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VENDOR-BUYER COORDINATION AND SUPPLY CHAIN OPTIMIZATION WITH DETERMINISTIC DEMAND FUNCTION

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Abstract: This paper presents a model that deals with a vendor-buyer multi-product, multi-facility and multi-customer location selection problem, which subsume a set of manufacturer with limited production capacities situated within a geographical area. We assume that the vendor and the buyer are coordinated by mutually sharing information. We formulate Mixed Integer Linear Fractional Programming (MILFP) model that maximize the ratio of return on investment of the distribution network, and a Mixed Integer Program (MIP), used for the comparison. The performance of the model is illustrated by a numerical example. In addition, product distribution and allocation of different customers along with the sensitivity of the key parameters are analyzed. It can be observed that the increment of the opening cost decreases the profit in both MILFP and MIP models. If the opening cost of a location decreases or increases, the demand and the capacity of that location changes accordingly.

Keywords: Vendor and Buyer, Coordination, Optimization, Deterministic Demand.

MSC: 90B06.

1. INTRODUCTION

In the global competitive market, the importance of Supply Chain Management (SCM) increases day by day. To maximize the profit and minimize the cost are the main goals, so it is important to make the model optimal for both consumer and a manufacturer. An efficient supply chain system operates under a strategy to minimize costs by integrating the different functions inside the system and by meeting customer demands in time.

A vast amount of literature available on SCM research, was dealing with the different aspects of the subject. Numerous models, conceptual as well as quantitative, refer to planning and quantitative aspects of different business functions location, production, inventory and transportation considering these areas for combined optimization. Proposed models include combination of two, or more of these areas. Facility Location Problems (FLP), which are typically used to design distribution networks, involve determining the sites to install resources, as well as the assignment of potential consumers to those resources. Drezner et al.[1] briefly described FLP the location of manufacturing plants, the assignment of ware houses to these plants, and finally the assignment of retailers to each warehouse. Other than geographical boundaries, Hung et al. [2] described the location allocation with balancing requirements among Distribution Centre (DC). They formulated a bi-level programming model to minimize the total cost of the distribution network, and balanced the work load of each DC for the delivery of products to its customer, solving the model by the genetic algorithm.

Further, considering customer's responsiveness, a two-echelon distribution network was modeled by Azad et al.[3], and a hybrid heuristic, combining Tabu search with Simulated Annealing (SA) sharing the same tabu list, was developed for solving the problem by Azad. In addition, a two-echelon inventory system was explained by Jakor and Seifbarghy [4], where the independent Poisson demand with constant transportation and lead time were considered. Finally, they developed an approximate cost function to find the optimal reorder points for given batch sizes in all installations and accuracy was assessed by simulation. Moreover, Nagurney [5] derived a relationship between supply chain network equilibrium and transportation network equilibrium.

Jose et al. [6] presented mixed integer type linear programming to solve a capacitated vehicle routing problem minimizing number of vehicle and travelling time. They implemented the model to a real life problem of a distribution company and solved it numerically. They obtained a feasible solution to the formulated model considering six delivery points with some characteristics. Yamada et al.[7] investigated super network equilibrium model. They combined super network with supply chain networks and transport a network. They considered not only the behavior of freight carriers but also the transport network users, and determined the transport costs generated in the supply chain networks. They also investigated the interaction between transport networks and supply chain networks. By numerical example, they showed that by the development of transport network it is possible to improve the efficiency of supply chain networks.

On the other hand, Dhaenens-Flipoand Finke [8] considered an integrated production-distribution problem in multi-facility, multi-product and multi-period environment. They formulated a network flow problem with an objective to match products with production lines to minimize the related costs generated randomly, and solved it by using CPLEX software. Moreover, a MIP model for production, transportation, and distribution problem was developed, representing a multi-product tri-echelon capacitated plant and warehouse location problem by Pirkul and Jayaraman [9]. They minimized the sum of fixed costs of operating the plants and warehouses, and the variable costs of transporting multiple products from the plants to the warehouses and finally to the customers. In addition, a solution procedure was provided based on lagrangian relaxation to find the lower bound, followed by a heuristic to solve the problem. There were copious researches on LFP to find the best solution approach.

Among these, Charnes andCooper [10] described a transformation technique, which transforms the Linear Fractional Program (LFP) into equivalent linear program. This method is quite simple but needs to solve two-transformed model to obtain the optimal solution. Fractional programming problems have become a subject of wide interest since they arise in many fields like agricultural planning, financial analysis of a firm, location theory, capital budgeting problem, supply chain, portfolio selection problem, cutting stock problem, stochastic processes problem. From time to time survey papers on applications and algorithms on fractional programming were developed by various authors. In addition, fractional programming has benefited from advances in generalized convexity and vice versa. Further, Charnes and Cooper transformation reduces the linear fractional program into linear program and then an optimal solution to the problem could be obtained easily.

In this study, vendor-buyer multi-product, multi-facility, and multi-customer location production problem is formulated as a MILFP which maximizes the ratio of return on investment, and at the same time optimizes location, transportation cost, and the investment. Further, a MIP model is derived from the same model so that the model determines the sites for vendor and the best allocation for both the buyer and the vendor. Using the suitable transformation of Charnes and Cooper [10], the formulated MILFP was solved by AMPL. Finally, a numerical example along with the sensitivity of opening cost is considered to estimate the performance of the models

As described above, in previous research, the MILFP in vendor buyer system was not considered. Therefore, we believe that effect of coordination among the members, especially between vendor and buyer, should be introduced in the literature. Consequently, we have formulated coordinated vendor and buyer model that could improve the whole system, the individual profitability, the benefit for the end consumers. This integrated coordinated model, allow vendor and buyer to fully cooperate with each other when making decisions to maximize total system profit.

The paper has introduced coordination mechanism along with MILFP in the literature. The main aim is to demonstrate the effect of coordination among the members, especially between vendor and buyer. For each vendor-buyer system studied, we investigate how the cooperation could improve the whole system, the individual profitability, the benefit for the end consumers, and the facility location problem. This integrated coordinated model, enable vendor and buyer to fully cooperate when making decisions that maximize total system profit. It deals with an integrated multi-product, multi-facility and multi-customer problem with deterministic demand function.

