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MULTI-PRODUCT DYNAMIC ADVERTISEMENT PLANNING IN A SEGMENTED MARKET

Sugandha AGGARWAL
Amity School of Business, Amity University, Noida, Uttar Pradesh 201313, India
sugandha_or@yahoo.com

Anshu GUPTA
SBPPSE, Ambedkar University Delhi, Delhi 110006, India
anshu@aud.ac.in

Arshia KAUL
Department of Operational Research, University of Delhi, Delhi 110007, India
arshia.kaul@gmail.com

Mohan KRISHNAMOORTHY
Department of Mechanical and Aerospace Engineering, Monash University, Melbourne, VIC, 3168, Australia
mohan.krishnamoorthy@monash.edu

P.C. JHA
Department of Operational Research, University of Delhi, Delhi 110007, India
jhapc@yahoo.com

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Abstract: In this paper, a dynamic multi-objective linear integer programming model is proposed to optimally distribute a firm's advertising budget among multiple products and media in a segmented market. To make the media plan responsive to the changes in the market, the distribution is carried out dynamically by dividing the planning horizon into smaller periods. The model incorporates the effect of the previous period advertising reach on the current period (taken through retention factor), and it also considers cross-product effect of simultaneously advertising different products. An application of the model is presented for an insurance firm that markets five different products, using goal programming approach.

Keywords: Media Planning, Segmented Market, Multiple Products, Cross-Product Effect, Retention Factor, Dynamic Model.

MSC: 90B60, 90C10, 90C29.

1. INTRODUCTION

Advertising is an integral part of a firm's marketing strategy. The impact of communication through advertising plays a vital role in the success or failure of products/services. The media acts as an intermediary that communicates the advertising messages to the potential customers. Television, radio, internet, newspapers, magazines, trade journals etc. are the popular media used for advertising. Choice of the media for advertising a particular product or a product line depends upon the type of products, media preferences of the potential customers, advertising resources available, availability of media alternatives, media popularity, past experience, expert and higher management opinion and the use of qualitative or quantitative evaluation methods, etc. For developing an advertising plan, a media planner selects a set of media, based on the above characteristics, and tries to allocate advertising resources among selected media in such a way that advertising objectives are met [1, 2]. Advertising resource allocation is carried out for a planning period and, in the present ever changing marketing environment, the media planner seeks to develop a media plan that is responsive to market changes.

Also, early advertising efforts in a product's life cycle aim to create awareness and persuade customers to buy the product, and in the later phases, advertising plays an important role in building customer loyalty. The amount of advertising efforts to be allocated in different stages of a product life cycle must vary as per these changes. In this paper, we have formulated a dynamic multi-criteria media-planning model to determine the optimal number of insertions in different media for multiple products. The objectives of the model are to maximise the reach for each product in a segmented market and, simultaneously, minimize the utilisation of advertising resources.

The proposed model takes into account some factors important to be considered while developing a media plan in the present market scenario such as the media appropriateness factor, the cross-product effect of advertising, and the retention factor of reach from the previous periods in successive time periods. The models developed in the existing literature have not accounted for these factors simultaneously. In the subsequent paragraphs we describe each of these factors.

The effectiveness of advertisements may vary in different media, even after the media selection process, as per the preset criteria. For instance, a media may be preferred in terms of frequency of advertisements due to its cost efficiency; however, the same media may have lower effectiveness on the target population when compared to other media. The cost per insertion and the media appropriateness factor play a major role in the allocation of resources to a media [1, 3]. The media appropriateness factor (MAF) of a media depends on several factors such as the editorial climate, product fit, technical capabilities, the competitive advertising strategy, and the receptiveness of the target population [4]. A media scheduler can determine which factor is more relevant and requires to be considered. Here we have considered the MAF that lies in the unit interval [0,1] in order to weigh the reach-per-insertion of an advertisement in a media. The formulation of the reach function is discussed in detail in the model development section.

A firm's product mix is characterised by the product lines offered in the market as groups of associated items that customers tend to use together or think of as part of a group of similar products [5]. Based on the market analysis, within each product line, a firm may offer several products/brands depending on the market structure, the demand and the firm's capabilities. In order to meet the changing demands of customers and to remain competitive, firms further customise their existing products and introduce innovative products within the existing product lines. They also extend the breadth of their product lines. Firms disperse the advertising budget among all the product lines and to products within the product lines. They try to advertise in such a way that advertisements reach the maximum number of members of the potential group, for all products. Further, each product is designed to meet the needs of a target group.

The potential customers themselves may not possess homogeneous characteristics. Based on geographic, demographic, psychographic and behaviouristic characteristics, marketers divide the potential customers of their products into homogeneous groups and develop a segment-driven advertisement mix to target the segments. Multiple products from a product's line/mix of a firm are often targeted towards the same group of customers with similar segmentation. For example, home and personal care product line such as soap, skin care, hair care, oral care are often targeted towards the same group of customers. In this paper, we have considered those products of a firm's product mix that share similar segmentation.

Since the products are marketed simultaneously and targeted towards the same group of customers, the advertising of one product shows the cross-product effect (CPE) of advertising on the reach of the other product. The CPE can either be measured in terms of sales or reach-through-advertising. The effect can be positive or negative and is termed as the complementarity (positive) or the substitutability (negative) effect of one product on the other [6]. The mathematical formulation of reach objectives in the proposed model takes into account the CPE of advertising of one product on the reach of the other product.

As discussed above, advertising plays different roles in a product's life cycle. In the initial phase, advertising spreads awareness about the product, while in the later phases, it is more about sales and loyalty. Media and consumer characteristics such as viewership, readership or listenership and population and size of the potential market also change with time. Therefore, different advertisement plans are formed in different phases to utilise the available resources optimally. This can be done by dividing the entire planning horizon into multiple time periods; the advertising plan can then be changed in each of the time periods depending on the market growth of the products in the previous period, and the changes in media and consumer characteristics. The multi-objective model developed in this paper is a dynamic optimisation model. After every period, changes in media and potential segment characteristics are incorporated before determining the advertising schedule for the next period. A fraction of advertisement viewers show retention (advertisement recall) of advertisements seen in the previous time period. The proposed model also considers the retention effect of advertisements of the previous period in the next time period.

In this paper, a constrained dynamic multi-objective model is formulated to maximise the product-wise reach from advertising of multiple products marketed in a segmented market through multiple media and minimise the utilisation of the budget. The mathematical model is subject to upper bound and lower bound constraints on the number of advertisements that can be placed in different media channels. The proposed model is a multi-criteria linear integer programming model, validated through a real life case study of the insurance industry, and is solved using goal programming methodology.

2. LITERATURE REVIEW

A number of researchers have worked in the area of media planning in terms of allocation of budget in different media, determining the optimal number of insertions, measuring media effectiveness and media planning considering market segmentation.

In the initial stages, Bass and Lonsdale [7] proposed a simple linear media-planning model to maximise weighted exposures under budgetary constraints. Media allocations are obtained for a single product for a single time period. They carried out sensitivity analysis on the model by including constraints on upper bounds on number of insertions, media budgets and minimum segment exposures bounds. Charnes *e*t al. [8] proposed a model for media selection for a single product by maximising the average frequency of reach to a fraction of the audience segment, and they suggested the use of goal programming to solve the model. Wiedey and Zimmermann [9] proposed a similar model and suggested the use of fuzzy linear programming as a solution methodology. De Kluyver [10] reformulated the Bass and Lonsdale [7] as a goal programming model by imposing rigid constraints on the upper bounds on the number of insertions, media budgets, and segment exposures and their soft equivalents, in order to allow a trade-off in the solution.

Basu and Batra [11] proposed a regression-based advertising sales response function to allocate a firm's advertising budget optimally to multi-brands under budget, upper and lower bound constraints on the advertisement spending for each brand. Doyle and Saunders [12] proposed a logarithmic linear regression model for the allocation of the advertising budget to promotion campaigns. Each campaign is directed to a specific product by considering the CPE of the promotion campaigns of other products that lag or lead this campaign for up to four periods. The model is specifically developed for a retail store that sells multiple products. The sales data of three years of advertisement is used to determine the optimal spending on promotional campaigns for multiple products. Nowak *e*t al. [13] published a survey-based study to examine media planning practices (media preferences, budgeting and media evaluation) among local businesses and retailers in Athens-Clarke County. Danaher and Rust [14] proposed a model to calculate the amount of spending on a media campaign with the objective of maximising the return on investment by considering a diminishing return on advertising. Fruchter and Kalish [15] developed a model to determine a multi-instrument promotion strategy for a firm that maximises the present value of a profit stream over a planning horizon in a dynamic oligopoly market.

Rojas and Peterson [16] proposed a regression based market share model to study the effect of the total advertising expenditure, price and market expenditure of multiple brands, on individual brand market share. The coefficient of the causal relationship of a brand's market share and the total market advertising is defined as the CPE of advertising, which may be complementary or substitutive. Hsu *e*t al. [17] proposed a model to determine the optimum promotional mix and the number of insertions in each instrument, based on linguistic preferences of the domain experts. They suggested the collection of data for linguistic preferences of instruments in order to satisfy different business objectives from domain experts, and then they determined the promotional mix by using fuzzy linguistic quantification operator within a genetic algorithm.

In media planning, it is observed that individuals in the market are not similar. Each individual responds to different promotional vehicles differently. A heterogeneous market can be divided into homogeneous groups through segmentation. Albadvi and Koosha [18] gave a mathematical programming model for optimal allocation of marketing resources in different segments. The objective was to maximize the customer equity, subject to budget constraints under an uncertain environment. Bhattacharya [19] developed a multi-objective media planning model to determine the optimal number of insertions in different media for a single product. The objective was to maximise the reach of each media in the target segments within the available advertising resources. Jha *e*t al. [20] further explored the model for media planning of multiple products. The dynamic behaviour of the market was considered in the media-planning model by Aggarwal *e*t al. [21]. A multi-objective model to maximise the reach of a single product in different media for a segmented market was proposed. They dynamically divided the planning horizon into multiple periods and studied the changing parameters in each time period for media planning. Royo *e*t al. [6] proposed an advertising budget allocation model to optimise the investment on advertising in multiple media for multiple products by considering cross product elasticity. Royo *e*t al. [22] extended the previous model [6] to stochastic modelling.

Research gap and motivation

In the literature, most of the studies in the area of media planning focus on advertisement budget allocation for single products. Some studies examine media planning by considering the maximisation of the exposure or reach of advertisements, while others consider the maximisation of sales or profits [7, 8, 10, 15, 19, 17]. These models support a media planner in making decisions that are related to the disbursement of advertisement resources among several media that maximise the reach or sales of the product. Several firms offer their customers a product mix characterised by the product lines, rather than a single product.

