Yugoslav Journal of Operations Research
25 (2015), Number 3, 457-470
DOI: 10.2298/YJOR130517023P

OPTIMALITY TEST IN FUZZY INVENTORY MODEL FOR RESTRICTED BUDGET AND SPACE: MOVE FORWARD TO A NON-LINEAR PROGRAMMING APPROACH

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Received: May 2013 / Accepted: June 2014

Abstract: In this paper the concept of fuzzy Non-Linear Programming Technique is applied to solve an economic order quantity (EOQ) model for restricted budget and space. Since various types of uncertainties and imprecision are inherent in real inventory problems, they are classically modeled by using the approaches from the probability theory. However, there are uncertainties that cannot be appropriately treated by usual probabilistic models. The questions which arise are how to define inventory optimization tasks in such environment and how to interpret the optimal solutions. This paper allows the modification of the Single item EOQ model in presence of fuzzy decision making process where demand is related to the unit price and the setup cost varies with the quantity produced/Purchased. We consider the modification of objective function, budget and storage area in the presence of imprecisely estimated parameters. The model is developed for the problem by employing different modeling approaches over an infinite planning horizon. It incorporates all the concepts of a fuzzy arithmetic approach, the quantity ordered and the demand per unit compares both fuzzy non linear and other models. Investigation of the properties of an optimal solution allows developing an algorithm whose validity is illustrated through an example problem by using MATLAB (R2009a) version software; the two and three dimensional diagrams are represented to the application. Sensitivity analysis of the optimal solution is also studied with respect to the changes in different parameter values for obtaining managerial insights of the decision problem.

Keywords: Fuzzy, NLP, Budget, Space, Optimal decision.

MSC: 90B05.

1. INTRODUCTION

Although attempts were made to study the problem of control and maintenance of inventory by using analytic techniques since the turn of century, and its formulation in 1915, the square root formula for the economic order quantity (EOQ) was also used in the inventory literature for a pretty long time. Ever since its introduction in the second decade of the past century, the EOQ model has been the subject of extensive investigations and extensions by academicians. Although the EOQ formula has been widely used and accepted by many industries, some practitioners have questioned its practical application. For several years, classical EOQ problems with different variations were solved by many researchers and separated in reference books and survey papers e.g. Taha [21], Urgeletti [31]; recently, for a single product with demand related to unit price, and for multi products with several constraints by Cheng [2]. His treatments are fully analytical and much computational efforts were needed there to get the optimal solution.

Operations Research (OR), first coined by Mcclosky and Trefther in 1940, was used in a wider sense to solve the complex executive strategic and tactical problems of military teams. Since then, the subject has been enlarged in importance in the field of Economics, Management Sciences, Public Administration, Behavioral Science, Social Work Commerce Engineering, and different branches of Mathematics, etc. But various Paradigmatic changes in science and mathematics concern the concept of uncertainty. In Science, this change has been manifested by a gradual transition from the traditional view, which insists that uncertainty is undesirable and should be avoided by all possible means. According to the traditional view, science should strive for certainty in all its manifestations; hence uncertainty is regarded as unscientific. According to the modern view, uncertainty is considered essential to science; it is not an unavoidable plague but has, in fact, a great utility. But to tackle non-random uncertainty, no other mathematics was developed but fuzzy set theory with its intention to accommodate uncertainty in the presence of random variables. The application of fuzzy set concepts on EOQ inventory model has been proposed by many authors. Following Zadeh [34], significant contributions in this direction have been made in many fields, including production related areas. Consequently, investment in introducing fuzzy is the key to avoid uncertain decision space. Many studies have modified inventory policies by considering the issues of nonrandom uncertain and fuzzy based EOQ models. Widyadana et al. [33] explained the economic order quantity model for deteriorating items and planned the back order level. Hamacher et al. [5] discussed the sensitivity analysis in fuzzy linear programmin*g*. Pattnaik [10] explored some fuzzy and crisp inventory models. Vujosevic et al. [32] presented a theoretical EOQ formula when inventory cost is fuzzy. Lee et al. [8] studied an inventory model for fuzzy demand quantity and fuzzy production quantity. Tripathy et al. [24, 26, 30] introduced the concept, and developed the framework, for investing fuzzy in holding cost and setup cost in EOQ model. Tripathy et al. [23, 25] suggested improvements to production systems by employing entropy in the fuzzy model. Tripathy et al. [29] explains the effect of promotional factor in inventory model with units lost due to deterioration; and Pattnaik [13] extends this model in fuzzy decision space with units lost due to deterioration under promotional factor. Pattnaik [10] studied non linear profit-maximization entropic order quantity (EnOQ) model for deteriorating items with stock dependent demand rate. Pattnaik [11] extended an EOQ model for perishable items with constant demand and instant Deterioration. Pattnaik [13] investigated fuzzy NLP for a single item EOQ model with demand-dependent unit price and variable setup cost; and Pattnaik [12] extended a single item EOQ model for demand-dependent unit cost and dynamic setup cost. Tripathy et al. [28] again reinvestigated a single item EOQ model with two constraints. Tripathy et al. [27] explored optimal inventory policy with reliability consideration and instantaneous receipt under imperfect production process.