The reminder of this paper is organized as follows. In Section 2, a mathematical formulation of the model as MILFP and MIP are presented. The section has four subsections, describing the concept of mixed integer linear fractional programming problem, notations, assumptions, prerequisites, and finally the MILFP and MIP model. In Section 3, a numerical example is considered. In Section 4, the results of these models are discussed. Finally, in Section 5, the conclusions and contributions of this study are discussed

2. MODEL FORMULATION

In this section, we have formulated an integrated model that explores the trade off among the location, transportation cost and distribution, considering a multi-product, multi-facility, and multi-customer location-production-distribution system. Assume that a logistics center seeks to determine an integrated plan of a set of L locations of the vendor with production capability of m products and n buyers destinations as shown in Figure 1.In Figure 1, the solid arrows represent the commodity flow, and the dotted arrows stand for the information flow. Each source has an available supply of the commodity to distribute in various destinations, and each destination has a forecast demand of the commodity to be received from various sources. The coordination contains a set of manufacturing facilities with limited production capacities situated within a geographical area. Each of these facilities can produce one or all of the products in the company's portfolio. The buyer's demands for multiple products are to be satisfied from this set of manufacturing facilities. Therefore, the production capacities of each of the facilities effectively represent its current and potential capacities. This work focuses on developing a MILFP and MIP programs to optimize the capacitated facility location and buyer allocation decisions, and production quantities at these locations to satisfy customer demands.

2.1. Mixed integer linear fractional program

Recently various optimization problems, involving the optimization of the ratio of functions, (for instance; time/cost, volume/cost, profit/cost, loss/cost), measuring the efficiency of the system were the subject of wide interest in non-linear programming problem.

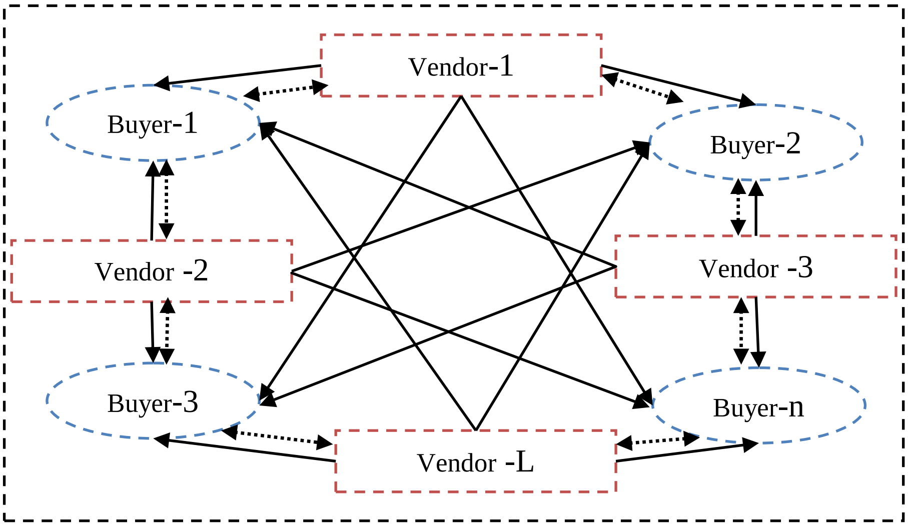

Figure 1: Distribution pattern of a coordinated supply chain

Fractional Programming problem is a mathematical programming problem in which the objective function is the ratio of two functions. If the numerator and denominator of the objective function and the constraints set are linear, then the fractional programming problem is called LFP problem.

Mathematically the LFP problem can be represented as: $Z = \frac{C^T x + \alpha}{D^T x + \beta}$

Subject to

$$x \in X = \{x \in R^n : Ax = B, x \ge 0\}$$

Where,

x is the set of decision variables of $n \times 1$

A is the constraint matrix of order $m \times n$

C and D are the contribution coefficient vector of order $n \times 1$

B is the constant or resource vector of order $m \times n$

α , β are scalar , which determines some constant profit and cost respectively

n and m are the number of variables and constraints respectively.

A MIP problem results when some variables in the model are real valued (can take on fractional values) and some are integer valued, the model is therefore mixed. When the objective function and the constraints set are all linear, then it is MIP. On the other hand, if the problem is of LFP types, then it is called MILFP problem.

Charnes and Coorper Transformation Technique:

There are numerous methods, such as iterative method, parametric method, genetic algorithm technique and fuzzy techniques, available in the literature, to solve LFP problem. In this work, we used the Charnes and Coorper transformation technique. Charnes and Cooper [11] considered the LFP problem defining that

1) The feasible region X is non–empty and bounded,

2) $C\ x + \alpha$ and $D\ x + \beta$ do not vanish simultaneously in X

Introducing the variable transformation y $= t\ x$, *w*here t ≥ 0, Charnes and Cooper proved that LFP problem could be reduced to either of the following two equivalent linear programs.

