

APPLICATION OF INTERVAL VALUED FUZZY-ROUGH NUMBERS IN MULTI-CRITERIA DECISION MAKING: THE IVFRN-MAIRCA MODEL

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Abstract: This paper presents a new approach for the treatment of uncertainty and imprecision based on interval-valued fuzzy-rough numbers (IVFRNs). IVFRNs make a decision making possible using only the internal knowledge from the data, using objective indeterminacy without the need to rely on models of any assumption. Namely, instead of subjectively entering external uncertainties, the structure of the given data is used. Taking into account the given assumptions, we developed an original multi-criteria model based upon the IVFR approach. In the multi-criteria model the traditional MAIRCA (Multi-Attribute Ideal-Real Comparative Analysis) method was modified. The model was tested and validated on a case study, considering selection of the optimal landing operations point for overcoming water obstacles. The sensitivity analysis of the IVFRN MAIRCA model was carried out through 24 scenarios which showed that our results are of a high stability degree.

Keywords: Rough Numbers, Fuzzy Sets, Interval-Valued Fuzzy-Rough Numbers, Multi-Criteria Decision Making, MAIRCA, MABAC, VIKOR, TOPSIS.

MSC: 91B, 93C.

1. INTRODUCTION

Multi-criteria decision-making models that contain qualitative and quantitative values of the attributes have wide application in the fields of operational research, management science, urban planning, natural sciences, military fields, etc. Due to ambiguity and complexity of attributes in multi-criteria decision-making problems (MCDM), the attributes cannot always be expressed with crisp numbers. In classic MCDM methods, such as TOPSIS [13, 16, 17], VIKOR [20, 25], MABAC [6, 22] and CODAS [1, 7], the weight of each attribute and the ranking of the alternatives are represented by crisp numbers. However, in the real world, the decision maker may prefer to assess the attributes using linguistic variables instead of crisp values because of his partial knowledge about the attributes or the lack of information in from the domain of a problem. In such a situation, priority information about the alternatives provided by the decision makers can be unclear, imprecise or incomplete. The fuzzy set, introduced by Zadeh [34], is one of the tools used to present such imprecision in a mathematical form. Multi-criteria decision-making (MCDM) problems with imprecise information can be successfully modeled using the theory of fuzzy sets in the field of decision making. However, the fuzzy set can only focus on the degree of belonging of unclear parameters or events.

Unlike fuzzy theory, a very convenient tool for treating imprecision without the effect of subjectivism is the theory of rough sets, introduced by Pavlak [29]. In nowadays literature rough set theory is successfully applied to a wide range of different areas of human activity [23, 31]. Knowing the advantages of rough set theory, it is completely justifiable to carry out the decision-making process using rough sets when it contains indeterminate and inaccessible data [24].

In the decision-making process the interval fuzzy technique is used to transform crisp numbers into fuzzy numbers which, with the help of the membership function, show the degree of belonging of elements to a given set. According to Zadeh [34], linguistic expressions (linguistic variables) can be used very successfully to quantify uncertainty in complex and uncertain situations [5]. Here, linguistic variables are variables whose values are linguistic terms that can be used in an intuitive and simple way to express the subjectivity and/or qualitative imprecision in the assessments of decision makers. Karnik and Mendel [14] regarded introducing linguistic expressions by using classical fuzzy sets (type-1 fuzzy sets) as not sufficiently clear and precise. Further, they consider that it is much more natural and precise to introduce linguistic expressions by using interval-valued fuzzy sets (type-2 fuzzy sets). Interval-valued fuzzy sets can provide greater flexibility in presenting imprecise and unclear information, especially in the process of group decision making, which is characterized by a high degree of uncertainty [3]. For this reason, the application of interval-valued fuzzy sets in multi-criteria decision making (MCDM) is a logical step, to ensure a clear enough presentation of linguistic expressions.

However, as with type-1 fuzzy sets, interval-valued fuzzy sets are also characterized by subjectivism when defining the borders of the sets and the footprint of

uncertainty [2]. In order to eliminate this subjectivity, the authors of this paper present a new approach which is a modification of fuzzy sets by using rough numbers. Interval-valued fuzzy-rough numbers (IVFRN) utilize the benefits of both theories: fuzzy sets and rough sets (numbers). In the IVFRN approach the borders are determined based on the border approximation areas and the uncertainty in them. The IVFRN approach uses only internal knowledge, that is, the operational data, and there is no need to rely on models of any assumption. This means that with the application of IVFRN, instead of different additional/external parameters, only the structure of the given data is used. This IVFRN-based approach combines the benefits of the fuzzy and rough concepts.

This paper has several objectives. The first objective is to improve the methodology for dealing with uncertainties in the field of group multi-criteria decision making. The second goal is to affirm the idea of interval-valued fuzzy-rough numbers through a detailed presentation of arithmetical operations with IVFRN that are characteristic of multi-criteria decision making. The third goal is to introduce other authors to the wider application of IVFRN in MCDM, since the benefits of IVFRN that are emphasized in this paper are a logical motive for their wider application. Finally, the fourth goal is to bridge the gap that exists in the methodology for evaluating landing operations points for overcoming water obstacles by means of a new approach to the treatment of uncertainties based on IVFRN.

In the multi-criteria model presented in this paper an original modification of the MAIRCA and TOPSIS methods was carried out by applying the IVFRN approach. The proposed models provide an evaluation of the alternatives in spite of doubts in the decision-making process and a lack of quantitative information. The authors hope that these modifications will make a significant contribution to the literature that discusses multi-criteria decision making. Since the IVFRN approach is essentially a unification of the advantages of the fuzzy and rough approaches, a logical scenario for validating the IVFRN model is the application of fuzzy and rough theory. Therefore, the fuzzy and rough modifications of the MAIRCA, VIKOR and TOPSIS methods are used to validate the basic IVFRN-MAIRCA model. The authors specifically highlight the original IVFRN modification of the TOPSIS model that was developed for validating the MCDM model and which has not been considered in the literature so far. In addition to the above contributions, the authors emphasize the contribution of the paper in the field of selecting the optimum landing operations point for overcoming water obstacles. The authors did not come across any MCDM model in the literature that considers the selection of the optimum landing operations point for overcoming water obstacles and they hope that the IVFR-MAIRCA model will make a significant contribution to decision makers in this field.

The rest of the paper is organized into six sections. After the introduction, the second section presents the basic concept of interval-valued fuzzy-rough numbers. The third section presents the algorithm for the hybrid IVFRN-MAIRCA model, tested in the fourth section on a case study, considering the optimal place for overcoming water obstacles and forming a landing operations point for Serbian Army units to cross. The fifth section is a discussion of the results and a validation

of the IVFRN-MAIRCA model. Finally, the sixth section presents the concluding considerations and directions for further research.

2. INTERVAL-VALUED FUZZY-ROUGH NUMBERS

We define fuzzy set A as a set of arranged pairs

$$A = \{(x, \mu_A(x)) \mid x \in X, 0 \leq \mu_A(x) \leq 1\},$$

which is described by a triangular membership function. Then we can represent a fuzzy number A as $A = (a_1, a_2, a_3)$, where a_1 and a_3 respectively represent the left and right borders of the interval of fuzzy number A , and a_2 represents the value at which fuzzy number A reaches its maximum value.

Suppose that U is a universe that contains all objects and let Y be an arbitrary object from U . Suppose that there is a set of k classes representing preferences of the DM, $G^* = (A_1, A_2, \dots, A_k)$, under the condition that they belong to a series that satisfies the condition that $A_1 < A_2 < \dots < A_k$. All objects are defined in the universe and connected with the preferences of the DM. Each element A_i ($1 \leq i \leq k$) represents a fuzzy number defined as $A_q = (a_{1q}, a_{2q}, a_{3q})$. Since element A_i from the class of objects G^* is represented as a fuzzy number $A_q = (a_{1q}, a_{2q}, a_{3q})$, for each of the values a_{1q} , a_{2q} and a_{3q} we obtain one class of objects, represented in the intervals $I(a_1)_q = \{I(a_1)_{lq}, I(a_1)_{uq}\}$, $I(a_2)_q = \{I(a_2)_{lq}, I(a_2)_{uq}\}$ and $I(a_3)_q = \{I(a_3)_{lq}, I(a_3)_{uq}\}$ where the condition that $I(a_j)_{lq} \leq I(a_j)_{uq}$ ($j = 1, 2, 3; 1 \leq q \leq k$) is satisfied, as well as the condition $I(a_1)_q, I(a_2)_q, I(a_3)_q \in G^*$. Then $I(a_j)_{lq}$ and $I(a_j)_{uq}$ ($j = 1, 2, 3; 1 \leq q \leq k$) respectively represents the lower and the upper limit of the interval of the q -th class of objects. If both limits of the class of objects (upper and lower limits) respectively are arranged so that $I^*(a_j)_{l1} < I^*(a_j)_{l2} < \dots < I^*(a_j)_{ls}; I^*(a_j)_{u1} < I^*(a_j)_{u2} < \dots < I^*(a_j)_{um}$ ($j = 1, 2, 3; 1 \leq s, m \leq k$), then for any class of objects $I^*(a_j)_{lq} \in G^*$ and $I^*(a_j)_{uq} \in G^*$ ($j = 1, 2, 3; 1 \leq q \leq k$) we can define the lower approximation $I^*(a_j)_{lq}$ using the following expressions

$$\underline{Apr}(I^*(a_1)_{lq}) = \bigcup \{Y \in U/G^*(Y) \leq I^*(a_1)_{lq}\}; \quad (1 \leq q \leq k) \quad (1)$$

$$\underline{Apr}(I^*(a_2)_{lq}) = \bigcup \{Y \in U/G^*(Y) \leq I^*(a_2)_{lq}\}; \quad (1 \leq q \leq k) \quad (2)$$

$$\underline{Apr}(I^*(a_3)_{lq}) = \bigcup \{Y \in U/G^*(Y) \leq I^*(a_3)_{lq}\}; \quad (1 \leq q \leq k) \quad (3)$$