Various products or brands may be offered within a product line. The promotion budget available to a firm is always limited, and only a partial budget is released for the advertising. This budget is further distributed among the product lines and to each product within the line. Therefore, judicious planning of the allocation of this budget among the products such that each product gets maximum exposure or reach is very important for the media planners. Single-product models are not applicable in this situation.

Apart from multi-product consideration, firms usually advertise their products throughout their lifecycle, although, with different intensities. In the initial phases, the intensity of advertisements is kept high in order to spread awareness about the product. In the later phases, media planning usually responds to the changes in market growth, the remaining potential, product acceptance, the product life cycle stage, and changes in the media and consumer characteristics. At the end of a product's life, only price promotions are carried in order to sell the inventory before the launch of a new product.

It is important for the media planner to keep a watch on changing market and media parameters and then revise media allocation decisions according to these changes. Most of the media allocation models discussed in the literature are developed only for single period applications [7, 19, 17]. Among the multi-product models, Basu and Batra's [11] model is applicable for advertising budget allocation among multi-brands for a single planning period. The model is not applicable for determining the distribution of this budget among the media for each product. Doyle and Saunders [12] model also considers multi-product advertisement budget allocation. The model is applicable to a situation in which a media planner wants to allocate a budget to a promotion campaign for a specific product by considering the CPE of advertising of the campaign launched for other products.

The studies due to Royo *e*t al. [6, 22] provide models that offer a wider scope of application. These models can be used for media planning for multi-products from among chosen media for several planning periods jointly. Though the models are developed for multi-period media planning, they apply to joint optimisation of multi-stages. Joint optimisation in the initial stages of media planning may not be able to account for the changes in the market and media parameters in the later stages. These models also ignore the market segmentation aspect, which is important in media planning decisions considering that there is a possible heterogeneity of potential markets, as discussed in the last section. It may be noted that most of the models discussed in the literature consider market segmentation [7, 8, 10, 18].

The product, the market and the media characteristics change with time. Therefore, a media planner looks for dynamic models in which a multi-period media planning model can be applied in stages, dividing the longer time horizon into smaller time periods; then media planning decisions can be made with respect to these changes. The model proposed in this study is a dynamic media planning model that a media planner can use to allocate advertising resources optimally among multiple products; the model also determines the number of insertions to be made in chosen media for each product. In contrast to the models suggested by Royo *e*t al. [6, 22] which are based on sales response functions, our model maximises the reach, measured in exposures for each product. Exposures are determined by market and media parameters.

Our model has the following specific features:

- The media planning model allocates the advertising budget of a firm among several products that compete for this fund by considering the CPE of products on each other.
- The model determines the distribution of the budget that is allocated to multiple media for each product such that the exposure is maximized for each product.
- The model can also be used to determine the advertising exposure in segments as well as in media. If the segment-wise reach is known, one can determine which segment responds better to which product. Similarly, the determination of media-wise reach allows one to analyse the effectiveness of the media.
- The dynamic nature of the proposed model allows the advertising budget allocation based on the current market, product, and media parameters. For example, media planning is adjusted dynamically with respect to the recent estimates of media viewership, listenership or readership, and the profile characteristics of the potential population.
- In the subsequent time periods, the retention of reach from the previous time periods advertisements is taken into account.
- Using the goal programming approach to solve the model, we present tradeoffs between reach and budget objectives in order to provide alternative solutions to the decision makers and to choose the one that best serves the firm's overall objective.
- The MAF is incorporated into the reach function to weigh the media as per their appropriateness to the products.

3. MODEL DEVELOPMENT

3.1. Notation

We provide the following notations before providing a mathematical formulation of the problem:

i	index for segments ($i = 1, ..., N$)
j	index for advertising media ($j = 1, 2, ..., M$)
p	index for products ($p = 1, 2, ..., P$)
k_j	index for media options of j^{th} media ($k_j = 1, 2, ..., K$)
l_j	index for slot in j^{th} media ($l_j = 1, 2, ..., L$)
q	index for time ($q = 1, 2, ..., D$)
r	index for potential customer profile characteristics ($r = 1, 2, ..., G$)
jk_jl_j	j^{th} media, k_j^{th} media option, l_j^{th} slot called as media driver
T	length of the total planning horizon
$a^p_{ijk_jl_jq}$	reach per advertisement for p^{th} product in i^{th} segment, $jk_jl_j^{th}$ media driver, q^{th} time period
$C_{ijk_jl_jq}$	average number of readers/viewers/listeners of $jk_jl_j^{th}$ media driver in segment i, q^{th} time period
$c_{ijk_jl_jq}$	cost per advertisement in $jk_jl_j^{th}$ media driver of segment i in q^{th} time period
$v^p_{ijk_jl_jq}$	lower bound on number of advertisements in $jk_jl_j^{th}$ media driver of segment i for p^{th} product in q^{th} time period
$u^p_{ijk_jl_jq}$	upper bound on number of advertisements in $jk_jl_j^{th}$ media driver of segment i for p^{th} product in q^{th} time period
$x^p_{ijk_jl_jq}$	decision variable representing number of advertisements to be given in $jk_jl_j^{th}$ media driver of segment i for p^{th} product in q^{th} time period
$g^p_{irjk_jl_jq}$	percentage of people who follow $jk_jl_j^{th}$ media driver in segment i and possess r^{th} profile characteristic of p^{th} product's potential customers in q^{th} time period
$\alpha^p_{ijk_jl_jq}$	percentage retention of advertising reach in q^{th} time period coming from $q-1^{th}$ period for p^{th} product, from $jk_jl_j^{th}$ media driver in segment i
w_{rp}	relative importance of r^{th} customer profile characteristic for p^{th} product
e_{ij}	media appropriateness factor in i^{th} segment for j^{th} media
A_q	advertisement budget in q^{th} time period
R_{pq}	total reach of product p in q^{th} time period
R^*_{pq}	reach aspiration for product p in q^{th} time period
A_{pq}	reach due to advertisement of product p in q^{th} time period
θ_{pf}	constant of proportionality representing CPE of advertising of product p on reach of product f in a particular time period

3.2. Model Formulation

The multi-objective mathematical model to determine the advertisement allocations to different media drivers for multiple products in a segmented market is formulated here, assuming that the total planning horizon is divided into smaller time periods. In the beginning of every time period, changes in the market growth of products in previous period and changes in media and consumer characteristics are re-analysed, model parameters are adjusted and media plan is determined for the following period. The objectives are to maximise the total reach from advertising for each product over all segments, while at the same time, minimising the expenditure on advertising. The problem is formulated for q^{th} time period as follows:

(P1):

$$\text{Maximize } R_q = \left[R_{1q}, R_{2q}, ..., R_{Pq}\right] \tag{1}$$

$$\text{Minimize } C_q = \sum_{p=1}^{P}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k_j=1}^{K}\sum_{l_j=1}^{L} c_{ijk_jl_jq} x^p_{ijk_jl_jq} \tag{2}$$

subject to

$$x^p_{ijk_jl_jq} \geq v^p_{ijk_jl_jq} \ \forall \ p, i, j, k_j, l_j \tag{3}$$

$$x^p_{ijk_jl_jq} \leq u^p_{ijk_jl_jq} \ \forall \ p, i, j, k_j, l_j \tag{4}$$

$$x^p_{ijk_jl_jq} \geq 0 \text{ \& integers } \forall \ p, i, j, k_j, l_j$$

where

$$R_{pq} = A_{pq} + \sum_{\substack{f=1 \\ f\neq p}}^{P} \theta_{pf} A_{fq} \tag{5}$$

$$A_{pq} = \sum_{i=1}^{N}\sum_{j=1}^{M}\left(e_{ij} * \left(\sum_{k_j=1}^{K}\sum_{l_j=1}^{L}\left(a^p_{ijk_jl_jq} + \alpha^p_{ijk_jl_jq} a^p_{ijk_jl_jq-1}\right) * x^p_{ijk_jl_jq}\right)\right) \tag{6}$$

$$a^p_{ijk_jl_jq} = \left\{\sum_{r=1}^{G} w_{rp} g^p_{irjk_jl_jq}\right\} C_{ijk_jl_jq} \tag{7}$$

R_q is a P-dimensional vector. R_{pq} represents the total advertising reach of the p^{th} product ($p = 1, 2, , P$) in the q^{th} time period. The vector R_q represents the P reach maximisation objectives for P products. The total reach (individual and cross-product) of a product in any time period q that is, R_{pq} (Equation 5 above) is the sum of reach from advertising for the product p and reach contribution from the CPE of advertising of other products in the same time period. As discussed earlier, the CPE can be positive or negative.

The reach obtained from a unit advertisement (Equation 7) in the $jk_jl_j^{th}$ media driver for the p^{th} product, in segment i, period q that is, $a^p_{ijk_jl_jq}$ is calculated as the product of the readership/viewership/listenership of the $jk_jl_j^{th}$ media driver and the relative proportion of potential customers among them for product p in the respective segment and time period. The individual reach (Equation 6) due to advertising of product p in q^{th} time period, that is, A_{pq}, is taken as sum of the reach from advertising for product p in the q^{th} time period and the retention effect from the previous time period. C_q (Equation 2) is the cost objective, which represents the total expenditure on advertisement. By keeping C_q as the objective to be minimised instead of using a fixed level of advertising spending as a constraint, we can calculate the advertising reach as well as the optimal level of spending.

Constraints in Equations (3) and (4) specify lower and upper bounds on the number of advertisements in any specific media driver. Bounds are introduced on the number of insertions so that product can be advertised through diverse media. They restrict too much and/or too less spending on any media.

3.3. Solution Methodology

The model formulated in this paper is a multi-objective linear integer programming model that can be solved using any of the approaches such as weighted sum approach, lexicographic approach, and interactive approach. The objectives of this study are to maximize the reach for each product due to advertising simultaneously minimising the advertising budget utilization. Since, in any industry, organization has prior knowledge of the relative importance of their products to be promoted in the market based on their respective performance, management is generally in position of providing their relative importance (weights). The management may give higher weights to the product which is weaker and promote it more to improve its reach. Here the weights are decided by the decision maker and weighted goal programming approach is used to solve the model. In this approach the weighted sum of deviations from the goals of the objective functions are minimized. Goal programming is suitable to solve this model as the decision maker can trade off the solution for different values of targets and choose an appropriate solution. In goal programming, first the targets on the goals are determined and then, deviations from the goals are minimized [23, 24].

Here we assume that an upper bound on budget is given by the management for media planning decisions. Using this budget as a base value, the targets on reach can be determined for each product. This value of budget can also be determined based on past data of advertisement for the products (or a similar product).

