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OPTIMAL SHORT–TERM OPERATION PLANNING OF INTERCONNECTED HYDRO–THERMAL POWER SYSTEMS

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Abstract: A method for short term operation planning and power interchange scheduling of interconnected hydro–thermal power systems is proposed in the paper. The criterion function is the total variable profit obtained by sales of energy to customers within the system, interchange of energy with other systems and system operation. The method is based on Lagrangian relaxation, decomposition technique and successive solution of dual and primal problems. The dual and primal problems are decomposed and solved using appropriate optimization methods, while Lagrangian multipliers are corrected using subgradient method. The efficiency of the method is illustrated on a test power system.

Key words: Lagrangian relaxation, decomposition, power system operation planning, interchange scheduling.

1. INTRODUCTION

The problem of short–term operation planning of hydro–thermal power systems consists of determining the operation plans of all thermal power units (including unit commitment), hydro power plants and pumped storage hydro power plants in the system 24–168 hours in advance. Many authors have based their solutions to operation planning problems on Lagrangian relaxation and decomposition. Merlin and Sandrin [11] proposed a method for short–term unit commitment and implemented it on a 172–unit thermal system. This method used dynamic programming to solve subproblems for each thermal unit. Geromel and Baptistella [7] used feasible direction method to determine the optimal operation schedule of a hydroelectric power system, while Soares et al. [17], Habibollahzadeh et al. [8,9] and Chao–an Li and Streiffert [3,4] treated the short–term hydro–thermal scheduling problem. Monti et al. [12] gave the basic idea of solving the

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problem of joint operation of thermal units and pumped storage hydro plants in the system, while Rakić et al. [15] and Ferreira et al. [6] treated the problem of short–term operation planning of all resources in the system. Latest papers such as Wu et al [18] and Rakić and Marković [16] extend this problem to include short–term interchange scheduling.

This paper jointly treats the problems of short–term operation planning and interchange scheduling of interconnected hydro–thermal power systems. Considering the structure of the hydro subsystem, two types of power systems were treated in this paper and, accordingly, two approaches were applied to solve the overall problem. The optimal operation plans of hydro power plants in hydro–thermal power systems with small numbers of cascade–coupled hydro power plants were determined using feasible direction method, whereas the primal problem was defined as a hydro–thermal coordination problem. On the other hand, the operation plans of hydro power plants in hydro–thermal power systems with great numbers of cascade–coupled hydro plants were determined using a method based on linearization and network flow algorithm [1] and the primal problem was defined as a thermal optimization problem.

1.1. NOTATION

T – time horizon, number of discrete time steps used in the model

J – number of thermal units

I – number of hydro power plants

K – number of pumped storage hydro power plants

N – number of high voltage tie lines, connecting the power system with other power systems

N_{zn},N_{yn} – number of export and import transactions performed over tie line n, respectively

D^t – system power demand in hour t

R^t – system spinning reserve requirement in hour t

S_j^t – loading of thermal unit j in hour t

$\underline{S}_j$, $\overline{S}_j$ – minimum and maximum capacity of unit j, respectively

Δ_{uj}, Δ_{dj} – maximum ramp–up and ramp–down rate of unit j, respectively

$\overline{R}_{Tj}$ – maximum spinning reserve of unit j

u_j^t – up/down state of unit j in hour t (1–unit is up, 0–unit is down)

τ_j^t – duration of up/down state of unit j in hour t

a_j, b_j, c_j – production cost curve coefficients for unit j

$\alpha_j, \beta_j, \delta_j$ – start–up cost curve coefficients for unit j

γ_j – shut–down cost for unit j

Q_i^t – water release from reservoir i in hour t

$\underline{Q}_i, \overline{Q}_i$ – minimum and maximum water release from reservoir i, respectively

G_i^t – spilling from reservoir i in hour t

V_i^t – content of reservoir i in hour t

$\underline{V}_i, \overline{V}_i$ – minimum and maximum content of reservoir i, respectively

A_i, B_i, C_i – power production curve coefficients of hydro plant i (depending on reservoir contents)

W_i^t – local water inflow to reservoir i in hour t

l_k^t – number of pumps operating in pumped storage hydro plant k in hour t

Q_{pk}^t – pumping flow in pumped storage hydro plant k in hour t

U_n – maximum capacity of tie line n

$\overline{p}_{ni}$ – power limit of energy transaction i over tie line n

$\underline{Z}_{ni}, \overline{Z}_{ni}$ – lower and upper bound of exported energy in transaction i over tie line n, respectively

$\underline{Y}_{ni}, \overline{Y}_{ni}$ – lower and upper bound of imported energy in transaction i over tie line n, respectively

z_A^t, y_A^t – maximum power export and import in hour t through a subset A of all tie lines, respectively

2. CHARACTERISTICS OF POWER UNITS

The power production sources of an electric power system are thermal power units, hydro power plants and pumped storage (reversible) hydro power plants. The characteristics of power units are given below.