EQP Maximize, $Z_1 = cy + \alpha t$

Subject to

$Ay + Bt = 0$

$Dy + \beta t = 1$

$y, t \ge 0$

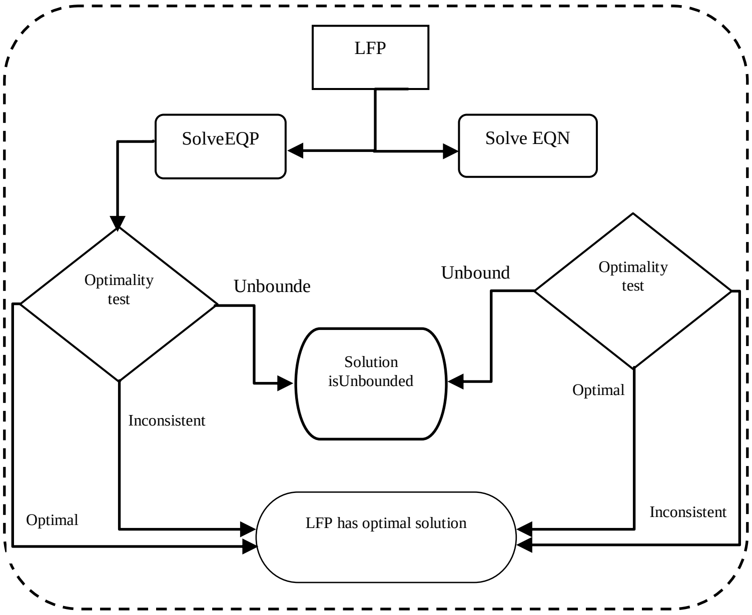

Figure 2: Flow chart for Charnes and Cooper algorithm

$$EQN \quad Maximize, \qquad Z_2 = -Cy - \alpha t$$

Subject to

$$Ay - Bt = 0$$

$$Dy + \beta t = -1$$

$$y, t \geq 0$$

The Equivalent Positive (EQP) or Equivalent Negative (EQN) problems were solved by the well-known Dantzig [11] simplex method. If one of the problems, EQP and EQN, has an optimal solution *(y*, t*)* and the other is inconsistent, then the LFP problem has an optimal solution which can be obtained simply by $x^* = y^* / t^*$. If any of the two problems is unbounded, then the LFP problem is unbounded. So, if the first problem is found unbounded, then one can avoid solving the other as described in Figure 2.

2.2. Notations and assumptions

In order to get the formulation of the model several assumptions, parameters declaration, decision variables and notations are required. In this subsection, we have described the notations, assumptions, parameters declaration and decision variables for the MILFP based vendor-buyer coordinated model. The notations are as follows.

Table 1: Notation for the multiproduct multicustomer and multi-facility vendor-buyer system

Index and Parameters

i	Index for product, for all $i = 1, 2, \dots\dots\dots\dots\dots\dots.., m$.
j	Index for buyer, for all $j = 1, 2, \dots\dots\dots\dots\ .., n$.
l	Index for location of the vendor, for all $l = 1, 2, \dots\dots\dots\dots\ .., L$.
c_{ij}	The price of i^{th} product to j^{th} buyer ($/unit).
α_l	The fixed cost for opening the vendor at location l ($).
ß	Any positive scalar.
c_i^l	The price of unit raw materials for i^{th} product at l^{th}vendor ($/unit).
a_i^l	The amount of raw materials need to producei^{th} product at l^{th}vendor($/unit). .
t_i^l	Unit transportation costof raw materials for i^{th} product at l^{th} vendor ($/unit).
p_{ij}^l	The production cost of i^{th} product to j^{th} buyer at l^{th} vendor ($/unit).
h_{ij}^l	Unit holding cost of i^{th} product from l^{th} vendor to buyer j for some given unit of time ($/unit-time).
cc_{ij}^l	The shipment cost of i^{th} product from l^{th} vendor to j^{th} buyer ($/unit).
d_{ij}	The total demand of i^{th} product by j^{th} buyer (unit).
w_i^l	The capacity for i^{th}product at l^{th} vendor (unit).
t_j^l	The time spent toreach of products from l^{th} vendor to buyer j (unit).
t_j^{*l}	The actual time should required to deliverthe products from l^{th} vendor to buyer j (unit).
p	Penalty cost for delay in delivery for one unit of demand in one unit of time ($/unit).
c_j^{*l}	The transportation cost per unit product from l^{th} vendor to buyer j ($/unit).

Penalty defining function

The function could be defined as

$g^l_j = \begin{cases} 1, \ if \ \ t^l_j > t^{*l}_j, \\ 0, \ else \end{cases}$, where $t_j^l$is the time spent toreach of products from l^{th}vendor to buyer j and $t_j^{*l}$is the actual time should required to deliverthe products from l^{th} vendor to buyer$j$.

Decision Variables

$$y^l_j = \begin{cases} 1, \ if \ \ customer \ \ j \ \ is \ \ assaign \ \ to \ \ manufacturer \ \ l \ , \\ 0, \ else \end{cases}$$

$$x_l = \begin{cases} 1, if \ \ location \ l \ is \ used, \\ 0, else \end{cases}$$

Q_{ij}^l = the production quantity of product i for buyer j at l^{th} vendor (unit).

Assumptions

1. Each manufacturing facility is able to produce all products. The company may have different plants situated at different locations. Each location can produce same types of all the products of the company.

2. The selling price for a product may vary from buyer to buyer depending on the discussions, order sizes, discounts, historical relationships, etc. Although the same inputs are required to produce a product at any plant, the costs required to obtain those inputs may vary for different plants depending on the location of the plant, its distance from the input sources, market rates in that area. As in the case of input costs, the manufacturing costs for the same product also may vary for different plants. This is because these costs depend on factors such as labor rates, overheads, etc. that may vary significantly for each plant. However, the transportation costs may or may not be exactly proportional to the travel times because the transportation costs per unit time per shipment may vary for each plant-customer pair depending on the route conditions, climate conditions, geographical factors, etc. Therefore, sales price for a product may vary from customer to customer.

3. The company and the buyer have agreed beforehand on the inventory distribution pattern so the shipping plans would be formulated accordingly. Production/distribution supply chain is such that the products are manufactured at the plants and shipped to customers in multiple shipments at regular intervals until the demand is satisfied. It is possible to store the whole order and ship it at the end of production. However, this option would incur higher inventory cost for storing a large number of products for a long time. It will also incur penalty costs because the customer would have to wait till the end of production to receive the products. It is assumed here that the customer is ready to accept the products as and when the shipment takes place. The products would be stored in the inventory if the shipment is not possible immediately. There can be different cases of inventory distribution patterns based on the difference between production rate and shipping rate, continuous or intermittent production and/or shipping, and instantaneous or gradual shipping. These patterns will in turn influence the inventory costs, penalty costs and transportation costs. Hence, the player should agree with a certain distribution pattern.

MILFP Model

In this subsection, we have formulated the MILFP considering all prerequisites terms.