In order to obtain reach targets R^*_{pq} for all products in every time period, the model (P1) can be solved p times ($p = 1, 2, .., P$) as a single objective model with the reach objective of the p^{th} product ($p = 1, 2, , P$) and the cost minimisation objective as the budgetary constraint in every run. The mathematical model to obtain a reach target for the p^{th} product can be expressed as

(P2):

Maximize R_{pq}

subject to

$$\sum_{p=1}^{P}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k_j=1}^{K}\sum_{l_j=1}^{L} c^{p}_{ijk_jl_jq} x^{p}_{ijk_jl_jq} \le A_q$$

$$x^{p}_{ijk_jl_jq} \ge v^{p}_{ijk_jl_jq} \ \forall \ p, i, j, k_j, l_j$$

$$x^{p}_{ijk_jl_jq} \le u^{p}_{ijk_jl_jq} \ \forall \ p, i, j, k_j, l_j$$

$$x^{p}_{ijk_jl_jq} \ge 0 \text{ \& integers } \forall \ p, i, j, k_j, l_j$$

It should be noted that the value of A_q is taken to be greater than $A_{pq} = \sum_{p=1}^{P}\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k_j=1}^{K}\sum_{l_j=1}^{L} c^{p}_{ijk_jl_jq} x^{p}_{ijk_jl_jq}$ that is the expenditure, when advertising is carried out at a lower bound for all products, otherwise, the model (P2) will result in an infeasible solution. It is reasonable to take $A_q \ge A_{vq}$ because both of the lower bounds $v^{p}_{ijk_jl_jq}$ and A_q are the parameters obtained from the decision maker. In case the available budget is less than A_{vq}, either of the lower bounds can be reduced or removed from the model; the rest of the solution method will remain the same.

(P2) is a single objective linear integer programming model. Various methods such as cutting plane methods, branch and bound method, and heuristic methods can be used to solve this class of problems. These methods are iterative search methods over the feasible solutions and require either explicitly or implicitly complete enumeration. These algorithms can guarantee global optimality only through an enumerative search. (P1) model finds application in real life problems where the budget needs to be allocated in different media categories for different products in a segmented market over a planning horizon. Complete enumeration of such large problems is very difficult and sometimes not possible due to scale of the problem. Complete enumeration of large scale real life problems is also time consuming. Several optimization software such as LINGO, CPLEX, MATLAB, Mathematica with inbuilt modules are available for solving integer programming models. In the literature several authors have used optimization software to obtain solution of this class of problems. Here, we have coded the model on LINGO software version 14.0 in order to obtain the solution. The software uses branch and bound algorithm to solve the linear integer models.

The goal model for the proposed multi-objective linear integer model for time period q (P1) using goal targets obtained in (P2) can now be formulated as follows:

(P3):

$$\text{Minimize } g_q(\eta, \rho) = \gamma_q \left(\sum_{p=1}^{P} \lambda_{pq} \eta_{pq} \right) + (1 - \gamma_q) \rho_{P+1,q} R_{pq}$$

subject to

$$Z_{pq} + \eta_{pq} - \rho_{pq} = 1 \ \forall \ p = 1, 2, ...P \quad (8)$$

$$B_q + \eta_{P+1} - \rho_{P+1} = 1 \tag{9}$$

$$x^p_{ijk_jl_jq} \geq v^p_{ijk_jl_jq} \ \forall \ p, i, j, k_j, l_j$$

$$x^p_{ijk_jl_jq} \leq u^p_{ijk_jl_jq} \ \forall \ p, i, j, k_j, l_j$$

$$x^p_{ijk_jl_jq} \geq 0 \ \& \text{ integers } \forall \ p, i, j, k_j, l_j$$

$$\sum_{p=1}^{P} \lambda_{pq} = 1$$

$$\eta_{pq}, \rho_{pq} \geq 0 \ \forall \ p = 1, 2, ..., P + 1$$

where,

$$Z_{pq} = R_{pq}/R^*_{pq} \tag{10}$$

$$B_q = \left(\sum_{p=1}^{P} \sum_{i=1}^{N} \sum_{j=1}^{M} \sum_{k_j=1}^{K} \sum_{l_j=1}^{L} c^p_{ijk_jl_jq} x^p_{ijk_jl_jq} \right) \Bigg/ A_q \tag{11}$$

The objective in the linear-integer goal model (P3) minimises the weighted sum of deviations from the reach and cost goals. Equations (8) and (9) are the normalised goal constraints on the reach and advertisement cost objectives in which the reach goals are set at the reach aspirations obtained on solving (P2) for each product; the advertisement cost is bounded by the budget given by the decision maker. Z_{pq} and B_q, expressed as in equations (10) and (11), are the normalised values of the reach and budget goals.

$0 \leq \gamma_q \leq 1$ is weight assigned to the budget objective, and $(1-\gamma_q)$ is the weight assigned to the total reach objective in the goal model. By introducing weights λ_{pq}, the effective weight assigned to the p^{th} product becomes $(1 - \gamma_q)\lambda_{pq}$. If equal priority is to be assigned to all the products, then one can assume $\lambda_{pq} = 1/P$ for all $p = 1, 2, ..., P$. Weights γ_q and λ_{pq} could either be provided by the decision maker; in case the weights are not known, one can use interactive goal programming to solve the model. The goal model (P3) can be solved for different values of the bound on the budget to trade-off between reach aspirations and the budget. Alternative solutions can be presented to the decision maker for selection of solution to be used. Again, using the standard goal programming procedure, and coding the model in LINGO software, the solution to the model (P3) is obtained. The optimal solution obtained from (P3) is an efficient solution for (P1) [25].

4. CASE STUDY

We test the media-planning model that has been developed in the preceding section on a case study involving the media-planning decisions of an insurance firm. The actual name of the firm, media, geographic segments, and other information is not shared here because this information is commercial-in-confidence. The firm requires a media plan for the five types of insurance policies that it offers, which are marketed in four geographic segments over a planning horizon of one year.

Media	Media Options	Slots
Newspaper(NP)	NP1,NP2	Front Page (FP), Other Page (OP)
Television(TV)	CH1,CH2	Prime Time (PT), Other time (OT)
Radio(R)	RCH1,RCH2,RCH3,RCH4	Prime Time (PT), Other Time (OT)

Table 1: Description for Media

4.1. Data Description

The unit of analysis in our study is the insurance policies which need to be advertised in the potential market. The media vehicles/channels have been chosen by the firm for advertising based on past experience and previous market surveys conducted by the firm in order to study potential customers and their media preferences. The target market for all products is defined on the basis of two demographic criteria: age, and income. The media chosen for advertising are newspaper, television, and radio. For every media, certain media options are chosen for each geographical segment based on potential market demographics. A media option is further explored in terms of media slots. For example, in the newspaper advertising cost and effectiveness of an insertion in the front page is different for an advertisement of the same size in other (inner) pages. In our case, the firm has provided a choice of media slots for every chosen media option. Descriptions of the media are given in Table 1, above. After seeking the opinions of a media planner and other experts, and studying the market and media parameters, the total planning horizon of one year is divided into three intervals of four months each. Before determining the media plan for any period, recent data for the potential market (media preferences) and media parameters (cost per insertion, readership/viewership/listenership) are collected from primary and secondary sources.

Currently, firms or market research agencies continuously conduct market analysis, collect and record data in order to make decisions that are based on evidence, data and facts. Data related to the media parameters that are required for our model can also be obtained from the respective media agencies. The media planner has proposed a budget of Rs. 560 million (Indian rupees, abbreviated as Rs.) for the media planning for a year. Again, it is proposed that this budget be divided into Rs. 200, Rs. 180, and Rs. 180 million among the three planning periods of four months each. However, in practice this distribution is not rigid. Therefore, we plan to carry out a sensitivity analysis on budget allocations in order to see if there may be alternative solutions.

Readership, viewership or listenership (audience membership), and the cost-per-insertion data for all segments in newspaper, television, and radio media are given in Tables 4, 5, and 6 (see Appendix) for all three planning periods. The figures in brackets are the cost per insertion, and the other value in each cell represents the audience membership. In case of newspaper advertising, the cost given in Table 4 is per one square centimetre.

The firm has decided to place a four-by-six square centimetre advertisement in all newspapers, both on the front page and other pages. For television and radio, the advertising cost is given per 10 second slot, and a slot of 30 seconds is used for advertising the product in all media options.

The data for the percentage profile matrix represented by $g^{p}_{irjk_jl_jq}$ $(r = 1, 2)$ are criteria denoted as CR1 and CR2 in the table) used in the study is given in Tables 7, 8, 9, 10, and 11 in Appendix. This data is based on the market survey before the media planning for every period for all the products in each segment, taking a sample size of 1000. The relative importance of the r^{th} customer profile characteristic $(r = 1, 2)$ for the p^{th} product, that is, w_{rp}, is given in Table 12. The data for MAF and CPE are given in Tables 13, 14, and the retention factor data is given in Tables 15, 16, and 17.

The cross product effect is taken to be constant across different time periods. Lower bounds and upper bounds on the number of insertions in each media category for every product for all segments and time periods are given in Tables 18-21. The values of the lower and upper bounds are provided by the media planner, based on the market survey results of media preferences, type and cost of the media and technical constraints.

4.2. Data Analysis

Using the data provided in the APPENDIX first, the linear integer programming model (P2) is coded in LINGO software version 14.0 for each product and solution is obtained. All the calculations are carried out on a Windows 7 system, Intel Core Duo 1.40 GHz processor and 4 GB RAM.

The reach aspirations obtained on solving (P2) are set as the reach goals while solving model (P3). Equal weights are assigned to both reach and budget goals, that is, $\gamma_q = 0.5$ as per decision makers choice. In case the weights are not available, it can be generated through the interactive approach [26].

Similarly, an equal weight is attached to the reach goal of each product, that is, $\lambda_{pq} = 0.2$. The data for the reach and budget goals, the bounds on the number of insertions and weights $\gamma_q = 0.5$ and $\lambda_{pq} = 0.2$ for all $p = 1, 2, ..., 5$ and $q = 1, 2, 3$, are used to solve the linear integer goal model (P3), coded and executed in LINGO software using global solver and branch and bound method, as discussed in solution methodology. Since a dynamic market environment is considered, media planning is done for three sub-periods, the model parameters are updated in every time period and revised media schedule is obtained.

4.3. Results and Discussion

In every period, a sensitivity analysis is done on the model (P3) by changing the upper bound on the budget goal by decreasing and increasing it by 20 million to obtain alternative solutions that are presented to the decision maker for final implementation. The solutions with reach goals, the reach that can be attained and the budget consumed, are presented in Table 2. Media-wise distribution of the reach and budget allocation are given in Table 3.

The solution given in Table 2 shows the distribution of the advertising budget among the products and the respective value of the reach that can be obtained if the product is advertised according to the media plan obtained on solving the model (P3). Table 23, Table 24, and Table 25 show the number of insertions in all media for all segments and products. The solution is shown only for the budget goals Rs. 200, Rs. 180, and Rs.180 million for the time periods 1, 2, and 3, respectively.