2.1. THERMAL POWER UNITS

Thermal power units are characterized by their spinning reserve characteristics, and production cost, start–up and shut–down cost functions. The available spinning reserve of a thermal power unit is presented as a piecewise linear function of the output power

$$R_{Tj}(S_j^t,u_j^t) = u_j^t \cdot \min(\overline{R}_{Tj}, \overline{S}_j - S_j^t), \qquad j=1,...,J,\ t=1,...,T. \qquad (1)$$

The production cost function of each unit is represented as a quadratic function of the loading

$$f_{1j}(S_j^t) = a_j \cdot (S_j^t)^2 + b_j \cdot S_j^t + c_j, \qquad j=1,...,J,\ t=1,...,T. \qquad (2)$$

The start–up cost is represented as an exponential function of the time τ_j^t the unit had been down before the start–up in hour t:

$$f_{2j}(\tau_j^t) = \alpha_j \cdot \left[1 - \exp(-\frac{\tau_j^t}{\delta_j})\right] + \beta_j, \qquad j=1,..., J,\ \ t=1,...,T \qquad (3)$$

The shut–down cost is assumed constant:

$$f_{3j} = \gamma_j, \qquad j = 1,...,J. \qquad (4)$$

The total operation cost of thermal unit j in hour t is the sum of production, start–up and shut–down costs in that hour, i.e.:

$$f_j^t = u_j^t \cdot f_{1j}(S_j^t) + u_j^t \cdot (1-u_j^{t-1}) \cdot f_{2j}(\tau_j^t) + u_j^{t-1} \cdot (1-u_j^t) \cdot f_{3j}, \qquad (5)$$

$$j = 1,..., J,\ \ t = 1,...,T.$$

The operation of thermal units is subject to the following constraints:

(a) constraints on the loading:

$$\underline{S}_j \le S_j^t \le \overline{S}_j, \qquad j=1,...,J,\ t=1,...,T, \qquad (6)$$

(b) constraints on the ramp–up and ramp–down rates:

$$-\Delta_{dj} \le S_j^t - S_j^{t-1} \le \Delta_{uj}, \qquad j=1,\dots,J, \;\; t=1,\dots,T, \qquad (7)$$

(c) minimum up and down times and

(d) maximum number of start–ups.

2.2. Hydro Power Plants

The power production characteristic of a hydro power plant is modeled as a quadratic function of water release, where the coefficients depend on the reservoir contents at the beginning of hour t:

$$H_i(Q_i^t, V_i^{t-1}) = A_i(V_i^{t-1}) + B_i(V_i^{t-1})\cdot Q_i^t + C_i(V_i^{t-1})\cdot (Q_i^t)^2, \qquad (8)$$

$$i=1,\dots,I \;\; t=1,\dots,T.$$

The available spinning reserve function of a hydro power plant represents the difference between maximum power production and current power production of the hydro power plant:

$$R_{Hi}(Q_i^t, V_i^{t-1}) = H_i(\overline{Q}_i, V_i^{t-1}) - H_i(Q_i^t, V_i^{t-1}), \qquad i=1,\dots,I, \;\; t=1,\dots,T. \; (9)$$

The operation of hydro power plants is subject to the following constraints:

(a) constraints on the water release of hydro power plant:

$$\underline{Q}_i \le Q_i^t \le \overline{Q}_i, \qquad i=1,\dots,I, \;\; t=1,\dots,T, \qquad (10)$$

(b) constraints on the reservoir contents:

$$\underline{V}_i \le V_i^t \le \overline{V}_i, \qquad i=1,\dots,I, \;\; t=1,\dots,T, \qquad (11)$$

(c) constraints on nonnegative spilling:

$$0 \le G_i^t, \qquad i=1,\dots,I, \;\; t=1,\dots,T, \qquad (12)$$

(d) flow conservation constraints:

$$V_i^t = V_i^{t-1} - Q_i^t - G_i^t + W_i^t + \sum_{j\in\Omega_{ui}} (Q_j^{t-\theta_{ji}} + G_j^{t-\theta_{ji}}), \qquad i=1,\dots,I, \;\; t=1,\dots,T. \; (13)$$

In (13) Ω_{ui} represents the set of neighboring upstream hydro power plants to the i–th one, and θ_{ji} is the wave propagation delay time from the j–th upstream hydro power plant to the i–th one. The last term in (13) represents the inflow to water reservoir i in hour t due to water release and spilling from all neighboring upstream reservoirs.

The flow conservation constraint (13) suggests network flow representation to be used for the hydro subsystem if it consists of a large number of coupled hydro power plants. Figure 1 illustrates the network model of a river basin with *M* cascade–coupled hydro power plants with no time delay in water wave propagation between upstream and downstream hydro plants.

