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PORTFOLIO OPTIMIZATION WITH STRUCTURED PRODUCTS UNDER RETURN CONSTRAINT

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Abstract: A new approach for optimizing risk in a portfolio of financial instruments involving structured products is presented. This paper deals with a portfolio selection model, which uses optimization methodology to minimize conditional Value-at-Risk (CVaR) under return constraint. It focuses on minimizing CVaR rather than on minimizing value-at-Risk VaR , as portfolios with low CVaR necessarily have low VaR as well. We consider a simple investment problem where besides stocks and bonds, the investor can also include structured products into the investment portfolio. Due to possible intermediate payments from structured product, we have to deal with a re-investment problem modeled as a linear optimization problem.

Keywords: Linear optimization, Risk measures, Linearization.

MSC: 49N05, 65K10.

1. INTRODUCTION

Risk management is a core activity in asset allocation conducted by almost all financial institutions. Portfolio optimization is the process of analyzing a portfolio and managing the assets within it. Martinelli *et al.* [6] assumed the financial market with investment possibilities bonds, stocks and options to mature exactly at investorôs horizon. In the portfolio optimization strategy, the aim of an investor is to minimize CVaR under expected return constraint. Korn and Serkon [10] considered an investment problem where the structured product was assumed to mature before horizon time, and the aim of investor was to maximize return under CVaR constraint. In the present paper, the utility criterion considered is the minimization of CVaR for a given level of expected return with intermediate payments for structured product and further, we consider another investment problem in which aim of investor is to minimize CVaR as well as negative return *i.e.* loss with intermediate payment from option (structured product).

The use of CVaR in portfolio optimization problems as a measure of allowed risk reduces the optimization problem to a linear optimization problem with linear constraints. This problem then can be solved by standard methods.

2. SELECTION OF RISK MEASURE

Several risk measures have been considered in the literature. It is important to decide which risk measure should be taken into account.

- Markowitz [5] used mean-variance model in Portfolio Selection to measure risk. Since variance is a measure of volatility, the risk of an investor is to face with a large negative return i.e. loss, but variance only takes into account the positive return *i.e.* profit desired by the investors.
- Value-at-risk (VaR) is a measure which takes only negative return into account. VaR is the amount of money that expresses the maximum expected loss from an investment over a specific investment horizon for a given confidence level. It has great popularity among banks, insurance companies and other financial institutions. But VaR does not give any information beyond this amount of money. Also, it has undesirable mathematical characteristics such as lack of subadditivity and convexity. For example, VaR corresponding to a combination of two portfolios can be deemed greater than the sum of risks of the individual portfolios, and also VaR is difficult to optimize when it is calculated from scenarios.
- Conditional Value-at-risk (CVaR) is an extension of VaR and expresses the expected loss of an investment beyond its VaR value. CVaR, which is quite similar to VaR has more attractive properties than VaR. CVaR is a sub-additive and convex function as proved by Rockafellar and Uryasev [7]. Moreover CVaR is a coherent measure of risk, [1]. A coherent risk measure is a risk measure that satisfies some desired properties, namely, monotonicity, sub-additivity, homogeneity, translational invariance. Numerical experiments indicate that usually minimization of CVaR leads to near optimal solutions in VaR terms because VaR never exceeds CVaR [7]. Therefore, the portfolios with low CVaR must have low VaR as well. Moreover, when the return-loss distribution is normal, these two measures are equivalent, i.e. they provide the same optimal portfolio.
- The aim of this paper is to minimize CVaR under expected return constraint. Some definitions and results given by Rockafellar and Uryasev [1, 2, 7] are used to achieve the aim.

3. DEFINITION OF CONDITIONAL VALUE-AT-RISK

Let $L(w,y)$ be the loss associated with decision vector w in $\mathbb{R}^n$ and random vector y is $\mathbb{R}^m$. The vector w can be interpreted as a portfolio, such that $w \in X \subseteq \mathbb{R}^n$, where X is the set of available portfolios, whereas vector y stands for uncertainties, like market prices that can affect the loss. For each w, loss $L(w,y)$ is a random variable having a distribution in $\mathbb{R}$ induced by that of y. The probability distribution of y in $\mathbb{R}^m$ will be assumed to have density denoted by $p(y)$. The probability of $L(w,y)$ not exceeding a threshold α is given by

$$\phi(w,\alpha) = \int_{L(w,y) \le \alpha} P(y)\,dy,$$

for loss associated with w, $\phi(w,\alpha)$ is the cumulative distribution function (here w is fixed). We assume $\phi(w,\alpha)$ to be continuous with respect to α (this assumption is made for simplicity) and the function $\phi(w,\alpha)$ is non-decreasing with respect to α.