Dutta et al. [4] studied the effect of tolerance in fuzzy linear fractional models. Sommer [31] applied fuzzy dynamic programming to an inventory, product and then withdraw from the market. Kacprzyk et al. [6] introduced the determination of optimal of firms from a global view point of top management in a fuzzy environment with fuzzy constraints improved on reappointments, and a fuzzy goal for preferable inventory levels to be attained. Park [9] examined the EOQ formula in the fuzzy set theoretic perspective, associating fuzziness with cost data. Here, inventory costs were represented by trapezoidal fuzzy numbers (TrFN) and the EOQ model was transformed to a fuzzy optimization problem. Pattnaik [14, 16] introduced fuzzy method for selecting suppliers for manufacturing system. Pattnaik [17] presented a fuzzy EOQ model with promotional effort cost for units lost due to deterioration. Similarly Lee et al. [8] and Vujosevic et al. [32] have applied fuzzy arithmetic approach in EOQ model without constraints.

Table 1: Summary of the Related Researches

Authors	Demand	Setup cost	Holding cost	Unit cost of production	Constraints	Planning horizon	Structure of the Model	Model class
Vujosevic et al. [32]	Constant	Constant	$\frac{\tilde{c}_h c_p Q}{2 \times 100}$	Constant	No	Finite	Fuzzy	Defuzzification
Tripathy et al. [26]	Constant	Constant	$\frac{Hr^2q^2}{2\lambda}$	Reliability and demand	Reliability	Infinite	Fuzzy	NLP
Tripathy et al. [24]	Constant	Constant	$\frac{H\lambda q^2}{2r^2}$	Reliability and demand	Reliability	Infinite	Fuzzy	NLP
Tripathy et al. [30]	Constant	Constant	$\frac{Hq^2}{2r^2\lambda}$	Reliability and demand	Reliability	Infinite	Fuzzy	NLP
Present paper (2015)	Constant	Variable	$\frac{1}{2}C_1q$	Demand	Budget and Storage Capacities	Infinite	Fuzzy	NLP

In this paper, a single item EOQ model is developed where unit price varies inversely with demand and setup cost increases, with the increase of production. In company or industry, total expenditure for production and storage area are normally limited but imprecise, uncertain, non-specificity, inconsistency vagueness and flexible, defined within some ranges. However, the no stochastic and ill formed inventory models can be realistically represented in the fuzzy environment. The problem is reduced to a fuzzy optimization problem, associating fuzziness with the storage area and total expenditure. The optimum order quantity is evaluated by both fuzzy non linear programming (FNLP) method and for linear membership functions. The model is illustrated with numerical example and the variation in tolerance limits for both shortage area and total expenditure. A sensitivity analysis is presented. The numerical results for fuzzy and crisp models are compared. The remainder of this paper is organized as follows. In section 2, assumptions and notations are provided for the development of the model, and the developed mathematical model is presented. In section 3, mathematical analysis of fuzzy non linear programming (FNLP) is formulated. The solution of the FNLP inventory is derived in section 4. The numerical example is presented to illustrate the development of the model in section 5. The sensitivity analysis is carried out in section 6 to observe the changes in parameters in the optimal solution. Finally section 7 deals with the summary and the concluding remarks.