And the upper approximation of $I^*(a_j)_{uq}$ using the following expressions

$$\overline{Apr}(I^*(a_1)_{uq}) = \bigcup \{Y \in U/G^*(Y) \geq I^*(a_1)_{uq}\}; \quad (1 \leq q \leq k) \quad (4)$$

$$\overline{Apr}(I^*(a_2)_{uq}) = \bigcup \{Y \in U/G^*(Y) \geq I^*(a_2)_{uq}\}; \quad (1 \leq q \leq k) \quad (5)$$

$$\overline{Apr}(I^*(a_3)_{uq}) = \bigcup \{Y \in U/G^*(Y) \geq I^*(a_3)_{uq}\}; \quad (1 \leq q \leq k) \quad (6)$$

Both classes of objects (object classes $I^*(a_j)_{lq}$ and $I^*(a_j)_{uq}$) are defined by their lower limit $\underline{Lim}(I^*(a_j)_{lq})$; $j = 1, 2, 3$, and their upper limit $\overline{Lim}(I^*(a_j)_{uq})$; $j = 1, 2, 3$. The lower limits are defined by the following expressions:

$$\underline{Lim}(I^*(a_1)_{lq}) = \frac{1}{M_{L(a_1)}} \sum G^*(Y) | Y \in \underline{Apr}(I^*(a_1)_{lq}); \quad (1 \leq q \leq k) \quad (7)$$

$$\underline{Lim}(I^*(a_2)_{lq}) = \frac{1}{M_{L(a_2)}} \sum G^*(Y) | Y \in \underline{Apr}(I^*(a_2)_{lq}); \quad (1 \leq q \leq k) \quad (8)$$

$$\underline{Lim}(I^*(a_3)_{lq}) = \frac{1}{M_{L(a_3)}} \sum G^*(Y) | Y \in \underline{Apr}(I^*(a_3)_{lq}); \quad (1 \leq q \leq k) \quad (9)$$

where $M_{L(a_1)}$, $M_{L(a_2)}$ and $M_{L(a_3)}$ represents the sum of the objects contained in the lower approximation of the classes of objects $I^*(a_1)_{lq}$, $I^*(a_2)_{lq}$ and $I^*(a_3)_{lq}$. The upper limits $\overline{Lim}(I^*(a_j)_{uq})$; $j = 1, 2, 3$, are defined by expressions (10)-(12)

$$\overline{Lim}(I^*(a_1)_{uq}) = \frac{1}{M_{U(a_1)}} \sum G^*(Y) | Y \in \overline{Apr}(I^*(a_1)_{uq}); \quad (1 \leq q \leq k) \quad (10)$$

$$\overline{Lim}(I^*(a_2)_{uq}) = \frac{1}{M_{U(a_2)}} \sum G^*(Y) | Y \in \overline{Apr}(I^*(a_2)_{uq}); \quad (1 \leq q \leq k) \quad (11)$$

$$\overline{Lim}(I^*(a_3)_{uq}) = \frac{1}{M_{U(a_3)}} \sum G^*(Y) | Y \in \overline{Apr}(I^*(a_3)_{uq}); \quad (1 \leq q \leq k) \quad (12)$$

where $M_{U(a_1)}$, $M_{U(a_2)}$ and $M_{U(a_3)}$ respectively represent the sum of the objects contained in the upper approximation of the classes of objects $I^*(a_1)_{uq}$, $I^*(a_2)_{uq}$ and $I^*(a_3)_{uq}$.

Both limits of the objects (the lower limit and upper limit) $\underline{Lim}(I^*(a_j)_{lq})$ and $\overline{Lim}(I^*(a_j)_{uq})$; $j = 1, 2, 3$ should satisfy the condition that

$$\frac{\underline{Lim}(I^*(a_1)_{lq})}{\underline{Lim}(I^*(a_2)_{uq})} \leq \frac{\overline{Lim}(I^*(a_1)_{uq})}{\underline{Lim}(I^*(a_3)_{lq})} \leq \frac{\underline{Lim}(I^*(a_2)_{lq})}{\overline{Lim}(I^*(a_3)_{uq})} \leq \quad (13)$$

If because of a great amount of uncertainty (disagreement) in the expert decision making and the characteristics of the predefined fuzzy linguistic scales, condition (13) is not satisfied, that is $\overline{Lim}(I^*(a_1)_{uq}) > \underline{Lim}(I^*(a_2)_{lq})$ or $\overline{Lim}(I^*(a_2)_{uq}) > \underline{Lim}(I^*(a_3)_{lq})$, then equations (14) and (15) apply

$$\overline{Lim}(I^*(a_1)_{uq}) = \underline{Lim}(I^*(a_2)_{lq}). \quad (14)$$

$$\underline{Lim}(I^*(a_3)_{lq}) = \overline{Lim}(I^*(a_2)_{uq}). \quad (15)$$

We can determine the rough boundary interval for each class of objects from $I(a_1)_q$ represented as $RB(I(a_j)_q)$; $j = 1, 2, 3$, which denotes the interval between the lower and upper limits:

$$RB(I(a_1)_q) = \overline{Lim}(I^*(a_1)_{uq}) - \underline{Lim}(I^*(a_2)_{lq}); \quad (j = 1, 2, 3; 1 \leq q \leq k). \quad (16)$$

As we can see, each class of objects $I(a_1)_q$, $I(a_2)_q$ and $I(a_3)_q$ is defined by its lower and upper limits, which make up interval fuzzy-rough number $\overline{\overline{A}}$, (Figure 1), which is defined as

$$\begin{aligned} \overline{\overline{A}} &= [A_q^L, A_q^U] = \\ &= \left[\left(\overline{\underline{Lim}}(I^*(a_1)_{uq}), \underline{Lim}(I^*(a_2)_{lq}), \overline{\underline{Lim}}(I^*(a_2)_{uq}), \underline{Lim}(I^*(a_3)_{lq}); w_1(A_q^L) \right), \right. \\ &\quad \left. \left(\underline{\overline{Lim}}(I^*(a_1)_{lq}), \underline{\overline{Lim}}(I^*(a_2)_{lq}), \overline{Lim}(I^*(a_2)_{uq}), \overline{Lim}(I^*(a_3)_{uq}); w_2(A_q^U) \right) \right] \end{aligned} \tag{17}$$

where A_q^L and A_q^U respectively represent the upper and lower trapezoidal fuzzy-rough number that satisfies the condition $A_q^L \subset A_q^U$, while $w_1(A_q^L)$ and $w_2(A_q^U)$ respectively represents maximum values of interval fuzzy-rough number $\overline{\overline{A}}$.

From Figure 1 we can see that for interval-valued fuzzy-rough number $\overline{\overline{A}}$ it is true that $w_1(A_q^L) = w_2(A_q^U) = 1$. On this basis, we can write expression (17) in the following form

$$\overline{\overline{A}} = [A_q^L, A_q^U] = [(a_{1q}^L, a_{1q}^U), (a_{2q}^L, a_{2q}^U), (a_{3q}^L, a_{3q}^U)] \tag{18}$$

where $a_{jq}^L = \underline{Lim}(I^*(a_j)_{lq})$ and $a_{jq}^U = \overline{\underline{Lim}}(I^*(a_j)_{uq})$; ($j = 1, 2, 3; 1 \leq q \leq k$).

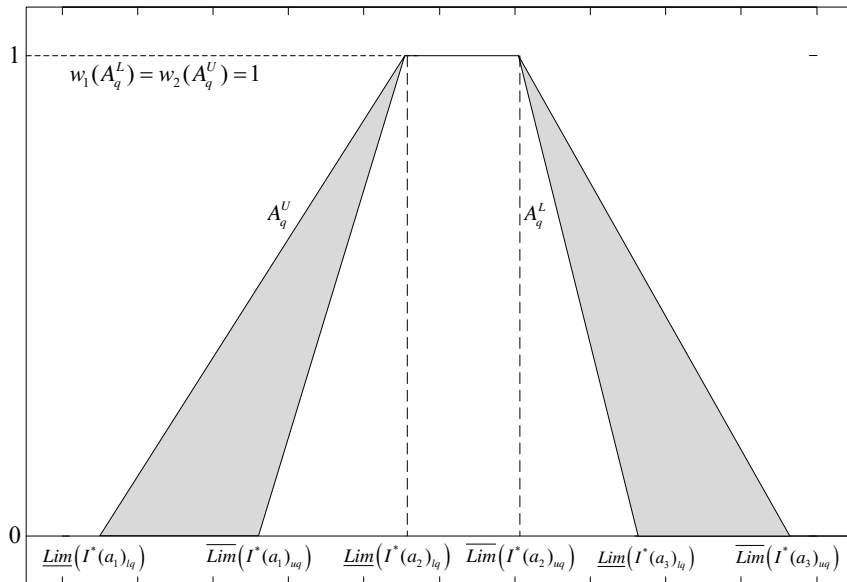


Figure 1: Interval-valued fuzzy-rough number $\overline{\overline{A}}$

If there is consensus among the decision makers (DMs) regarding assigning particular values from a linguistic fuzzy scale then $a_{1q}^L = a_{1q}^U, a_{2q}^L = a_{2q}^U$ i $a_{3q}^L = a_{3q}^U$. Then interval-valued fuzzy-rough number $\overline{\overline{A}}$ becomes fuzzy number A type-1.