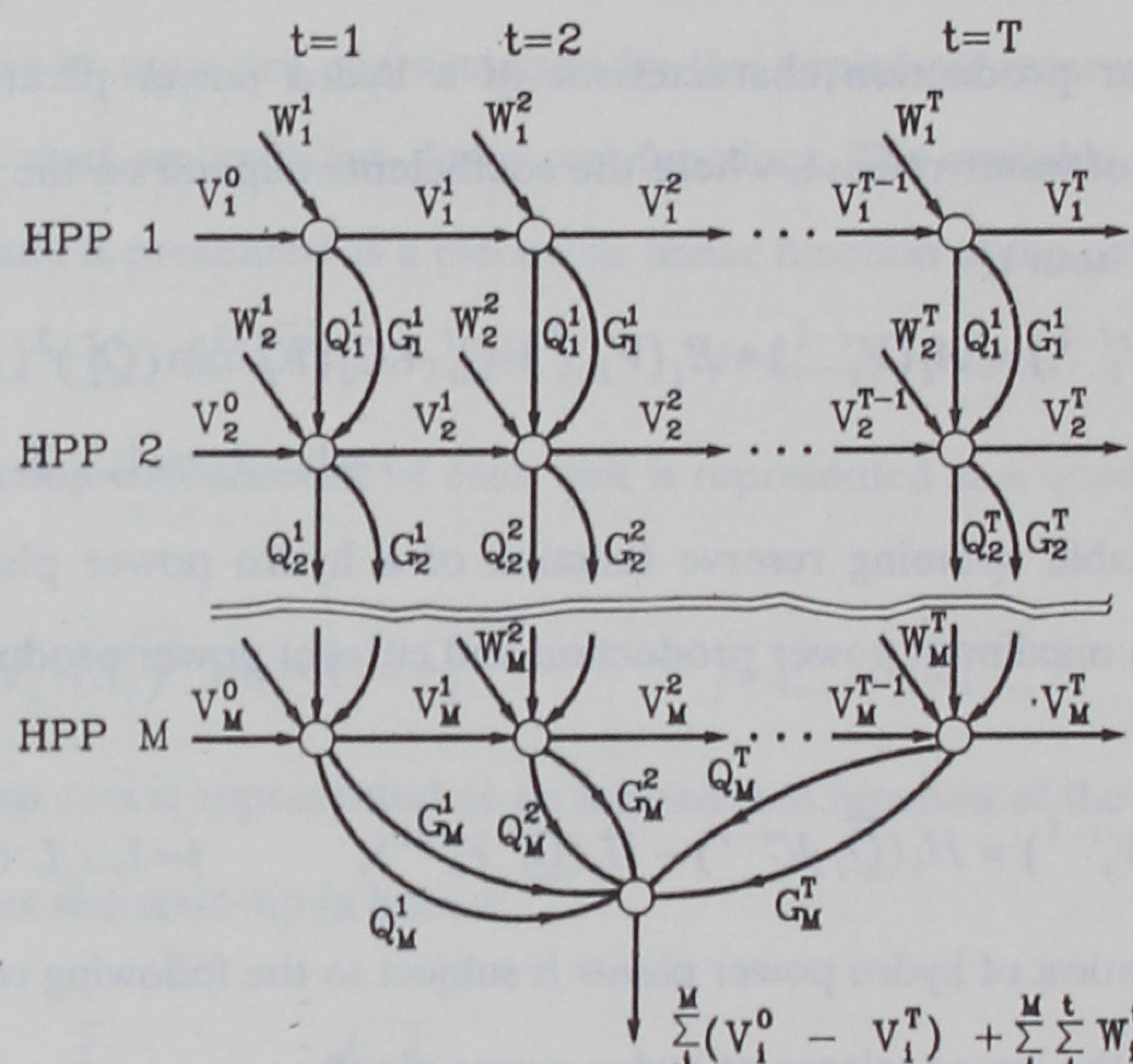

Figure 1. Network flow representation of cascade–coupled hydro power plants

2.3. Pumped Storage Hydro Power Plants

Pumped storage hydro power plants may operate in two modes: production (generation) mode and pumping mode. Constraints (11) have to be satisfied regardless of the operation mode of pumped storage hydro power plants. Since pumped storage hydro power plants are always located so that they have no upstream hydro power plants, and presumably there is no local water inflow to the reservoir of the pumped storage hydro plant, constraints (13), modified so as to take into account the pumping mode of operation, get the following form:

$$V_k^t = V_k^{t-1} - Q_k^t + Q_{pk}^t, \qquad k=1,\dots,K, \; t=1,\dots,T. \qquad (14)$$

Since pumped storage hydro power plants can operate either in the production or in the pumping mode, in expression (14) either water release Q_k^t, or water pumping flow Q_{pk}^t, equals zero.

Production mode: There is no formal difference between hydro power plants and pumped storage hydro power plants operating in production mode. The maximum and minimum water release constraints (10) and power production curve (8) are also applicable for pumped storage hydro power plants operating in this mode.

Pumping mode: The only control variable in this mode is the number of pumps in operation, while the upper reservoir content is treated as a parameter. In pumping mode, pumping flow may be changed in discrete steps only, each step corresponding to a number of pumps turned on. Hence, the pumping flow can be represented as:

$$Q_{pk}^t = Q_{pk}(l_k^t, V_k^{t-1}), \qquad k=1,\ldots,K, \; t=1,\ldots,T, \tag{15}$$

and the pumping power consumption can be represented by discrete values in a similar way:

$$P_k^t = P_k(l_k^t, V_k^{t-1}), \qquad k=1,\ldots,K, \; t=1,\ldots,T. \tag{16}$$

The available spinning reserve function of a pumped storage hydro plant could therefore be presented as the difference between the maximum power production and current hydro production (or "negative" pumping power consumption) of the pumped storage hydro plant:

$$R_{Pk}(Q_k^t, V_k^{t-1}, l_k^t) = H_k(\overline{Q}_k, V_k^{t-1}) - \left[H_k(Q_k^t, V_k^{t-1}) - P_k(l_k^t, V_k^{t-1})\right], \tag{17}$$
$$k=1,\ldots,K, \; t=1,\ldots,T,$$

where in (17) one of the last two terms equals zero.

3. CHARACTERISTICS OF POWER SYSTEM OPERATION

3.1. POWER EXCHANGE

The interconnection of power systems implies power and energy exchange through high voltage tie lines. These input and output energy interchanges consist of energy transactions.

An energy transaction i, through tie line n is a sequence of hourly power output or power input nonnegative values $z_{ni}^1, z_{ni}^2, \ldots, z_{ni}^T$, denoting hourly values of energy exported to, or $y_{ni}^1, y_{ni}^2, \ldots, y_{ni}^T$, denoting hourly values of energy imported from a power company within a certain agreement. It is supposed that the values of power exchange in hour t, z_{ni}^t and y_{ni}^t, may have corresponding hourly per–megawatt prices c_{zni}^t and c_{yni}^t, respectively.