2. MATHEMATICAL MODEL

A single item inventory model with demand dependent unit price and variable setup cost under storage constraint is formulated as

Min C (D,q) = $C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2}C_1$q

s.t. $\frac{1}{2}uq \leq U$

Aq ≤ B

∀ D, q > 0 (1)

Where, q = number of order quantity,
D = demand per unit time
C_1 = holding cost per item per unit time.
C_3 = Setup cost = C03 q^{ν}, (C_{03} (> 0) and ν (0< ν < 1) are constants)
p = Unit production cost = $KD^{-\beta}$, K (> 0) and β (> 1) are constants. Lead time is zero, no back order is permitted, and replenishment rate is infinite. U, u, A and B are nonnegative real numbers, U is the capital investment goal and B is the space constraint the goal. The above model in a fuzzy environment is

$\widetilde{Min}$ C (D,q) = $C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2}C_1$q

s. t. $\frac{1}{2}uq \leq \tilde{U}$

Aq ≤ $\tilde{B}$

∀ D, q > 0 (2)

(A wavy bar (~) represents fuzzification of the parameters).

3. MATHEMATICAL ANALYSIS OF FUZZY NON LINEAR PROGRAMMING (FNLP)

A fuzzy non linear programming problem with fuzzy resources and objective are defined as

$\widetilde{Min}$ g_0(x)

s.t. $g_i(x) \leq \tilde{b}_i$ i=1, 2, 3, ……m (3)

$g_j(x) \leq \tilde{u}_j$ j=1,2.3, ……..m

In fuzzy set theory, the fuzzy objective and fuzzy resources are obtained by their membership functions, which may be linear or nonlinear. Here μ_0 and μ_i (i = 1, 2 ...., m) are assumed to be non increasing continuous linear membership functions for objective and resources, respectively, such as

$$\mu_i\,(g_i(x)) = \begin{cases} 1 \;\; if\; g_i(x) < b_i, \\ 1 - \frac{g_i(x)-b_i}{P_i}\; if\; b_i \le g_i(x) \le b_i + P_i, \\ 0\; if\; g_i > b_i + P_i, \end{cases} \qquad i = 0, 1, 2, ..., m.$$

$$\mu_j\,(g_j(x)) = \begin{cases} 1 \;\; if\; g_j(x) < u_i, \\ 1 - \frac{g_j(x)-b_j}{P_j}\; if\; u_j \le g_j(x) \le u_j + P_j, \\ 0\; if\; g_j > b_j + P_j, \end{cases} \qquad j = 0,1, 2, ...m$$

In this formulation, the fuzzy objective goal is b_0 and its corresponding tolerance is $P_{0;}$ and for the fuzzy constraints, the goals are b_i's and their corresponding tolerances are P_i's (i = 1, 2, ..., m). To solve the problem (3), the max - min operator of Bellman et al. [32], and the approach of Zimmermann [6] are implemented.

The membership function of the decision set, μ_D (x), is μ_D (x) = min {μ_0 (x), μ_1 (x), ..., μ_m (x)},$\forall$ x ∈ X.

The min operator is used here to model the intersection of the fuzzy sets of objective and constraints. Since the decision maker wants to have a crisp decision proposal, the maximizing decision will correspond to the value of x, x_{max} that has the highest degree of membership in the decision set.