The objective function is: $$Maximize = \frac{Z_1}{Z_2} \tag{1}$$

Where,

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L} Q^{l}_{ij} c_{ij} = Z_1$$

$$\sum_{l=1}^{L} x_l \alpha_l + \sum_{l=1}^{L}\sum_{i=1}^{m} c^l{}_i a^l{}_i + \sum_{l=1}^{L}\sum_{i=1}^{m} t^l{}_i \ \ a^l{}_i + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} Q^l{}_{ij} \ \ p^l{}_{ij} + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} Q^l{}_{ij} \ cc^l{}_{ij} +$$

$$\sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} Q^l{}_{ij} \ \ h^l{}_{ij} / 2 + \sum_{i=1}^{m}\sum_{l=}^{L}\sum_{j=1}^{n} p \ \ d_{ij} z^l{}_j \ \ (t^l{}_j - t^{*l}{}_j) \ \ g^l{}_j + \sum_{l=1}^{L}\sum_{j=1}^{n} t^l{}_j \ \ c{*}^l{}_j = Z_2$$

Subject to

$$\sum_{l=1}^{L}\sum_{i=1}^{m} Q^l{}_{ij} = \sum_{i=1}^{m} d_{ij} \ \ , \forall j \tag{2}$$

$$\sum_{l=1}^{L}\sum_{j}^{n} Q^l{}_{ij} = \sum_{j=1}^{n} d_{ij}, \ \forall i \tag{3}$$

$$\sum_{l=1}^{L} Q^l{}_{ij} = d_{ij} \ \ , \ \forall i, j \tag{4}$$

$$\sum_{j=1}^{n} Q^l{}_{ij} \le w^l{}_i \ \ \forall i, l \tag{5}$$

$$\sum_{j=1}^{n}\sum_{i=1}^{m} Q^l{}_{ij} \le \beta \ \ x_l \ , \forall l \tag{6}$$

$$\sum_{l=1}^{L} y^l{}_j = 1, \ \forall j \tag{7}$$

$$\begin{aligned} & Q^l{}_{ij} \ , c_{ij} \ , \alpha_l \ \ , d_{ij}, \ w^l{}_i, cc^l{}_{ij}, \ h^l{}_{ij}, p^l{}_{ij}, t^l{}_i, t^l{}_j, t{*}^l{}_j, p, c^{*l}{}_j, a^l{}_i, c^l{}_i \ge 0, \ x_l \ \ , y^l{}_j \\ & are \quad binary \ \ \forall i, j, l \end{aligned} \tag{8}$$

The objective function (1) estimates the ratio of return and investment. Constraints (2) ensure that the total amount of products being manufactured at all plants for a particular buyer is equal to the total demand of that buyer. Similarly, constraints (3) guarantee that the total amount of a particular product being manufactured at all plants for all buyers is equal to the total demand of that product from all buyers. It is important to note here that the first two constraints are stated separately to show better accountability of the total demands from all buyers and for all products respectively. Constraints (4) assurance that the total amount of a specific product being manufactured for a particular buyer at all plants is equal to the demand of the specific product from that buyer. Constraints (5) present the capacity constraint. Constraints (6) premise that a plant is located when and only if there is a demand for any product. Constraints (7) show that each buyer is assigned to exactly one vendor. The last equation (8) is the nonnegative constraints.

MIP Model

In this subsection, we have formulated the equivalent mixed integer programming problem that estimate the total profit as well as optimal allocation and distribution. The objective function is the difference between return and investment.
The objective function is: $Maximize = Z_1 - Z_2$

Subject to
The set of constraints described in the previous subsection.

3. SOLUTION APPROACH

In order to solve the formulated MILFP, we need to apply suitable transformation. In this section, we have applied the Charnes and Cooper transformation to solve the formulated MILFP as described in subsection (2.1).

For any nonnegative r the new decision could be redefined as follows:

$$z_l = rx_l\,,\ \ for \quad r \geq 0 \quad and \quad l = 1,.....,L$$

$$z^l{}_j = ry^l{}_j\,,\ \ for \quad r \geq 0 \quad and \quad j = 1,....,n\ ,\ l = 1,........,L$$

$$z^l{}_{ij} = rQ^l{}_{ij}\,,\ \ for \quad r \geq 0 \quad and \quad i = 1,....,m,\ j = 1,....,n\ ,\ l = 1,........,L$$

Since $r \geq 0$, y_j^l and x_l are binary; as a result, z_l and z_j^l become either zero or r. Further, since, Q_{ij}^l is non negative, consequently, z_{ij}^l are also remaining non-negative. Therefore, MILFP can be reformulated into two equivalent linear problems as follows:

$$(EQP)\,Maximize \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L} z^l{}_{ij}\,c_{ij}$$

Subject to

$$\sum_{l=1}^{L}\sum_{i=1}^{m} z^l{}_{ij} = r\sum_{i=1}^{m} d_{ij} \quad ,\forall j \tag{9}$$

$$\sum_{l=1}^{L}\sum_{j}^{n} z^l{}_{ij} = r\sum_{j=1}^{n} d_{ij}\,,\ \forall i \tag{10}$$

$$\sum_{l=1}^{L} z^l{}_{ij} = r \quad d_{ij} \quad ,\ \forall i,j \tag{11}$$

$$\sum_{j=1}^{n} z^l{}_{ij} \leq r \quad w^l{}_i \quad \forall i,l \tag{12}$$

$$\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}_{ij} \leq 10000 \quad z_{l} \ ,\forall l \tag{13}$$

$$\sum_{l=1}^{L} z^{l}_{j} = r, \ \forall j \tag{14}$$

$$\sum_{l=1}^{L} z_{l}\alpha_{l} + \sum_{l=1}^{L}\sum_{i=1}^{m} r \quad t^{l}_{i} \quad d_{i} + \sum_{l=1}^{L}\sum_{i=1}^{m} r \quad c^{l}_{i} a^{l}_{i} + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}_{ij} \quad p^{l}_{ij} + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}_{ij} \ cc^{l}_{ij} + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}_{ij} \quad h^{l}_{ij} / 2 + \sum_{i=1}^{m}\sum_{l=}^{L}\sum_{j=1}^{n} p \quad d_{ij} z^{l}_{j} \quad (t^{l}_{j} - t^{*l}_{j}) \quad g^{l}_{j} + \sum_{l=1}^{L}\sum_{j=1}^{n} r \quad t^{l}_{j} \quad c^{*l}_{j} = 1 \tag{15}$$

$$\left(EQN\right) Maximize: \ \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{L} -z^{l}_{ij} c_{ij}$$

