

## A FUZZY BI-LINEAR MANAGEMENT MODEL IN REVERSE LOGISTIC CHAINS

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**Abstract:** The management of the electrical and electronic waste (WEEE) problem in the uncertain environment has a critical effect on the economy and environmental protection of each region. The considered problem can be stated as a fuzzy non-convex optimization problem with linear objective function and a set of linear and non-linear constraints. The original problem is reformulated by using linear relaxation into a fuzzy linear programming problem. The fuzzy rating of collecting point capacities and fix costs of recycling centers are modeled by triangular fuzzy numbers. The optimal solution of the reformulation model is found by using optimality concept. The proposed model is verified through an illustrative example with real-life data. The obtained results represent an input for future research which should include a good benchmark base for tested reverse logistic chains and their continuous improvement.

**Keywords:** Reverse logistic chain, Electrical and electronic waste, Linear relaxation, Fuzzy linear programming.

MSC: 90C70.

## 1. INTRODUCTION

Nowadays, design and management of reverse logistics chains (RLCs) are one of the most important research topics in the logistics area. Results of good practice show that the companies that use products made in processes of reverse logistics have a lot of benefits, for instance: reduced use of raw materials, reduced waste disposal costs, hence, the profit increase and better competitive positioning. In addition to the economic importance, which is measurable, RLCs have a big non-measurable influence in ecological domain such as: environmental protection, resource conservation, health care, etc. Some authors suggest that by returning products which are at the end of its life cycle, companies create value flow and no financial loss [7]. In general, the effective management of reverse logistics activities is essential, because if the total costs associated with the reclamation efforts exceeded the total cost of new materials or products, companies would have no financial incentive for implementing a reverse logistics system. Regulatory constraints, products characteristics, return volumes, transportation costs, disposal costs and viable disposition alternatives all have a direct impact on the strategic priorities of the reverse logistics systems.

In the literature, terms like waste management and reverse logistics management are often synonymous. However, there is a fundamental difference between these two terms. Waste management can be defined as the collection, transport, processing or disposal, managing and monitoring of waste materials (products that can no longer be used). In the developed and in the large number of developing countries, waste management is regulated by laws.

There are numerous definitions of RLC management. According to [14], RSC management is defined as the effective and efficient management of the series of activities required for retrieving a product from a customer, and either disposing it or recovering its value. The definition of reverse logistics by other authors ([17], [2]) is quite similar. Rogers and Tibben-Lembke [17] gave the definition of RLC management as follows: the process of planning, implementing and controlling the efficient and effective flow of raw materials, goods in process, finished goods, and related information from the point of consumption to the point of origin in order to re-obtain the value or dispose properly. It can be mentioned that, reverse logistics term differs from green logistics, which treats environmental aspects of logistics activities and focuses on the flow of goods from producers to customers.

This paper deals with the problem of managing electrical and electronic waste (WEEE) in RLC. The use of electrical and electronic equipment has grown immensely in the past few decades, so the volume of discarded devices is "globally huge" ([15], [16]). In the United States (USA) the amount of WEEE per year is more than 65 thousand tons. According to the EU report, published in 2005, its amount increases every five years by 16% -28%. It can be mentioned that this type of waste contains gold, platinum, copper, lead, chromium, mercury, cadmium, and other heavy metals which can lead to serious

environmental accidents [6]. Problem of managing WEEE is very important in developing county. According to the research results presented in [20] over 500 thousand disposing of PCs is on the territory of Serbia, (all are WEEE), and over 1.6 million PCs are in use, which are latent computer waste. The inventory cost of recycling materials was over 3.8 million USD ten years ago.

The management of this waste is not adequate, and most of the dangerous ingredients are easily available. Waste management is very important management problem with respect to environment protection, optimal use of natural resources, climate change [9].

The mathematical base of the proposed fuzzy model is similar to the developed mathematical models in ([5],[13],[22]). The short retrospective of these papers is presented further.