A similar solution is obtained for other budget goals (all solutions are not given in the manuscript because the tables are very large and will increase the size of paper considerably). It can be seen that the budget is distributed fairly among all the products.

Time Period 1							
Product		Achieved reach with budget			Product wise budget utilised		
	Reach Goal	180 million	200 million	220 million	180 million	200 million	220 million
P1	54490560	45023274	46455976	50854311	46171747	49077097	58856347
P2	44040110	38070686	40844423	41755708	40869236	48127136	50653736
P3	36910040	30437303	32246781	33661799	37958702	43105952	47905952
P4	9715330	9610233	9612703	9650981	20730909	20730909	21174909
P5	31217690	27582399	29152635	29989641	34269289	38958789	41408789
Total	176373730	150723895	158312518	165912441	179999883	199999883	219999733
Time Period 2							
Product		Achieved reach with budget			Product wise budget utilised		
	Reach Goal	180 million	200 million	220 million	180 million	200 million	220 million
P1	71547790	44548896	47798797	56010656	36058743	39306303	48651549
P2	55615100	40984767	46625243	47018327	37722438	45013266	45411864
P3	50283990	34551589	37328284	41455367	34589970	38539383	44887836
P4	16103410	15953871	15965945	16005152	22658136	22688496	22850064
P5	39704030	30046514	33338656	35709952	28970169	34452456	38198556
Total	233254320	166085636	181056926	196199454	159999456	179999904	199999869
Time Period 3							
Product		Achieved reach with budget			Product wise budget utilised		
	Reach Goal	180 million	200 million	220 million	180 million	200 million	220 million
P1	84041960	36557130	51313137	59484589	25295904	37964604	45397839
P2	64455650	46415722	47314780	54370866	39591168	40460322	48795942
P3	60882710	42474784	43258271	45630318	38780703	39485973	42146613
P4	21279500	20541262	20904032	20970452	26837142	27957162	28170762
P5	45004050	33305409	36955794	38309478	29495142	34131390	35488830
Total	275663870	179294307	199746014	218765702	160000059	179999451	199999986

Table 2: Product-Wise Reach Aspired, Achieved and the Budget Utilised for Different Budget Limits

Consistent with the literature [27], from Table 3, it can be seen that the overall maximum reach is obtained from advertising in television media with maximum utilisation of budget. However, due to the lower bounds on the number of insertions, all products will be advertised through all media. Media planners use different media to advertise the product so that the advertisement reaches maximum number of the potential customers. With the increase in the bound on the budget goal, both reach and budget allocations increase for almost all media and vice versa.

Media	Achieved Reach with Budget			Media Wise Budget Utilised		
	180 million	200 million	220 million	180 million	200 million	220 million
Time Period 1						
NP	33482530	33776185	33776185	47595813	48474813	48474813
TV	69121255	76290292	83874544	102222540	121025540	140989540
R	48120110	48246041	48261712	30181530	30499530	30535380
Time Period 2						
NP	43781622	49884762	50817504	45187368	52629624	53847624
TV	78092836	86645198	100187656	88547619	100687575	118572369
R	44211179	44526966	45194294	26264469	26682705	27579876
Time Period 3						
NP	40060999	47042796	55912010	39684144	46061424	55528944
TV	87812512	100888855	110687154	93211479	106338099	116504007
R	51420796	51814363	52168248	27104436	27599928	27967035

Table 3: Media-Wise Reach Attained and the Budget Utilised for Different Budget Limits

5. CONCLUSION

In this paper, a dynamic multi-objective media planning model is proposed to allocate the advertising budget to different competing products of a firm, marketed in a segmented market. The model also determines the number of insertions for all products in the different media that are selected for advertising, maximising the reach for each product and minimising the budget utilisation. The CPE of advertising of one product on another one and the media effectiveness factor are incorporated in the model formulation.

The model provides solutions dynamically over several periods of time based on the recent data related to the market and the media parameters used in the model. The solutions are provided by considering the retention of reach from the previous periods. A case study is presented to show the application of the proposed model.

The following are certain limitations of the proposed model: the reach function is linear, the CPE of advertising from competitors products is not considered and audience duplication in the reach function is not removed because we have considered reach in terms of exposures. These limitations could be addressed in further studies on the topic. We can also explore the model performance using other solution methods.

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Appendix

Segment	NP1		NP2	
	FP	OP	FP	OP
Time Period 1				
S1	943223 (1920)	943223 (917)	764736 (1344)	764736 (688)
S2	828000 (2000)	828000 (917)	600000 (1167)	600000 (417)
S3	825944 (1728)	825944 (833)	414155 (1056)	414155 (458)
S4	595000 (1152)	595000 (589)	399000 (704)	399000 (333)
Time Period 2				
S1	960000 (2079)	960000 (950)	777989 (1400)	777989 (750)
S2	900000 (2376)	900000 (1100)	559104 (1167)	559104 (520)
S3	858384 (1750)	858384 (950)	379277 (1000)	379277 (450)
S4	445370 (990)	445370 (490)	456000 (800)	456000 (450)
Time Period 3				
S1	990000 (2170)	990000 (975)	793600 (1500)	793600 (1050)
S2	965000 (2400)	965000 (450)	525000 (1250)	525000 (1080)
S3	885104 (1950)	885104 (400)	319746 (1200)	319746 (900)
S4	522690 (1300)	522690 (416)	366750 (975)	366750 (740)

Table 4: Average Readership and Cost (in Rs. Per sq cm) in Newspaper

Segment	CH1		CH2	
	PT	OT	PT	OT
Time Period 1				
S1	1280000 (36000)	704000 (18500)	832000 (31000)	560000 (16667)
S2	1254400 (34500)	576000 (16500)	558900 (20750)	332800 (9600)
S3	896000 (24000)	420000 (10000)	608000 (17500)	441600 (7800)
S4	650000 (32000)	384000 (14780)	550400 (21000)	307200 (10250)
Time Period 2				
S1	1042800 (33500)	652740 (17000)	1650099 (39000)	981823 (18000)
S2	1705000 (35800)	945000 (18000)	509850 (18800)	257400 (10000)
S3	979000 (25000)	473000 (10600)	940720 (25434)	594000 (11817)
S4	838530 (33000)	457380 (16000)	594000 (20000)	287496 (10200)
Time Period 3				
S1	980000 (30100)	595000 (16250)	1700000 (41020)	1090000 (19950)
S2	1800000 (38000)	912000 (20000)	449200 (16800)	228000 (8000)
S3	1024800 (29000)	480200 (12000)	1188000 (27368)	762210 (12430)
S4	780000 (33000)	378000 (15500)	480700 (19789)	244500 (9000)

Table 5: Average Viewership and Cost (in Rs. Per 10 sec) in TV

Segment	CH1		CH2		CH3		CH4	
	PT	OT	PT	OT	PT	OT	PT	OT
Time Period 1								
S1	476280 (1600)	190260 (440)	411936 (1300)	123480 (300)	283500 (1200)	99120 (325)	207480 (675)	82992 (250)
S2	365820 (1180)	164220 (300)	331800 (800)	149310 (250)	229320 (600)	126126 (250)	163170 (500)	48951 (225)
S3	169260 (1475)	64260 (500)	131712 (1250)	52500 (350)	94290 (1150)	37716 (300)	73836 (625)	31011 (250)
S4	253260 (1500)	79380 (450)	180600 (1325)	63210 (500)	146832 (1200)	58380 (330)	122346 (700)	55056 (275)
Time Period 2								
S1	391992 (1423)	142553 (374)	477817 (1837)	192246 (824)	344995 (1408)	110147 (495)	205406 (660)	82163 (231)
S2	284846 (1100)	110294 (319)	237482 (825)	75717 (259)	432441 (990)	192343 (385)	252539 (1210)	98000 (561)
S3	200000 (1760)	98000 (737)	130395 (1122)	51975 (352)	91462 (1023)	36585 (308)	73098 (495)	30702 (248)
S4	230000 (1485)	112000 (385)	108794 (1320)	62578 (451)	72428 (1122)	56629 (341)	191123 (979)	89000 (440)
Time Period 3								
S1	368472 (1268)	133999 (323)	513312 (1900)	206526 (978)	417937 (1882)	125882 (840)	198363 (600)	79345 (280)
S2	251478 (1029)	94537 (370)	191867 (1900)	61182 (980)	586883 (960)	247297 (468)	279957 (1810)	98000 (875)
S3	220661 (1882)	104718 (918)	124713 (1094)	49710 (280)	93291 (910)	36585 (350)	73620 (450)	30920 (250)
S4	219121 (1345)	98008 (360)	95117 (1300)	70299 (1100)	54109 (355)	54711 (400)	200679 (1020)	92386 (500)