μ_D (x_{max}) = $\max_{x\ge 0}[\min\{\mu_0\,(x), \mu_1\,(x) \ldots, \mu_m\,(x)\}]$. It is equivalent to solving the following crisp non linear programming problem.

$$\begin{aligned} &\text{Max } \alpha \\ &\text{s.t.} \quad \mu_0(x) \ge \alpha \\ &\mu_i(x) \ge \alpha \;(\text{ i}= 1, 2, \ldots, \text{m}) \\ &\forall\, x \ge 0,\ \alpha \in (0, 1) \end{aligned} \tag{4}$$

A new function, i.e the Lagrangian function L (α, x, λ) is formed by introducing (m +1) Lagrangian multipliers λ = (λ_0, λ_1, ...λ_m).

L $(\alpha, x, \lambda) = \alpha - \sum_{i=0}^{m} \lambda_i(g_i(x) - b_i - (1-\alpha)P_i) - \sum_{i=0}^{m} \lambda_i(g_i(x) - u_i - 1 - \alpha P i)$. The necessary condition of Kuhn et al. [8] for the optimal solution to this problem implies that optimal values $x_1^*, x_2^*, x_3^*, \ldots x_n^*$ and $\lambda_1^*, \lambda_2^*, \lambda_3^*, \ldots \lambda_n^*$ should satisfy

$$\frac{\partial L}{\partial xj} = 0 \qquad j = 1, 2, ... n \tag{5}$$

$\frac{\partial L}{\partial \alpha} = 0$

λ_i (g_i (x) - b_i - (1 - α) P_i) = 0

λ_i (g_i (x) $- u_i - (1-\alpha)P_i$) = 0,

$g_i(x) \leq b_i + (1-\alpha)P_i,$
$g_i(x) \leq u_i + (1-\alpha)P_i,$

$\lambda_i \leq 0, i = 0, 1, \ldots., m$

Moreover, Kuhn-Tucker's sufficient condition demands that the objective function for maximization and the constraints should be respectively colane and convex. In this formulation, it can be shown that both objective function and constraints satisfy the required sufficient conditions. Now, solving (5), the optimal solution for the FNLP problem is obtained.

4. SOLUTION OF THE PROPOSED INVENTORY

The proposed inventory model depicted by equation (2)

$\widetilde{Min}$ C (D,q) = $C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2}C_1$q

s. t. $\frac{1}{2}\,uq \leq \tilde{U}$

Aq ≤ $\tilde{B}$

∀ D, q > 0, reduces to following equation (4),

Max α

s.t. $C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2}C_1$q ≤ C_0 + (1 - α) P_0,

$\frac{1}{2}\,uq \leq U + (1-\alpha)P_1$

Aq ≤ B + (1 - α)P_2, (6)

∀ D, q > 0 & α ∈ (0, 1)

Here, the objective goal is C_0 with tolerance P_0, the capital investment constraint goal with tolerance $P_{1,}$ and space constraint goal is B with tolerance P_2. So, the corresponding Lagrangian function is

L (α, D, q, λ_1, λ_2, λ_3) $= \alpha - \lambda_1\left(C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2}C_1 q - C_0 - (1-\right.$ $\alpha)P0$ -$\lambda 212 uq{-}U{-}1{-}\alpha P1$ -$\lambda 3 Aq{-}B{-}1{-}\alpha P2$

From Kuhn - Tucker's necessary conditions,

$\frac{\partial L}{\partial \propto} = 0,\ \frac{\partial L}{\partial D} = 0, \frac{\partial L}{\partial q} = 0, \frac{\partial L}{\partial \lambda_1} = 0, \frac{\partial L}{\partial \lambda_2} = 0, \frac{\partial L}{\partial \lambda_3} = 0, \forall\, \lambda_1, \lambda_2, \lambda_3 \leq 0$