The $\beta-\text{VaR}$ and $\beta-\text{CVaR}$ values for a loss random variable associated with w and specified probability level $\beta \in (0,1)$, are denoted by $\alpha_\beta(w)$ and $\psi_\beta(w)$ defined as

$$\alpha_\beta(w) = \min\{\alpha \in \mathbb{R}: \phi(w,\alpha) \ge \beta\},$$

$$\psi_\beta(w) = (1-\beta)^{-1} \int_{L(w,y) \ge \alpha_\beta(w)} L(w,y)\,P(y)\,dy,$$

$\alpha_\beta(w)$ is the left-end-point of the non-empty interval consisting the values of β such that $\phi(w,\alpha) = \beta$, and the probability that $L(w,y) \ge \alpha_\beta(w)$ is equal to $(1-\beta)$. Thus, $\psi_\beta(w)$ gives the conditional expectation of the loss associated with w relative to that loss being $\alpha_\beta(w)$ or greater.

The characterization of $\psi_\beta(w)$ and $\alpha_\beta(w)$ in terms of function F_β on $X \times \mathbb{R}$ is defined as

$$F_\beta(w,\alpha) = \alpha + (1-\beta)^{-1} \int_{y \in \mathbb{R}^m} [L(w,y)-\alpha]^+ P(y)\,dy, \tag{1}$$

where $[\delta]^+ = \max\{\delta, 0\}$.

Theorem 1. [7]

As a function of α, $F_\beta(w, \alpha)$ is convex and continuously differentiable. The formula

$$\psi_\beta(w) = \min_{\alpha \in \mathbb{R}} \left[F_\beta(w,\alpha)\right], \tag{2}$$

determines $\beta-\text{CVaR}$ loss associated with $w \in X$.

The set consisting of the values of α for which the minimum is attained, namely

$$A_\beta(w) = \arg\min_{\alpha \in \mathbb{R}} \left[F_\beta(w,\alpha) \right],$$

is a non-empty, closed, bounded interval (or a single point), and α-VaR of the loss is given by

$$\alpha_\beta(w) = \text{ Left end point of } A_\beta(w),$$

i.e.

$$\alpha_\beta(w) \in A_\beta(w) \text{ and } \psi_\beta(w) = F_\beta(w, \alpha_\beta(w)),$$

Now in the next theorem, Rockofellar and Urysev [7] proved the advantages of defining β-VaR and β-CVaR through the formula in Theorem 1.

Theorem 2. [7]

Minimizing β-CVaR of the loss associated with $w \in X$ is equivalent to minimizing $F_\beta(w,\alpha)$ over all $(w,\alpha) \in X \times \mathbb{R}$, in the sense that

$$\min_{w \in X} \left[\psi_\beta(w) \right] = \min_{(w,\alpha) \in X \times \mathbb{R}} \left[F_\beta(w,\alpha) \right],$$

where a pair (w^*, α^*) achieves the right hand side minimum if and only if w^* achieves the left hand side minimum and $\alpha^* \in A_\beta(w^*)$.

In particular, in circumstances where interval $A_\beta(w^*)$ reduces to a single point, the minimization of $F_\beta(w,\alpha)$ over $(w,\alpha) \in X \times \mathbb{R}$ produces a pair (w^*, α^*), not necessarily unique such that w^* minimizes the β-CVaR and α^* gives corresponding β-VaR.

Note that $F_\beta(w,\alpha)$ is convex with respect to (w,α) and $\psi_\beta(w)$ is convex with respect to w, when $L(w,y)$ is convex with respect to w, in which case, if the constraints are such that X is a convex set, the joint minimization is an instance of convex programming.