Since energy transactions are commonly power limited, power exchange variables are bounded:

$$0 \le z_{ni}^t \le \overline{p}_{ni}, \quad 0 \le y_{ni}^t \le \overline{p}_{ni}, \qquad t=1,\ldots,T. \tag{18}$$

Various constraints may be introduced to power exchange relations. Some of them are:

(a) Constraints on the tie line capacity

$$-U_n \le \sum_{i=1}^{N_{zn}} z_{ni}^t - \sum_{i=1}^{N_{yn}} y_{ni}^t \le U_n, \qquad t=1,\ldots,T. \tag{19}$$

(b) Constraints on the energy to be exchanged within a transaction:

$$\underline{Z}_{ni} \le \sum_{t=1}^{T} z_{ni}^t \le \overline{Z}_{ki}, \quad \underline{Y}_{ni} \le \sum_{t=1}^{T} y_{ni}^t \le \overline{Y}_{ni}. \tag{20}$$

(c) Constraints on the power to be exported or imported through a subset of tie lines:

$$\sum_{n \in A} \sum_{i=1}^{N_{zn}} z_{ni}^t \le z_A^t, \quad \sum_{n \in A} \sum_{i=1}^{N_{yn}} y_{ni}^t \le y_A^t, \quad A \subseteq \{1,\ldots,N\}. \tag{21}$$

Although various types of constraints could be introduced, they all have forms of linear inequalities.

3.2. System Constraints

Constraints on the total power system operation have to be satisfied throughout the whole optimization period. Two types of constraints were taken into account:

The first one represents the set of equality constraints on the balance of power production of all power plants and power import, on one side, and the power consumption by customers, pumps of pumped storage hydro power plants and export of power, on the other, as represented by the expression:

$$\Phi_D^t = D^t - \sum_{j=1}^{J} S_j^t - \sum_{i=1}^{I} H_i(Q_i^t, V_i^{t-1}) - \\ - \sum_{k=1}^{K}\left[H_k(Q_k^t, V_k^{t-1}) - P_k(l_k^t, V_k^{t-1})\right] + \sum_{n=1}^{N}\left[\sum_{i=1}^{N_{zn}} z_{ni}^t - \sum_{i=1}^{N_{yn}} y_{ni}^t\right] = 0. \tag{22}$$

$$t = 1,...,T$$

The second one represents the set of inequality constraints on the total power spinning reserve requirements:

$$\Phi_R^t = R^t - \sum_{j=1}^{J} R_{Tj}(S_j^t, u_j^t) - \sum_{i=1}^{I} R_{Hi}(Q_i^t, V_i^{t-1}) - \\ - \sum_{k=1}^{K} R_{Pk}(Q_k^t, V_k^{t-1}, l_k^t) \le 0, \qquad t = 1,\dots,T \tag{23}$$

where $R_{Tj}(\cdot,\cdot)$, $R_{Hi}(\cdot,\cdot)$, and $R_{Pk}(\cdot,\cdot,\cdot)$ are defined by (1), (9) and (17) respectively.

4. STATEMENT OF THE OPERATION PLANNING PROBLEM

The problem of short–term power system operation planning is stated as follows:

Find the optimal operation plan of a complex power system, including commitment of thermal power units, power production of hydro power plants, operation of pumped storage hydro power plants and power exchange with other companies, so as to maximize the total variable profit of power system operation, power delivery and power exchange defined by:

$$\pi = \sum_{t=1}^{T}\left[c_D^t \cdot D^t + \sum_{n=1}^{N}\sum_{i=1}^{N_{zn}} c_{zni}^t \cdot z_{ni}^t\right] - \sum_{t=1}^{T}\left[\sum_{j=1}^{J} f_j^t + \sum_{n=1}^{N}\sum_{i=1}^{N_{yn}} c_{yni}^t \cdot y_{ni}^t\right], \tag{24}$$

subject to all local and system constraints.

In (24), c_D^t represents the internal price of the power consumed by customers in the power system, and f_j^t is defined by (5). The first portion in (24) represents the total revenue from energy sales to the customers in the power system, so as to satisfy forecasted demand, and sales of energy to other companies, while the second part represents total cost of power production and purchase of energy from other companies. The profit function terms which are independent of the production level and operation schedule were omitted from (24).

Since the part of the revenue obtained by energy sales to the customers $\sum_{t=1}^{T} c_D^t \cdot D^t$ is constant, it may be omitted from (24), and hence the problem stated above may be reformulated as:

Minimize:

$$F = \sum_{t=1}^{T}\left[\sum_{j=1}^{J} f_j^t + \sum_{n=1}^{N}\sum_{i=1}^{N_{yn}} c_{yni}^t \cdot y_{ni}^t - \sum_{n=1}^{N}\sum_{i=1}^{N_{zn}} c_{zni}^t \cdot z_{ni}^t\right], \tag{25}$$

under the same conditions as in the above problem statement.

Let us note that the criterion function (25) differs from the usual formulation in references [5,6,11,15] where the criterion function is the total cost of energy production. Due to energy exchange, two new members appear in (25). The first one represents the cost of energy import and the second one represents the revenue ("negative cost") from energy export.