Table 6: Number of Listeners and Advertisement cost (in Rs. Per 10 sec) in Radio

Segment	Time Period 1								Time Period 2								Time Period 3							
	NP1				NP2				NP1				NP2				NP1				NP2			
	CR1		CR2		CR1		CR2		CR1		CR2		CR1		CR2		CR1		CR2		CR1		CR2	
	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP
Product 1																								
S1	.13	.05	.15	.05	.08	.03	.09	.05	.14	.07	.1	. 05	.12	.06	.1	.04	.15	.08	.12	.06	.13	.07	.11	.03
S2	.12	.08	.07	.04	.07	.04	.1	.06	.16	.07	.13	.08	.14	.07	.12	.08	.17	.09	.14	.09	.15	.08	.13	.08
S3	.09	.04	.12	.05	.08	.04	.09	.06	.15	.09	.13	.07	.13	.07	.13	.06	.16	.09	.14	.08	.14	.07	.13	.07
S4	.07	.03	.09	.06	.09	.04	.07	.04	.1	4 .07	.12	.05	.12	.06	.11	.06	.13	.07	.12	.05	.11	.06	.11	.06
Product 2																								
S1	.11	.08	.07	.04	.07	.03	.1	.04	.13	.06	.13	.05	.1	.07	.09	.05	.12	.06	.14	.08	.14	.08	.11	.05
S2	.07	.04	.11	.05	.08	.03	.1	.04	.12	.06	.11	.06	.11	.05	.1	.05	.14	.06	.12	.07	.11	.06	.12	.06
S3	.08	.03	.1	.04	.07	.04	.09	.03	.13	.07	.11	.06	.11	.04	.11	.05	.15	.09	.13	.07	.1	.06	.09	.06
S4	.09	.05	.09	.04	.05	.02	.06	.03	.13	.07	.12	.04	.11	.04	.11	.05	.12	.07	.11	.04	.12	.05	.11	.05
Product 3																								
S1	.07	.03	.1	.04	.04	.02	.05	.02	.09	.04	.1	.07	.11	.05	.1	.05	.11	.05	.12	.08	.12	.06	.11	.06
S2	.1	.03	.07	.02	.05	.02	.03	.01	.1	.05	.1	.06	.1	.07	.11	.06	.12	.07	.13	.06	.09	.06	.09	.05
S3	.08	.03	.1	.06	.07	.03	.05	.02	.12	.07	.1	.05	.1	.05	.1	.04	.12	.08	.11	.05	.08	.05	.1	.04
S4	.08	.04	.1	.03	.04	.02	.06	.03	.09	.03	.1	.05	.12	.07	.1	.05	.1	.03	.08	.05	.13	.08	.12	.08
Product 4																								
S1	.04	.02	.06	.02	.04	.03	.04	.01	.06	.03	.07	.04	.09	.05	.07	.04	.09	.04	.09	.05	.1	.06	.08	.05
S2	.05	.03	.07	.03	.04	.02	.05	.04	.08	.03	.07	.03	.08	.04	.09	.04	.08	.04	.08	.05	.07	.04	.06	.03
S3	.04	.02	.06	.02	.08	.03	.07	.03	.1	.05	.07	.03	.08	.03	.09	.05	.11	.06	.1	.06	.07	.03	.06	.04
S4	.05	.02	.06	.02	.02	.01	.03	.02	.08	.05	.09	.04	.1	.04	.08	.04	.07	.04	.06	.03	.11	.05	.09	.05
Product 5																								
S1	.08	.03	.09	.03	.04	.02	.06	.03	.08	.04	.09	.04	.1	.04	.08	.04	.1	.04	.09	.04	.1	.04	.08	.04
S2	.07	.02	.08	.04	.03	.01	.05	.01	.1	.04	.09	.04	.1	.04	.1	.04	.1	.04	.09	.04	.1	.04	.1	.04
S3	.06	.03	.04	.02	.04	.02	.06	.02	.11	.08	.09	.04	.1	.04	.11	.05	.11	.08	.09	.04	.1	.04	.11	.05
S4	.06	.04	.08	.04	.06	.02	.03	.02	.1	.04	.1	.03	.1	.03	.1	.04	.1	.04	.1	.03	.1	.03	.1	.04

Table 7: Readership Profile Matrix for Newspaper

Segment	Time Period 1								Time Period 2								Time Period 3							
	CH1				CH2				CH1				CH2				CH1				CH2			
	CR1		CR2		CR1		CR2		CR1		CR2		CR1		CR2		CR1		CR2		CR1		CR2	
	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT
Product 1																								
S1	.16	.06	.15	.08	.1	.05	.1	.06	.15	.07	.14	.08	.16	.07	.13	.06	.16	.08	.15	.08	.16	.07	.15	.08
S2	.16	.07	.12	.05	.1	.06	.09	.03	.14	.07	.14	.06	.12	.08	.13	.08	.15	.08	.14	.07	.13	.09	.1	.06
S3	.15	.06	.2	.13	.13	.06	.09	.04	.12	.06	.11	.05	.14	.08	.14	.07	.13	.06	.12	.05	.15	.09	.11	.07
S4	.15	.08	.1	.04	.07	.03	.07	.02	.14	.07	.13	.07	.13	.06	.1	.05	.1	.05	.1	.04	.1	.04	.13	.05
Product 2																								
S1	.13	.04	.1	.05	.07	.03	.06	.03	.14	.08	.13	.07	.14	.08	.14	.04	.15	.08	.14	.07	.14	.07	.15	.07
S2	.15	.05	.17	.11	.09	.05	.08	.06	.13	.06	.12	.07	.13	.05	.14	.06	.14	.07	.14	.08	.14	.06	.14	.04
S3	.19	.1	.18	.09	.1	.05	.08	.03	.13	.06	.13	.06	.13	.06	.13	.06	.13	.05	.13	.06	.13	.06	.13	.06
S4	.13	.08	.08	.04	.08	.05	.1	.05	.13	.05	.13	.06	.14	.06	.12	.06	.09	.05	.1	.05	.12	.06	.11	.06
Product 3																								
S1	.13	.05	.12	.04	.06	.03	.05	.02	.12	.06	.11	.06	.13	.06	.13	.06	.13	.07	.12	.05	.12	.06	.14	.06
S2	.14	.05	.1	.06	.1	.06	.08	.04	.12	.05	.11	.05	.11	.06	.13	.05	.13	.06	.13	.04	.09	.06	.13	.03
S3	.09	.03	.12	.04	.14	.08	.1	.05	.11	.05	.11	.05	.11	.05	.11	.05	.11	.06	.11	.05	.08	.06	.15	.06
S4	.09	.05	.08	.04	.07	.03	.05	.02	.13	.06	.12	.05	.13	.06	.11	.05	.1	.06	.09	.04	.11	.05	.09	.05
Product 4																								
S1	.03	.02	.03	.01	.04	.03	.06	.02	.09	.04	.07	.04	.1	.05	.11	.05	.1	.05	.08	.04	.1	.05	.1	.05
S2	.05	.04	.05	.02	.05	.04	.06	.05	.11	.04	.1	.05	.09	.04	.11	.07	.12	.06	.1	.05	.12	.03	.09	.05
S3	.08	.03	.06	.03	.05	.02	.03	.02	.09	.04	.09	.04	.11	.05	.1	.04	.13	.05	.01	.04	.1	.04	.09	.04
S4	.07	.04	.06	.02	.02	.01	.01	.01	.09	.04	.09	.03	.11	.04	.09	.04	.08	.04	.08	.03	.09	.06	.12	.06
Product 5																								
S1	.1	.03	.08	.03	.08	.04	.06	.02	.1	.04	.1	.05	.12	.05	.13	.05	.12	.05	.12	.04	.12	.05	.14	.05
S2	.15	.09	.09	.06	.06	.03	.07	.05	.13	.04	.1	.06	.11	.04	.12	.05	.14	.05	.13	.06	.1	.04	.09	.04
S3	.14	.07	.13	.05	.07	.04	.07	.03	.11	.04	.11	.04	.1	.05	.11	.04	.12	.04	.11	.05	.1	.05	.1	.06
S4	.09	.05	.09	.04	.06	.03	.05	.04	.11	.03	.11	.05	.12	.05	.11	.05	.09	.03	.08	.04	.1	.05	.12	.06

Table 8: Viewership Profile Matrix for Television

Time Period 1																
Segment	CH1				CH2				CH3				CH4			
	CR1		CR2		CR1		CR2		CR1		CR2		CR1		CR2	
	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT
Product 1																
S1	.13	.06	.12	.08	.10	.03	.10	.07	.07	.03	.06	.02	.08	.04	.05	.02
S2	.11	.05	.13	.05	.09	.04	.10	.03	.08	.04	.07	.02	.04	.02	.06	.02
S3	.17	.07	.19	.07	.17	.09	.15	.06	.12	.04	.14	.08	.07	.05	.10	.06
S4	.13	.08	.13	.08	.09	.06	.14	.04	.10	.06	.07	.03	.05	.02	.05	.02
Product 2																
S1	.16	.05	.14	.05	.14	.06	.15	.05	.12	.04	.10	.03	.05	.03	.06	.03
S2	.19	.10	.09	.04	.16	.05	.11	.09	.10	.04	.06	.03	.06	.05	.05	.03
S3	.18	.11	.18	.09	.18	.05	.16	.05	.09	.05	.10	.04	.09	.02	.05	.04
S4	.18	.10	.14	.07	.12	.05	.17	.05	.13	.05	.06	.05	.05	.03	.02	.01
Product 3																
S1	.11	.05	.10	.06	.08	.03	.06	.04	.09	.04	.08	.04	.05	.03	.05	.02
S2	.14	.09	.13	.06	.15	.09	.14	.04	.10	.04	.08	.05	.05	.03	.06	.04
S3	.20	.08	.19	.06	.18	.07	.16	.08	.16	.06	.15	.09	.05	.04	.04	.02
S4	.13	.06	.14	.07	.10	.05	.14	.06	.08	.05	.10	.04	.05	.04	.04	.03
Product 4																
S1	.08	.03	.07	.05	.06	.03	.07	.04	.05	.02	.05	.03	.05	.03	.06	.03
S2	.09	.05	.09	.04	.07	.05	.08	.04	.06	.03	.07	.03	.04	.01	.05	.02
S3	.11	.04	.12	.05	.07	.04	.06	.04	.05	.04	.06	.05	.04	.02	.03	.02
S4	.06	.04	.06	.05	.06	.04	.07	.03	.04	.03	.03	.02	.03	.01	.04	.02
Product 5																
S1	.12	.06	.13	.04	.08	.04	.09	.05	.11	.04	.09	.04	.06	.03	.07	.04
S2	.13	.06	.14	.08	.10	.06	.15	.04	.12	.05	.09	.04	.08	.03	.08	.03
S3	.21	.09	.15	.07	.14	.07	.15	.07	.19	.06	.20	.09	.15	.05	.12	.09
S4	.12	.03	.14	.06	.09	.03	.14	.04	.10	.05	.09	.06	.07	.03	.05	.03

Table 9: Listenership Profile Matrix in Radio for Time Period 1

Time Period 2																
Segment	CH1				CH2				CH3				CH4			
	CR1		CR2		CR1		CR2		CR1		CR2		CR1		CR2	
	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT
Product 1																
S1	.14	.07	.12	.07	.13	.04	.11	.07	.08	.03	.07	.03	.09	.03	.06	.02
S2	.12	.05	.13	.06	.1	.05	.12	.05	.09	.04	.08	.03	.05	.02	.07	.03
S3	.16	.08	.18	.07	.19	.09	.15	.07	.13	.05	.14	.08	.07	.03	.09	.04
S4	.11	.06	.15	.05	.15	.08	.16	.08	.11	.05	.08	.02	.06	.02	.07	.02
Product 2																
S1	.15	.06	.13	.05	.13	.07	.14	.06	.12	.05	.1	.02	.07	.02	.09	.03
S2	.17	.11	.11	.04	.17	.06	.13	.1	.09	.06	.08	.04	.08	.04	.06	.03
S3	.15	.1	.16	.09	.18	.04	.17	.04	.1	.07	.11	.04	.11	.04	.05	.03
S4	.16	.09	.15	.06	.11	.06	.16	.06	.14	.04	.07	.03	.06	.05	.04	.02
Product 3																
S1	.13	.06	.11	.04	.09	.04	.08	.06	.1	.06	.09	.04	.07	.04	.08	.04
S2	.12	.05	.14	.05	.14	.1	.13	.05	.11	.05	.1	.06	.06	.03	.06	.04
S3	.16	.07	.17	.06	.19	.06	.14	.07	.14	.07	.14	.08	.08	.03	.05	.03
S4	.14	.05	.12	.04	.12	.06	.13	.05	.09	.05	.09	.04	.1	.05	.07	.04
Product 4																
S1	.1	.05	.08	.04	.09	.05	.09	.05	.07	.02	.07	.03	.06	.03	.06	.04
S2	.08	.04	.09	.04	.08	.03	.1	.06	.06	.02	.09	.03	.08	.02	.07	.03
S3	.11	.06	.13	.07	.1	.05	.07	.04	.07	.04	.08	.04	.06	.02	.05	.02
S4	.08	.06	.1	.05	.1	.04	.08	.03	.05	.02	.05	.03	.04	.01	.06	.02
Product 5																
S1	.13	.07	.11	.05	.09	.04	.11	.05	.09	.03	.08	.03	.08	.04	.1	.05
S2	.11	.06	.12	.06	.11	.05	.14	.06	.13	.07	.1	.06	.09	.04	.08	.04
S3	.18	.08	.13	.07	.12	.06	.13	.07	.17	.09	.18	.1	.12	.06	.13	.07
S4	.12	.07	.15	.07	.1	.05	.12	.05	.09	.04	.11	.06	.09	.04	.07	.03