The optimization of a global supply problem that maximizes after tax profits of a multinational corporation includes transfer prices and allocation of transportation costs as considered in [19] (further this model is denoted as VG). In this model, the four variable types exist: (1) flow of material, (2) transfer cost, (3) net profit before paying taxes, (4) proportion of transportation costs. The two bi-linear terms are defined; (1) product material flow and transfer of price, and (2) product material flow and transportation costs. The proposed model is a non-convex optimization problem with a linear objective function, a set of linear constraints, and a set of bilinear constraints. The solving procedure is developed as a heuristic solution algorithm that applies successive linear programming based on the reformulation and relaxation of the original problem. In this paper, the computational experiments are performed to investigate the impact of using different starting points. The algorithm produces feasible solutions with very small gaps between the solutions and their upper bound.

In [13], an optimization model of a multinational corporation is developed to maximize its global after tax profits by determining the flow of goods, the transfer prices, and the transportation cost allocation between each of its subsidiaries. A new relaxation procedure for reducing the number of bilinear terms is given. Authors present three other solution methods: an implementation of Variable Neighborhood Search (VNS) designed for any bilinear model, an implementation of VNS specifically designed for the problem considered here, and an exact method based on a branch and cut algorithm. The solution methods are tested on artificial instances. These results show that implementation of VNS outperforms the two other heuristics. The exact method found the optimal solution of all small instances and of 26% of medium instances.

Many and varied types of uncertainty exist in the treated problem. The term uncertainty implies that in a certain situation, a person does not have appropriate information which quantitatively and qualitatively describes, prescribes or predicts deterministically and numerically a system, its behaviour or other characteristic [25]. It is assumed that these uncertainties are far better judged by using linguistic expressions than by representing them in terms of precise numbers. The concept of linguistic variables is introduced by Zadeh [24]. Linguistic variables are variables whose values are not numbers but words, or sentences in a natural or artificial language [24]. It is very useful when dealing with situations which are too complex or not well defined, to describe them in conventional quantitative expressions [25]. There are numerous theories claiming to be the only proper tool to model linguistic expressions modelled by a triangular fuzzy set

([10], [25]). According to Zimmermann [25], fuzzy sets theory can be the most appropriate way for modelling linguistic expressions.

In modified model of [5] VG, which is presented in [5], the three variable types exist: (1) flow of material, (2) transfer cost, and (3) net profit before paying taxes. It is assumed that the total transportation costs between two entities of RLS can be divided between them. In this way, in modified model there is one bi-linear term- product material flow and a transfer price. Relaxation of the proposed model is performed by outer approximation in [1]. It can be shown that linear relaxation, which is used in [22], presents the special case of the outer approximation in [1]. As RLC exists in changed environment, it can be supposed that customs value of each product cannot be described by precise numbers. In this paper, these parameters are described by discrete fuzzy numbers. The determining values in domains of these discrete fuzzy numbers are based on evidence data. The treated problem in outer approximation in [5] can be stated as FLP task with fuzzy values of objective function coefficients. The solution of the treated problem is obtained by using the principle of equal possibilities in [4].

The aim of this paper is to find optimal flow of WEEE in RLC with the target that profit in recycling process is the highest. The proposed model is based on analyzed models in this Section. The RLC consists of two entities: collecting centers and recycling centers. The objective function is defined as the total profit of treated RLC. Constrains are defined with respect to type of the considered problem. There are some differences between constrains of analyzed models and the proposed model. The relaxation of bi-linear term is performed by using linear relaxation (by analogy VG model). It is supposed that fix costs of recycling centers can not be described by precise numbers. The management of RLC is stated as FLP task with fuzzy values of constrains. By using the optimality principle (by analogy [21]), the optimal solution is obtained.

The paper is organized in the following way. Section 2 describes the evaluation framework. A new fuzzy bilinear model for determining the optimal flows in RLC is proposed in Section 3. In Section 4, a proposed model is illustrated by an example with real-life data. Conclusions are presented in Section 5.