Interval-valued fuzzy-rough number $\overline{\overline{A}}$ which is defined at interval $(-\infty, +\infty)$ can be represented using expressions (19) and (20)

$$\overline{\overline{A}} = \left\{ x, \left[\mu_{A_q^L}(x), \mu_{A_q^U}(x) \right] \right\},$$

$$x \in (-\infty, +\infty), \mu_{A_q^L}(x), \mu_{A_q^U}(x) : (-\infty, +\infty) \rightarrow [0, 1]. \tag{19}$$

$$\mu_{\overline{\overline{A}}}(x) = \left[\mu_{A_q^L}(x), \mu_{A_q^U}(x) \right], \mu_{A_q^L}(x) \leq \mu_{A_q^U}(x), \forall x \in (-\infty, +\infty). \tag{20}$$

where $\mu_{A_q^L}(x)$ and $\mu_{A_q^U}(x)$ represent the degree of membership to the upper and lower function of interval fuzzy-rough number $\overline{\overline{A}}$.

Based on the above we can define arithmetic operations between two interval-valued fuzzy-rough numbers $\overline{\overline{A}} = [(a_1^L, a_1^U), (a_2^L, a_2^U), (a_3^L, a_3^U)]$ and $\overline{\overline{B}} = [(b_1^L, b_1^U), (b_2^L, b_2^U), (b_3^L, b_3^U)]$ [26]:

(1) Adding interval-valued fuzzy-rough numbers ”+”

$$\overline{\overline{A}} + \overline{\overline{B}} = [(a_1^U, a_1^L), (a_2^L, a_2^U), (a_3^L, a_3^U)] + [(b_1^L, b_1^U), (b_2^L, b_2^U), (b_3^L, b_3^U)] =$$

$$= [(a_1^U + b_1^U, a_1^L + b_1^L), (a_2^L + b_2^L, a_2^U + b_2^U), (a_3^L + b_3^L, a_3^U + b_3^U)] \tag{21}$$

(2) Subtracting interval-valued fuzzy-rough numbers ”-”

$$\overline{\overline{A}} - \overline{\overline{B}} = [(a_1^L, a_1^U), (a_2^L, a_2^U), (a_3^L, a_3^U)] - [(b_1^L, b_1^U), (b_2^L, b_2^U), (b_3^L, b_3^U)] =$$

$$= [(a_1^L - b_3^L, a_1^U - b_3^U), (a_2^L - b_2^L, a_2^U - b_2^U), (a_3^L - b_1^L, a_3^U - b_1^U)] \tag{22}$$

(3) Multiplying interval-valued fuzzy-rough numbers ”×”

$$\overline{\overline{A}} \times \overline{\overline{B}} = [(a_1^U, a_1^L), (a_2^L, a_2^U), (a_3^L, a_3^U)] \times [(b_1^L, b_1^U), (b_2^L, b_2^U), (b_3^L, b_3^U)] =$$

$$= [(a_1^U \times b_1^U, a_1^L \times b_1^L), (a_2^L \times b_2^L, a_2^U \times b_2^U), (a_3^L \times b_3^L, a_3^U \times b_3^U)] \tag{23}$$

(4) Dividing interval-valued fuzzy-rough numbers ”÷”

$$\overline{\overline{A}} \div \overline{\overline{B}} = [(a_1^L, a_1^U), (a_2^L, a_2^U), (a_3^L, a_3^U)] \div [(b_1^L, b_1^U), (b_2^L, b_2^U), (b_3^L, b_3^U)] =$$

$$= [(a_1^L \div b_3^L, a_1^U \div b_3^U), (a_2^L \div b_2^L, a_2^U \div b_2^U), (a_3^L \div b_1^L, a_3^U \div b_1^U)] \tag{24}$$

If two interval-valued fuzzy-rough numbers $\overline{\overline{A}}$ and $\overline{\overline{B}}$ are given, they are represented as

$$\overline{\overline{A}} = [(a_1^L, a_1^U), (a_2^L, a_2^U), (a_3^L, a_3^U)] \text{ and } \overline{\overline{B}} = [(b_1^L, b_1^U), (b_2^L, b_2^U), (b_3^L, b_3^U)].$$

Let

$$h(\overline{\overline{A}}) = \frac{a_1^L + a_1^U + a_2^L + a_2^U + a_3^L + a_3^U}{6} \tag{25}$$

$$h(\overline{\overline{B}}) = \frac{b_1^L + b_1^U + b_2^L + b_2^U + b_3^L + b_3^U}{6} \tag{26}$$

Then we can say that $\overline{\overline{A}} > \overline{\overline{B}}$ if the condition is met that $h(\overline{\overline{A}}) > h(\overline{\overline{B}})$.

Example 1:

Based on the above section, we can very easily determine the upper and lower approximations of the IVFRN. The process of determining the IVFRN will be explained using the example of evaluating alternative A_i according to evaluation criterion C_j . The evaluation of the alternative was carried out by five experts. Triangular fuzzy numbers in the form $M=(l,m,u)$ were used for the evaluation, where m represents the value at which the membership function reaches its maximum, while l and u represent the left and right limits of the fuzzy set respectively. The fuzzy scale used to evaluate alternative A_i is represented with the values: Very little (VL) – (0,1,2); Little (L) – (1,2,3); Medium (M) – (2,3,4); Large (L) – (3,4,5) and Very large (VH) – (4,5,6). The expert evaluations are presented in Table 1.

Alternative	Experts				
	E1	E2	E3	E4	E5
A_i	(2,3,4)	(4,5,6)	(3,4,5)	(2,3,4)	(4,5,6)

Table 1: Expert evaluation of alternative A_i according to evaluation criterion C_j

The expert evaluations in Table 1 show that the experts do not have a united position on the value of this alternative according to the evaluation criterion.

In addition to the fuzzy approach, the uncertainties described can also be represented by IVFRN. By means of expressions (1)-(12), it is defined that IVFRNs consist of three rough sequences. So, based on the values from Table 1, we select three classes of objects l , m and u : $l = \{2; 4; 3; 2; 4\}$, $m = \{3; 5; 4; 3; 5\}$ and $u = \{4; 6; 5; 4; 6\}$. Using expressions (1)-(12) rough sequences are formed for each class of objects l , m and u . So for the class of objects l we determine the upper and lower approximation for each object:

$$\begin{aligned} \underline{Lim}(2) &= 2, \overline{Lim}(2) = \frac{1}{5}(2 + 4 + 3 + 2 + 4) = 3; \\ \underline{Lim}(3) &= \frac{1}{3}(2 + 3 + 2) = 2.33, \overline{Lim}(3) = \frac{1}{3}(4 + 3 + 4) = 3.67; \\ \underline{Lim}(4) &= \frac{1}{5}(2 + 4 + 3 + 2 + 4) = 3, \overline{Lim}(4) = 4. \end{aligned}$$

For the second class of objects m we obtain:

$$\begin{aligned} \underline{Lim}(3) &= 3, \overline{Lim}(3) = \frac{1}{5}(3 + 5 + 4 + 3 + 5) = 4; \\ \underline{Lim}(4) &= \frac{1}{3}(3 + 4 + 3) = 3.33, \overline{Lim}(4) = \frac{1}{3}(5 + 4 + 5) = 4.67; \\ \underline{Lim}(5) &= \frac{1}{5}(3 + 5 + 4 + 3 + 5) = 4, \overline{Lim}(5) = 5. \end{aligned}$$

In an identical way, we determine the upper and lower approximations for each object from the class of objects u :

$$\begin{aligned} \underline{Lim}(4) &= 4, \overline{Lim}(4) = \frac{1}{5}(4 + 6 + 5 + 4 + 6) = 5; \\ \underline{Lim}(5) &= \frac{1}{3}(4 + 5 + 4) = 4.33, \overline{Lim}(5) = \frac{1}{3}(6 + 5 + 6) = 5.67; \\ \underline{Lim}(6) &= \frac{1}{5}(4 + 6 + 5 + 4 + 6) = 5, \overline{Lim}(6) = 6. \end{aligned}$$

Since when defining the upper and lower approximations for $\overline{\overline{A}}(E3)$ condition (13) was not met, we use equations (14) and (15). In this way we obtain interval-valued fuzzy-rough numbers (17): $\overline{\overline{A}}(E1) = [(2, 3), (3, 4), (4, 5)]$, $\overline{\overline{A}}(E2) = [(3, 4), (4, 5), (5, 6)]$, $\overline{\overline{A}}(E3) = [(2.33, 3.33), (3.33, 4.33), (4.33, 5.67)]$, $\overline{\overline{A}}(E4) = [(2, 3), (3, 4), (4, 5)]$ and $IRN(E5) = [(3, 4), (4, 5), (5, 6)]$. A comparative presentation of the expert evaluation using crisp, fuzzy and IVFRN is presented in Table 2.