Subject to

$$\sum_{l=1}^{L}\sum_{i=1}^{m} z^{l}_{ij} = -r \sum_{i=1}^{m} d_{ij} \ ,\forall j \tag{16}$$

$$\sum_{l=1}^{L}\sum_{j}^{n} z^{l}_{ij} = -r \sum_{j=1}^{n} d_{ij}, \ \forall i \tag{17}$$

$$\sum_{l=1}^{L} z^{l}_{ij} = - \quad r \quad d_{ij} \quad , \ \forall i, j \tag{18}$$

$$\sum_{j=1}^{n} z^{l}_{ij} \leq -r \quad w^{l}_{i} \quad \forall i,l \tag{19}$$

$$\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}_{ij} \leq -10000 \quad z_{l} \ ,\forall l \tag{20}$$

$$\sum_{l=1}^{L} z^{l}_{j} = -r, \ \forall j \tag{21}$$

$$\sum_{l=1}^{L} z_{l}\alpha_{l} + \sum_{l=1}^{L}\sum_{i=1}^{m} r \quad t^{l}_{i} \quad d_{i} + \sum_{l=1}^{L}\sum_{i=1}^{m} r \quad c^{l}_{i} a^{l}_{i} + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}_{ij} \quad p^{l}_{ij} + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}_{ij} \ cc^{l}_{ij} + \sum_{l=1}^{L}\sum_{j=1}^{n}\sum_{i=1}^{m} z^{l}_{ij} \quad h^{l}_{ij} / 2 + \sum_{i=1}^{m}\sum_{l=}^{L}\sum_{j=1}^{n} p \quad d_{ij} z^{l}_{j} \quad (t^{l}_{j} - t^{*l}_{j}) \quad g^{l}_{j} + \sum_{l=1}^{L}\sum_{j=1}^{n} r \quad t^{l}_{j} \quad c^{*l}_{j} = -1 \tag{22}$$

$$z^l_{ij}, c_{ij}, \alpha_l, d_{ij}, w^l_i, cc^l_{ij}, h^l_{ij}, p^l_{ij}, t^l_i, t^l_j, t*^l_j, p, c^{*l}_j, a^l_i, c^l_i, z_l, z^l_j \geq 0, \ \forall i, i, l \qquad (23)$$

In order to find the solution of the formulated MILFP model, first the EQP and EQN models were solved by employing AMPL (AMPL Student Version 20121021) with Bonmin and Couenne. The program was written according to the flow chart illustrated in Figure 2 for AMPL. The program consists of two main parts; the main module containing the actual program and the data file containing data of the various parameters. Further, the formulated MIP model was solved by the method of branch and bound algorithm deploying AMPL with CPLEX accordingly. Eventually, the solution of the EQN model became inconsistent, whereas, the solution of the EQP model is optimal. Therefore, by Charnes and Cooper algorithm, it is concluded that the optimal solution of the MILFP could be obtained by the optimal solution of the EQP problem. The program was executed on Pentium IV personal machine with a 1.73 GHz processor and 2.0 GB RAM.

4. COMPUTATIONAL ANALYSIS

In order to analyze the effectiveness of the proposed models, a numerical example was considered. It is assumed that a vendor has 5 locations sets, with 3 production forecast for 2 buyers. The deterministic demand of unit productsof buyers are (1700, 3500, 2200) and (2300, 1500, 2800), selling prices of per unit products (in $) of buyers are (40, 56, 82) and (42, 58, 75), penalty cost of per unit products (in $) of buyers are (0.50, 0.60, 0.60) and (0.25, 0.40, 0.30) respectively. Further, Table 2 describes additional information regarding the parameters of the MILFP and MIP models.

Table 2: Parameters of the MILFP model

Parameters	Locations of the vendor				
	1	2	3	4	5
Raw Materials(units)	(130,120,130)	(120,180,200)	(150,200,170)	(100,100,100)	(100,100,100)
Trans. Cost (input) ($)	(0.3 ,0.2, 0.3)	(0.2, 0.25, 0.2)	(0.5, 0.45,0.6)	(0.1, 0.1, 0.1)	(0.1, 0.1, 0.1)
Production cost ($)	(10 ,17,15)	(12,12,18)	(14,15,16)	(20,25,30)	(5,10,15)
Holding cost ($)	(1 ,2,3)	(3,2,2)	(3,4,3)	(5,4,5)	(2,3,1)
Shipping cost ($)	(11,23,36)	(25,27,32)	(13,26,35)	(25,27,32)	(10,15,20)
Capacity(in hund.) units	(13,12,13)	(12,18,20)	(15,20,17)	(10,10,10)	(10,10,10)
Travel time units	(5,7)	(9,10)	(12,8)	(15,20)	(10,10)
Required Delivery time	(5,7)	(10,10)	(12,8)	(15,20)	(10,10)
Obligatory Delivery time	(5,7)	(9,10)	(12,8)	(10,10)	(15,20)
Trans. cost ($/unit time)	(0.5,0.7)	(0.6,0.4)	(0.6,0.5)	(1.0,1.2)	(0.5,0.5)

In order to observe the effect of the key parameters, six sets of the vendor's opening costs (in $) with same average value such as (50000, 30000, 40000, 60000, 20000), (40000, 40000, 40000, 40000, 40000), (60000, 30000, 40000, 50000, 20000), (50000, 60000, 40000, 30000, 20000), (50000, 30000, 60000, 40000, 20000) and (50000, 30000, 40000, 20000, 60000) are considered. Whereas rest o the parameters are unchanged as shown in Table 2. Significant finding regarding the numerical example of the proposed MILFP and MIP models as well as the allocations and the distribution of different products for the two buyers has estimated. The values Return on Investment (RI in %) represent the gap in percentage between the total return on investment incurred by MILFP and MIP model, that is,

$$RI(\%) = |RI1 - RI2| \times \frac{100}{RI2}$$, where,

RI1 and *RI2* is the ratio of return on investment obtained by MILFP and MIP algorithms respectively. Finally, in order to estimate the effect of the sensitivity of opening cost parameter we employ sensitivity on the opening cost ($) of location 3. It is assumed that the opening cost of the vendor located at location 3 are 50, 100, 1000,2000, 3000, 4000, 5000 and 10000 while all other remaining parameters are unchanged.