5. PROBLEM SOLUTION

The variables representing on/off states of thermal power units and pumps of pumped storage hydro plants are integer, and hence, the problem stated in Section 4 represents a large-scale, mixed-integer, nonconvex (due to the nonconvexity of the solution set) nonlinear optimization problem with a large number of variables. The solution of the hydro-thermal operation planning problem without power exchange, using Lagrangian relaxation method was considered in papers [5,6,15]. A similar approach is applied in this paper. By incorporating system constraints (22) and (23) into the criterion (25) using vectors of Lagrange multipliers λ and μ, the following Lagrangian function is obtained:

$$\begin{aligned} L = & \sum_{t=1}^{T}\sum_{j=1}^{J} f_j^t + \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=1}^{N_{yn}} c_{yni}^t \cdot y_{ni}^t - \\ & -\sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=1}^{N_{zn}} c_{zni}^t \cdot z_{ni}^t + \sum_{t=1}^{t}(\lambda^t \cdot \Phi_D^t + \mu^t \cdot \Phi_R^t) \end{aligned} \tag{26}$$

where in (26), the first three terms represent the total cost of power system operation and power exchange, as defined by (25), and Φ'_D and Φ'_R are defined by (22) and (23), respectively. The following dual problem is formulated:

$$\max_{\lambda,\mu} \left(\min_{S,u,Q,l,z,y} \mathcal{L} \right), \tag{27}$$

where in (27) $\mathcal{L}$ is represented by relation (26).

The technique to solve the problem of minimization of (26) subject to all local constraints is the successive solving of the mixed–integer dual problem for given (fixed) values of Lagrange multipliers λ and μ and the primal problem with all integer and some continuous variables fixed. If the solution of the problem is found, this technique guarantees that the global optimum is within the given accuracy range of the obtained solution.

5.1. DUAL PROBLEM SOLUTION

The dual problem, defined by (27) and all local constraints, is solved using a two–level algorithm. For fixed values of Lagrange multipliers, the problem is decomposed into a number of subproblems: (a) one subproblem for each thermal power plant, (b) one subproblem for each river basin or independent hydro power plant, (c) one subproblem for each pumped storage hydro power plant and (d) one subproblem for optimal power exchange planning. These subproblems are solved as the first level optimizations independently for fixed values of λ and μ multipliers. On the second level λ and μ multipliers are corrected by the coordination algorithm, until the dual problem is solved.

Thermal unit subproblems: The thermal unit subproblems are defined by local constraints on minimum up and down times and maximum number of start–ups, upper and lower bound constraints (6), ramping constraints (7), and the optimization criterion defined by:

$$\mathcal{L}_{Tj} = \sum_{t=1}^{T} \left[f_j^t - \lambda^t \cdot S_j^t - \mu^t \cdot R_{Tj}(S_j^t, u_j^t) \right]. \tag{28}$$

Each thermal unit subproblem is solved using backward dynamic programming method. The state of the j–th thermal unit at time interval t, X_j^t, consists of 4 components: (a) on/off state of thermal unit u_j^t; (b) duration of the on/off state the unit has been operating in, τ_j^t, (c) power output S_j^t and (d) the number of start-ups. The optimal value of (28) over period t, $t+1$, ... ,T, Ψ_j^t depends on the initial state X_j^{t-1}:

$$\Psi_j^t(X_j^{t-1}) = \min_{\{S_j^\vartheta, u_j^\vartheta\}} \left\{ \sum_{\vartheta=t}^{T} \left[f_j^\vartheta - \lambda^\vartheta \cdot S_j^\vartheta - \mu^\vartheta \cdot R_{Tj}(S_j^\vartheta, u_j^\vartheta) \right] \right\}, \tag{29}$$

$t=T$, $T-1$, ... ,1.

The optimal value of the criterion function at the beginning of the optimization period depends on the initial state X_j^0:

$$\min \mathcal{L}_{Tj} = \Psi_j^1(X_j^0). \tag{30}$$

The recurrence relation for the solution of the j–th thermal unit subproblem using dynamic programming is:

$$\Psi_j^t(X_j^{t-1}) = \min_{S_j^t, u_j^t} \left\{ f_j^t - \lambda^t \cdot S_j^t - \mu^t \cdot R_{Tj}(S_j^t, u_j^{t-1}) + \Psi_j^{t+1}(X_j^t) \right\}, \tag{31}$$

$t=T-1$, $T-2$, ... ,1,

where in (31) $\Psi_j^{t+1}(X_j^t)$ is determined by using (31) in the previous dynamic programming step for $t=t+1$, $t+2$, ... ,$T-1$, while Ψ_j^T is determined by (29) for $t=T$. For each feasible state X_j^{t-1} the values of u_j^t and S_j^t which minimize (31) have to be determined.

Hydro subproblems: The hydro subproblems have local constraints (10)–(13) and the optimization criterion:

$$\mathcal{L}_{Hm} = -\sum_{t=1}^{T} \sum_{i \in \Omega_m} \left\{ \lambda^t \cdot H_i(Q_i^t, V_i^{t-1}) + \mu^t \cdot R_{Hi}(Q_i^t, V_i^{t-1}) \right\} \tag{32}$$

where in (32) Ω_m represents the set of indices of hydro plants belonging to river basin m. One should note that due to constraints (13), for each river basin an optimization subproblem has to be established. Such a decomposition of the hydro subsystem is natural, since there is no interaction between river basins. Isolated hydro power plants are considered as separate optimization subproblems. If the hydro subsystem predominantly consists of isolated hydro plants, or the number of hydro plants in river basins is rather small, it is suitable to solve the problem using feasible direction method of nonlinear programming. This solution method is shown in Appendix A. If the hydro subsystem consists of a great number of cascade–coupled hydro plants, nonlinear network flow algorithm is more appropriate to solve the hydro subproblems. This method is shown in Appendix B.