$\frac{\partial L}{\partial \alpha} = 1 - \lambda_1 P_0 - \lambda_2 P_1 - \lambda_3 P_2 \geq 0$

$$\frac{\partial L}{\partial D} = \lambda_1\left(C_{03}q^{\nu-1} + (1-\beta)KD^{-\beta}\right) \leq 0$$

$$\frac{\partial L}{\partial q} = \lambda_1\left(C_{03}(\nu-1)q^{(\nu-2)}D + \frac{1}{2}C_1\right) + \frac{u}{2}\lambda_2 + A\lambda_3 \leq 0$$

$$\frac{\partial L}{\partial \lambda_1} = \left(C_{03}q^{(\nu-1)}D + KD^{1-\beta} + \frac{1}{2}C_1 q\right) - C_0 - (1{-}\propto)P_0 \geq 0$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{1}{2}uq - U - (1{-}\propto)P_1 \geq 0$$

$$\frac{\partial L}{\partial \lambda_3} = Aq - B - (1{-}\propto)P_2 \geq 0$$

and $\alpha(1{-}\lambda_1 P_0 - \lambda_2 P_1 - \lambda_3 P_2) = 0$

$$\lambda_1 D\left(C_{03}q^{(\nu-1)} + (1-\beta)KD^{-\beta}\right) = 0$$

$$\lambda_1 q\left(C_{03}(\nu-1)q^{(\nu-2)}D + \frac{1}{2}C_1\right) + \frac{u}{2}\lambda_2 q + A\lambda_3 q = 0$$

$$\lambda_1\left(C_{03}q^{(\nu-1)}D + KD^{1-\beta} + \frac{1}{2}C_1 q - C_0 - (1-\alpha)P_0\right) = 0$$

$\lambda_2(Aq - B - (1-\alpha)P_1) = 0$, $\forall\, \alpha$, D, q ≥ 0 and $\forall\, \lambda_1, \lambda_2, \lambda_3 \leq 0$, solving these equations, optimum quantities are $q = \frac{B+(1-\alpha)P_2}{A} = \frac{2(U+(1-\alpha)P_1)}{u}$ $D^* = \left[\frac{C_1 q - C_{03}q^{(\nu-1)}}{(1-\beta)K}\right]^{-1/\beta}$

q = f (α) and D = f (q) where α^* is a root of K $\beta D^{*(1-\beta)} + \frac{1}{2}C_1\, q^* - C_0 - (1-\alpha^*)P_0 = 0$

$$C^*(D^*, q^*) = C_{03}q^{*\nu-1}D^* + KD^{*1-\beta} + \frac{1}{2}C_1 q^*$$

So, by both FNLP and NLP techniques, the optimal values of q^* and D^* and the corresponding minimum cost are evaluated for the known values of other parameters.

5. NUMERICAL EXAMPLE

For a particular EOQ problem, let C_{03} = \$4, K = 100, C_1 = \$2, ν = 0.5, β = 1.5, u = \$0.5, U = \$3.5, A = 5 units, B = 90 units, C_0 = \$40 and $P_0 = \$20$ and P_1 =\$15 and $P_2 = 25$ units. For these values, the optimal value of productions batch quantity q*, optimal demand rate D*, minimum average total cost C* (D*, q*), $\frac{1}{2}uq^*$ and Aq* obtained by FNLP are given in Table 2.

After 28 iterations, Table-2 reveals the optimal replenishment policy for a single item with demand dependent unit cost and dynamic setup cost. In this table, the optimal numerical results of fuzzy model are compared with the results of crisp model and fuzzy model of Roy et al. [34]. The optimum replenishment quantity q^* and Aq^* are both 34.82% and 30.37% more than that of other crisp model, respectively; and 21.19% and 57.61% more than that of the fuzzy model, respectively. The optimum quantity demand D^* is 10.62227, but 9.21 and 9.81 are for comparing models, hence 13.29% and 7.63% more from the demand of the given model, respectively. The minimum total average cost $C^*(D^*, q^*)$ is 53.69, but 54.43 and 53.93 are for comparing models, hence 1.37% and 0.44% less than the cost of the other crisp and fuzzy models, respectively. It permits better use of present fuzzy model as compared to the crisp model and the fuzzy model. The results are justified and agree with the present model. It shows the present fuzzy EOQ model is consistent and cost effective than that of the compared models of Roy et al. [34].

