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STOCHASTIC INVENTORY MODELS WITH REWORKS

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Abstract: In this paper, two stocks, for fresh and the returned things, are considered for the efficient stock management. Hence, we give two models: the first is for non-perishable and the second for perishable things.In addition, inventories kept in the stock may lose their fairly estimated worth, which we additionally viewed in model-II, for example, PC and versatile embellishments, or most current engine autos. In model-II, the stock decay (a loss of significant worth) in a steady rate θ is chosen arbitrarily. Though the models are more fitting where guarantees are accommodated to a settled time length after the deal for new things was made, they can be used to separate characteristics of a stock system for a broad scale production firm.It is expected that the stock level for both new and the returned things are pre-decided. When the stock level scopes at the re-order point s, a request for renewal is put with parameter γ for new things. The requests for both new and the returned things take after the Poisson process with parameter λ & δ, respectively. Service will be given according to Poisson process for returned things with parameter μ. The joint probability distribution for both returned and new things are derived in the steady state examination. A few system characteristics of two models are inferred here and the outcomes are outlined, based on some numerical cases.

Keywords: Inventory, Guarantee, Reworks, Poisson process, Joint probability distribution.

MSC: 90B05, 90B30.

1. INTRODUCTION

1.1. Back ground of the study

In the client driven business world, one of the key challenges in merchandise exchange is the returns management,comprehended as an agreement between the maker and forward positions in the supply chain (retailers, providers, clients), concerning the system of tolerating back items in the wake of having sold them, either utilized or being as good as new. Clients restore their arranged things if they do not satisfy their desire, the practice drastically expanded in the ongoing years. A few investigations demonstrated that the conceivable reason for a high number of the returns was not that the objects were demaged or broken; e.g, in 2007, Americans returned between 11% - 20% of electronic things, which indicates the amazing measure of $13.8 billion, out of which only 5% were really broken and the rest did not meet the clients expectations.
On the other hand, most items lose their reasonable worth within the time flow, where so called transitory items lose their walue even more quickly, as for example, dairy items, meat, medications or vitamins. Though nowadays numerous items are not transitory in the customary sense (do not rot), we can consider them at present as perishable. Such items lose their usage for changes of innovative advances, for example, PCs, PC parts, (smaller scale processors, memory, information stockpiling units), mobile phones or advanced cameras. The existence cycles of such items are getting shorter constantly because of innovative advances.

We focus on stochastic stock management and on the papers that are essential, in our opinion, and establish the framework for some future work. Recent developments in this field may be found in the work of **Huel-Hsin et al. [6]**, where they studied the optimal inventory replenishment policy and the long-run production inventory costs. **Biswaranjan Mandal [1]** developed an EOQ inventory model for Weibull distributed, where deterioration items under ramp type demand and shortages were considered. **Chung [4]** developed a supply chain management model and presented a solution procedure to find the optimal production quantity with rework process. **Chiu et al. [3]** developed a Mathematical modeling for determining the replenishment policy for a EMQ model with rework and multiple shipments. **Sing et al. [10]** developed an inventory model for decaying items with selling price depending on demand in inflationary environment. **Ghosh et al. [5]** studied an optimal inventory replenishment policy for a deteriorating time-quadratic demand and time-dependent partial backlogging, which depends on the length of the waiting time for the next replenishment over a finite time horizon and variable replenishment cycle. **Zeinab et al. [13]** developed an inventory model for a main class of deteriorating items, under stochastic lead time assumption, and considered a non-linear holding cost. **Brojeswar et al.** [**2**] developed a multi-echelon supply chain model for multiple-markets with different selling seasons and the manufacturer produces a random proportion of defective items, reworked after regular production and sold in a lot to another market just after completion of rework. **Vinod et al.** [**12**] considered a deterministic inventory model with time-dependent demand and time-varying holding cost where deteriorating is time proportional and the model allows shortages and the demand partially backlogged. **Krisnamoorthi et al.** [**7**] developed a single stage production process where the produced defective items are reworked, and two models of rework processes are considered, EPQs without and with shortages. **Sivashankari** & **Panayappan** [**11**] proposed a Production inventory model where reworking of imperfect production was considered, so as scrap and shortages. **Sanjai et al.** [**9**] considered the production rate as demand dependent, which is more realistic. The time dependent rate of deterioration is taken into consideration and demand rate is price dependent. **Mandal et al.** [**8**] developed an Optimal production inventory problem in a specific time period, which is the basis of our study.

1.2. Justification of models formulation

The purpose of this paper is three folded:(1) to give a stochastic stock model utilizing Markov process for makers who give benefit guarantee for their item for a settled time duration after the deal. (2) to decide about framework attributes with numerical illustrations. (3)to survey the effect of modify process on associations benefit margin.