Presentation of the evaluation	Experts				
	E1	E1	E3	E4	E5
Crisp	3	5	4	3	5
Fuzzy	(2,3,4)	(4,5,6)	(3,4,5)	(2,3,4)	(4,5,6)
IVFRN	[(2,3),(3,4), (4,5)]	[(3,4),(4,5), (5,6)]	[(2.3,3.3), (3.3,4.3), (4.3,5.6)]	[(2,3),(3,4), (4,5)]	[(3,4),(4,5), (5,6)]

Table 2: Expert evaluation by means of crisp, fuzzy and interval-valued fuzzy-rough numbers

The uncertainties that exist in the decision making in the example from Table 1 are presented by means of the fuzzy concept, which includes the fuzzification of crisp values 1, 2, 3, 4 and 5. The traditional representation of the expert evaluations from tables 1 and 2 using crisp values includes averaging the expert evaluations. Most commonly, arithmetic averaging is used for the aggregation of expert opinions. By arithmetic averaging of the expert opinions from Table 1 we obtain the value of 4. The crisp expert evaluations are found between 3 and 5, so we can intuitively conclude that the "real perception" that the other approaches (fuzzy numbers and IVFRN) should have a value of 4. In Figure 2 the "real perception" is represented by an intermittent vertical line.

From Figure 2 we see that the "real perception" is found in the composition of the maximum values of all three IVFRN functions (Figures 2b, 2c and 2d). On the other hand, with fuzzy numbers (Figure 2a) the "real perception" belongs to only one of three fuzzy numbers (fuzzy number $H(3,4,5)$). The remaining two fuzzy numbers $M(2,3,4)$ and $VH(4,5,6)$ with their functions do not include the "real perception".

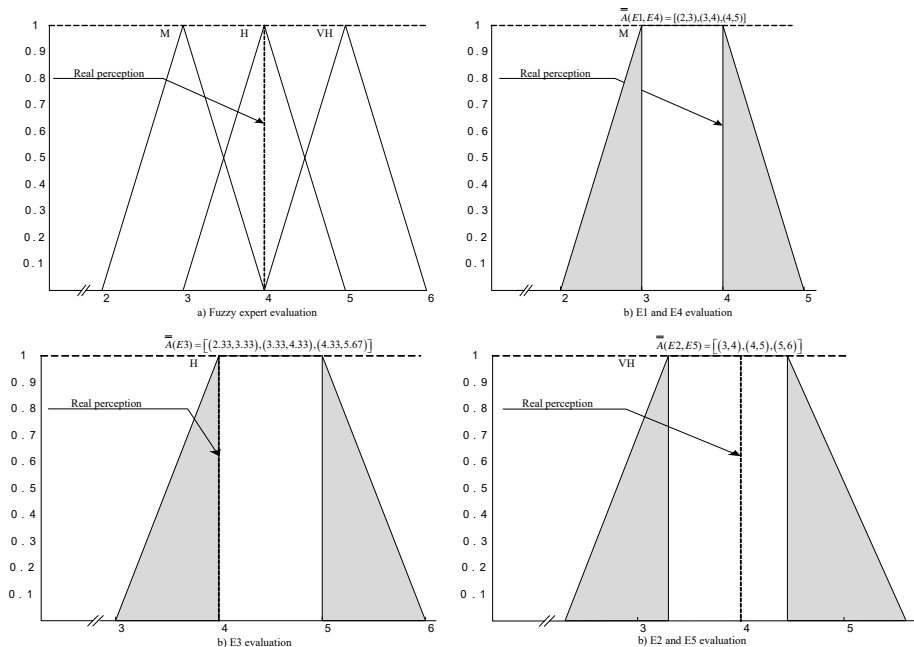


Figure 2: Expert evaluation – the crisp, fuzzy and IVFRN approach

Based on these analyses we can conclude that IVFRNs more accurately describe decision makers perception with fuzzy numbers because they are much closer to the "real perceptions".

3. INTERVAL-VALUED FUZZY-ROUGH NUMBERS MAIRCA MODEL

The IVFR approach was tested by means of an MCDM model in which an original modification of the MAIRCA method was used to evaluate the alternatives [28, 25] based on an IVFR approach. The authors chose to use the MAIRCA method because of these numerous advantages: (1) the mathematical framework of the method remains the same regardless of the number of alternatives and criteria; (2) it can be applied in cases in which there are a large number of alternatives and criteria; (3) it has a clearly defined ranking of the alternatives that is expressed in numerical values, which makes it possible to understand the results more easily; (4) it is applicable to both qualitative and quantitative types of criteria and (5) it provides stable solutions regardless of any changes in the measurement scale for qualitative criteria and changes in the method of formulating quantitative criteria [28]. The algorithm for the IVFR-MAIRCA method is presented in detail in the following section.

The MAIRCA method is one of the more recent methods of multi-criteria decision making (MCDM) [7]. The MAIRCA method was developed at the Centre

for Research in the field of defence logistics at the University of Defence in Belgrade [28]. To date, it has found wide application and modification with the purpose of solving numerous problems in the field of multi-criteria decision making [7, 9, 10, 21, 25, 26, 33]. The basic MAIRCA method defines the gap between the ideal and real parameters. The following section presents the algorithm for the modified IVFRN-MAIRCA method.

Step 1. Forming the initial decision matrix (Y). The first step is to evaluate l alternatives according to n criteria. The alternatives are evaluated based on a predefined fuzzy scale represented by triangular fuzzy numbers $\tilde{y} = (y^L, y^U, y^U)$. Based on the matrices of answers $Y_k = \begin{bmatrix} \%e \\ y_{ij} \end{bmatrix}_{l \times n}$ ($1 \leq e \leq m$), we obtain three matrices of aggregated sequences for experts Y^{*L} , Y^{*S} and Y^{*U} .

$$Y^{*L} = \begin{bmatrix} y_{11}^{1L}, y_{11}^{2L}, \dots, y_{11}^{mL} & y_{12}^{1L}; y_{12}^{2L}; \dots; y_{12}^{mL} & \dots & y_{1n}^{1L}; y_{1n}^{2L}, \dots, y_{1n}^{mL} \\ y_{21}^{1L}, y_{21}^{2L}, \dots, y_{21}^{mL} & y_{22}^{1L}; y_{22}^{2L}; \dots; y_{22}^{mL} & \dots & y_{2n}^{1L}; y_{2n}^{2L}, \dots, y_{2n}^{mL} \\ \dots & \dots & \ddots & \dots \\ y_{n1}^{1L}, y_{n1}^{2L}, \dots, y_{n1}^{mL} & y_{n2}^{1L}; y_{n2}^{2L}; \dots; y_{n2}^{mL} & \dots & y_{nn}^{1L}; y_{nn}^{2L}, \dots, y_{nn}^{mL} \end{bmatrix} \tag{27}$$

$$Y^{*S} = \begin{bmatrix} y_{11}^{1S}, y_{11}^{2S}, \dots, y_{11}^{mS} & y_{12}^{1S}; y_{12}^{2S}; \dots; y_{12}^{mS} & \dots & y_{1n}^{1S}; y_{1n}^{2S}, \dots, y_{1n}^{mS} \\ y_{21}^{1S}, y_{21}^{2S}, \dots, y_{21}^{mS} & y_{22}^{1S}; y_{22}^{2S}; \dots; y_{22}^{mS} & \dots & y_{2n}^{1S}; y_{2n}^{2S}, \dots, y_{2n}^{mS} \\ \dots & \dots & \ddots & \dots \\ y_{n1}^{1S}, y_{n1}^{2S}, \dots, y_{n1}^{mS} & y_{n2}^{1S}; y_{n2}^{2S}; \dots; y_{n2}^{mS} & \dots & y_{nn}^{1S}; y_{nn}^{2S}, \dots, y_{nn}^{mS} \end{bmatrix} \tag{28}$$

$$Y^{*U} = \begin{bmatrix} y_{11}^{1U}, y_{11}^{2U}, \dots, y_{11}^{mU} & y_{12}^{1U}; y_{12}^{2U}; \dots; y_{12}^{mU} & \dots & y_{1n}^{1U}; y_{1n}^{2U}, \dots, y_{1n}^{mU} \\ y_{21}^{1U}, y_{21}^{2U}, \dots, y_{21}^{mU} & y_{22}^{1U}; y_{22}^{2U}; \dots; y_{22}^{mU} & \dots & y_{2n}^{1U}; y_{2n}^{2U}, \dots, y_{2n}^{mU} \\ \dots & \dots & \ddots & \dots \\ y_{n1}^{1U}, y_{n1}^{2U}, \dots, y_{n1}^{mU} & y_{n2}^{1U}; y_{n2}^{2U}; \dots; y_{n2}^{mU} & \dots & y_{nn}^{1U}; y_{nn}^{2U}, \dots, y_{nn}^{mU} \end{bmatrix} \tag{29}$$

where $y_{ij}^L = \{y_{ij}^{1L}, y_{ij}^{2L}, \dots, y_{ij}^{mL}\}$, $y_{ij}^S = \{y_{ij}^{1S}, y_{ij}^{2S}, \dots, y_{ij}^{mS}\}$ and $y_{ij}^U = \{y_{ij}^{1U}, y_{ij}^{2U}, \dots, y_{ij}^{mU}\}$ represent sequences of triangular fuzzy number \tilde{y} which describe the relative significance of criterion i in relation to alternative j . Using expressions (1)-(13) each sequence y_{ij}^{mL} , y_{ij}^{mS} and y_{ij}^{mU} is transformed into a rough sequence $RN(y_{ij}^{mL})$, $RN(y_{ij}^{mS})$ and $RN(y_{ij}^{mU})$. Thus we obtain rough matrices $Y^{1L}, Y^{2L}, \dots, Y^{mL}$; $Y^{1S}, Y^{2S}, \dots, Y^{mS}$; $Y^{1U}, Y^{2U}, \dots, Y^{mU}$ for each rough sequence $RN(y_{ij}^{mL})$, $RN(y_{ij}^{mS})$ and $RN(y_{ij}^{mU})$ respectively. For each group of rough matrices obtained we get rough sequences

$$RN(y_{ij}^L) = \{[\underline{Lim}(y_{ij}^{1L}), \overline{Lim}(y_{ij}^{1L})], [\underline{Lim}(y_{ij}^{2L}), \overline{Lim}(y_{ij}^{2L})], \dots, [\underline{Lim}(y_{ij}^{mL}), \overline{Lim}(y_{ij}^{mL})]\},$$

$$\begin{aligned}
 RN(y_{ij}^S) &= \\
 &= \{ [\underline{Lim}(y_{ij}^{1S}), \overline{Lim}(y_{ij}^{1S})], [\underline{Lim}(y_{ij}^{2S}), \overline{Lim}(y_{ij}^{2S})], \dots, [\underline{Lim}(y_{ij}^{mS}), \overline{Lim}(y_{ij}^{mS})] \}, \\
 RN(y_{ij}^U) &= \\
 &= \{ [\underline{Lim}(y_{ij}^{1U}), \overline{Lim}(y_{ij}^{1U})], [\underline{Lim}(y_{ij}^{2U}), \overline{Lim}(y_{ij}^{2U})], \dots, [\underline{Lim}(y_{ij}^{mU}), \overline{Lim}(y_{ij}^{mU})] \}.
 \end{aligned}$$