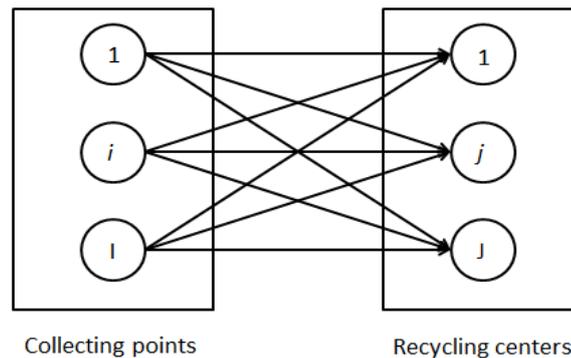
## 2. FRAMEWORK EVALUATION

Growing public pressure for environmental protection, and the fact that the resources are limited and not renewable influenced waste disposal costs increase, thus promoting the development of recycling technologies. Electrical and electronic waste (WEEE) specifics are: complexity, speed of product obsolescence, the rate of appearance of new types of electrical and electronic devices. Recycling means the extraction of materials from the waste, obtaining the undamaged electronic components as well as the expensive precious metals (which are included in printed circuit boards as an essential component of all electronic products) and its reuse. The EU adopted two directives for WEEE management: (1) Waste of Electrical and Electronic Equipment (WEEE) - the rules of collecting, storing, selling and recycling of these materials are defined in Directive [8], and (2) Restriction of the use of hazardous substances (RoHS), which amends WEEE Directive by limiting the amount of potentially hazardous materials contained in electrical appliances.

On the other hand, recycling enables a large number of companies to gain additional revenue and increase competitiveness. However, it can be mentioned that the value of products that are recycled will be usually lower than the same value of goods produced for the first time.

The proposed evaluation procedure is shown further.

*Step 1.* In this step design and processes of reverse logistic chain are described. The structure reverse logistic chain is presented in Fig. 1.



**Figure 1:** Reverse logistics chain

The collection points consist of large landfills which are equipped according to the rules of statutory provisions. Formally, collecting points are presented by index set  $I = \{1...i...I\}$ , where  $i$  is the index for collecting point, while  $I$  is the total number of collecting points. According to results of developed EU countries, it can be said that design of collecting points have the strongest influence on customer motivation (households and small enterprises; large enterprise, government, etc.; manufactories of original electrical and electrical devices) to become part of reverse logistic chains.

The process of collection of WEEE which initiates other processes of reverse logistic chains must be planned in accordance to applicable regulation which include, inter alia, that there should be a selective collecting points and users of electrical and electrical devices motivated to WEEE relate in collecting points. At the collecting points, WEEE can be sorted into many different WEEE types. Formally, they are denoted by set  $E = \{1...e...E\}$ , where  $e$  is index for WEEE type and  $E$  is a total number of WEEE types.

The process of disposal of the waste is realized after completing the process of collecting. Disposal of the waste in a landfill is the least desirable solution and poses a real threat, since no soil is completely impermeable. There may be a leakage of hazardous substances into the soil and groundwater. WEEE collection and disposal is a difficult challenge for environmentally sound waste - especially in developing countries [3]. Due to the hazardous composition of WEEE, a special landfill should be made. Local, state and national governments make laws that restrict or prohibit the storage of WEEE on conventional landfills [18]. The need to establish a selective collection system and to encourage the participation of end-users in these systems is defined by the EU

Directive. WEEE collection points are the key element of the system, but they are not defined by the Directive.

When the collection process is completed, process of transport WEEE to recycling centers starts. Transportation is usually the largest component of reverse logistics costs [19]. Transportation of hazardous materials from storage point to recycling centers is usually carried out by specialized companies, which have specialized equipment and personnel.

Recycling centers are formally presented as set  $J = \{1 \dots j \dots J\}$ , where  $j$  is the index for recycling center and  $J$  is the total number of recycling centers. In general, to recycle center  $j$ ,  $j=1, \dots, J$  can be delivered all types of WEEE  $E$  with each collecting point  $i$ ,  $i=1, \dots, I$ . Dismantling and sorting operations are firstly performed in recycling centers. When dismantling WEEE many different components are obtained. All materials obtained can be non-recyclable and recyclable materials. The non-recyclable materials are those which do not have the character of raw materials and are temporarily stored in environmentally friendly and safe conditions. In this paper, this kind of materials is not considered.