In section 1, we displayed some past related works. In section 2, we build up our stock models and understand it to get state likelihood vectors. Utilizing state likelihood vectors, numerical consequences of the proposed stock models are given in section 3. We talk about the genuine Application and affect-ability investigation of created models in section 4. Conclusion and future research proposals are introduced in section 5.

2. MATHEMATICAL MODELS

2.1. Assumptions and notations

2.1.1. Assumptions

- Initially the inventory level for fresh items is S and for return items is φ which are the highest inventory levels respectively .
- Arrival rate of demands follows Poisson process with parameter λ for fresh items and δ for return items.
- Lead-time is exponentially distributed with parameter γ for fresh items.

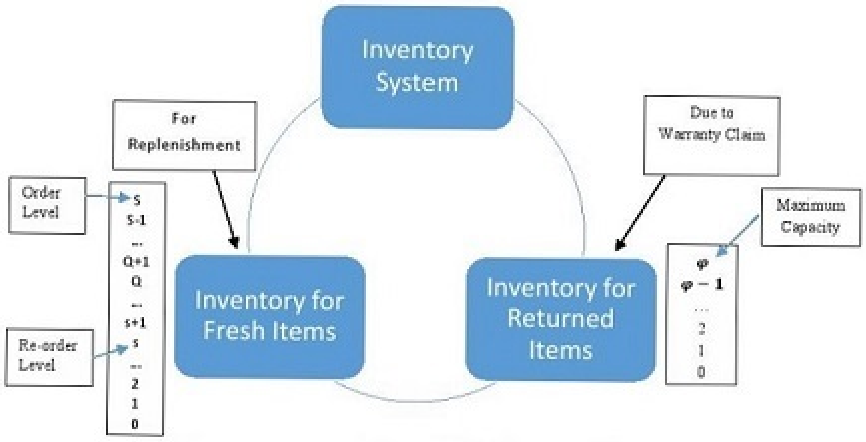

Figure 1: Stochastic Inventory models with Reworks

- If the inventory level for return item is in φ, then service for the return items will be provided at customer's home.
- Items will decay at a constant rate θ.
- Service for the return items follow exponential distribution with parameter μ.

2.1.2. Notations

$S \to$ Maximum inventory level for fresh items.
$\varphi \to$ Maximum inventory level for returned items.
$\lambda \to$ Arrival rate of demands for fresh items.
$\delta \to$ Arrival rate of demands for returned items.
$\gamma \to$ Replenishment rate for fresh items.
$\theta \to$ Decay rate for fresh items.
$\mu \to$ Service rate for returned items.
I(t) $\to$ Inventory level at time t for fresh items.
$x(t) \to$ Inventory level at time t for returned items
The state space of the process $E = E_1 \times E_2$.
where, $E_1 = \{0, 1, 2, \ldots, S\}, \quad E_2 = \{0, 1, 2, \ldots, \varphi\};$ and
$e_{(\varphi+1)} = (1, 1, 1, ., 1)'$ is a $(S+1)(\varphi+1)$-Components column vector of $1'$s.

It is assumed that maximum inventory levels for fresh items and return items are at S and at φ, respectively. The inter-arrival time between two successive demands is assumed to be exponentially distributed with parameter λ for fresh items and δ for return items . Each demand is for exactly one unit for each item.

When inventory level for fresh items reduces to the re-order point s, an order for replenishment is placed. Lead-time is exponentially distributed with parameter γ. When inventory level for the return items reaches at φ ,home service is provided because of space limitation.

2.2. Model-I: Non-Perishable Inventory Model With Reworks

In this model, it is expected that new things will not disintegrate because of time passes.

Now, the infinitesimal generator of the two dimensional Markov process $\{I(t), x(t), t \geq 0\}$ can be defined $\tilde{A} = (a(i,j,k,l)); (i,j),(k,l) \in E$

Hence, we get

$\tilde{A}(i,j,l,k)$= { $\lambda\ : i = (s+1),(s+2),\ldots,S; k = i-1, j = 0,12\ldots,\varphi, l = j$
$-(\lambda+\mu)\ : i = (s+1),(s+2),\ldots,S; k = i, j = 0,1,2\ldots,\varphi-1, l = j$
$-(\lambda+\delta)\ : i = (s+1),(s+2),\ldots,S; k = i, j = 0, l = j$
$-(\lambda+\mu)\ : i = (s+1),(s+2),\ldots,S; k = i, j = \varphi, l = j$
$\lambda\ : i = 1,2,\ldots,s; k = i-1, j = 0,1,2\ldots,\varphi, l = j$
$-(\lambda+\delta+\gamma)\ : i = 1,2,\ldots,s; k = i, j = 0, l = j$
$-\mu\ : i = 0; k = i, j = 0,1,2\ldots,\varphi-1, l = j$
$\delta\ : i = 0,1,2,\ldots,s; k = i, j = 0,1,2\ldots,\varphi-1, l = j+1$
$\mu\ : i = 0,1,2,\ldots,s; k = i, j = 12\ldots,\varphi, l = j-1$
$\gamma\ : i = 0,1,2,\ldots,s; k = i+Q, j = 0,12\ldots,\varphi, l = j$