Figure 1 represents the relationship between demand per unit time D and unit cost of production P and Figure 2 depicts the mesh plot of demand per unit time D, the number of order quantity q, and the average total cost C.

Table 2: Optimal Values for the Proposed Inventory Model

Model	Method	Iteration	q^*	D^*	$C^*(D^*,q^*)$	α^*	$\frac{1}{2}uq^*$	Aq^*
Fuzzy model	FNLP	28	7.670636	10.62227	53.69446	0.3152770	1.917659	38.35318
Crisp model, Roy et al. [18]	NLP	-	5	9.21	54.43	1	-	50
Relative Error	-	-	34.816	13.29537	1.369861	217.1814	-	30.367286
Fuzzy model, Roy et al. [18]	FNLP	-	6.0449	9.8115	53.9324	0.3033	-	60.449
Relative Error	-	-	21.1943	7.632737	0.443136	3.79888	-	57.6114418

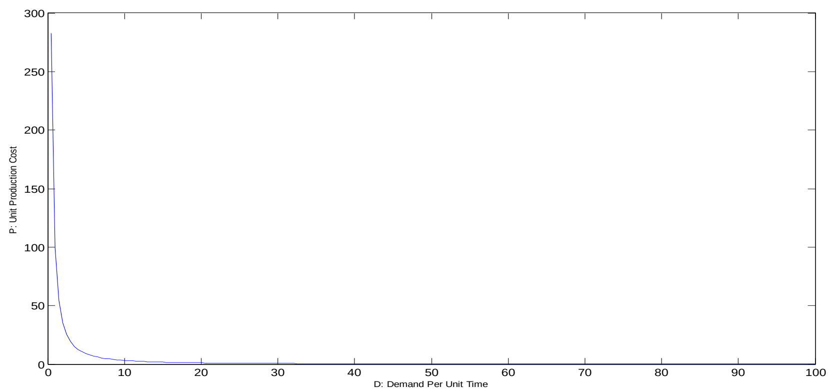

Figure 1: Demand per Unit Time D and Unit Production Cost P.

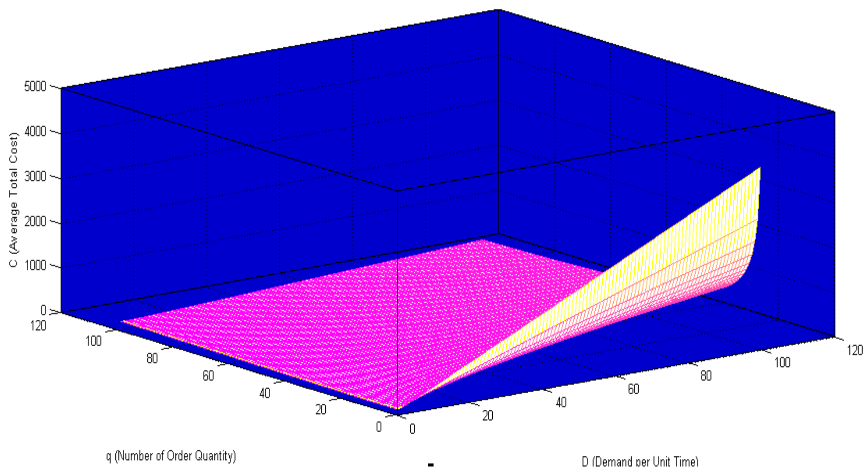

Figure 2: Mesh plot of Demand per Unit Time D, Number of Order Quantity q and Average Total Cost C.