According to Theorem 2, it is not necessary, for the purpose of determining vector w that yields the minimum β-CVaR, to work directly with the function $\psi_\beta(w)$. Moreover it is hard to work with $\psi_\beta(w)$ because of the nature of its definition in terms of β-VaR value $\alpha_\beta(w)$ and the often troublesome mathematical properties. Instead, one can operate on the far simpler expression $F_\beta(w,\alpha)$ with its convexity in the variable α.

4. DIFFERENT FORMULATIONS FOR OPTIMIZATION PROBLEM

Theorem 3 shows that equivalent formulation of three optimization problems in the sense that they produce the same efficient frontier [4].

Theorem 3. [4]

Let us consider function $\psi(w)$ and $R(w)$ dependent on the decision vector w. For the following three problems:

$$\min_{w\in X}[\psi(w)-\mu R(w)],\quad \mu\geq 0,\ w\in X\ , \tag{3}$$

$$\min_{w\in X}\left[\psi(w)\right],\quad R(w)\geq c,\ w\in X\ , \tag{4}$$

$$\min_{w}\left[-R(w)\right],\ \psi(w)\leq d,\ w\in X\ , \tag{5}$$

if $\psi(w)$ is convex, $R(w)$ is concave and set X is convex, then (3), (4), (5) generate the same efficient frontier.

Here μ, c, d are parameters and constraints $R(w)\geq c$ in (4) and $\psi(w)\leq d$ in (5) have internal points.

The equivalence of the problems (3)-(5) holds for any concave reward function and convex risk function with convex constraints.

We consider that the loss function $L(w,y)$ is linear with respect to w, and Theorem 2 shows that CVaR function $\psi(w)$ is convex with respect to w. Suppose that reward function $R(w)$ and other constraints are linear. So by Theorem 3, maximization of $R(w)$ (*i.e.* minimization of - $R(w)$) under CVaR constraint (problem (5) in Theorem 3) generates the same efficient frontier as the minimization of CVaR under reward constraint (problem (4) in Theorem 3). Theorem 2 shows that the function $F_\beta(w,\alpha)$ can be used instead of $\psi_\beta(w)$ to solve (4). Similarly, it can be shown that $F_\beta(w,\alpha)$ be used instead of $\psi_\beta(w)$ in problem (5) of Theorem 3

Theorem 4. [4]

The two minimization problems

$\min_{w}\left[-R(w)\right]$, (6)

subject to $\psi_\beta(w)\leq d,\ w\in X$ and

$\min_{w}\left[-R(w)\right]$, (7)

subject to $F_\beta(w,\alpha)\leq d,\ w\in X$

are equivalent in the sense that their objective functions achieve the same values. Moreover, if CVaR constraint in (6) is active, the pair (w^*, α^*) achieves minimum of (7) if and only if w^* achieves the minimum of (6) and $\alpha^* \le A_\beta(w^*)$. In particular, if the interval $A_\beta(w^*)$ reduces to a single point, then the minimization of $-R(w)$ over $(w,\alpha) \in X \times \mathbb{R}$ produces the pair (w^*, α^*) such that w^* minimizes the return and α^* gives corresponding β-VaR .

Next Theorem shows that $F_\beta(w, \alpha)$ can be used instead of $\psi_\beta(w)$ in problem (3) in Theorem 3.

Theorem 5. [4] The two minimization problems

$$\min_{w}[\psi_\beta(w) - \mu R(w)], \qquad \mu \ge 0,\; w \in X , \tag{8}$$

$$\min_{(w,\,\alpha)}[F_\beta(w, \alpha) - \mu R(w)], \quad \mu \ge 0,\; (w,\alpha) \in X \times \mathbb{R} , \tag{9}$$

are equivalent in the sense that their objective functions achieve the same values. Moreover, the pair (w^*, α^*) achieves minimum of (9) if and only if w^* achieves the minimum of (8) and $\alpha^* \in A_\beta(w^*)$. In particular, when the interval $A_\beta(w^*)$ reduces to a single point, then the minimization of $F_\beta(w, \alpha) - \mu R(w)$ over $(w,\alpha) \in X \times \mathbb{R}$ produces a pair (w^*, α^*) such that w^* minimizes $\psi_\beta(w) - \mu R(w)$ and α^* gives corresponding β-VaR .

In the next section, we explain the process of discretization and linearization defined in [4].