The recycled materials (for instance, plastics, metal, rubber, cables, batteries, Nickel Metal Hydride, lithium, lead, screens, electronic components, etc.) are obtained in the process of dismantling and sorting of each WEEE type  $e$ ,  $e=1, \dots, E$  and are formally presented by set  $K = \{1, \dots, k, \dots, K\}$ . The index for a product which can be further recycled is denoted as  $k$ , and  $K$  is total number of these products. Some types of recycled materials cannot be decomposed into constituent parts, so they are sold as finished products. This class recycled material are presented by set of indices  $L = \{1, \dots, l, \dots, L\}$ . The index for a product which can be further solved is denoted as  $l$ , and  $L$  is total number of final products of recycled materials. Types of products produced in the recycling process are most dependent on the technological level of recycling center. The recycling process is quite different at different recycling centers. The recycling process can be classified into: (1) dismantling of equipment that is carried out by hand, (2) mechanical treatments like cutting or chopping, (3) incineration and refinement by which combustible parts are burnt and metals are extracted, and (4) extraction of precious metals by chemical processing. For example, plastics are sorted into several categories, and then milled or granulated and as such stored in the warehouse recyclable. Cables are treated by machines for stripping insulation material, etc. The combined metals that can be used again and contain ferrous iron, are re-processed in the traditional smelters and factories for processing steel scrap. Non-ferrous metals can be used as second-class raw materials. By the use of mills for grinding, up to 99% of pure copper can be isolated.

*Step 2.* The assessment of the value of fix costs at the recycling center  $j$ ,  $j=1, \dots, J$  is stated as fuzzy decision making problem. Decision makers use the pre-defined linguistic expressions, which are modelled by triangular fuzzy numbers (TFNs). The shape of the membership functions can be obtained based on one's experience, the subjective belief of decision makers, their knowledge, etc. Jointly used, shapes of triangular function offer a good compromise between descriptive power and computational simplicity. Fuzzy sets of higher types and levels have not yet played a significant role in the applications of the fuzzy sets theory [11]. Granularity is defined as the number of fuzzy numbers assigned to the fix costs at the recycling centers. According to [12], human being can have only seven categories at the most. The lower, upper bounds and modal value of defined TFNs

are defined by decision makers. It is assumed that all decision makers have equal importance.

*Step 3.* The fuzzy bilinear model is defined. It contains a set of linear constraints and one set of bilinear constraint (constraint 3). The bilinear term corresponds to the product of two decision variables representing the flow of products and the proportion of transportation costs. The model which is developed in this paper is similar to the one obtained in [22]. It is known that a fuzzy bilinear model solution is very hard to achieve.

*Step 4.* The proposed fuzzy bilinear model is transformed into fuzzy LP (FLP) problem by applying the linear relaxation procedure. The way to obtain the relaxation for proposed model is basically the same as in [22]. The solution of this relaxation model is calculated by using the concept of fuzzy optimality [23]. Optimal solutions are defined by using discrete fuzzy set. The optimal solution of the proposed FLP problem is called *satisfaction level* [18], and it is calculated by using LINDO linear programming software.

### 3. THE PROPOSED MODEL

In this Section the proposed Algorithm is realized through the following steps. At the beginning, the notation is presented.