6. SENSITIVITY ANALYSIS

The effect of changes in the system parameters on the optimal values of q, D, C (D, q) and $\frac{1}{2}uq$ and Aq when only one parameter changes and others remain unchanged, the computational results are described in Table 3. As a result, it is:

q^*,D^* and $C^*(D^*, q^*)$ are insensitive to the parameters P_0, P_1 and P_2 but α^* is less sensitive to these parameters. Following Dutta et al. [1], and Hamacher et al. [4], it is observed that the effect of tolerance in the said EOQ model, with the earlier numerical values and constructing Table 3 for the degrees of violation $T_0(= (1-\alpha)P_0), T_1$(=(1-$\alpha)P_1$) and T_2 (=(1-$\alpha)P_2$) for the two constraints is given by equation (6).

From Table 3, it is shown that: (i) for different values of P, degrees of violations $T_0\ , T_1, and\ T_2$ are never zero, i.e. different optimal solutions are obtained. (ii) As $P_0, P_1,$ and P_2 increase from original values, the minimum average cost $C^*(D^*, q^*)$, q^*, and D^* are remaining insensitive, respectively.

Table 3: Sensitivity Analysis of the Parameters P_0, P_1 and P_2

P	Value	Iteration	α^*	q^*	D^*	T_0	T_1	T_2	$C^*(D^*,q^*)$	Relative Error in $C^*(D^*,q^*)$
P_0	25	28	0.4522216	7.670637	10.62227	10.9557	8.216676	13.6944	53.69446	1.5603E-06
	50	27	0.7261108	7.670635	10.62227	5.47778	4.108338	6.84723	53.69446	1.5603E-06
	100	52	0.8630554	7.670636	10.62226	2.73889	2.054169	3.42361	53.69446	1.5603E-06
	200	46	0.9315277	7.670637	10.62226	1.36944	1.027085	1.71180	53.69446	1.5603E-06
	1000	44	0.9863055	7.670663	10.62226	0.27389	0.205418	0.34236	53.69446	1.5601E-06
P_1	16	28	0.3152770	7.670636	10.62227	13.6944	10.27085	17.1180	53.69446	1.5603E-06
	20	28	0.3152770	7.670636	10.62227	13.6944	10.27085	17.1180	53.69446	1.5603E-06
	23	28	0.3152770	7.670636	10.62227	13.6944	10.27085	17.1180	53.69446	1.5603E-06
	38	28	0.3152770	7.670636	10.62227	13.6944	10.27085	17.1180	53.69446	1.5603E-06
	40	28	0.3152770	7.670636	10.62227	13.6944	10.27085	17.1180	53.69446	1.5603E-06
P_2	30	28	0.3152770	7.670636	10.62227	13.6944	10.27085	17.1180	53.69446	1.5603E-06
	40	28	0.3152770	7.670636	10.62227	13.6944	10.27085	17.1180	53.69446	1.5603E-06
	50	28	0.3152770	7.670636	10.62227	13.6944	10.27085	17.1180	53.69446	1.5603E-06
	80	28	0.3152770	7.670636	10.62227	13.6944	10.27085	17.1180	53.69446	1.5603E-06
	100	28	0.3152770	7.670636	10.62227	13.6944	10.27085	17.1180	53.69446	1.5603E-06

Now, the effect of changes in the system parameters on the optimal values of q, D, and C (D, q) when only one parameter changes and others remain unchanged, the computational results are described in Table 4. As a result, it is:

q^*,D^* and $C^*(D^*,q^*)$ are less sensitive to the parameter ‘u’.
q^*,D^* and $C^*(D^*,q^*)$ are insensitive to the parameter ‘U’.
q^*,D^* and $C^*(D^*,q^*)$ are moderately sensitive to the parameter ‘A’.
q^*,D^* and $C^*(D^*,q^*)$ are insensitive to the parameter ‘B’.
q^*,D^* and $C^*(D^*,q^*)$ are sensitive to the parameter ‘$C_1{}'$.
q^*,D^* and $C^*(D^*,q^*)$are sensitive to the parameter ‘C_{03}’.
q^*,D^* and $C^*(D^*,q^*)$ are sensitive to the parameter ‘K’.