Now, the infinitesimal generator $\tilde{A}$ can be conveniently expressed as a partition matrix $A_{i,k} = A_{(ik)}$, where A_{ik} is a $(\varphi+1)\times(\varphi+1)$ sub-matrix which is given by :

$A_{i,k}$= { $A_1\, if\, k = i-1 : i = (s+1),(s+2),\ldots,S$
$A_2\, if\, k = i : i = (s+1),(s+2),\ldots,S$
$A_3\, if\, k = i : i = 1,2,\ldots,s$
$A_4\, if\, k = i : i = 0$
$A_5\, if\, k = i-1 : i = 1,2,\ldots,s$
$A_6\, if\, k = i+Q : i = 0,1,2,\ldots,s$
$0 : Otherwise$

with
$A_1 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$
$= diag(\lambda,\lambda,\ldots,\lambda),\ Where\,(i,j) \rightarrow (i-1,j), \forall\, i = (s+1),(s+2),\ldots,S; j = 0,1,2,\ldots,\varphi$

$A_2 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$
={ (i,j)→ $(i,j)\, is\, -(\lambda+\mu)\ : \forall, i = (s+1),(s+2),\ldots,S; j = \varphi$
$(i,j) \rightarrow (i,j)\, is\, -(\lambda+\mu+\delta)\ : \forall, i = (s+1),(s+2),\ldots,S; j = 1,2,\ldots,(\varphi-1)$

$(i,j) \to (i,j)\, is\, -(\lambda+\delta)\ : \forall\, , i=(s+1),(s+2),\ldots,S; j=0$
$(i,j) \to (i,j-1)\, is\, \mu\ : \forall\, , i=(s+1),(s+2),\ldots,S; j=1,2,\ldots,\varphi$
$(i,j) \to (i,j+1)\, is\, \delta\ : \forall\, , i=(s+1),(s+2),\ldots,S; j=0,1,2,\ldots,(\varphi-1)$
Other elements are zero

$A_3 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$
$=\{$ (i,j)$\to (i,j)\, is\, -(\lambda+\mu+\gamma)\ : \forall\, , i=1,2,\ldots,s; j=\varphi$
$(i,j) \to (i,j)\, is\, -(\lambda+\mu+\gamma+\delta)\ : \forall\, , i=1,2,\ldots,s; j=1,2,\ldots,(\varphi-1)$
$(i,j) \to (i,j)\, is\, -(\lambda+\gamma+\delta)\ : \forall\, , i=1,2,\ldots,s; j=0$
$(i,j) \to (i,j-1)\, is\, \mu\ : \forall\, , i=1,2,\ldots,s; j=1,2,\ldots,\varphi$
$(i,j) \to (i,j+1)\, is\, \delta\ : \forall\, , i=1,2,\ldots,s; j=0,1,2,\ldots,(\varphi-1)$
Other elements are zero

$A_4 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$
$=\{$ (i,j)$\to (i,j)\, is\, -(\mu+\gamma)\ : \forall\, , i=0; j=\varphi$
$(i,j) \to (i,j)\, is\, -(\mu+\delta+\gamma)\ : \forall\, , i=0; j=1,2,\ldots,(\varphi-1)$
$(i,j) \to (i,j)\, is\, -(\delta+\gamma)\ : \forall\, , i=0; j=0$
$(i,j) \to (i,j-1)\, is\, \mu\ : \forall\, , i=0; j=1,2,\ldots,\varphi$
$(i,j) \to (i,j+1)\, is\, \delta\ : \forall\, , i=0; j=0,1,2,\ldots,(\varphi-1)$
Other elements are zero

$\mathrm{A}_5 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$
$= diag(\lambda,\lambda,\ldots,\lambda), Where(i,j) \to (i-1,j); \forall\, , i=1,2,\ldots,s; j=0,1,2,\ldots,\varphi$
$A_6 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$
$= diag((\gamma,\gamma,\ldots,\gamma), Where(i,j) \to (i{+}Q,j); \forall\, , li=0,1,2,\ldots,s; j=0,1,2,\ldots,\varphi$