Pumped storage hydro subproblems: An optimization subproblem corresponds to each pumped storage hydro power plant. It is defined by initial and final states, local constraints (10), (11) and (14) and the optimization criterion:

$$\mathcal{L}_{Pk} = -\sum_{t=1}^{T}\Big\{\lambda^t\cdot\Big[H_k(Q_k^t,V_k^{t-1}) - P_k(l_k^t,V_k^{t-1})\Big] + +\mu^t\cdot R_{Pk}(Q_k^t,V_k^{t-1},l_k^t)\Big\}. \tag{33}$$

The pumped storage subproblems are solved using T-step backward dynamic programming method. The state X_k^t of pumped storage hydro power plant at time interval t is the water reservoir content. Only the states X_k^t, t=1,...,T-1, feasible in respect to the initial and final states, minimum and maximum reservoir contents and maximum water flow in both operation modes (production and pumping mode) are tested. If the optimal value of (33) over period t, t+1, ... ,T, is defined as a function of the initial state X_k^{t-1}:

$$\Psi_k^t(X_k^{t-1}) = \max_{\{Q_k^\vartheta,l_k^\vartheta\}}\left\{\left[\sum_{\vartheta=t}^{T}\lambda^\vartheta\cdot(H_k(Q_k^\vartheta,V_k^{\vartheta-1}) - P_k(l_k^\vartheta,V_k^{\vartheta-1})) + +\mu^\vartheta\cdot R_{Pk}(Q_k^\vartheta,V_k^{\vartheta-1},l_k^\vartheta)\right]\right\} \tag{34}$$

$$t=T,T-1,...,1$$

one could write the following recurrence relation:

$$\Psi_k^t(X_k^{t-1}) = \max_{Q_k^t,l_k^t}\Big\{\lambda^t\cdot\Big[H_k(Q_k^t,V_k^{t-1}) - P_k(l_k^t,V_k^{t-1})\Big] + +\mu^t\cdot R_{Pk}(Q_k^t,V_k^{t-1},l_k^t) + \Psi_k^{t+1}(X_k^t)\Big\} \tag{35}$$

$$t=T-1,T-2,...,1$$

where in (35) $\Psi_k^{t+1}(X_k^t)$ is determined using (35) in the previous dynamic programming step for $t=t+1,t+2,...,T-1$ and Ψ_k^T is determined by (34) for $t=T$. Hence, the optimal value of the criterion function for the whole optimization period is obtained from (35) for $t=1$:

$$\min\mathcal{L}_{Pk} = -\Psi_k^1(X_k^0) \tag{36}$$

where X_k^0 represents the initial reservoir content.

Solution of the power exchange subproblem: As a result of the dual problem decomposition, a power exchange subproblem is obtained with the optimization criterion:

$$\mathcal{L}_E = \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=1}^{N_{yn}} (\lambda^t - c^t_{yni})\cdot y^t_{ni} + \sum_{t=1}^{T}\sum_{n=1}^{N}\sum_{i=1}^{N_{zn}} (c^t_{zni} - \lambda^t)\cdot z^t_{ni}\,. \quad (37)$$

The goal is to maximize the criterion function (37) subject to constraints (18)–(21).

The power exchange subproblem is a linear programming problem with upper bounds. The problem is easily solved using simplex method for given power exchange prices c^t_{yni} and c^t_{zni}, and λ parameters determined by the coordination algorithm.

Coordination algorithm: The Lagrangian multipliers λ and μ are modified according to the following rules:

a) If the primal problem solution was not obtained in the current iteration, λ^t and μ^t multipliers are modified according to the calculated values of Φ^t_D and Φ^t_R functions, defined by (22) and (23) respectively:

$$\lambda^t_{new} = \lambda^t_{old} + k_1 \cdot \Phi^t_D, \quad (38)$$

$$\mu^t_{new} = \mu^t_{old} + k_2 \cdot (\Phi^t_R + \varepsilon^t), \quad (39)$$

where coefficients k_1 and k_2 depend on the iteration index and ε^t is a constant.

b) If the primal problem solution was obtained in the current iteration, Lagrangian multipliers λ are modified according to the values of incremental production costs determined in the primal problem solution, ρ^t_{opt}:

$$\lambda^t_{new} = x\cdot\lambda^t_{old} + (1-x)\rho^t_{opt} \quad (40)$$

where x is empirically determined as $0<x<1$.

5.2. Primal Problem Solution

According to the technique described above, the primal problem can be formulated in two ways. The first one is to fix all commitments of thermal units, pumping schedules of pumped storage hydro power plants and power exchange schedules with other companies obtained in the dual problem solution, defining the primal problem as a hydro–thermal coordination problem. The second approach includes the fixing policy as in the first case, plus fixing the operation schedule of all hydro power plants in the system, yielding a primal problem defined as a thermal cost minimization problem.

