methods with lower precision and than use SUMT method to get optimal solutions. This approach required longer CPU time.

Advantage of using the SUMT method for this purposes is possibility of getting optimal solutions without using minimal spanning three algorithm as it is the case in Hardy Cross method. Besides, this method has a possibility to give alternative solutions of the flow regulator locations, which are very important in mining practice.

At the end it should be emphasised, that application of Hardy Cross method is much simpler for the practical purposes of solving air distribution in complex mine ventilation networks. However, application of SUMT method can be used to define some alternative solutions.

5. CONCLUSION

Determination of partially controlled air distribution in mine ventilation networks may be carried out very successfully by reducing the problem to nonlinear programming and solution by SUMT optimization. The mathematical model presented in this paper, based on Telegan' s theorem as a criteria function and Kirchhoff' s current and voltage laws as constraints, is applicable for solving air distribution in networks containing branches with fixed air flows.

The mathematical method of optimization with constraints, i.e. SUMT, described in this paper is applied for solving above mentioned ventilation problems, while the advantage of using this method compared with those usually applied in mining practice lies in the possibility of obtaining optimum solutions for stated problems, and getting alternative solutions.

Optimization of the mining ventilation networks using SUMT method has some advantages as well as some disadvantages. Complex ventilation networks have a number of function constraints which after transformation of problem into optimization model without constraints become very complicated.

Complex models like this are solved using gradient methods, where rate of convergence depends on initial solution.

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