Figures 3 and 4 describe the optimum allocation of different products for the first case and for both buyers. From the distribution pattern of different products, it is clear that MILFP provides optimal locations of the vendor for buyer-1 are 1, 2 and 5, whereas, MIP provides the optimal locations of the vendor for buyer-1 are 1, 2, 3 and 5. The optimal locations are achieved by MILFP model of the vendor for buyer-2 is 1, 2, 3 and 5. Similarly, MIP model provides the optimal locations of the vendor for buyer-2 are 1, 2, 3 and 5. Therefore, from the distribution of different products by MILFP and MIP models, it is apparently recommended that vendor-4 is not remained optimal for the first case.

From the sensitivity analysis, it seems that by MILFP model for buyer-1 the vendor located at location 3 does not remain optimal except the second case. In addition, MILFP model has no optimum product distribution from the location 4 for buyer-1 for all six cases. At the same way, the entire six cases the vendor located at location-4 is not profitable for buyer-2 by MILFP model. Similarly, by MIP model for the both buyers the unselected vendor is located at location 4 for all cases. Therefore, the results of these algorithms indicate that vendors 1, 2, 3 and 5 should be located to satisfy buyer's demands and vendor-4 could be removed without loss of the optimality. Further, it could be concluded that the optimal solutions of the MILFP algorithm are as good as the MIP algorithm. In addition, for the six cases, all the differences of the return on investment for both solutions are less than 0.94.

Figures 5 and 6 describe the average demand of different products achieved by MILFP and MIP models for buyer-1. By both MILFP and MIP models, the highest demand of the product for buyer-1 is product 2 which is followed by product 3 and product 1 as shown in Figures 5 and 6. The MILFP and MIP models satisfy the optimal demand of buyer-1 by the manufactures located at the location points 1, 2, 3 and 5. MILFP model illustrates that vendor located at locations 1 is profitable for all three products. MIP model illustrates that locations 1 and 3 are profitable for all three products. Further, both MILFP and MIP models describe that vendor-4 is not anyhow optimum for buyer-1 for all three products.

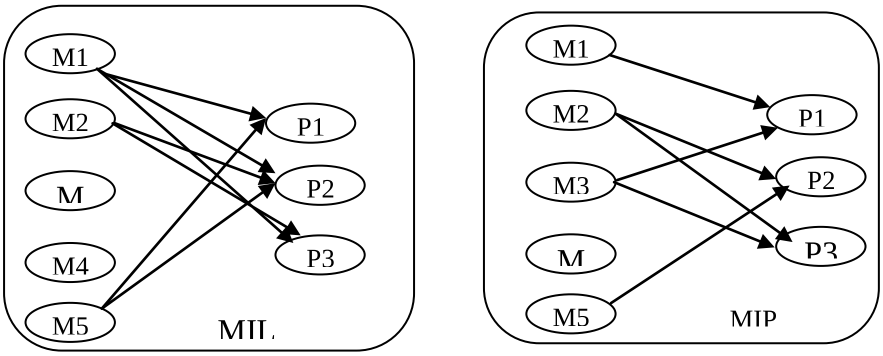

Figure 3: Allocations for buyer-1 by (MILFP) and (MIP) models

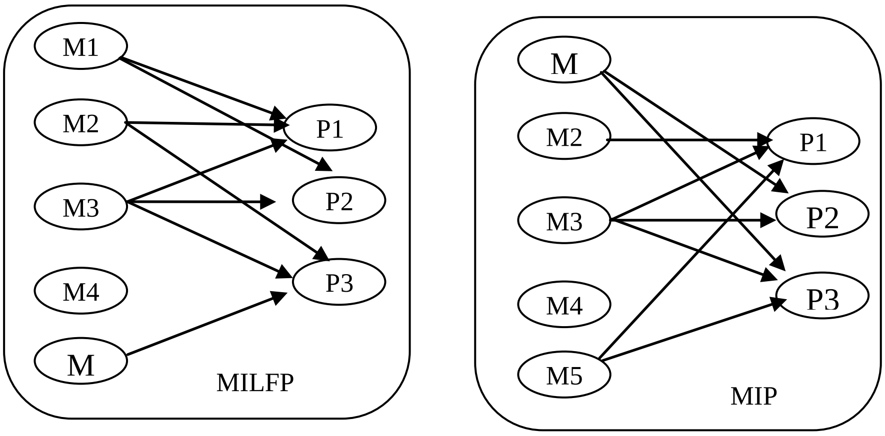

Figure 4: Allocations for buyer-2 by (MILFP) and (MIP) Models

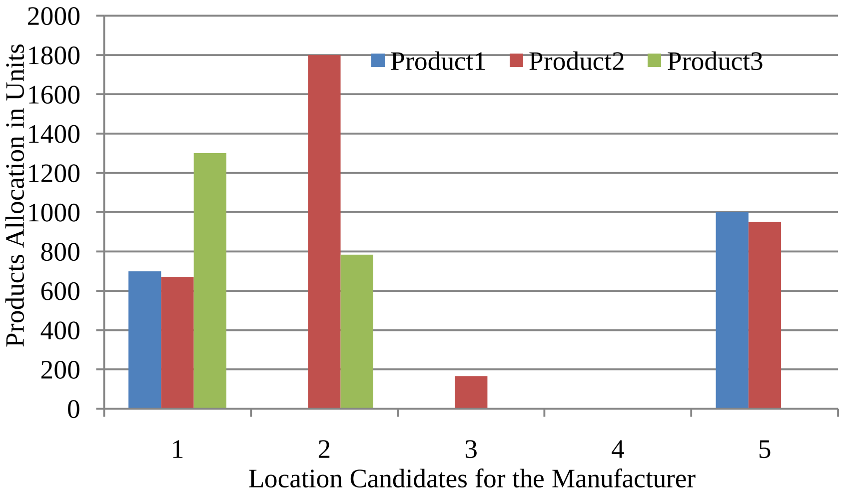

Figure 5: The demand of different products at different locations for buyer-1 by MILFP