Using equations (30)-(32) we obtain the averaged rough sequences

$$RN(y_{ij}^L) = RN(y_{ij}^{1L}, y_{ij}^{2L}, \dots, y_{ij}^{mL}) = \begin{cases} \underline{Lim}(y_{ij}^L) = \frac{1}{m} \sum_{e=1}^m \underline{Lim}(y_{ij}^{eL}) \\ \overline{Lim}(y_{ij}^L) = \frac{1}{m} \sum_{e=1}^m \overline{Lim}(y_{ij}^{eL}) \end{cases} \quad (30)$$

$$RN(y_{ij}^S) = RN(y_{ij}^{1S}, y_{ij}^{2S}, \dots, y_{ij}^{mS}) = \begin{cases} \underline{Lim}(y_{ij}^S) = \frac{1}{m} \sum_{e=1}^m \underline{Lim}(y_{ij}^{eS}) \\ \overline{Lim}(y_{ij}^S) = \frac{1}{m} \sum_{e=1}^m \overline{Lim}(y_{ij}^{eS}) \end{cases} \quad (31)$$

$$RN(y_{ij}^U) = RN(y_{ij}^{1U}, y_{ij}^{2U}, \dots, y_{ij}^{mU}) = \begin{cases} \underline{Lim}(y_{ij}^U) = \frac{1}{m} \sum_{e=1}^m \underline{Lim}(y_{ij}^{eU}) \\ \overline{Lim}(y_{ij}^U) = \frac{1}{m} \sum_{e=1}^m \overline{Lim}(y_{ij}^{eU}) \end{cases} \quad (32)$$

where $RN(y_{ij}^L)$, $RN(y_{ij}^S)$ and $RN(y_{ij}^U)$ represent rough sequences of interval-valued fuzzy-rough number

$$\overline{y}_{ij} = [RN(y_{ij}^L), RN(y_{ij}^S), RN(y_{ij}^U)] = [(l_{y_{ij}}^L, l_{y_{ij}}^U), (s_{y_{ij}}^L, s_{y_{ij}}^U), (u_{y_{ij}}^L, u_{y_{ij}}^U)].$$

Thus we obtain interval valued fuzzy-rough vectors $A_i = (\overline{y}_{i1}, \overline{y}_{i2}, \dots, \overline{y}_{in})$ of the averaged initial decision matrix, where $\overline{y}_{ij} = [(l_{y_{ij}}^L, l_{y_{ij}}^U), (s_{y_{ij}}^L, s_{y_{ij}}^U), (u_{y_{ij}}^L, u_{y_{ij}}^U)]$ represents the value of the i -th alternative according to the j -th criterion ($i = 1, 2, \dots, l; j = 1, 2, \dots, n$).

$$Y = \begin{matrix} A_1 \\ A_2 \\ \dots \\ A_l \end{matrix} \begin{bmatrix} C_1 & C_2 & \dots & C_n \\ \overline{y}_{11} & \overline{y}_{12} & \dots & \overline{y}_{1n} \\ \overline{y}_{21} & \overline{y}_{22} & \dots & \overline{y}_{2n} \\ \dots & \dots & \ddots & \dots \\ \overline{y}_{l1} & \overline{y}_{l2} & \dots & \overline{y}_{ln} \end{bmatrix}_{l \times n} \quad (33)$$

where l denotes the number of alternatives, n denotes the total number of criteria.

Step 2. Determining the preferences according to the selection of alternative P_{A_i} . When selecting an alternative the decision maker (DM) is neutral towards

selecting an alternative, i.e., he or she has no preference towards any of the alternatives. This makes it possible for each alternative to be considered with equal probability, and so the preference towards selecting one of l possible alternatives is

$$P_{A_i} = \frac{1}{l}; \sum_{i=1}^l P_{A_i} = 1, i = 1, 2, \dots, l \tag{34}$$

where l represents the total number of alternatives that the choice is made from.

Step 3. Normalizing the elements of the initial matrix (33). By normalizing the elements of the initial decision matrix we obtain normalized matrix Z

$$Z = \begin{matrix} A_1 \\ A_2 \\ \dots \\ A_l \end{matrix} \begin{bmatrix} C_1 & C_2 & \dots & C_n \\ \bar{z}_{11} & \bar{z}_{12} & \dots & \bar{z}_{1n} \\ \bar{z}_{21} & \bar{z}_{22} & \dots & \bar{z}_{2n} \\ \dots & \dots & \ddots & \dots \\ \bar{z}_{l1} & \bar{z}_{l2} & \dots & \bar{z}_{ln} \end{bmatrix}_{l \times n} \tag{35}$$

The elements \bar{z}_{ij} of the normalized matrix (Z) are determined using expressions (36) and (37):

a) For “benefit” type criteria (higher values of the criteria are desirable)

$$\bar{z}_{ij} = \left\{ \left[\left(\frac{l_{ij}^L - l_{i-}^L}{u_{i+}^U - l_{i-}^L}, \frac{l_{ij}^U - l_{i-}^L}{u_{i+}^U - l_{i-}^L} \right), \left(\frac{s_{ij}^L - l_{i-}^L}{u_{i+}^U - l_{i-}^L}, \frac{s_{ij}^U - l_{i-}^L}{u_{i+}^U - l_{i-}^L} \right), \left(\frac{u_{ij}^L - l_{i-}^L}{u_{i+}^U - l_{i-}^L}, \frac{u_{ij}^U - l_{i-}^L}{u_{i+}^U - l_{i-}^L} \right) \right] \right\} \tag{36}$$

b) For “cost” type criteria (lower values of the criteria are desirable)

$$\bar{z}_{ij} = \left[\left(\frac{u_{ij}^U - u_{i-}^U}{l_{i-}^L - u_{i+}^U}, \frac{u_{ij}^L - u_{i-}^U}{l_{i-}^L - u_{i+}^U} \right), \left(\frac{s_{ij}^U - u_{i-}^U}{l_{i-}^L - u_{i+}^U}, \frac{s_{ij}^L - u_{i-}^U}{l_{i-}^L - u_{i+}^U} \right), \left(\frac{l_{ij}^U - u_{i-}^U}{l_{i-}^L - u_{i+}^U}, \frac{l_{ij}^L - u_{i-}^U}{l_{i-}^L - u_{i+}^U} \right) \right] \tag{37}$$

where u_{j+}^U and l_{j-}^L are defined as $u_{j+}^U = \max_{1 \leq i \leq n} (u_{ij}^U)$ and $l_{j-}^L = \min_{1 \leq i \leq n} (l_{ij}^L)$.

Step 4. Calculating the elements of the theoretical assessment matrix (T_p). A theoretical assessment matrix (T_p) is formed with a format of $l \times n$ (l represents the total number of alternatives, n represents the total number of criteria). The elements of the theoretical assessment matrix (\bar{t}_{pij}) are IVFRNs and they are calculated as the product of the preferences in the selection of alternatives P_{A_i} and weight coefficients of the criteria (\bar{w}_j , $i = 1, 2, \dots, n$).

$$T_p = \begin{matrix} P_{A_1} \\ P_{A_2} \\ \dots \\ P_{A_l} \end{matrix} \begin{bmatrix} \bar{w}_1 & \bar{w}_2 & \dots & \bar{w}_n \\ \bar{t}_{p11} & \bar{t}_{p12} & \dots & \bar{t}_{p1n} \\ \bar{t}_{p21} & \bar{t}_{p22} & \dots & \bar{t}_{p2n} \\ \dots & \dots & \dots & \dots \\ \bar{t}_{pl1} & \bar{t}_{pl2} & \dots & \bar{t}_{pln} \end{bmatrix}_{l \times n} \tag{38}$$

where P_{A_i} represents the preferences in the selection of alternatives, \bar{w}_j the weight coefficients of the evaluation criteria, and \bar{t}_{pij} the theoretical assessment of an alternative for the observed evaluation criterion. The elements of matrix T_p are determined using expression (39)

$$\bar{t}_{pij} = P_{A_i} \cdot \bar{w}_j = P_{A_i} \cdot [(w_{lj}^L, w_{lj}^U), (w_{sj}^L, w_{sj}^U), (w_{uj}^L, w_{uj}^U)] \tag{39}$$

Since the DM is neutral in relation to the initial selection of an alternative, then the preferences (P_{A_i}) are the same for all alternatives. Since the preferences (P_{A_i}) are the same for all alternatives, then the matrix (31) can be displayed in the format $1 \times n$ (n represents the total number of criteria).

$$T_p = P_{A_i} [[(\bar{w}_1; (t_{pl1}^L, t_{pl1}^U), (t_{ps1}^L, t_{ps1}^U), (t_{pu1}^L, t_{pu1}^U)) \dots ((t_{pln}^L, t_{pln}^U), (t_{psn}^L, t_{psn}^U), (t_{pun}^L, t_{pun}^U))]_{1 \times n} \tag{40}$$

where n represents the total number of criteria, P_{A_i} the preferences in the selection of the alternatives, \bar{w}_j the weight coefficients of the criteria.