#### 3.1. Notation

$a_k$	unit profit per raw material $k$ , $k=1,\dots,K$ ; it can be supposed that this value depend on market conditions and is equal for all recycling centers,
$b_l$	unit profit per final product $l$ , $l=1,\dots,L$ ; the values of final products from recycling processes are determined by market demand,
$q_{ke}$	percentage share of product $k$ , $k=1,\dots,K$ in WEEE $e$ , $e=1,\dots,E$ ,
$n_{le}$	number of parts WEEE type $e$ , $e=1,\dots,E$ that after dismantling at the recycling centers, they are sold as final products,
$m_{je}$	unit costs of product dismantling $e$ , $e=1,\dots,E$ at the recycling center $j$ , $j=1,\dots,J$ ,
$r_{jk}$	unit costs of getting raw material $k$ , $k=1,\dots,K$ by using recycling process at the recycling center $j$ , $j=1,\dots,J$ ,
$t_{jl}$	unit transportation costs of final product $l$ , $l=1,\dots,L$ from recycling center $j$ , $j=1,\dots,J$ ,
$p_{le}$	percentage share of product $l$ , $l=1,\dots,L$ , which is transported to customers
$\tilde{X}_i$	triangular fuzzy number describing the total quantity of all WEEE types that may be stored at the collecting point $i$ , $i=1,\dots,I$ ,
$\tilde{F}_j$	triangular fuzzy number describing the value of fix costs at the recycling center $j$ , $j=1,\dots,J$

$x_{ije}$  amount of WEEE product  $e$ ,  $e=1,\dots,E$  which is transported to the recycling center  $j$ ,  $j=1,\dots,J$  from collecting point  $i$ ,  $i=1,\dots,I$ ,

### 3.2. Formal problem statement

Formal problem statement of WEEE management in RLS is presented as follows.  
Objective function:

$$\sum_{j=1}^J a_{kj} \cdot \sum_{i=1}^I \sum_{e=1}^E q_{ke} \cdot x_{ije} + \sum_{j=1}^J \sum_{l=1}^L b_{lj} \sum_{i=1}^I \sum_{e=1}^E n_{le} \cdot x_{ije} \quad (1)$$

Subject to:

$$\sum_{j=1}^J \sum_{e=1}^E x_{ije} \leq \tilde{X}_i, \quad i = 1, \dots, I; j = 1, \dots, J \quad (2)$$

Availability collecting center  $i$

$$\sum_{e=1}^E m_{je} \sum_{i=1}^I x_{ije} + \sum_{k=1}^K r_{jk} \sum_{i=1}^I \sum_{e=1}^E q_{ke} \cdot x_{ije} + \sum_{l=1}^L t_{jl} \cdot p_l \cdot \sum_{i=1}^I \sum_{e=1}^E n_{le} \cdot x_{ije} \leq \tilde{F}_j, \quad j = 1, \dots, K; k = 1, \dots, K; l = 1, \dots, L \quad (3)$$

Bounds:

$$x_{ije} \geq 0 \quad (4)$$

$$0 \leq p_{jl} \leq 1 \quad (5)$$

The proportion of transportation costs from the specific origin to different destinations, represented by the variables  $p_{le}$  may differ, using the given transportation mode. Each subsidiary of RLC can be paid a different transportation costs. Determining transportation costs for subsidiaries is very flexible. As a consequence, the bilinear term can be linearized. There are many papers in the literature in which the following substitution is used and it represents a special case of linearization, as suggested in [1]:

$$p_l \cdot \sum_{i=1}^I \sum_{e=1}^E n_{le} \cdot x_{ije} = z_l, \text{ and} \quad (6)$$

$$\text{and } 0 \leq p_l = \frac{z_l}{\sum_{i=1}^I \sum_{e=1}^E n_{le} \cdot x_{ije}} \leq 1 \quad (7)$$

$$\text{so that } 0 \leq z_l \leq \sum_{i=1}^I \sum_{e=1}^E n_{ie} \cdot x_{ije} \quad (8)$$

After relaxation of fuzzy bi-linear model, the fuzzy LP task can be defined:

Objective function:

$$\sum_{j=1}^J a_{kj} \cdot \sum_{i=1}^I \sum_{e=1}^E q_{ke} \cdot x_{ije} + \sum_{j=1}^J \sum_{l=1}^L b_{lj} \sum_{i=1}^I \sum_{e=1}^E n_{ie} \cdot x_{ije} \quad (9)$$

Subject to:

$$\sum_{e=1}^E x_{ije} \leq \tilde{X}_i, \quad i = 1, \dots, I \quad (10)$$

$$\sum_{e=1}^E m_{je} \sum_{i=1}^I x_{ije} + \sum_{k=1}^K r_{jk} \sum_{i=1}^I \sum_{e=1}^E q_{ke} \cdot x_{ije} + \sum_{l=1}^L t_{jl} \cdot z_l \leq \tilde{F}_j, \quad (11)$$