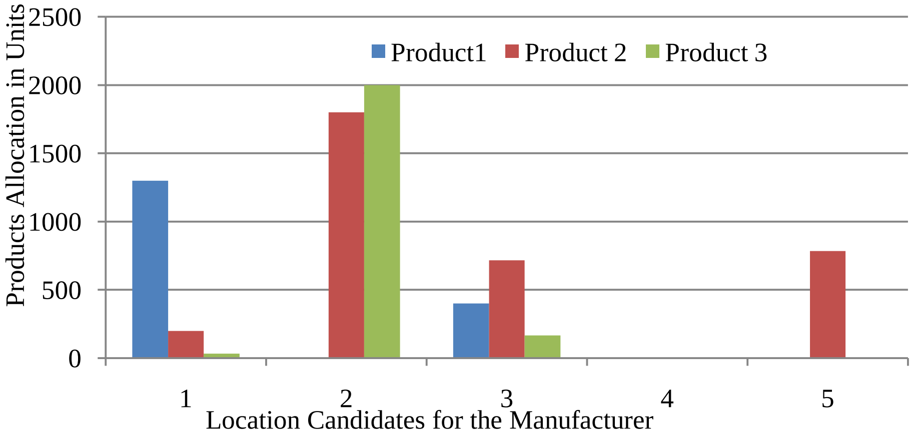

Figure 6: The demand of different products at different locations for buyer-1 by MIP

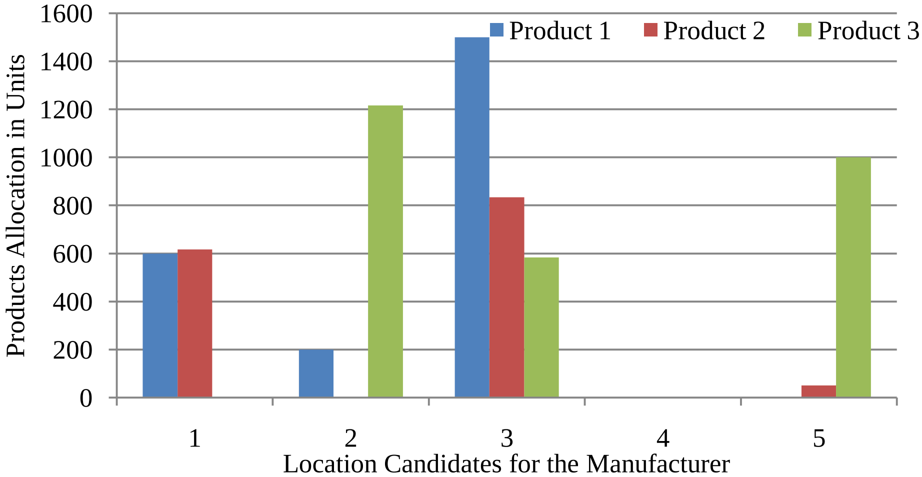

Figure 7: The demand of different products at different locations for buyer-2 by MILFP

Figures 7 and 8 depict the average demand of different products have obtained by MILFP and MIP models for buyer-2. By both MILFP and MIP models, the maximum demand for the product of buyer-2 is product 2 which is followed by product 3 and product1 as shown in Figures 7 and 8. The MILFP and MIP models perform the optimal demand of buyer-2 by the manufactures located at the location points 1, 2, 3 and 5. MILFP model illustrates that vendor located at 3 is profitable and can satisfy the optimal demand of all three products. MIP model illustrates that location 3 and 5 are profitable for all three products. Further, both MILFP and MIP models explain that vendor-4 is not anyway profitable for buyer-2 for all the products.

Figure 9 describes the sensitive of the opening cost on the total ratio of return on investment has obtained different cases by the MILFP and MIP models. The proportion of the return and investment obtained by both MILFP and MIP models are not differing much. In addition, all cases the profit achieved by MILFP model is slightly higher than that of by MIP model as shown in Figure 9 because the methodology of MILFP and MIP. The sensitivity of the opening cost demonstrates that all the cases the increases of the opening cost decreases the profit by both MILFP and MIP models since this additional cost increases the investment as well as cost. Figures 10 and 11 illustrate the influence of the opening cost on the demand and capacity of the each location. If the opening cost of a location decreases or increases, the demand and capacity of that location changes accordingly. The opening cost changes the demand dramatically than the capacity of the product. This can be interpreted that by additional opening cost, additional advertise and promotion can be offered that increase the demand. Similarly, opening cost that also concerns the reconstruction and expansion activity, so the capacity can be increased by increasing the opening cost.