Hydro–thermal coordination: As in the dual problem statement, local constraints on thermal unit and hydro power plant operation have to be taken into account. The criterion function is the total cost of power production. Thermal power units commitment, pumping schedules of pumped storage hydro power plants and power exchange schedules are adopted from the dual problem solution and fixed. The problem is solved using Lagrangian relaxation and feasible direction method. If the system constraint (22) is treated as a soft one, and the problem is relaxed, the Lagrangian function of the primal problem will have the following form:

$$\mathcal{P}=\sum_{t=1}^{T}\left[\sum_{j=1}^{J}u_j^t\cdot f_{1j}(S_j^t)+\rho^t\Phi_D^t\right], \tag{41}$$

where in (41) $f_{1j}(S_j^t)$ represents the operation cost of thermal power units, defined by (2), with on/off switching of thermal power units u_j^t fixed to the values 0 or 1 obtained in the corresponding dual problem solution, and Φ_D^t represents the current value of the power balance function (22) in the primal problem solving process with all integer variables fixed. The problem with the criterion function (41) and all local constraints on thermal power units and hydro power plants operation, can be decomposed into: (a) hydro subproblems, solved independently for each river basin, as in the dual problem solution, and (b) thermal subproblems, solved independently for each thermal power unit.

In hydro subproblems the criterion function is similar to (32) (with the exception that the last term in (32), concerning the spinning reserve constraint is omitted, since spinning reserve is not treated in the primal problem solution). Since the pumping schedules of pumped storage hydro power plants are fixed, they are treated as regular hydro power plants, but all the constraints, including (14), have to be satisfied. Hydro subproblems are solved using feasible direction method, starting from a feasible solution, satisfying local constraints.

Thermal subproblems are solved using feasible direction method also, with the criterion function similar to (28), with the exception that the last term in (28), concerning the spinning reserve constraints, is omitted. Thermal subproblems are solved starting from a feasible solution, satisfying all local constraints.

The coordination is achieved by modifying multipliers ρ^t, using gradient method of ρ^t multipliers modification:

$$\rho^t_{new} = \rho^t_{old} + \kappa \cdot \Phi^t_D . \tag{42}$$

Thermal production cost minimization: Another way to solve the whole problem is to fix all control and state variables to the values obtained in the dual problem solution, except the output power of thermal units. The primal problem is then reduced to solving thermal subproblems with local constraints on thermal units operation and modifying Lagrangian multipliers in succession. Since all hydro variables are fixed, the CPU time for finding the primal solution will be significantly reduced compared to the hydro–thermal statement of the primal problem. However, the computational efficiency of the overall problem solution is not necessarily increased, due to the difficulties in achieving smooth modification of λ and μ multipliers. This is why the number of iterations in the dual problem solution increases.

This approach could be recommended in power systems where the hydro subsystem consists predominantly of cascade–coupled hydro plants, since the primal problem solution method based on hydro–thermal coordination may be highly time consuming in this case.

6. PROBLEM TYPES THAT CAN BE SOLVED WITH THE PROPOSED METHOD

The overall problem solution gives the optimal power system operation plan and optimal schedules for all transactions so as to minimize the criterion function (25). Incremental production costs, obtained in the problem solution, represent boundary values of transaction hourly prices: to obtain economic power exchange, the agreed hourly per–megawatt export prices should not be less than these values, nor should the agreed hourly per–megawatt import prices be greater than these values. The practical value of the proposed method is that it could solve several types of power exchange problems along with the operation planning problem. Some typical power exchange problems are defined in the latter as type–a, type–b and type–c problems.

Type–a: Given agreed transaction hourly per–megawatt prices c^t_{zni} and c^t_{yni}, together with the power exchange constraints (18)–(21), the power exchange schedule optimal in respect to (25) should be determined. This type of problem may consist of several power exchange contracts, each one having its own hourly per–megawatt transaction prices.

Type–b: Given total energy to be exported and/or imported without specifying transaction hourly prices, the power exchange schedule optimal in respect to (25) should be determined together with boundary values of transaction hourly prices. If energy export is concerned, this type of problem is defined by setting $\underline{Z}_{ni}$ in (20) to the value of energy to be exported, by setting $\overline{Z}_{ni}$ to a very great value, while transaction hourly prices are set to $c^t_{zni} = 0$. Conversely, if energy import is concerned, $\overline{Y}_{ni}$ is set to the value of energy to be imported, $\underline{Y}_{ni}$ is set to zero and $c^t_{yni} = 0$.

Due to the nature of the problem, where the objective is the maximization of profit (or minimization of costs of operation and power import decreased by the revenue of power export), the problem solution with zero transaction prices will tend to maximize power import and minimize power export (minimizing operation costs in both cases), giving the solution of the power exchange problem with fixed energy export/import on the bounds $\underline{Z}_{ni}$ and $\overline{Y}_{ni}$ for energy export and energy import respectively, provided that such a solution is feasible. The same effect could be achieved if inequality constraints (20) are replaced by equality constraints:

$$Z_{ni} = \sum_{t=1}^{T} z^t_{ni}, \quad Y_{ni} = \sum_{t=1}^{T} y^t_{ni}, \tag{43}$$

where Z_{ni} and Y_{ni} represent the values of energy to be exported and imported, respectively. However, the problem may arise if constraints (43) could not be satisfied even if the system operates on its capacity bounds (e.g. when the power export constraint could not be satisfied due to limited total capacity of power units).

Type–c: The schedules of some agreed transactions are given. The boundary values of transaction hourly prices should be determined, so that all these transactions are profitable.

The proposed method could solve various combinations of type–a, type–b and type–c problems subject to constraints (18)–(21).