$j = 1, \dots, K; k = 1, \dots, K; l = 1, \dots, L$

Bounds:

$$x_{ije} \geq 0 \quad (12)$$

$$z_l \geq 0 \quad (13)$$

$$z_l \leq \sum_{i=1}^I \sum_{e=1}^E n_{ie} \cdot x_{ije} \quad (14)$$

#### 4. ILLUSTRATIVE EXAMPLE

The developed fuzzy bilinear model is tested on a real life data obtained from literature [20]. In this case, authors consider one type WEEE- Personal Computers (PCs) and computer equipments (Category 3, the EU Directive). This type of WEEE is very important in economic domain, and in the domain of environmental protection. It can be illustrated by data that 1.8 million kilogram of considered WEEE type is recycled in Hewelett Packard recycling centers for one month. By using the statistical methods, it can be forecasted that about 1.5 million PCs, which are denoted as WEEE, are in the Republic of Serbia. Inventory costs of recyclable materials are about 11.5 million US dollars.

The considered RLC consists of three collecting points (I=3), which are in the 3 biggest cities of Serbia. There is one point at which sorting, dismantling and recycling of

products is done ( $J=1$ ). The RLC structure is designed with respect to European Recycling Platform (<http://www.scheideanstalt.de/>).

After dismantling the waste collected in a recycling center, there is a kind of product which cannot be further recycled, such as processor board, ( $L=1$ ). The three recyclable products ( $R=3$ ) are considered: non-ferrous metals ( $r=1$ ), aluminum and precious metals ( $r=2$ ), and specific groups of plastics ( $r=3$ ). The unit profit of the treated recyclable products is: 2.06 \$ per kilogram, 0.33 \$ per kilogram, and 1.2 \$ per kilogram, respectively. The unit profit of processor board is 0.18\$ per unit product. The percentage share of product  $k$ ,  $k=1, \dots, 3$ , in one kilogram of treated WEEE type is: 26.1%, 3.4%, and 0.8%, respectively.

The capacity of each collecting point, and fix cost value of recycling center are estimated by using evidence data. The measurement unit for capacities of collecting centers is thousand kilograms. The fix cost values are measure by 100 US dollar.

Objective function:

$$\max \{0.538 \cdot x_1 + 0.011 \cdot x_2 + 0.010 \cdot x_3\}$$

Subject to:

$$x_1 \leq (460, 480, 500)$$

$$x_2 \leq (45, 50, 55)$$

$$x_3 \leq (7075, 80)$$

$$1.2 \cdot (x_1 + x_2 + x_3) + 0.18 \cdot 26.1\% \cdot (x_1 + x_2 + x_3) + 1.8 \cdot 3.4\% \cdot (x_1 + x_2 + x_3) + 0.33 \cdot 0.8\% \cdot (x_1 + x_2 + x_3) + 3.4 \cdot (x_1 + x_2 + x_3) \leq (1000, 1200, 1400)$$

Bounds:

$$x_1, x_2, x_3 \geq 0$$

$$\leq 0 p_{ik} \geq 1$$

Introducing the relaxation of the proposed fuzzy bi-linear model (see eq. 6-8), the fuzzy LP problem can be presented as:

Objective function:

$$\max \{0.538 \cdot x_1 + 0.011 \cdot x_2 + 0.010 \cdot x_3 + 0.18 \cdot x_4\}$$

Subject to:

$$x_1 \leq (460, 480, 500)$$

$$x_2 \leq (45, 50, 55)$$

$$x_3 \leq (7075, 80)$$

$$1.2 \cdot (x_1 + x_2 + x_3) + 0.18 \cdot 26.1\% \cdot (x_1 + x_2 + x_3) + 1.8 \cdot 3.4\% \cdot (x_1 + x_2 + x_3) \\ + 0.33 \cdot 0.8\% \cdot (x_1 + x_2 + x_3) + 3.4 \cdot (x_1 + x_2 + x_3) \leq (1000, 1200, 1400)$$

Bounds:

$$x_4 \leq 602$$

$$x_1, x_2, x_3 \geq 0$$

Extreme values optimality criteria which belong to the field of permissible solutions are determined by using the concept of fuzzy optimality [23]. In this case, the minimum and maximum value of objective function in permissible domain is:  $f_L = 268.5188$  and  $f_U = 308.4412$ .