Step 5. Determining the elements of the actual assessment matrix (T_r). The elements of the actual assessment matrix (T_r) are calculated by multiplying the elements of the theoretical assessment matrix (T_p) and the elements of the normalized matrix (Z) according to the expression:

$$\begin{aligned} \bar{t}_{rij} &= \bar{t}_{pij} \cdot \bar{z}_{ij} = \\ &= [(t_{plij}^L, t_{plij}^U), (t_{psij}^L, t_{psij}^U), (t_{puij}^L, t_{puij}^U)] \cdot [(z_{lij}^L, z_{lij}^U), (z_{sij}^L, z_{sij}^U), (z_{uij}^L, z_{uij}^U)] \end{aligned} \tag{41}$$

where \bar{t}_{pij} represents the elements of the theoretical assessment matrix, and \bar{z}_{ij} represents the elements of normalized matrix $Z = [\bar{z}_{ij}]_{l \times n}$.

Step 6. Calculating the total gap matrix (G). The elements of matrix G are obtained as the difference (gap) between the theoretical (\bar{t}_{pij}) and actual assessment (\bar{t}_{rij}), that is by subtracting the elements of the theoretical weight matrix (T_p) and the elements of the actual weight matrix (T_r)

$$G = T_p - T_r = \begin{bmatrix} \bar{g}_{11} & \bar{g}_{12} & \dots & \bar{g}_{1n} \\ \bar{g}_{21} & \bar{g}_{22} & \dots & \bar{g}_{2n} \\ \dots & \dots & \dots & \dots \\ \bar{g}_{l1} & \bar{g}_{l2} & \dots & \bar{g}_{ln} \end{bmatrix}_{l \times n} \tag{42}$$

where n represents the total number of criteria, l represents the total number of alternatives, wherefrom the selection is made, and \bar{g}_{ij} represents the gap obtained for alternative i according to criterion j . Gap \bar{g}_{ij} represents the IVFRN and is obtained using expression (43)

$$\begin{aligned} \bar{g}_{ij} &= \bar{t}_{pij} - \bar{t}_{rij} = \\ &= \left[(t_{pl_{ij}}^L, t_{pl_{ij}}^U), (t_{ps_{ij}}^L, t_{ps_{ij}}^U), (t_{pu_{ij}}^L, t_{pu_{ij}}^U) \right] - \left[(t_{rl_{ij}}^L, t_{rl_{ij}}^U), (t_{rs_{ij}}^L, t_{rs_{ij}}^U), (t_{ru_{ij}}^L, t_{ru_{ij}}^U) \right] \end{aligned} \tag{43}$$

It is desirable that the value of \bar{g}_{ij} inclines towards zero ($\bar{g}_{ij} \rightarrow 0$) since we choose an alternative that has the smallest difference between the theoretical (\bar{t}_{pij}) and actual assessment (\bar{t}_{rij}). If alternative A_i for criterion C_i has a theoretical assessment value which is equal to the value of the actual assessment ($\bar{t}_{pij} = \bar{t}_{rij}$) then the gap for alternative A_i according to criterion C_i inclines towards zero. That is, alternative A_i according to criterion C_i is the best (ideal) alternative.

If alternative A_i for criterion C_i has a theoretical assessment value of \bar{t}_{pij} and an actual assessment value that inclines towards zero, then the gap for alternative A_i according to criterion C_i is $\bar{g}_{ij} \approx \bar{t}_{pij}$. This means that alternative A_i according to criterion C_i is the worst (anti-ideal) alternative.

Step 7. Calculating the values of the criterion functions (Q_i) for the alternatives. The values of the criterion functions are obtained by summing the gap from the matrix (42) for each alternative according to the evaluation criteria, expression (44)

$$\bar{Q}_i = \sum_{j=1}^n \bar{g}_{ij}, \quad i = 1, 2, \dots, l \tag{44}$$

The alternatives are ranked based on the value obtained for Q_i . It is desirable for an alternative to have the value as low as possible for Q_i . Comparison of the alternatives is carried out using expressions (25) and (26).

4. APPLICATION OF THE IVFRN-MAIRCA MODEL: SELECTING THE OPTIMUM PLACE FOR OVERCOMING WATER OBSTACLES

During combat operations, Serbian Army units are used in various operations. In situations when there is one or more water obstacles in the zone of operation [30], and when it is impossible or inappropriate to evade them, the water obstacles need to be negotiated. This involves action, which in the course of combat operations, ensures transition of individuals and units through natural and artificial water obstacles, in order to accomplish the given task [19]. Water obstacles are overcome by crossing or by forced crossing [18]. They are crossed when the firepower of the enemy is eliminated on existing bridges or other types of crossing [18], that is, when the opposite shore is not being defended by enemy forces [30]. A forced crossing of the water obstacle means that the opposite shore is being defended by the enemy [30], which is more characteristic for an offensive peration. Crossing water obstacles by force begins at a landing operations point (LOP). When a sufficiently deep bridgehead is established, access is gained to establish a raft or pontoon crossing point.

A landing operations point is "part of a river, coast and coastline on its own or the opposite shore which should be used in order to ensure passage across the water obstacle by means of a landing operation" [30]. A forced river crossing at an LOP is a key segment of successfully overcoming water obstacles in an offensive operation. If there is no other alternative, then failure at a forced river crossing means the failure of the entire operation. Therefore, planning and organizing activities at an LOP require special attention, whereby the possibility of any error must be excluded [4]. In this sense, the choice of the place where the LOP will be organized is one of the most important elements of the success of an entire operation.

Six criteria are key to the selection of an LOP [4]: Relative combat power (C1), Combat capabilities for preparing for a forced crossing under fire (C2), Secrecy of the preparations (C3), Level of exposure to enemy fire (C4), Characteristics of the opposite shore (C5) and the characteristics of the present shore (C6). Criteria C1, C2, C3, C5 and C6 belong to the group of benefit criteria (*max* type of criteria), and C4 belongs to a group of cost criteria (*min* type of criteria). Evaluation of the criteria, that is, determining the weight values of the criteria was carried out by four experts. The criteria are evaluated using the fuzzy scale shown in Table 3 [26].

No.	Linguistic terms	Triangular fuzzy numbers
1.	Very poor (VP)	(1,1,1)
2.	Poor (P)	(2/3,1,3/2)
3.	Medium (M)	(3/2,2,5/2)
4.	Good (G)	(5/2,3,7/2)
5.	Very good (VG)	(7/2,4,9/2)

Table 3: Fuzzy scale for evaluating the alternatives

By analyzing the expert assessments, the weight coefficients of the criteria are obtained using expressions (1)-(24), Table 4.

Linguistic terms	IVFRN weight coefficient	Rank
C1	[(0.240,0.267),(0.303,0.316),(0.362,0.383)]	1
C2	[(0.122,0.273),(0.293,0.244),(0.327,0.333)]	3
C3	[(0.063,0.079),(0.090,0.101),(0.103,0.110)]	6
C4	[(0.197,0.208),(0.219,0.247),(0.341,0.386)]	2
C5	[(0.065,0.078),(0.089,0.098),(0.126,0.149)]	5
C6	[(0.117,0.157),(0.182,0.133),(0.197,0.114)]	4

Table 4: IVFRN weight coefficients of the evaluation criteria

After calculating the weight coefficients of the criteria, expert evaluation of the alternatives was carried out (landing operations crossing points for units of the Serbian Army) according to the predefined evaluation criteria. The IVFRN-MAIRCA model was used to evaluate four places for landing operations points in the South Morava Valley between Bujanovac and Vladičin Han: Vranjska Banja (A1), Korbevac (A2), Toplac (A3), Gramada (A4), Bujkovac (A5) and Vrbovo (A6). The expert assessment of the alternatives according to the evaluation criteria is shown in Table 5.

Alter.	C1	C2	C3	C4	C5	C6
A1	F	VG	G	VP	F	F
	F	G	G	P	F	F
	G	G	F	P	P	F
	G	VG	F	F	G	G
A2	F	G	F	G	F	F
	P	G	F	G	F	F
	F	F	G	F	G	F
	G	F	G	G	F	P
A3	G	F	G	F	G	G
	F	G	F	F	G	VG
	F	G	VG	G	VG	F
	F	F	G	G	VG	VG
A4	G	F	VG	P	G	F
	G	F	G	F	G	F
	VG	P	F	P	F	G
	G	F	F	P	G	G
A5	VG	P	VG	F	VG	G
	VG	P	G	F	G	G
	G	VP	VG	G	G	G
	G	P	G	G	G	F
A6	F	VG	VG	G	G	P
	P	G	VG	G	F	P
	P	G	VG	G	F	F
	F	F	G	F	F	F

Table 5: Expert assessment of the alternatives

Expert evaluation of the alternatives was carried out using a fuzzy scale, Table 3.