5. DISCRETIZATION AND LINEARIZATION USING DUMMY VARIABLES

The integral in (4) can be approximated in various ways. This can be done by sampling the probability distribution of y according to its density $P(y)$. If the sampling generates a collection of vectors $y_1, y_2, y_3, y_4 ......y_N$ then, the corresponding approximation to $F_\beta(w,\alpha)$ is

$$\left.\begin{aligned} F_\beta(w,\alpha) &= \alpha + (1-\beta)^{-1}\int_{y\in\mathbb{R}^m}[L(w,y)-\alpha]^+ P(y)dy, \\ F'_\beta(w,\alpha) &= \alpha + (1-\beta)^{-1}\sum_{i=1}^{N} p_i[L(w,y)-\alpha]^+ , \end{aligned}\right\} \tag{10}$$

where p_i's are the probabilities of scenarios y_i's . If the loss function $L(w,y)$ is linear w. r. t w, then the function $F'_\beta(w,\alpha)$ is convex and piecewise linear. So, the function

$F_\beta(w,\alpha)$ in optimization problems of Theorems 2, 4, 5 can be approximated by function $F'_\beta(w,\alpha)$ given by (10).

By using dummy variables z_i's $i = 1, 2,..., N$, function $F'_\beta(w,\alpha)$ can be further replaced by the linear function

$$\alpha + (1-\beta)^{-1}\sum_{i=1}^{N} p_i z_i$$

where $z_i \geq L(w, y_i) - \alpha,\ i = 1, 2,..., N,\ z_i \geq 0,\ i = 1, 2,..., N$.

In Theorem 4, $\psi_\beta(w) \leq d$ can be replaced by $F_\beta(w,\alpha) \leq d$. This constraint can be approximated by $F'_\beta(w,\alpha) \leq d$

i.e.

$$\alpha + (1-\beta)^{-1}\sum_{i=1}^{N} p_i z_i \leq d\ , \tag{11}$$

$$z_i \geq L(w, y_i) - \alpha,\ i = 1, 2,..., N,\quad z_i \geq 0,\ i = 1, 2,..., N,\ \alpha \in \mathbb{R}.$$

Now if we assume that all scenarios y_i's are equally probable *i.e.* $p_i = \frac{1}{N}\quad \forall\ i = 1, 2, ..., N$, then (11) becomes $\alpha + \frac{1}{N(1-\beta)}\sum_{i=1}^{N} z_i \leq d$.

We consider a simple investment problem where besides stocks and bonds, an investor can also include options (or structured products) into the investment portfolio.

Let L^w be the loss of an investor where investment portfolio w is given by $w = [w_S,\ w_B, w_p]$ where w_S is the weight of stock, w_B is weight of bond, w_p is weight of structured product β is confidence level, α is value at risk and d is a constant.

$$L^w = -R^w\ ,$$

where R^w is return associated with portfolio vector *w*. R^w is given by

$$R^w = \frac{\text{final wealth - initial wealth}}{\text{initial wealth}}.$$

Now assume that an investor invests in stock, bond and structured product with return r_T^S, r_T^B, r_T^P, respectively at time T. Return from portfolio w is R_T^w given by:

$$R_T^w = w_S r_T^S + w_B r_T^B + w_p r_T^p\ ,$$

$$w_S + w_B + w_p = 1\ ,$$

$$w_S, w_B, w_p \geq 0\,.$$

We attempt to minimize CVaR of a portfolio with constraint on expected return that has lower bound "d".

Problem 1.

$$\min_{w\in\mathbb{R}^3} CVaR(-R_T^w,\beta),$$

subject to

$$R_T^w = w_S r_T^S + w_B r_T^B + w_p r_T^p,$$

$$w_S + w_B + w_p = 1,$$

$$w_S, w_B, w_p \geq 0,$$

$$E(R_T^w) \geq d,$$

Using linearization procedure [4] for CVaR, the above problem is converted into linear optimization problem as follows:

Problem 2.

$$\min_{w\in\mathbb{R}^3}\left(\alpha+\frac{1}{N(1-\beta)}\sum_{i=1}^{N} z_i^w\right)$$

subject to

$$R_{T,i}^w = w_S r_{T,i}^S + w_B r_{T,i}^B + w_p r_{T,i}^B, \qquad i=1,2,\ldots,N,$$

$$w_S + w_B + w_p = 1,$$

$$w_S, w_B, w_p \geq 0,$$

$$\frac{1}{N}\sum_{i=1}^{N} R_{T,i}^w \geq d, \quad i=1,2,\ldots,N,$$

$$z_i^w \geq -R_{T,i}^w - \alpha, \quad i=1,2,\ldots,N,$$

$$z_i^w \geq 0, \qquad i=1,2,\ldots,N.$$

This problem mainly consists of two steps:

i. Simulate N paths of the market prices of the stock, bond and the structured product.
ii. This linear problem on those simulated paths can be solved by the well-known simplex method.