When transforming FLP model into LP model, the criterion of optimality is defined as maximization *satisfaction level*,  $\lambda$ . The transformed model is:

Objective function:

$$\max \{\lambda\}$$

Subject to:

Subject to:

$$10 \cdot \lambda + x_1 \leq 490$$

$$10 \cdot \lambda + x_2 \leq 60$$

$$10 \cdot \lambda + x_3 \leq 85$$

$$1.31x_1 + 1.31 \cdot x_2 + 1.31 \cdot x_3 + 3.4 \cdot x_{4 \leq 1210}$$

Bounds:

$$10 \cdot \lambda + x_4 \leq 612$$

$$x_1, x_2, x_3 \geq 0$$

By using LINDO software the optimal solutions are given:

$$x_1^* = 467.432, x_2^* = 37.432, x_3^* = 62.432, x_4^* = 589.432, \lambda^* = 2.257$$

The obtained values represent the optimal amount of WEEE that must be transported from each treated collecting point to recycling center. In this plan, the total profit is considered to be RSC 35758.62 UD dollars. It can be mentioned that at the optimal amounts of WEEE, recycling and the recycling center can provide about 154 tons of non-ferrous metals, about 20 tons of rims and precious metals, and less than 5 tons of special groups of plastics. The results show that recycling is not only important from economic point of view but also from the aspect of natural resources conservation.

## 5. CONCLUSION AND FURTHER RESEARCH

Industrial management practice shows that reverse logistic management represents one of the most relevant issues of economic sustainability in every country. In this paper, a fuzzy bi-linear model for the assessment of optimal WEEE amounts in reverse logistic chain is proposed. To successfully apply the proposed model, the appropriate management strategy should be defined.

In general, many data which existing in RSC management problem cannot be expressed by crisp values. The fuzzy assessment of their values is performed by RSC management team. These linguistic expressions are modelled by using fuzzy sets theory. The fuzzy set theory is very suitable mathematical tool for the model development since it is flexible and tolerant to imprecise data, which are used for modelling linguistic expression.

This paper contributes to both practice and research. As a contribution to real-life practice, the method could be very useful for: management teams of RSCs, and other stakeholders to increase efficiency of its business, environmental protection, etc.

The main contribution of this paper is the introduction of fuzzy model for determining the WEE optimal amounts in RSC that exists in uncertain environment. The proposed fuzzy bi-linear model is based on developed models that can be found in the literature ([13],[22]). It includes the allocation of transportation costs as explicit decision variables. Therefore, the resulting problem is nonlinear and can hardly be solved. By using linear relaxation procedure, which is developed in [22], the proposed model is transformed into fuzzy linear programming (FLP) model. The optimal solution of FLP task is given by using the procedure proposed in [23]. We tested the model on small instances.

Main advantages of the presented model are user friendly assessment methodology, accomplished by the proposed linguistic expressions, and general output of the model which may significantly facilitate the decisions of managers. The optimal WEEE amount is determined by using LINDO software.

The proposed model is flexible:

- It is possible to describe the considered problem by formal language that enables the calculating of a solution by an exact method.
- The uncertainties that exist in the model can be described by fuzzy numbers.
- All the changes, including the changes in the number of collecting points, recycling centres, capacities of entities of RSC can be easily incorporated into the model.

General limitation of the model is the need for well structured RSC. The need for defined organizational entities comes from the fact that RSC is based on the requirements of local administration.

Future research will be directed to the analysis of complex RSCs, which may have different WEEE types. It should be mentioned that the proposed model presented in this paper can be easily extended to the analysis of other management decision problems in different research areas.

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