So, we can write the partitioned matrix as follows:
$\tilde{A} = \{$ (i,j)$\to (i-1,j)\ is\ A_1 : \forall i=(s+1),(s+2),\ldots,S$
$(i,j) \to (i,j)\ is\ A_2 : \forall i=(s+1),(s+2),\ldots,S$
$(i,j) \to (i,j)\ is\ A_3 : \forall i=1,2,\ldots,s$
$(i,j) \to (i,j)\ is\ A_4 : \forall i=0$
$(i,j) \to (i-1,j)\ is\ A_5 : \forall i=1,2,\ldots,s$
$(i,j) \to (i+Q,j)\ is\ A_6 : \forall i=0,1,2,\ldots,s$

2.2.1. Steady State Analysis

It can be seen from the structure of matrix $\tilde{A}$ that the state space E is irreducible. Let the limiting distribution be denoted by $\Pi^{(i,j)}$:

$$\Pi^{(i,j)} = \lim_{x\to\infty} Pr[I(t),N(t)=(i,j)], (i,j)\in E$$

Let, Π=$(\Pi^{(S)},\Pi^{(S-1)},\Pi^{(S-2)},\ldots,,\Pi^{(2)},\Pi^{(1)},\Pi^{(0)})$ with
$\Pi^{(k)}$=$(\Pi^{(k,\varphi)}),\Pi^{(k,\varphi-1)},\Pi^{(k,\varphi-2)},\ldots,\Pi^{(k,2)},\Pi^{(k,1)},\Pi^{(k,0)}),\forall\, k=0,1,2,\ldots,S.$
The limiting distribution exists and satisfies the following equations:
$\Pi\tilde{A}=0\, and\, \sum_{i=0}^{S}\sum_{j=0}^{\varphi}\Pi^{(i,j)}=1\ldots(1)$
The first equation of the above yields the sets of equations:
$\Pi^{(1)}A_5 + \Pi^{(0)}A_4 = 0$

$\Pi^{(i+1)}A_5 + \Pi^{(i)}A_4 = 0\ : i = 0$
$\Pi^{(i+1)}A_5 + \Pi^{(i)}A_3 = 0\ : i = 1, 2, , s - 1$
$\Pi^{(i+1)}A_1 + \Pi^{(i)}A_3 = 0\ : i = s$
$\Pi^{(i+1)}A_1 + \Pi^{(i)}A_2 = 0\ : i = s + 1, s + 2, , Q - 1$
$\Pi^{(i+1)}A_1 + \Pi^{(i)}A_2 + \Pi^{(i-Q)}A_6 = 0\ : i = Q, Q + 1, , S - 1$
$\Pi^{(S)}A_2 + \Pi^{(s)}A_6 = 0$
The solution of the equations (except the last one) can be conveniently expressed as:
$\pi^{(i)} = \pi^{(0)}\beta_i;\ i = 0, 1, 2, \ldots, S.$
Where, β_i={ I : i=0
-$A_5A_4^{-1} : i = 1$
$(-1)^{i-1}\beta_i(A_5A_4^{-1})^{i-1} : i = 1, 2, 3, \ldots, s - 1$
$(-1)^{i-1}\beta_i(A_5A_4^{-1})^{-1}(A_1A_3^{-1}) : i = s$
$(-1)^{i-1}\beta_i(A_5A_4^{-1})^{i-1}(A_1A_3^{-1})(A_2A_1^{-1})^{i-1} : i = s + 1, s + 2, \ldots, Q$
$-\beta_{i-1}(A_2A_1^{-1}) - (A_4A_1^{-1})\beta_{i+Q-1} : i = Q + 1, Q + 2, \ldots, S$
$Tocompute\pi^{(0)}$, the following equations can be used:
$\pi^{(S)}A_2 + \pi^{(s)}A_6 = 0$and $\sum \pi^{(K)}e_{K+1} = 1$
Which yeilds respectively
$\pi^{(0)}(\beta_S A_2 + \beta_s A_6) = 0\ \ and\ \pi^{(0)}(I + \sum \beta_i)e_{K+1} = 1$

2.2.2. System Characteristics

(a) Mean inventory level: (i) Mean inventory level for fresh items Ł$_1$= $\sum_{i=1}^{S} i \sum_{j=1}^{\varphi} \Pi^{(i,j)}$
(ii) Mean inventory level for Return items Ł$_2$= $\sum_{j=1}^{\varphi} j \sum_{i=1}^{S} \Pi^{(i,j)}$
(b) Re-order rate for fresh items R=$(\lambda) \sum_{j=0}^{\varphi} \Pi^{(s+1,j)}$
(c) Average customer lost to the system CL=$\lambda \sum_{j=0}^{\varphi} \Pi^{(0,j)}$
(d)Sercivc rate for return items SR=$\mu \sum_{j=1}^{\varphi} \sum_{i=1}^{S} \Pi^{(i,j)}$
(e)Expected Total Cost(ETC)=$c_1L_1 + c_2L_2 + c_3CL + c_4R + c_5SR$
Where,
c_1= Holding cost per Unit for fresh items,
c_2=Holding cost per Unit for return items
c_3= Replenishment cost per order,
c_4= Cost for per unit lost sales
c_5= Sercice cost per unit for return items.