From Table 5 we can see that there is no complete agreement of the experts in the evaluation of the alternatives according to the criteria. In order to provide a more complete presentation of the imprecision in the expert assessments, the fuzzy evaluations from Table 5 using expressions (1)-(15) were converted into IVFRNs. Finally, in order to apply the IVFR-MAIRCA multi-criteria model for evaluating the alternatives, aggregation of the values was carried out using equations (30)-(32) and an initial decision matrix was obtained (Table 6).

Alt./ Crit.	C1	C2	...	C6
A1	[(2.75,3.25),(3.25,3.75),(3.75,4.25)]	[(3.75,4.25),...]		[(2.56,2.94),(3.06,3.44),(3.56,3.94)]
A2	[(2.08,2.92),(2.58,3.42),(3.08,3.92)]	[(2.75,3.25),...]		[(2.06,2.44),(2.56,2.94),(3.06,3.44)]
A3	[(2.56,2.94),(3.06,3.44),(3.56,3.94)]	[(2.75,3.25),...]		[(3.25,4.23),(3.75,4.73),(4.13,4.83)]
A4	[(3.56,3.94),(4.06,4.44),(4.53,4.72)]	[(2.06,2.44),...]	...	[(2.75,3.25),(3.25,3.75),(3.75,4.25)]
A5	[(3.75,4.25),(4.25,4.75),(4.63,4.87)]	[(1.28,1.47),...]		[(3.06,3.44),(3.56,3.94),(4.06,4.44)]
A6	[(1.75,2.25),(2.25,2.75),(2.75,3.25)]	[(3.08,3.92),...]		[(1.75,2.25),(2.25,2.75),(2.75,3.25)]

Table 6: Initial decision matrix

The IVFRN from Table 5 were aggregated using expressions (30)-(32). After obtaining the initial decision matrix, using expressions (36) and (37) the elements \bar{z}_{ij} of the normalized matrix (Z) were obtained, Table 7.

Alt./ Crit.	C1	C2	...	C6
A1	[(0.32,0.48),(0.48,0.64),(0.64,0.80)]	[(0.68,0.83),...]		[(0.26,0.38),(0.42,0.54),(0.58,0.70)]
A2	[(0.11,0.27),(0.27,0.53),(0.53,0.69)]	[(0.41,0.54),...]		[(0.10,0.22),(0.26,0.38),(0.42,0.54)]
A3	[(0.26,0.38),(0.42,0.54),(0.58,0.70)]	[(0.41,0.54),...]		[(0.48,0.64),(0.64,0.96),(0.96,1.00)]
A4	[(0.58,0.70),(0.74,0.86),(0.89,0.95)]	[(0.21,0.32),...]	...	[(0.32,0.48),(0.48,0.64),(0.64,0.81)]
A5	[(0.64,0.80),(0.80,0.96),(0.96,1.00)]	[(0.00,0.05),...]		[(0.42,0.54),(0.58,0.71),(0.75,0.87)]
A6	[(0.00,0.16),(0.16,0.32),(0.32,0.48)]	[(0.50,0.64),...]		[(0.00,0.16),(0.16,0.32),(0.32,0.48)]

Table 7: Normalized matrix

In the next steps, using expressions (38)-(41), the elements of the theoretical and actual assessment were calculated. Finally, using expressions (42) and (43) the gap between the theoretical and actual assessments was determined. In the final step (step 7), by applying expression (44) the final values of the gap were obtained for each alternative, Table 8.

Alternative	Q	Rang
A1	$[(0.122,0.150),(0.155,0.203),(0.211,0.260)]$	5
A2	$[(0.117,0.144),(0.150,0.194),(0.203,0.254)]$	3
A3	$[(0.111,0.137),(0.141,0.191),(0.197,0.245)]$	2
A4	$[(0.124,0.152),(0.157,0.206),(0.213,0.260)]$	6
A5	$[(0.111,0.135),(0.142,0.186),(0.194,0.238)]$	1
A6	$[(0.115,0.140),(0.146,0.191),(0.198,0.242)]$	4

Table 8: Ranking of the alternatives using the IVFR-MAIRCA model

It is desirable for an alternative to have between the theoretical and the actual assessment as small as possible, that is, for as many criteria as possible to be close to the ideal alternative. The alternatives were ranked using expressions for comparing IVFRNs, i.e., using expressions (25) and (26).

5. DISCUSSION OF THE RESULTS

The discussion of the results has two parts. In the first part, the results of the IVFRN-MAIRCA model are compared with the results given by other MCDM models: TOPSIS, MABAC, and VIKOR. These methods were selected because their application so far has shown that they give stable and reliable results [16, 17, 20]. The TOPSIS, MABAC and VIKOR methods were modified using fuzzy, rough and interval-valued fuzzy-rough techniques. The second part is a sensitivity analysis of the IVFRN-MAIRCA model through 24 scenarios. A more detailed analysis of the first and second parts of the discussion of the results is presented in the next section.

The ranking of the alternatives obtained by the IVFRN-MAIRCA model was compared with the ranking of the other MCDM techniques mentioned above. A comparative ranking of the different MCDM techniques is shown in Figure 3.

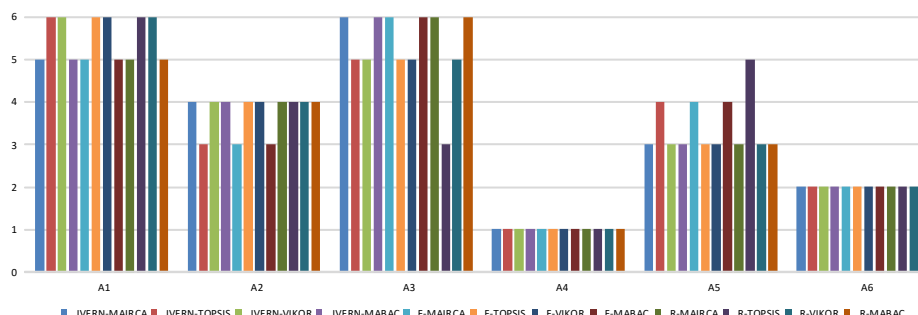


Figure 3: Ranks of the alternatives

The ranking of the alternatives by the methods presented shows that alternative A4 was ranked as the first by all of the methods. Alternative A6, which according to the IVFRN-MAIRCA method was ranked as the second, retained second position in all of the MCDM models shown. The main question to be answered before making the final decision is the assessment of the reliability of the results obtained when compared with other MCDM techniques. Spearman’s Rank correlation coefficient is a useful and important measure for determining the connection between the results obtained by different approaches [27, 32, 33, 11, 8]. In addition, it is a suitable coefficient when there are ordinal variables or ranked variables. In this paper, Spearman’s coefficient (SCC) was used to determine the statistical significance of the difference between the ranks obtained by the IVFRN-MAIRCA model and other approaches. The results show that there is a great correlation between the rankings of the MCDM methods compared. Based on the recommendations of [27], all *SCC* values greater than 0.8 show extremely high correlation. Since in this paper all *SCC* values are significantly greater than 0.8, and the mean value is 0.964, we can conclude that there is great correlation (closeness) between the proposed approach and other MCDM techniques tested, i.e., that the proposed ranking is confirmed and credible.

The results of MCDM methods depend to a great extent on the values of the weight coefficients of the evaluation criteria. Sometimes the ranks of the alternatives change with very small changes in the weight coefficients, because of which the results of MCDM methods, as by rule are followed by an analysis of their sensitivity to these changes. Therefore, this section of the paper shows an analysis of the sensitivity of the ranks of the alternatives to changes in the weight coefficients of the criteria. The sensitivity analysis is presented through 24 scenarios (Table 9) divided into three phases.

Phase	Scenario	Weights of criteria
Phase I	S1	$w_{c1} = 1.10 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, w_{c3} = 0.45 \times w_{c3(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S2	$w_{c2} = 1.25 \times w_{c2(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c3} = 0.45 \times w_{c3(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S3	$w_{c3} = 1.25 \times w_{c3(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S4	$w_{c4} = 1.25 \times w_{c4(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S5	$w_{c5} = 1.25 \times w_{c5(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S6	$w_{c6} = 1.25 \times w_{c6(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c5} = 0.45 \times w_{c5(oid)}$
Phase II	S7	$w_{c1} = 1.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, w_{c3} = 0.45 \times w_{c3(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S8	$w_{c2} = 1.45 \times w_{c2(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c3} = 0.45 \times w_{c3(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S9	$w_{c3} = 1.45 \times w_{c3(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S10	$w_{c4} = 1.45 \times w_{c4(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S11	$w_{c5} = 1.45 \times w_{c5(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S12	$w_{c6} = 1.45 \times w_{c6(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c5} = 0.45 \times w_{c5(oid)}$
Phase III	S13	$w_{c1} = 1.65 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, w_{c3} = 0.45 \times w_{c3(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S14	$w_{c2} = 1.65 \times w_{c2(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c3} = 0.45 \times w_{c3(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S15	$w_{c3} = 1.65 \times w_{c3(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S16	$w_{c4} = 1.65 \times w_{c4(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S17	$w_{c5} = 1.65 \times w_{c5(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S18	$w_{c6} = 1.65 \times w_{c6(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c5} = 0.45 \times w_{c5(oid)}$
Phase IV	S19	$w_{c1} = 1.85 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, w_{c3} = 0.45 \times w_{c3(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S20	$w_{c2} = 1.85 \times w_{c2(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c3} = 0.45 \times w_{c3(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S21	$w_{c3} = 1.85 \times w_{c3(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S22	$w_{c4} = 1.85 \times w_{c4(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S23	$w_{c5} = 1.85 \times w_{c5(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c6} = 0.45 \times w_{c6(oid)}$
	S24	$w_{c6} = 1.85 \times w_{c6(oid)}, w_{c1} = 0.45 \times w_{c1(oid)}, w_{c2} = 0.45 \times w_{c2(oid)}, \dots, w_{c5} = 0.45 \times w_{c5(oid)}$

Table 9: Scenarios for the sensitivity analysis

Based on the recommendations of Kirkwood [15] and Kahraman [12], the values were defined for the change in the weight coefficients of the criteria through the scenarios (i.e. 1.25, 1.45, 1.65 and 1.85). The weight coefficient of elasticity was defined, which expresses the relative compensation of the other values of the weight coefficients in relation to the given changes in the weight of the most important criteria in the scenario. Parameter Δx was defined, which represents the size of the change applied to the set of weight coefficients, depending on their coefficients of weight elasticity. This interval was divided into four values of 1.25, 1.45, 1.65 and 1.85. In their research, Kirkwood [15] and Kahraman [12] recommend only an impact analysis of the most important criterion. In order to comprehensively validate the results, the authors carried out an analysis of the impact of changing the weight of all the criteria in the defined range. In each of the four phases of the sensitivity analysis the weight coefficients of the criteria were increased by 1.25, 1.45, 1.65 and 1.85 respectively. At the same time, the weight coefficients of the remaining criteria were reduced by 0.45.