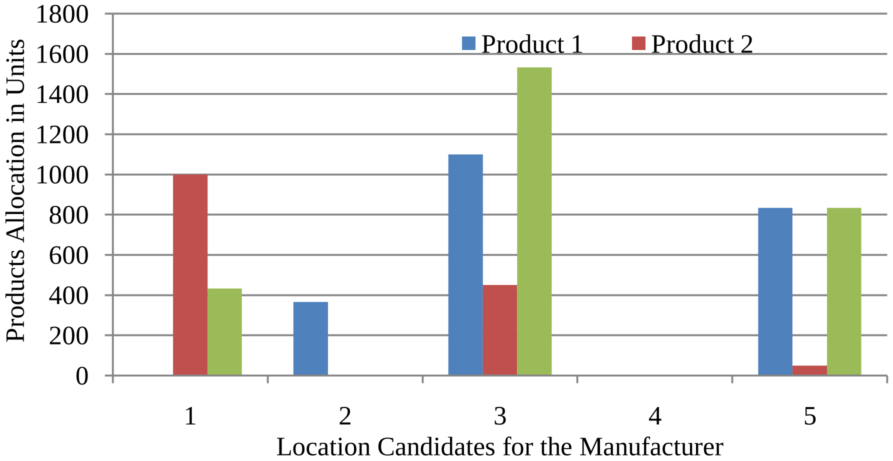

Figure 8: The demand of different products at different locations for buyer-2 by MIP

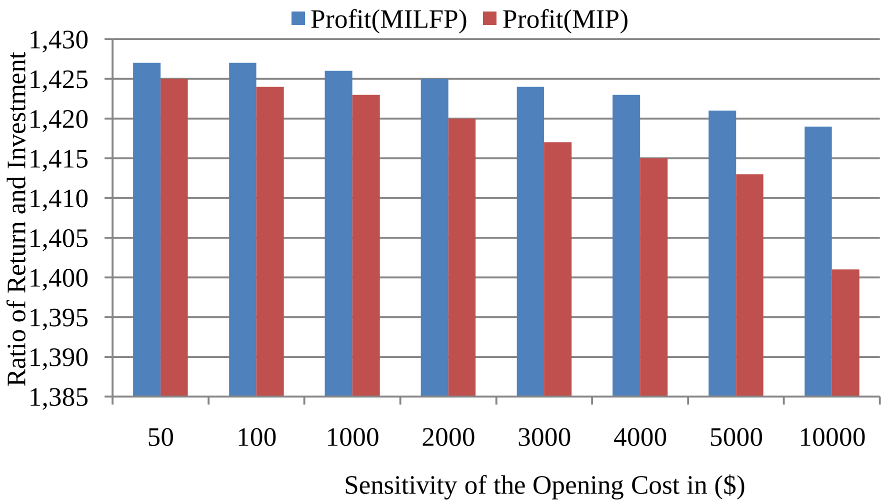

Figure 9: Comparison between return and investment obtained by MILFP and MIP models

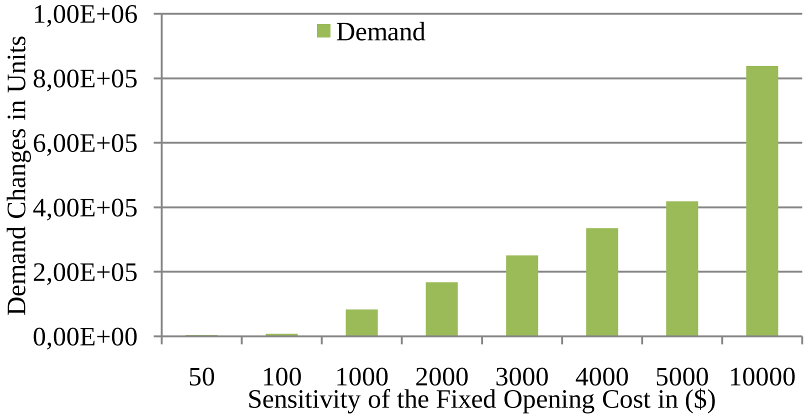

Figure 10: Effect of the sensitivity analysis of fixed opening cost on demand

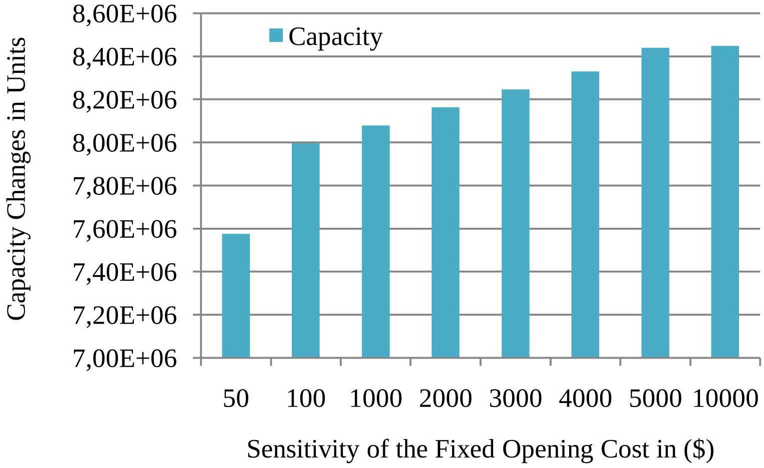

Figure 11: Effect of the sensitivity analysis of fixed opening cost on capacity

5.CONCLUSION

In this study, an MILFP based model is developed for the integrated supply chain network which is solved by AMPL using the suitable transformation. The formulated model simultaneously maximizes the ratio of return on investment. In order to demonstrate the significance of MILFP model, an MIP based model is also formulated. Some of the significance findings can be summarized as follows:

Firstly, the illustrated numerical example apparently shows that both MILFP and MIP have acheived very similar distribution pattern for the integrated multi-product, multi-facility, and multi-buyer location production supply chain network, which is worthy to the developed MILFP model. Secondly, the optimums locations of the warehouse are obtained by both of the models are very similar. Both models are also rejected the same location that is not optimum. The optimal demands for different products by the buyer are almost analogous by both MILFP and MIP models. The differences of the ratio of the return on investment achieved by the both models are less than 0.94%. Moreover, from the sensitivity analysis of the opening cost, it is concluded that opening cost is one of the momentous factors that increase and decrease the demand and capacity of a vendor. Further, the fixed opening cost has negative influence on the total profit. Therefore, MILFP model could be one of the relevant approaches in a logistic model which seeks to find the optimum manufacturer as well as optimum distribution with profit maximization and cost minimization.

Nonetheless, additional work may be needed to show the applicability of the model in real life problems. In future work, this model might be applied to calibrate and validate for the real industrial survey data considering the scale and complexity.

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