2.3. Model-II: Perishable Inventory Model With Reworks

In the present model, Perish-capacity of things with a steady rate θ is considered here and all others suppositions are the same as in the previous model.

Now, the infinitesimal generator of the two dimensional Markov process $\{I(t), x(t), t \geq 0\}$ can be defined $\tilde{A} = (a(i, j, k, l)); (i, j), (k, l) \in E$

Hence, we get

$\tilde{A}(i, j, l, k)$= { $(\lambda+\theta)\ : i = (s+1), (s+2), \ldots, S; k = i-1, j = 0, 12 \ldots, \varphi, l = j$

$-(\lambda+\theta+\mu)\ : i=(s+1),(s+2),\ldots,S; k=i, j=0,1,2\ldots,\varphi-1, l=j$
$-(\lambda+\theta+\delta)\ : i=(s+1),(s+2),\ldots,S; k=i, j=0, l=j$
$-(\lambda+\theta+\mu)\ : i=(s+1),(s+2),\ldots,S; k=i, j=\varphi, l=j$
$(\lambda+\theta)\ : i=1,2,\ldots,s; k=i-1, j=0,1,2\ldots,\varphi, l=j$
$-(\lambda+\theta+\delta+\gamma)\ : i=1,2,\ldots,s; k=i, j=0, l=j$
$-\mu\ : i=0; k=i, j=0,1,2\ldots,\varphi-1, l=j$
$\delta\ : i=0,1,2,\ldots,s; k=i, j=0,1,2\ldots,\varphi-1, l=j+1$
$\mu\ : i=0,1,2,\ldots,s; k=i, j=12\ldots,\varphi, l=j-1$
$\gamma\ : i=0,1,2,\ldots,s; k=i+Q, j=0,12\ldots,\varphi, l=j$

Now, the infinitesimal generator $\tilde{A}$ can be conveniently expressed as a partition matrix $A_{i,k}=A_{(ik)}$, where A_{ik} is a $(\varphi+1)\times(\varphi+1)$ sub-matrix which is given by :

$A_{i,k}$= { $A_1\, if\, k=i-1 : i=(s+1),(s+2),\ldots,S$
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$A_3\, if\, k=i : i=1,2,\ldots,s$
$A_4\, if\, k=i : i=0$
$A_5\, if\, k=i-1 : i=1,2,\ldots,s$
$A_6\, if\, k=i+Q : i=0,1,2,\ldots,s$
$0 : Otherwise$

with
$A_1=(a_{ij})_{(\varphi+1)\times(\varphi+1)}$
$= diag((\lambda+\theta),(\lambda+\theta),\ldots,(\lambda+\theta)), Where(i,j)\rightarrow(i-1,j)\forall, i=(s+1),(s+2),\ldots,S$
$A_2=(a_{ij})_{(\varphi+1)\times(\varphi+1)}$
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$(i,j)\rightarrow(i,j-1)\, is\, \mu\,\forall, i=(s+1),(s+2),\ldots,S; j=1,2,\ldots,\varphi$
$(i,j)\rightarrow(i,j+1)\, is\, \delta\,\forall, i=(s+1),(s+2),\ldots,S; j=0,1,2,\ldots,(\varphi-1)$

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={ (i,j)→ $(i,j)\, is\, -(\lambda+\theta+\mu+\gamma)\,\forall, i=1,2,\ldots,s; j=\varphi$
$(i,j)\rightarrow(i,j)\, is\, -(\lambda+\theta+\mu+\delta+\gamma)\,\forall, i=1,2,\ldots,s; j=1,2,\ldots,(\varphi-1)$
$(i,j)\rightarrow(i,j)\, is\, -(\lambda+\theta+\gamma)\,\forall,\forall, i=1,2,\ldots,s; j=0$
$(i,j)\rightarrow(i,j-1)\, is\, \mu\,\forall, i=1,2,\ldots,s; j=1,2,\ldots,\varphi$
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$A_4=(a_{ij})_{(\varphi+1)\times(\varphi+1)}$
={ (i,j)→ $(i,j)\, is\, -(\mu+\gamma)\,\forall, i=0; j=\varphi$
$(i,j)\rightarrow(i,j)\, is\, -(\mu+\delta+\gamma)\,\forall, i=0; j=1,2,\ldots,(\varphi-1)$
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$\mathrm{A}_5 = (a_{ij})_{(\varphi+1)\times(\varphi+1)}$
$= diag((\lambda+\theta),(\lambda+\theta),\ldots,(\lambda+\theta)), Where(i,j) \rightarrow (i-1,j); \forall\, , i=1,2,\ldots,s$
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$= diag((\gamma,\gamma,\ldots,\gamma), Where(i,j) \rightarrow (i+Q,j); \forall\, , li=0,1,2,\ldots,s$