Table 4: Sensitivity Analysis of the Parameters u, U, A, B, C_1, C_{03} and K

Parameter	Value	Iteration	D^*	q^*	α^*	$C^*(D^*, q^*)$	Relative Error in $C^*(D^*, q^*)$
u	1	28	10.62227	7.670636	0.3152770	53.69446	1.56E-06
	2	62	10.62227	7.670636	0.3152770	53.69446	1.56E-06
	3	26	10.62227	7.670636	0.3152770	53.69446	1.56E-06
	5	33	9.579972	5.626945	0.2955092	54.08982	0.73631
	10	33	7.915074	3.1735460	0.1754845	56.49031	5.20696
U	4	28	10.62227	7.670636	0.3152770	53.69446	1.56E-06
	5	28	10.62227	7.670636	0.3152770	53.69446	1.56E-06
	20	28	10.62227	7.670636	0.3152770	53.69446	1.56E-06
	30	28	10.62227	7.670636	0.3152770	53.69446	1.56E-06
	50	28	10.62227	7.670636	0.3152770	53.69446	1.56E-06
A	10	26	10.62227	7.670636	0.3152770	53.69446	1.56E-06
	15	25	10.62227	7.670636	0.3152770	53.69446	1.56E-06
	30	21	9.431145	5.368751	0.2893747	54.21251	0.9648
	50	24	8.106413	3.409308	0.1953459	56.09308	4.46717
	60	27	7.694779	2.915871	0.1504772	56.99046	6.13843
B	150	28	10.62227	7.670636	0.3152770	53.69446	1.56E-06
	200	28	10.62227	7.670636	0.3152770	53.69446	1.56E-06
	250	28	10.62227	7.670636	0.3152770	53.69446	1.56E-06
	400	28	10.62227	7.670636	0.3152770	53.69446	1.56E-06
	1000	28	10.62227	7.670636	0.3152770	53.69446	1.56E-06
C_1	2.5	28	9.966176	6.335278	0.2283157	53.84987	0.28943
	3	28	9.460310	5.418712	0.1551762	54.18712	0.91752
	3.5	42	9.052691	4.748026	0.09183397	54.6023	1.69075
	3.8	40	8.842463	4.424862	0.05746647	54.86829	2.18614
	4	38	8.713820	4.234535	0.03582553	55.04895	2.5226
C_{03}	3	22	12.52018	7.065366	0.5271220	54.1678	0.88155
	3.5	25	11.46451	7.383501	0.4157748	53.79407	0.18552
	4.5	27	9.930867	7.933164	0.2233926	53.76922	0.13924
	5	27	9.350611	8.175608	0.1385375	53.95901	0.49269
	5.5	27	8.854969	8.401300	0.05954503	54.22657	0.991
K	105	24	11.07591	7.887504	0.2393735	53.71015	0.02922
	110	26	11.52648	8.099988	0.1650041	53.75447	0.11176
	115	27	11.97413	8.308371	0.09206994	53.8238	0.24088
	118	25	12.24138	8.431539	0.04896127	53.87611	0.3383
	120	28	12.41900	8.512906	0.02048276	53.91507	0.41087

7. CONCLUSION

In contrast to Roy [35], this paper follows real life inventory model for single item in fuzzy environment by using FNLP technique, and this approach provides solutions better than those obtained by using properties with two constraints. Sensitivity analyses on the tolerance limits have been presented, too. The results of the fuzzy model are compared with that of the crisp model, which reveals that the fuzzy model gives better result than the usual crisp model. Inventory modelers have so far considered where the setup cost is fixed or constant, which rarely occurs in the real market. In the opinion, an alternative (and perhaps more realistic) approach is to consider the setup cost as a function quantity produced / purchased, which may represent the tractable decision making procedure in fuzzy environment. A new mathematical model is developed, and a numerical example is provided to illustrate the solution procedure. The new modified EOQ model was numerically compared with the traditional EOQ model. Finally, the budget and space strategy was demonstrated numerically in the model to have the adverse affect on the total average cost per unit. This method is quite general and can be extended to other similar inventory models including those with shortages, discounts, and deteriorated items.

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