Table 10: Listenership Profile Matrix in Radio for Time Period 2

Time Period 3																
Segment	CH1				CH2				CH3				CH4			
	CR1		CR2		CR1		CR2		CR1		CR2		CR1		CR2	
	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT
Product 1																
S1	.13	.06	.11	.06	.17	.04	.15	.07	.13	.08	.07	.05	.08	.05	.06	.03
S2	.11	.05	.10	.05	.09	.05	.08	.07	.15	.08	.16	.07	.10	.06	.09	.06
S3	.15	.08	.19	.10	.13	.06	.12	.09	.11	.08	.12	.09	.08	.03	.08	.03
S4	.13	.05	.11	.04	.10	.07	.15	.07	.11	.05	.08	.02	.06	.02	.07	.02
Product 2																
S1	.14	.05	.11	.04	.16	.08	.15	.07	.14	.05	.12	.05	.06	.03	.08	.05
S2	.11	.06	.09	.06	.10	.06	.10	.07	.16	.07	.15	.08	.09	.06	.11	.05
S3	.17	.11	.18	.06	.12	.05	.10	.08	.09	.04	.09	.05	.09	.04	.08	.05
S4	.11	.05	.10	.05	.09	.05	.09	.06	.08	.04	.06	.03	.08	.03	.14	.07
Product 3																
S1	.11	.06	.09	.05	.15	.07	.16	.06	.12	.06	.09	.04	.10	.05	.10	.05
S2	.09	.05	.08	.05	.09	.05	.10	.08	.13	.07	.13	.07	.11	.06	.10	.04
S3	.17	.06	.18	.06	.12	.05	.12	.07	.09	.06	.08	.07	.09	.05	.08	.04
S4	.09	.06	.08	.05	.08	.03	.07	.03	.09	.05	.07	.04	.15	.07	.13	.06
Product 4																
S1	.09	.04	.07	.04	.12	.05	.13	.05	.09	.04	.08	.03	.05	.03	.07	.03
S2	.09	.04	.01	.04	.08	.06	.09	.06	.12	.06	.11	.05	.09	.05	.01	.05
S3	.12	.05	.08	.04	.10	.04	.10	.04	.06	.05	.06	.05	.07	.04	.05	.03
S4	.07	.05	.08	.05	.08	.04	.07	.03	.05	.03	.04	.03	.04	.02	.05	.03
Product 5																
S1	.09	.05	.09	.05	.14	.06	.13	.05	.10	.06	.10	.04	.07	.04	.09	.03
S2	.09	.04	.08	.06	.10	.05	.09	.05	.15	.05	.13	.06	.12	.06	.13	.05
S3	.19	.04	.14	.05	.08	.04	.09	.06	.07	.05	.15	.05	.07	.06	.09	.05
S4	.10	.06	.12	.06	.09	.05	.08	.04	.08	.04	.09	.04	.10	.05	.17	.04

Table 11: Listenership Profile Matrix in Radio for Time Period 3

	CR 1	CR 2
P1	0.65	0.35
P2	0.6	0.4
P3	0.36	0.64
P4	0.3	0.7
P5	0.55	0.45

Table 12: w_{rp}

	MAF
M1	0.4
M2	0.5
M3	0.2

Table 13: MAF

Product	P1	P4	P3	P2	P5
P1	0	0.01417	0.02522	0.009638	0.007157
P2	0.00442	0	0.0152	0.00946	0.006278
P3	0.009856	0.01181	0	0.008873	0.00493
P4	0.000149	0.000171	0.000132	0	0.001101
P5	0.037	0.02198	0.01902	0.0113	0

Table 14: Cross-Product Effect Matrix

Segment	Time Period 2				Time Period 3			
	NP1		NP2		NP1		NP2	
	FP	OP	FP	OP	FP	OP	FP	OP
Product 1								
S1	8	4	6	3	8	4	7	3
S2	7	2	5	1	7	3	4	1
S3	7	3	6	2	8	4	5	1
S4	6	2	4	1	5	2	7	2
Product 2								
S1	7	3	5	2	7	3	6	2
S2	4	1	3	1	6	2	2	1
S3	6	2	5	1	6	2	5	1
S4	5	1	4	2	5	1	6	2
Product 3								
S1	6	2	4	2	7	3	5	2
S2	5	2	4	1	6	2	3	1
S3	4	1	4	1	5	2	3	1
S4	4	1	3	2	3	1	6	2
Product 4								
S1	2	1	2	2	2	1	2	2
S2	1	1	1	1	2	1	1	1
S3	2	1	2	1	2	1	1	1
S4	2	1	1	1	2	1	2	2
Product 5								
S1	4	2	3	2	4	2	4	2
S2	4	1	3	1	4	2	2	1
S3	4	2	3	1	4	2	2	1
S4	3	1	2	1	3	1	4	1

Table 15: Retention Factor in NP (%)

Segment	Time Period 2				Time Period 3			
	CH1		CH2		CH1		CH2	
	PT	OT	PT	OT	PT	OT	PT	OT
Product 1								
S1	9	4	7	3	7	3	9	5
S2	8	3	7	2	8	3	4	2
S2	6	3	5	2	8	3	9	4
S2	6	2	5	2	8	2	5	2
Product 2								
S1	8	3	5	2	6	3	7	3
S2	7	3	6	2	8	3	5	2
S3	5	2	4	1	8	3	6	3
S4	5	2	4	1	5	2	4	1
Product 3								
S1	6	2	4	1	4	2	6	3
S2	6	2	5	1	6	2	4	1
S3	4	1	3	2	5	2	6	2
S4	3	1	3	1	5	2	5	1
Product 4								
S1	2	1	1	1	2	1	2	1
S2	2	1	1	1	2	1	1	2
S3	1	1	1	1	2	2	2	1
S4	1	1	1	1	1	1	1	1
Product 5								
S1	3	2	2	1	2	1	3	1
S2	3	1	2	1	4	2	2	1
S3	3	1	2	1	4	2	3	1
S4	3	2	2	1	3	2	2	1

Table 16: Retention Factor in TV (%)

Segment	Time Period 2								Time Period 3							
	CH1		CH2		CH3		CH4		CH1		CH2		CH3		CH4	
	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT
Product 1																
S1	8	3	7	2	6	2	5	2	6	5	8	3	7	4	4	3
S2	9	4	8	3	8	2	6	2	6	3	5	3	8	4	7	2
S3	8	4	7	2	6	1	5	2	8	4	6	2	7	3	6	3
S4	7	3	6	2	5	1	6	3	5	3	5	2	3	2	7	5
Product 2																
S1	7	3	5	2	4	2	4	1	7	3	6	3	6	3	4	1
S2	7	3	6	2	5	1	4	1	6	2	5	2	6	2	4	1
S3	6	2	5	1	4	1	3	1	7	3	4	2	6	3	3	2
S4	6	2	5	1	4	2	5	2	7	2	7	4	4	2	5	3
Product 3																
S1	6	2	5	1	3	1	3	1	6	3	5	3	6	3	3	2
S2	6	2	6	1	5	1	5	2	6	2	6	1	6	3	5	2
S3	5	2	4	2	3	1	4	1	6	2	3	1	4	1	6	3
S4	5	1	4	1	3	1	4	1	4	2	4	1	3	1	5	2
Product 4																
S1	2	1	1	1	1	1	1	1	2	1	3	2	2	1	1	1
S2	2	2	1	2	1	1	2	2	1	1	1	2	2	2	3	2
S3	2	1	1	1	1	1	1	1	3	2	1	1	2	1	2	1
S4	2	1	1	1	1	2	1	1	1	1	1	1	1	2	2	2
Product 5																
S1	4	1	3	2	3	1	2	1	4	2	4	2	3	2	2	1
S2	4	2	4	1	3	1	4	2	4	2	4	1	3	2	5	2
S3	3	2	3	1	2	1	1	1	4	3	2	3	5	2	3	1
S4	3	1	3	2	2	1	2	1	2	1	2	2	1	1	3	2

Table 17: Retention Factor in Radio (%)