So we can write the partitioned matrix as follows:
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2.3.1. Steady State Analysis

It can be seen from the structure of matrix $\tilde{A}$ that the state space E is irreducible. Let the limiting distribution be denoted by $\Pi^{(i,j)}$:

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Let, Π=$(\Pi^{(S)},\Pi^{(S-1)},\Pi^{(S-2)},\ldots,,\Pi^{(2)},\Pi^{(1)},\Pi^{(0)})$ with
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The limiting distribution exists, satisfies the following equations:
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$\Pi^{(i+1)}A_5+\Pi^{(i)}A_4=0\ : i=0$
$\Pi^{(i+1)}A_5+\Pi^{(i)}A_3=0\ : i=1,2,,s-1$
$\Pi^{(i+1)}A_1+\Pi^{(i)}A_3=0\ : i=s$
$\Pi^{(i+1)}A_1+\Pi^{(i)}A_2=0\ : i=s+1,s+2,,Q-1$
$\Pi^{(i+1)}A_1+\Pi^{(i)}A_2+\Pi^{(i-Q)}A_6=0\ : i=Q,Q+1,,S-1$
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The solution of the equations (except the last one) can be conveniently expressed as:
$\pi^{(i)}=\pi^{(0)}\beta_i;\ i=0,1,2,\ldots,S.$
Where β_i={ I : i=0
-$\mathrm{A}_5A_4^{-1} : i=1$
$(-1)^{i-1}\beta_i(A_5A_4^{-1})^{i-1} : i=1,2,3,\ldots,s-1$
$(-1)^{i-1}\beta_i(A_5A_4^{-1})^{-1}(A_1A_3^{-1}) : i=s$
$(-1)^{i-1}\beta_i(A_5A_4^{-1})^{i-1}(A_1A_3^{-1})(A_2A_1^{-1})^{i-1} : i=s+1,s+2,\ldots,Q$
$-\beta_{i-1}(A_2A_1^{-1})-(A_4A_1^{-1})\beta_{i+Q-1} : i=Q+1,Q+2,\ldots,S$

$Tocompute \pi^{(0)}$, the following equations can be used
$\pi^{(S)}A_2 + \pi^{(s)}A_6 = 0$and $\sum \pi^{(K)} e_{K+1} = 1$
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2.3.2. System Characteristics

(a) Mean inventory level:
(i) Mean inventory level for fresh items Ł$_1$= $\sum_{i=1}^{S} i \sum_{j=1}^{\varphi} \Pi^{(i,j)}$
(ii) Mean inventory level for Return items Ł$_2$= $\sum_{j=1}^{\varphi} j \sum_{i=1}^{S} \Pi^{(i,j)}$
(b) Re-order rate for fresh items R=$(\lambda + \theta) \sum_{j=0}^{\varphi} \Pi^{(s+1,j)}$
(c) Average customer lost to the system CL=$\lambda \sum_{j=0}^{\varphi} \Pi^{(0,j)}$
(d)Sercivc rate for return items SR=$\mu \sum_{j=1}^{\varphi} \sum_{i=1}^{S} \Pi^{(i,j)}$
(e)Expected Total Cost(ETC)=$c_1 L_1 + c_2 L_2 + c_3 CL + c_4 R + c_5 SR$
Where,
c_1= Holding cost per Unit for fresh items,
c_2=Holding cost per Unit for return items
c_3= Replenishment cost per order,
c_4= Cost for per unit lost sales
c_5= Sercice cost per unit for return items.

3. RESEARCH OUTCOMES

3.1. Numerical Illustration OF Model-I

Putting, $S = 5, s = 2, \varphi = 3, Q = 3, \lambda = 0.35, \delta = 0.05, \mu = 0.01, \gamma = 0.45, c_1 = 1.5,$
$c_2 = 0.70, c_3 = 0.20, c_4 = 0.10 \, and \, c_5 = 0.25$
we get,

L_1	L_2	R	CL	SR	ETC
3.211720	0.733330	0.486285	0.004667	0.016547	5.432350

Table 1: Result of Non-perishable Inventory model with Reworks.