During the sensitivity analysis of the IVFRN-MAIRCA model to changes in the weight coefficients for all values of the interval rough weight coefficients of the criteria $\bar{w}_j = [(l_j^{wL}, l_j^{wU}), (s_j^{wL}, s_j^{wU}), (u_j^{wL}, u_j^{wU})]$ the condition $0 \leq l_j^{wL} \leq l_j^{wU} \leq s_j^{wL} \leq s_j^{wU} \leq u_j^{wL} \leq u_j^{wU} \leq 1$ was fulfilled for each evaluation criterion $c_j \in C$. Since these are interval numbers, weight coefficient w_j belongs to interval $[s_j^{wL}, s_j^{wU}]$, that is $l_j^{wL} \leq l_j^{wU} \leq s_j^{wL} \leq w_j \leq s_j^{wU} \leq u_j^{wL} \leq u_j^{wU}$ for each value $j = 1, 2, \dots, n$. Based on the previously defined, during the sensitivity analysis of the model to change in the weight coefficients of the criteria, the conditions that $\sum_{j=1}^n l_j^{wL}, \sum_{j=1}^n l_j^{wU}, \sum_{j=1}^n s_j^{wL} \leq 1$ and $\sum_{j=1}^n s_j^{wU}, \sum_{j=1}^n u_j^{wL}, \sum_{j=1}^n u_j^{wU} \geq 1$ were fulfilled. After reducing the values of the weight coefficients through the scenarios, the values were corrected to satisfy the previously presented conditions.

Changes in the ranking of the alternatives during the 24 scenarios are shown

in Figure 4.

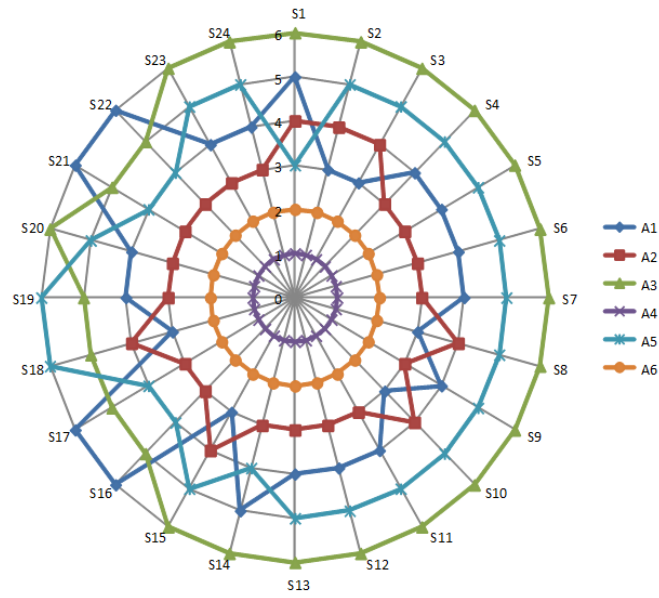


Figure 4: Sensitivity analysis of the ranking of the alternatives through 24 scenarios

The results (Figure 4) show that assigning different weights to the criteria through the scenarios leads to a change in the ranks of individual alternatives, which confirms that the model is sensitive to changes in the weight coefficients. By comparing the first-ranked alternatives (A4 and A6) in scenarios 1-24 with the results shown in Table 8 the ranking of alternatives A4 and A6 was confirmed. By analyzing the ranks through 24 scenarios we can see that alternatives A4 and A6 retain their ranks in all scenarios (they remained as the first-or second-ranked alternatives). During changes in the weights of the criteria through the scenarios there was a change in the ranks of the remaining alternatives. However, we can conclude that these changes were not drastic, which was confirmed by the correlation of the ranks through the scenarios, Figure 5.

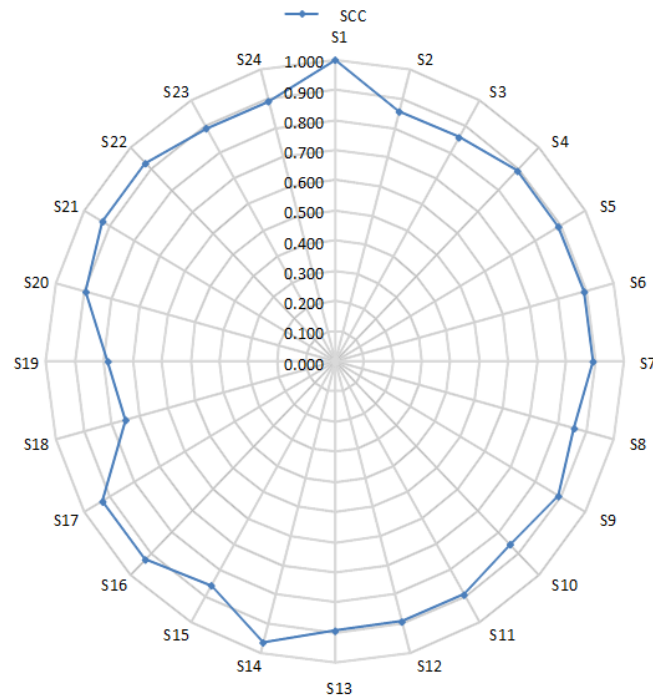


Figure 5: Correlation of the ranks through 24 scenarios

By analyzing the correlation of ranks through the scenarios a mean SCC value was obtained in all scenarios of 0.897, which shows an extremely high correlation. Since all *SCC* values are significantly higher than 0.850 we can conclude that there is a very high correlation of the ranks and that that the proposed ranking is confirmed and credible.

6. CONCLUSION

The development and application of tools that take uncertainty into account adequately is a significant area of MCDM. The decision makers are asked to objectively and impartially make decisions taking into account uncertainty and imprecision. Therefore, the use of these tools is a prerequisite for objective decision making. This paper presents a new model for the treatment of uncertainty based on the application of IVFRN. The benefits of using IVFRN are numerous. IVFRN exclusively uses internal knowledge for presenting the limit values of the attributes of a decision. This eliminates subjectivity and assumptions when defining the limit values of traditional fuzzy sets, which can affect the attribute values and the final selection of alternatives.

The IVFRN approach, which is presented in this paper, includes the definition of an initial fuzzy reference set that describes the uncertainties in MCDM. After defining the initial fuzzy set, indeterminacies in the assessment of the decision maker (DM) are measured by means of rough sets. This leads to objective indicators that are contained in the data. The basic logic of IVFRN is that the data should speak for themselves. IVFRNs use only the internal values from the data set for presenting the limit values of the attributes of a decision. This eliminates the shortcomings of the traditional fuzzy approach that relate to the interval boundaries, since for each rating by the decision maker (DM), unique interval boundaries are formed. This means that the boundaries of the intervals do not depend on subjective assessment but rather are defined based on imprecision in the data. In the case of less imprecision, IVFRN are transformed into type-1 fuzzy sets, while with greater imprecision the footprint of uncertainty increases and IVFRN are transformed into interval-valued fuzzy sets with rough boundaries. If there is disagreement in the assessment of the DMs, the interval boundaries of the IVFRN are increased, since the uncertainty and imprecision in decision making are greater. On the other hand, stronger consensus results in fewer changes in the boundaries and the IVFRN are transformed into traditional fuzzy numbers. This is reflected in less uncertainty in the assessments of the DMs. In the case of consensus among the DMs the boundaries of the initial fuzzy numbers are not mentioned and the assessments are described by a unique linguistic expression from the defined fuzzy scale, that is, by a type-1 fuzzy set.

The IVFR approach was tested by means of a case study in which IVFRNs were used in combination with the MAIRCA, MABAC, TOPSIS and VIKOR methods to evaluate landing operations points. In the multi-criteria model presented here, original modifications of the MAIRCA and TOPSIS methods were performed using IVFRN. In addition to these modifications, in this study an original rough modification of the TOPSIS model was also carried out which has not been considered in the literature so far. Finally, the model was validated by comparing the results given by the fuzzy and rough modifications of the given methods. The discussion of the results and validations show significant stability of the results and indicate the promising possibilities for the application of the IVFN MCDM model. Since this is a new approach, the directions for future research should focus on the

application of IVFRN in traditional MCDM models for determining the weight coefficients of criteria (e.g., the AHP/ANP model, the DEMATEL method).

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