Segment	Time Period 1				Time Period 2				Time Period 3			
	NP1		NP2		NP1		NP2		NP1		NP2	
	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP
Product 1												
S1	[2, 31]	[13, 90]	[1, 23]	[7, 94]	[3, 32]	[12, 98]	[2, 25]	[8, 90]	[4, 40]	[15, 102]	[3, 45]	[9, 80]
S2	[2, 27]	[15, 96]	[1, 20]	[5, 47]	[3, 29]	[12, 98]	[2, 19]	[6, 45]	[4, 37]	[14, 98]	[2, 25]	[5, 67]
S3	[1, 16]	[11, 70]	[1, 23]	[7, 70]	[2, 19]	[8, 76]	[1, 19]	[6, 68]	[4, 24]	[9, 80]	[1, 19]	[5, 68]
S4	[2, 31]	[9, 105]	[2, 27]	[4, 99]	[3, 25]	[10, 100]	[1, 34]	[4, 102]	[3, 30]	[10, 85]	[3, 40]	[5, 102]
Product 2												
S1	[2, 17]	[11, 76]	[1, 13]	[5, 51]	[3, 18]	[12, 78]	[2, 13]	[6, 52]	[5, 24]	[14, 82]	[3, 15]	[8, 56]
S2	[1, 11]	[7, 72]	[1, 6]	[4, 42]	[2, 14]	[8, 75]	[2, 5]	[5, 40]	[4, 16]	[9, 78]	[2, 10]	[4, 58]
S3	[2, 11]	[5, 68]	[1, 11]	[7, 59]	[2, 12]	[6, 70]	[2, 12]	[8, 60]	[3, 20]	[7, 70]	[2, 12]	[6, 60]
S4	[0, 13]	[9, 68]	[1, 15]	[5, 63]	[3, 14]	[10, 68]	[2, 16]	[8, 64]	[2, 18]	[10, 65]	[4, 12]	[9, 58]
Product 3												
S1	[2, 18]	[7, 71]	[2, 12]	[4, 47]	[2, 18]	[8, 72]	[2, 12]	[3, 50]	[3, 28]	[9, 75]	[3, 28]	[3, 50]
S2	[2, 18]	[5, 47]	[2, 12]	[2, 41]	[2, 19]	[6, 50]	[2, 13]	[4, 48]	[4, 24]	[8, 58]	[2, 15]	[4, 50]
S3	[2, 15]	[7, 53]	[2, 7]	[4, 35]	[3, 16]	[8, 55]	[2, 9]	[3, 38]	[4, 24]	[9, 56]	[2, 15]	[3, 34]
S4	[2, 21]	[5, 71]	[2, 18]	[4, 53]	[2, 20]	[6, 75]	[2, 20]	[5, 55]	[2, 18]	[5, 56]	[3, 25]	[6, 78]
Product 4												
S1	[1, 6]	[4, 25]	[1, 4]	[2, 17]	[1, 6]	[4, 26]	[0, 4]	[2, 18]	[3, 12]	[5, 28]	[1, 8]	[2, 24]
S2	[1, 8]	[4, 30]	[1, 6]	[2, 13]	[1, 9]	[4, 32]	[0, 6]	[2, 13]	[3, 12]	[5, 20]	[1, 8]	[2, 34]
S3	[1, 6]	[2, 21]	[1, 4]	[2, 17]	[1, 7]	[1, 22]	[1, 5]	[2, 17]	[3, 12]	[6, 25]	[1, 8]	[2, 15]
S4	[1, 6]	[4, 25]	[1, 4]	[4, 21]	[1, 7]	[3, 26]	[1, 4]	[3, 22]	[1, 6]	[3, 20]	[1, 10]	[3, 28]
Product 5												
S1	[1, 8]	[4, 34]	[1, 4]	[2, 34]	[1, 8]	[4, 34]	[1, 4]	[3, 34]	[1, 12]	[4, 38]	[1, 5]	[3, 30]
S2	[1, 8]	[2, 30]	[2, 4]	[4, 25]	[1, 8]	[2, 32]	[1, 4]	[4, 28]	[1, 12]	[3, 38]	[1, 5]	[2, 24]
S3	[1, 8]	[4, 25]	[1, 4]	[2, 21]	[0, 8]	[5, 27]	[1, 4]	[3, 24]	[0, 12]	[5, 34]	[1, 6]	[3, 25]
S4	[2, 8]	[2, 30]	[1, 4]	[4, 21]	[2, 8]	[1, 33]	[1, 4]	[3, 22]	[2, 9]	[1, 30]	[1, 15]	[3, 32]

Table 18: Upper and Lower Bounds on Advertisements in Newspaper

Segment	Time Period 1				Time Period 2				Time Period 3			
	CH1		CH2		CH1		CH2		CH1		CH2	
	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT
Product 1												
S1	[4,63]	[2,28]	[3,50]	[2,21]	[4,63]	[2,28]	[4,60]	[2,21]	[7,60]	[3,26]	[8,75]	[2,30]
S2	[4,59]	[1,25]	[4,59]	[3,28]	[4,60]	[3,27]	[3,60]	[2,28]	[8,62]	[3,34]	[6,56]	[2,28]
S3	[1,55]	[0,31]	[3,50]	[2,32]	[3,50]	[2,29]	[2,49]	[1,34]	[4,45]	[1,26]	[2,54]	[2,32]
S4	[4,67]	[2,35]	[2,42]	[1,25]	[5,64]	[2,35]	[3,42]	[1,22]	[5,60]	[1,30]	[5,45]	[2,35]
Product 2												
S1	[8,67]	[6,32]	[7,53]	[3,27]	[9, 66]	[6, 28]	[8, 52]	[4, 24]	[11, 56]	[5, 24]	[11, 68]	[7, 30]
S2	[7,56]	[4,25]	[6,42]	[2,17]	[8, 56]	[5, 24]	[6, 42]	[3, 16]	[9, 59]	[5, 26]	[6, 40]	[3, 16]
S3	[8,45]	[3,17]	[5,39]	[4,14]	[9, 42]	[4, 15]	[6, 37]	[4, 14]	[5, 35]	[4, 14]	[10, 50]	[3, 20]
S4	[8,62]	[5,21]	[8,36]	[3,18]	[8, 60]	[5, 21]	[8, 36]	[3, 16]	[10, 56]	[4, 24]	[8, 40]	[5, 15]
Product 3												
S1	[8,67]	[4,35]	[4,50]	[2,17]	[8, 65]	[4, 35]	[5, 50]	[2, 14]	[7, 54]	[3, 18]	[9, 67]	[5, 36]
S2	[6,62]	[3,21]	[3,53]	[2,22]	[8, 61]	[3, 21]	[3, 53]	[3, 22]	[10, 67]	[4, 24]	[3, 56]	[2, 15]
S3	[4,53]	[2,28]	[2,34]	[2,14]	[6, 53]	[2, 28]	[3, 31]	[2, 14]	[4, 34]	[2, 15]	[7, 58]	[4, 30]
S4	[8,67]	[4,28]	[4,56]	[2,25]	[8, 60]	[4, 28]	[4, 56]	[2, 23]	[6, 64]	[3, 26]	[6, 56]	[4, 26]
Product 4												
S1	[1, 20]	[0, 8]	[0, 14]	[0, 6]	[2, 18]	[0, 9]	[1, 14]	[0, 8]	[2, 15]	[0, 8]	[2, 20]	[1, 10]
S2	[1, 22]	[0, 10]	[0, 11]	[0, 4]	[2, 22]	[1, 8]	[0, 11	[1, 4]	[2, 24]	[1, 8]	[1, 12]	[1, 4]
S3	[0, 13]	[0, 7]	[1, 8]	[0, 6]	[1, 11]	[0, 7]	[1, 8]	[0, 5]	[2, 20]	[0, 5]	[1, 10]	[0, 7]
S4	[1, 17]	[0, 8]	[0, 15]	[0, 7]	[1, 16]	[1, 8]	[1, 14]	[0, 7]	[1, 28]	[1, 12]	[1, 15]	[0, 8]
Product 5												
S1	[4, 56]	[2, 22]	[4, 52]	[2, 20]	[5, 56]	[2, 24]	[4, 48]	[3, 18]	[6, 48]	[2, 18]	[6, 56]	[3, 24]
S2	[4, 53]	[0, 21]	[3, 38]	[0, 10]	[4, 52]	[1, 21]	[5, 34]	[2, 9]	[5, 52]	[2, 21]	[4, 34]	[2, 12]
S3	[4, 39]	[0, 20]	[3, 29]	[0, 8]	[5, 38]	[1, 19]	[5, 28]	[1, 8]	[5, 28]	[1, 12]	[6, 40]	[2, 19]
S4	[4, 50]	[2, 24]	[4, 39]	[2, 17]	[4, 48]	[2, 22]	[3, 38]	[2, 15]	[4, 48]	[3, 25]	[3, 40]	[1, 18]

Table 19: Upper and Lower Bounds on Advertisements in Television

Segment	CH1		CH2		CH3		CH4	
	PT	OT	PT	OT	PT	OT	PT	OT
Product 1								
S1	[28,166]	[12,72]	[24,130]	[10,48]	[20,115]	[8,43]	[12,108]	[4,50]
S2	[24,173]	[10,72]	[15,158]	[5,72]	[12,130]	[5,58]	[10,104]	[2,40]
S3	[12,115]	[4,68]	[10,104]	[4,50]	[8,90]	[4,43]	[6,76]	[3,32]
S4	[14,137]	[3,58]	[8,122]	[4,47]	[6,112]	[2,36]	[8,86]	[2,25]
Product 2								
S1	[28,151]	[12,49]	[24,122]	[10,49]	[18,112]	[8,43]	[10,96]	[4,50]
S2	[20,137]	[10,72]	[15,115]	[5,65]	[10,90]	[5,55]	[8,79]	[2,40]
S3	[10,115]	[4,68]	[10,91]	[4,43]	[8,79]	[4,40]	[6,65]	[3,27]
S4	[14,115]	[3,58]	[8,115]	[4,47]	[6,104]	[2,36]	[8,83]	[2,25]
Product 3								
S1	[26,144]	[12,65]	[20,122]	[10,40]	[16,101]	[9,43]	[10,89]	[4,52]
S2	[24,126]	[10,72]	[15,115]	[5,65]	[10,90]	[5,55]	[6,65]	[2,40]
S3	[8,92]	[4,68]	[10,86]	[4,43]	[8,79]	[4,40]	[6,52]	[3,47]
S4	[10,115]	[3,58]	[10,94]	[4,47]	[10,101]	[2,36]	[4,61]	[2,25]
Product 4								
S1	[22,130]	[8,65]	[18,108]	[6,50]	[15,104]	[9,53]	[10,79]	[4,18]
S2	[20,112]	[6,50]	[14,79]	[5,43]	[8,83]	[5,43]	[6,58]	[2,35]
S3	[14,95]	[4,68]	[10,76]	[4,43]	[6,58]	[4,40]	[6,95]	[2,22]
S4	[20,94]	[3,50]	[12,86]	[4,50]	[10,65]	[2,35]	[4,56]	[2,25]
Product 5								
S1	[20,130]	[12,61]	[14,104]	[10,54]	[12,94]	[9,46]	[8,79]	[4,25]
S2	[15,119]	[6,65]	[8,86]	[3,43]	[6,90]	[5,43]	[6,65]	[2,35]
S3	[8,104]	[4,68]	[10,83]	[4,43]	[6,96]	[4,40]	[4,53]	[3,22]
S4	[10,115]	[3,58]	[10,104]	[4,47]	[10,95]	[2,36]	[4,61]	[2,24]