3.2. Numerical Illustration OF Model-II

Putting, $S = 5, s = 2, \varphi = 3, Q = 3, \lambda = 0.25, \delta = 0.15, \mu = 0.20, \gamma = 0.30, \theta = 0.05$
$c_1 = 1.5, c_2 = 0.70, c_3 = 0.20, c_4 = 0.10 \; and \, c_5 = 0.15$
we get,

L_1	L_2	R	CL	SR	ETC
3.000000	1.148570	0.092307	0.023077	0.184615	5.352460

Table 2:Result of Perishable Inventory model with Reworks.

4. APPLICATION AND SENSITIVITY ANALYSIS

Since stock speaks for methods to address buyer needs and create proficiency, the management of stock rotates around the targets of consumer loyalty, stock speculation, and generation effectiveness. Our models can be fitted to separate the characteristics of a stock system for broad scale production firm in the forceful business world for better stock administration.

From the outline of our models, it might be observed that all cost related to stock structure raises add up to the total cost. Unit growth of holding cost increase indicates cost simply more rapidly in respect to various costs. Requesting expense and overhauling cost little affect add up to cost of the framework. This investigation proposes the administration to keep rework service efficient for branding image and requesting in short interim for keeping stock at least level.

5. CONCLUSION AND FUTURE RESEARCH DIRECTIONS

Cost optimization is the ultimate goal of a business organization. Rework process may increase total cost of the system slightly but it has a great effect on branding image in the long run. Rework process is an essential factor for item guarantee benefit. A business firm without having such kind of facilities will lose a major share of net revenue.

Additional research should be done for deferred request with revamp process. We have utilized Markov process in our investigation. One can utilize semi-Markov process or Markov arrival process for promoting changes of the models.

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Appendix-A

$\pi^{(0,0)}$	0.02521480	$\pi^{(3,0)}$	0.1693730
$\pi^{(0,1})$	0.01260740	$\pi^{(3,1)}$	0.0846864
$\pi^{(0,2)}$	0.00630370	$\pi^{(3,2)}$	0.0423432
$\pi^{(0,3)}$	0.00315185	$\pi^{(3,3)}$	0.0211716
$\pi^{(1,0)}$	0.03241900	$\pi^{(4,0})$	0.1369540
$\pi^{(1,1)}$	0.01620950	$\pi^{(4,1)}$	0.0684769
$\pi^{(1,2)}$	0.00810476	$\pi^{(4,2)}$	0.0342385
$\pi^{(1,3)}$	0.00405238	$\pi^{(4,3)}$	0.0171192
$\pi^{(2,0)}$	0.07410060	$\pi^{(5,0)}$	0.0952722
$\pi^{(2,1)}$	0.03705030	$\pi^{(5,1)}$	0.0476361
$\pi^{(2,2)}$	0.01852520	$\pi^{(5,2)}$	0.0238181
$\pi^{(2,3)}$	0.00926258	$\pi^{(5,3)}$	0.0119090

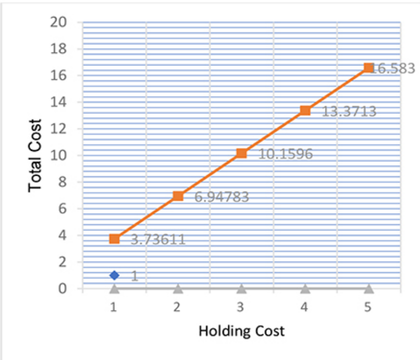
Table 03: State probability vectots of model-I

Appendix-B

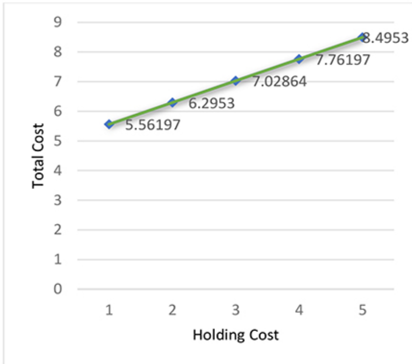
$\pi^{(0,0)}$	0.02521480	$\pi^{(3,0)}$	0.1693730
$\pi^{(0,1})$	0.01260740	$\pi^{(3,1)}$	0.0846864
$\pi^{(0,2)}$	0.00630370	$\pi^{(3,2)}$	0.0423432
$\pi^{(0,3)}$	0.00315185	$\pi^{(3,3)}$	0.0211716
$\pi^{(1,0)}$	0.03241900	$\pi^{(4,0})$	0.1369540
$\pi^{(1,1)}$	0.01620950	$\pi^{(4,1)}$	0.0684769
$\pi^{(1,2)}$	0.00810476	$\pi^{(4,2)}$	0.0342385
$\pi^{(1,3)}$	0.00405238	$\pi^{(4,3)}$	0.0171192
$\pi^{(2,0)}$	0.07410060	$\pi^{(5,0)}$	0.0952722
$\pi^{(2,1)}$	0.03705030	$\pi^{(5,1)}$	0.0476361
$\pi^{(2,2)}$	0.01852520	$\pi^{(5,2)}$	0.0238181
$\pi^{(2,3)}$	0.00926258	$\pi^{(5,3)}$	0.0119090