The index ñi ò corresponds to the values that occur in simulation run number. Note that dimension of the problem is of the order of number of simulated paths N. However, this also shows that the number of simulation runs determines the size of the problem, as considering more investment opportunities would only increase the dimension of the problem. In fact, one security more leads to one variable more, the corresponding component of the portfolio vector.

Suppose that our desired investment horizon time is T but structured product matures at time $T' < T$. The presence of such intermediate payments is main extension to problem 2. When investor receives these intermediate payments, he faces a problem of re-investment. We assume that an investor re-invests the intermediate payments in the remaining investment opportunities at the intermediate time T'. So, a single period problem becomes a multi-period one. At time $T' = (3/4)T$ (say), we choose a fixed reinvestment portfolio $x = [x_S, x_B]$, where $x_S, x_B \geq 0$ and $x_S + x_B = 1$.

Now, our problem is to find an optimal (initial) portfolio w (given fixed choice of x),

Here x_S = portfolio weight of stocks at time $(3/4)T$

x_B = portfolio weight of bond at time $(3/4)T$.

Let Π^0 denote the call option return at maturity time $(3/4)T$, S_0 denote initial stock price, K denote strike price and $(3/4)T$ is maturity time, $C\left(S_{0,} K, (3/4)T\right)$ is price of call option.

If $[K - S_{(3/4)T}]^+ = Max[0, K - S_{(3/4)T}]$

then $$\Pi^0 = \frac{[K - S_{(3/4)T}]^+ - C(S_0, K, (3/4)T)}{C(S_0, K, (3/4)T)},$$

$$i.e.\; 1 + \Pi^0 = \frac{[K - S_{(3/4)T}]^+}{C(S_0, K, (3/4)T)}.$$

If we use a call option with maturity at $(3/4)T$ as the structured product, then its return $r_T^{p,x}$ is given by

$$r_T^{p,x} = (1 + \Pi^0)\,[x_S(1 + R^S) + x_B(1 + R^B)] - 1$$

where R^S, R^B denote the return of the stock and the bond on the interval $\left[\frac{3}{4}T, T\right]$, respectively.

Problem 3.

$$\min_{w \in \mathbb{R}^3} \left(\alpha + \frac{1}{N(1-\beta)} \sum_{i=1}^{N} z_i^w \right), \tag{12}$$

subject to

$$R_{T,i}^{w,x} = w_S r_{T,i}^S + w_B r_{T,i}^B + w_p r_{T,i}^{p,x}, \qquad i = 1, 2, \ldots, N, \tag{13}$$

$$r_{T,i}^{p,x} = (1+\Pi_i^0)\,[x_S(1+R_i^S)+x_B(1+R_i^B)]-1, \tag{14}$$

$$w_S + w_B + w_p = 1, \tag{15}$$

$$w_S, w_B, w_p \geq 0, \tag{16}$$

$$x_S + x_B = 1, \tag{17}$$

$$x_S, x_B \geq 0, \tag{18}$$

$$z_i^w \geq -(R_{T,i}^{w,x} + \alpha), \qquad i=1,2,....,N, \tag{19}$$

$$\frac{1}{N}\sum_{i=1}^{N} R_{T,i}^{w,x} \geq d, \quad i=1,2,...,N, \tag{20}$$

$$z_i^w \geq 0, \quad i=1,2,...,N, \tag{21}$$

$$\alpha \text{ is free.} \tag{22}$$

Here again, the subscript i indicates the value of the indexed variable corresponding to the simulation run number i.

Note that the choice of the optimal re-investment strategy x mostly depends on the option structured product that is the alternative to the standard investment possibilities bond and stock.