Table 20: Upper and Lower Bounds on Advertisements in Radio for Time Period 1

Segment	CH1		CH2		CH3		CH4	
	PT	OT	PT	OT	PT	OT	PT	OT
Product 1								
S1	[30,153]	[12,71]	[24,126]	[10,45]	[21,87]	[9,46]	[12,83]	[5,38]
S2	[26,162]	[10,72]	[16,149]	[6,54]	[12,85]	[6,44]	[9,71]	[3,30]
S3	[13,112]	[4,65]	[11,89]	[5,38]	[8,71]	[3,33]	[5,72]	[3,41]
S4	[15,141]	[3,61]	[8,101]	[4,47]	[5,72]	[3,38]	[7,59]	[3,27]
Product 2								
S1	[29,141]	[12,63]	[25,99]	[9,37]	[17,108]	[7,61]	[9,81]	[3,45]
S2	[22,119]	[11,59]	[15,108]	[5,49]	[8,99]	[4,51]	[8,78]	[2,40]
S3	[11,108]	[5,52]	[11,81]	[5,33]	[6,71]	[3,30]	[6,71]	[3,31]
S4	[15,112]	[5,44]	[7,87]	[4,36]	[8,89]	[3,41]	[9,63]	[2,29]
Product 3								
S1	[26,137]	[13,61]	[21,117]	[11,61]	[15,104]	[8,49]	[8,85]	[4,39]
S2	[25,115]	[11,49]	[16,108]	[8,49]	[9,77]	[4,27]	[5,63]	[2,30]
S3	[10,81]	[4,44]	[11,81]	[4,33]	[7,71]	[3,30]	[6,40]	[3,36]
S4	[12,87]	[6,32]	[12,79]	[6,36]	[9,92]	[4,27]	[5,51]	[2,19]
Product 4								
S1	[23,125]	[10,60]	[16,98]	[4,45]	[14,87]	[8,46]	[8,72]	[4,38]
S2	[22,130]	[8,54]	[12,119]	[5,54]	[6,78]	[5,44]	[5,54]	[2,30]
S3	[16,87]	[8,63]	[8,79]	[4,38]	[6,54]	[4,33]	[6,57]	[2,25]
S4	[20,103]	[10,50]	[10,92]	[4,47]	[8,56]	[2,27]	[3,54]	[2,27]
Product 5								
S1	[22,122]	[11,63]	[12,89]	[9,37]	[12,84]	[8,45]	[8,73]	[4,38]
S2	[16,112]	[6,54]	[7,87]	[2,49]	[5,72]	[2,42]	[5,65]	[2,31]
S3	[10,97]	[3,52]	[8,81]	[3,33]	[6,71]	[4,30]	[4,49]	[3,21]
S4	[12,112]	[5,44]	[10,87]	[4,36]	[10,79]	[2,27]	[4,63]	[2,29]

Table 21: Upper and Lower Bounds on Advertisements in Radio for Time Period 2

Segment	CH1		CH2		CH3		CH4	
	PT	OT	PT	OT	PT	OT	PT	OT
Product 1								
S1	[24,90]	[9,40]	[32,154]	[14,84]	[26,130]	[10,64]	[12,80]	[6,40]
S2	[20,120]	[3,45]	[16,70]	[5,30]	[26,165]	[7,80]	[24,150]	[3,50]
S3	[14,115]	[4,40]	[12,90]	[6,67]	[10,65]	[3,30]	[6,75]	[4,38]
S4	[16,145]	[4,62]	[9,100]	[5,45]	[6,72]	[3,38]	[12,116]	[4,55]
Product 2								
S1	[18,98]	[7,33]	[30,142]	[9,56]	[25,104]	[8,40]	[9,86]	[4,33]
S2	[16,106]	[9,48]	[14,90]	[5,35]	[24,120]	[12,60]	[20,110]	[10,55]
S3	[11,110]	[6,52]	[12,84]	[5,44]	[7,74]	[4,34]	[7,67]	[4,30]
S4	[16,125]	[6,65]	[8,90]	[4,48]	[9,70]	[4,34]	[13,120]	[5,50]
Product 3								
S1	[19,98]	[10,52]	[28,134]	[13,68]	[25,100]	[11,55]	[10,90]	[4,48]
S2	[15,90]	[11,32]	[14,75]	[8,30]	[24,120]	[11,56]	[17,100]	[9,49]
S3	[12,90]	[5,40]	[11,84]	[5,56]	[8,78]	[4,32]	[7,86]	[4,30]
S4	[14,78]	[6,50]	[12,56]	[6,34]	[9,52]	[4,40]	[12,65]	[6,46]
Product 4								
S1	[20,78]	[6,40]	[24,127]	[10,67]	[22,98]	[8,48]	[17,70]	[4,30]
S2	[14,90]	[9,38]	[10,67]	[5,30]	[22,135]	[6,60]	[18,110]	[6,45]
S3	[16,90]	[9,47]	[8,78]	[4,32]	[6,65]	[5,38]	[7,54]	[3,27]
S4	[20,102]	[11,50]	[11,78]	[5,34]	[9,67]	[2,30]	[14,89]	[8,42]
Product 5								
S1	[12,92]	[8,46]	[24,124]	[9,50]	[20,118]	[9,38]	[10,89]	[4,44]
S2	[7,90]	[3,46]	[5,38]	[3,24]	[7,120]	[3,68]	[6,79]	[2,40]
S3	[12,98]	[4,42]	[8,72]	[4,28]	[7,64]	[5,48]	[5,54]	[4,38]
S4	[13,95]	[6,58]	[10,67]	[4,38]	[10,50]	[2,29]	[11,80]	[4,45]

Table 22: Upper and Lower Bounds on Advertisements in Radio for Time Period 3

Segment	Newspaper				Television				Radio							
	NP1		NP2		CH1		CH2		CH1		CH2		CH3		CH4	
	FP	OP	FP	OP	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT
Product 1																
S1	31	90	23	94	63	2	3	2	166	72	130	48	115	43	108	50
S2	27	96	20	47	59	1	4	3	173	72	158	72	130	58	104	2
S3	16	70	23	70	55	31	50	32	115	68	104	50	90	43	76	3
S4	31	105	27	99	26	2	2	1	137	58	122	42	112	36	86	2
Product 2																
S1	17	76	13	51	67	6	7	3	151	49	122	49	112	43	96	50
S2	11	72	6	42	56	25	6	2	137	72	115	65	90	55	79	40
S3	11	68	11	59	45	17	39	14	115	68	91	43	79	40	65	3
S4	13	68	15	63	62	5	36	3	115	58	115	47	104	36	83	2
Product 3																
S1	18	71	12	47	67	4	4	2	144	65	122	40	101	43	89	52
S2	18	47	12	41	62	21	53	2	126	72	115	65	90	55	65	4
S3	15	53	7	35	53	2	34	14	92	68	86	43	79	40	52	3
S4	21	71	18	53	8	4	4	2	115	58	94	47	101	36	61	25
Product 4																
S1	6	25	4	17	20	0	14	6	130	65	108	50	104	53	79	18
S2	8	30	6	13	22	10	11	4	112	50	79	43	83	43	58	35
S3	6	21	4	17	13	7	8	6	95	68	76	43	58	40	95	22
S4	6	25	4	21	17	8	0	0	94	50	86	50	65	35	56	25
Product 5																
S1	8	34	4	34	56	2	52	2	130	61	104	54	94	46	79	25
S2	8	30	4	4	53	21	3	0	119	65	86	43	90	43	65	35
S3	8	25	4	21	39	20	29	8	104	68	83	43	96	40	53	22
S4	8	30	4	21	50	2	4	2	115	58	104	47	95	36	61	24

Table 23: Optimal Number of Insertions in Each Media in Time Period 1

Segment	Newspaper				Television				Radio							
	NP1		NP2		CH1		CH2		CH1		CH2		CH3		CH4	
	FP	OP	FP	OP	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT
Product 1																
S1	32	98	25	90	4	2	60	2	153	71	126	10	87	9	83	5
S2	29	98	19	45	60	3	3	2	162	72	149	54	85	44	9	3
S3	19	76	1	68	3	2	2	1	112	4	89	5	8	3	5	3
S4	25	100	34	102	5	2	3	1	141	61	19	4	5	3	7	3
Product 2																
S1	18	78	13	52	66	6	52	4	141	63	99	37	108	7	81	3
S2	14	75	5	40	56	5	6	3	119	59	108	49	99	51	78	2
S3	12	70	2	8	42	4	37	4	108	52	81	5	6	3	71	3
S4	14	68	16	64	8	5	8	3	112	44	87	4	8	3	9	2
Product 3																
S1	18	72	12	50	8	4	50	2	137	61	117	61	104	49	85	39
S2	19	50	13	48	61	3	3	3	115	49	108	49	77	27	63	2
S3	16	55	2	3	53	2	31	2	81	4	81	33	71	3	7	3
S4	20	75	20	55	8	4	4	2	87	32	79	6	9	4	51	2
Product 4																
S1	6	26	4	18	18	9	14	8	125	60	98	45	87	46	72	38
S2	9	32	6	13	22	8	11	4	130	54	119	54	78	44	54	30
S3	7	22	5	17	11	7	8	5	87	63	79	38	54	33	57	2
S4	7	26	4	22	16	1	14	0	103	50	92	47	8	27	54	27
Product 5																
S1	8	34	4	34	35	2	48	3	122	63	89	37	84	8	73	38
S2	8	32	4	28	52	1	5	2	112	54	87	49	72	42	65	2
S3	8	27	4	24	38	1	28	1	97	52	81	33	71	30	49	21
S4	8	33	4	22	4	2	38	2	112	44	87	4	10	27	63	2

Table 24: Optimal Number of Insertions in Each Media in Time Period 2

Segment	Newspaper				Television				Radio							
	NP1		NP2		CH1		CH2		CH1		CH2		CH3		CH4	
	FP	OP	FP	OP	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT	PT	OT
Product 1																
S1	40	102	45	9	7	3	75	2	90	40	154	14	130	10	80	6
S2	22	98	25	67	62	3	6	2	120	3	16	5	165	80	24	3
S3	24	9	1	5	4	1	2	2	115	4	12	6	10	3	6	4
S4	3	10	3	5	5	1	5	2	145	4	9	5	6	3	12	4
Product 2																
S1	24	82	15	56	11	5	68	7	98	33	142	56	104	8	86	4
S2	16	9	10	58	59	5	6	3	106	48	14	5	120	60	110	10
S3	20	70	2	6	5	4	50	3	110	6	84	5	7	4	67	4
S4	2	10	4	9	10	4	8	5	125	65	8	4	9	4	120	5
Product 3																
S1	28	75	28	3	7	3	67	5	98	52	134	68	100	11	90	48
S2	24	8	2	50	67	4	3	2	90	32	14	8	120	56	100	9
S3	24	9	2	3	4	2	58	4	90	5	84	5	8	4	86	4
S4	2	5	25	78	6	3	6	4	78	50	12	6	9	4	65	6
Product 4																
S1	12	28	8	24	15	0	20	10	78	40	127	67	98	48	70	30
S2	12	20	8	34	24	8	12	1	90	38	67	5	135	60	110	45
S3	12	25	8	15	20	0	10	7	90	47	78	32	65	38	54	3
S4	6	20	10	28	28	1	15	0	102	50	78	6	9	30	89	42
Product 5																
S1	12	4	5	3	48	2	56	3	92	46	124	50	118	9	89	44
S2	12	3	5	24	52	2	4	2	90	46	38	3	120	68	79	2
S3	12	34	1	3	28	1	40	2	98	4	72	4	64	48	54	4
S4	2	1	15	3	4	3	3	1	95	58	10	4	10	2	80	4

Table 25: Optimal Number of Insertions in Each Media in Time Period 3