Table 04: State probability vectots of model-II

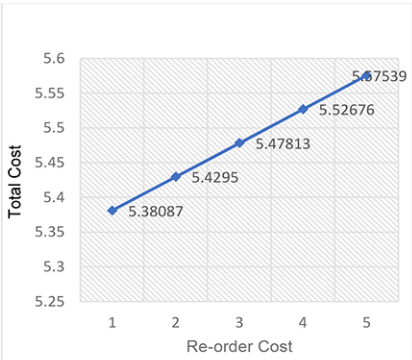
Appendix-C: Graphs of model-I



Graph-1(a): Total Cost vs holding Cost for Fresh Items.

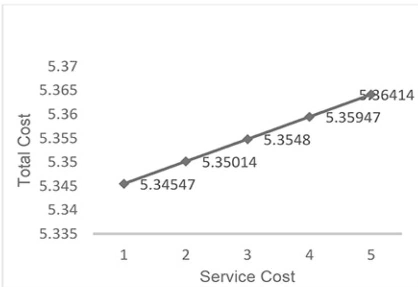


Graph-1(b): Total cost vs holding cost for rework items.



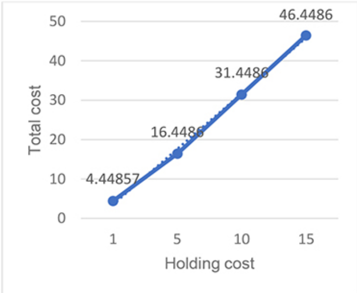
Graph-1(c): Total Cost vs Ordering Cost.

Graph-1(d): Total Cost vs Lost sale.

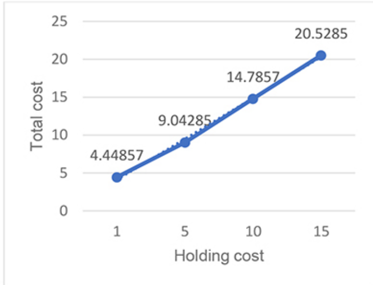


Graph-1(e): Total cost vs service cost.

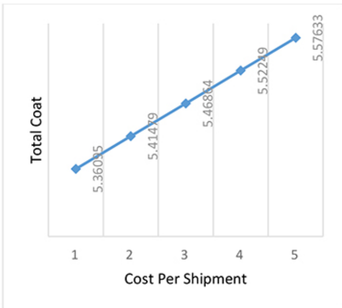
Appendix- D: Graphs of model-II



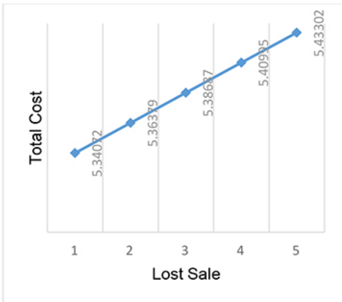
Graph-2(a): Total cost vs Holding cost for fresh items.



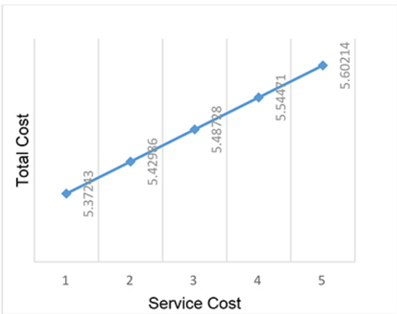
Graph-2(b): Total cost vs Holding cost for reworks items.



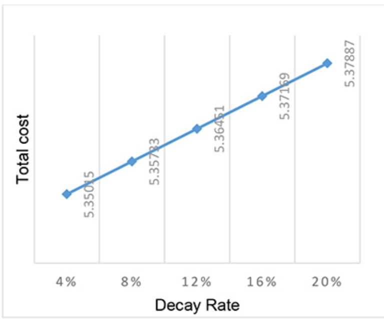
Graph-2(c):Total Cost vs Oredering Cost.



Graph-2(d):Total Cost vs Lost Sale.



Graph-2(e):Total Cost vs Service Cost.



Graph-2(f): Total cost vs Decay rate.