Here eq. (12) is the objective function with the goal to minimize CVaR, eq. (13) represents portfolio return at time T for each scenario, eq. (14) shows total return from structured product at time T for each scenario. The eq. (15) and eq. (17) enable that the portfolio weights add upto 1, eq. (16) and eq. (18) guarantee that short selling is not allowed, eq. (19), eq. (21) and eq. (22) are needed to control CVaR objective. These three constraints guarantee that optimal value of objective function gives CVaR and the corresponding value of α (if it is unique) will be equal to VaR**.** If there are many optimal values of α, then the required VaR is the left end-point of the optimal interval. Eq. (20) gives the expected return constraint with lower bound "d".

The optimization model (problem 3) can be reformulated by taking objective function as a combination of two objectives both CVaR and expected return. It will produce the same efficient frontier as explained in Theorem 3 using problems (3) and (4).

Problem 4.

$$\min_{w\in\mathbb{R}^3}\left[\delta\left(\alpha+\frac{1}{N(1-\beta)}\sum_{i=1}^{N} z_i^w\right)-(1-\delta)\left(\frac{1}{N}\sum_{i=1}^{N} R_{T,i}^{w,x}\right)\right], \tag{23}$$

subject to

$$R_{T,i}^{w,x} = w_S r_{T,i}^S + w_B r_{T,i}^B + w_p r_{T,i}^{p,x}, \qquad i=1,...,N, \tag{24}$$

$$r_{T,i}^{p,x} = (1+\Pi_i^0)\left[x_S(1+R_i^S)+x_B(1+R_i^B)\right]-1, \qquad i=1,2,...,N, \tag{25}$$

$$w_S + w_B + w_p = 1\,, \tag{26}$$

$$x_S + x_B = 1\,, \tag{27}$$

$$w_S, w_B, w_p, x_S, x_B \geq 0, \tag{28}$$

$$z_i^w \geq -(R_{T,i}^{w,x} + \alpha), \qquad i = 1, 2, ..., N, \tag{29}$$

$$z_i^w \geq 0, \;\; i = 1, 2, ..., N, \tag{30}$$

$$\alpha \text{ is free.} \tag{31}$$

The objective function in eq. (23) is a convex combination of two objectives CVaR , expected return CVaR is weight by δ, and expected return is weighted by $(1-\delta)$. If $\delta = 1$, the investor is interested in minimizing risk with no interest in the return objective; if $\delta = 0$, the investorôs only objective is to maximize the return. The minus sign before the return objective is because we minimize this weighted average.

Eq. (24) is return of the portfolio at time T for each scenario *i* (with portfolio weight-vector *w* and re-investment portfolio weight-vector *x*). Eq. (25) is return from structured product at time *T* for each scenario *i* (after re-investment of time $(3/4)T$).

Eq. (26) and Eq. (27) guarantee that portfolio weights add up to 1. Eq. (28) confirms that no short selling is allowed. Eq. (29) and Eq. (30) and Eq. (31) are needed to control the first part of the objective function i.e. CVaR part. These constraints guarantee that the optimal value of this term in the objective gives CVaR and that the corresponding optimal value of α (if unique) will be equal to VaR. If there are many optimal values of α then, VaR is the left end-point of the optimal interval.

7. CONCLUSIONS

In this paper, we discussed a particular investment problem, where besides stocks and bonds an investor can also include options (structured products) into the portfolio. We allow intermediate payments of the securities and are thus, faced with a re-investment problem which turns the originally one-period problem into a special kind of multi-period problem. We have considered an approach for simultaneous calculation of VaR and optimization of CVaR . We showed that CVaR can be efficiently minimized by using linear programming. Although, formally, the method minimizes only CVaR , but in fact, it also lowers VaR because CVaR $\geq$ VaR .

This approach can handle large number of instruments and scenarios. Structured products allow investors to make profit from the equity risk premium without being fully exposed to the downside risk associated with investing in stocks. Our investment problem can also be solved when we have more securities which can also have multiple internal payments. In particular, we can think of more than two periods in our optimization problem. However, then in that case the outer optimization loop for obtaining the optimal re-investment strategy gets more complicated. Each additional time period will add one more loop; and hence, finding the solution of the problem will take longer. Despite computational difficulties such problems are important to explore in future.

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