7. TEST EXAMPLE

Computer programs were developed in order to test both approaches: the first, in which feasible direction method was used to solve the hydro subproblem and the second, in which the hydro subproblem solution was based on nonlinear network flow programming. The test power system consisted of 13 thermal units, 11 hydro power plants and 1 pumped storage hydro power plant. The power system was connected using two interconnection tie lines of limited capacity, with 3 energy export transactions over one tie line, and 2 energy import transactions over the other.

In applying feasible direction method, four out of eleven hydro plants of the test power system were coupled in a cascade. The results obtained in this case were shown in [16].

In applying nonlinear network flow programming, a hypothetical hydro power subsystem was treated with all 11 hydro power plants coupled in a cascade. Maximum total export capacity was 120 MW, and maximum total import capacity was 80 MW. The production, start–up and shut–down costs in this example were given in gigajoules (GJ) and prices of power exchange and incremental costs of power production in gigajoules per megawatthour (GJ/MWh). The test was performed in two scenarios of water inflow to water reservoirs: "low" and "high" inflow.

Figure 2 presents the diagram of power production for the scenario of "low" local water inflows to the reservoirs of hydro power plants. One could see that as a result of optimization, the pumped storage hydro power plants are pumping in the periods of low demand and producing power in peak load periods. In Figure 3 the load diagram (given as a fixed input value) and its corrections due to optimal power import and power export are presented. Figure 3 shows that power is imported and exported in many hours, due to given values of power sales and purchase prices, which are higher and lower than the marginal costs, respectively. Power is poorly imported in hours with minimum demand, since marginal power production costs were lower than power import prices in these hours.

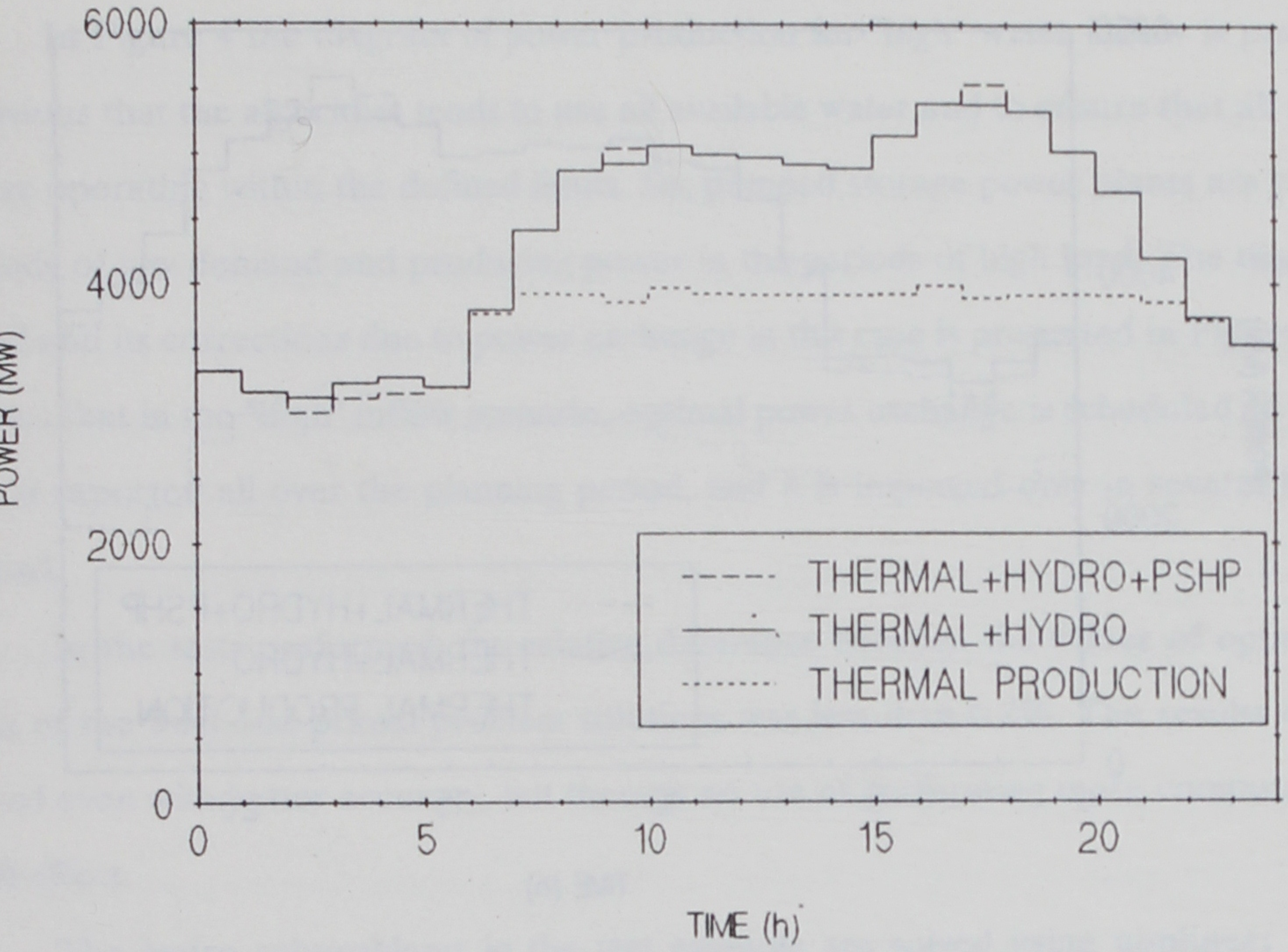

Figure 2. Operation plan of a power system in the "low" inflow scenario

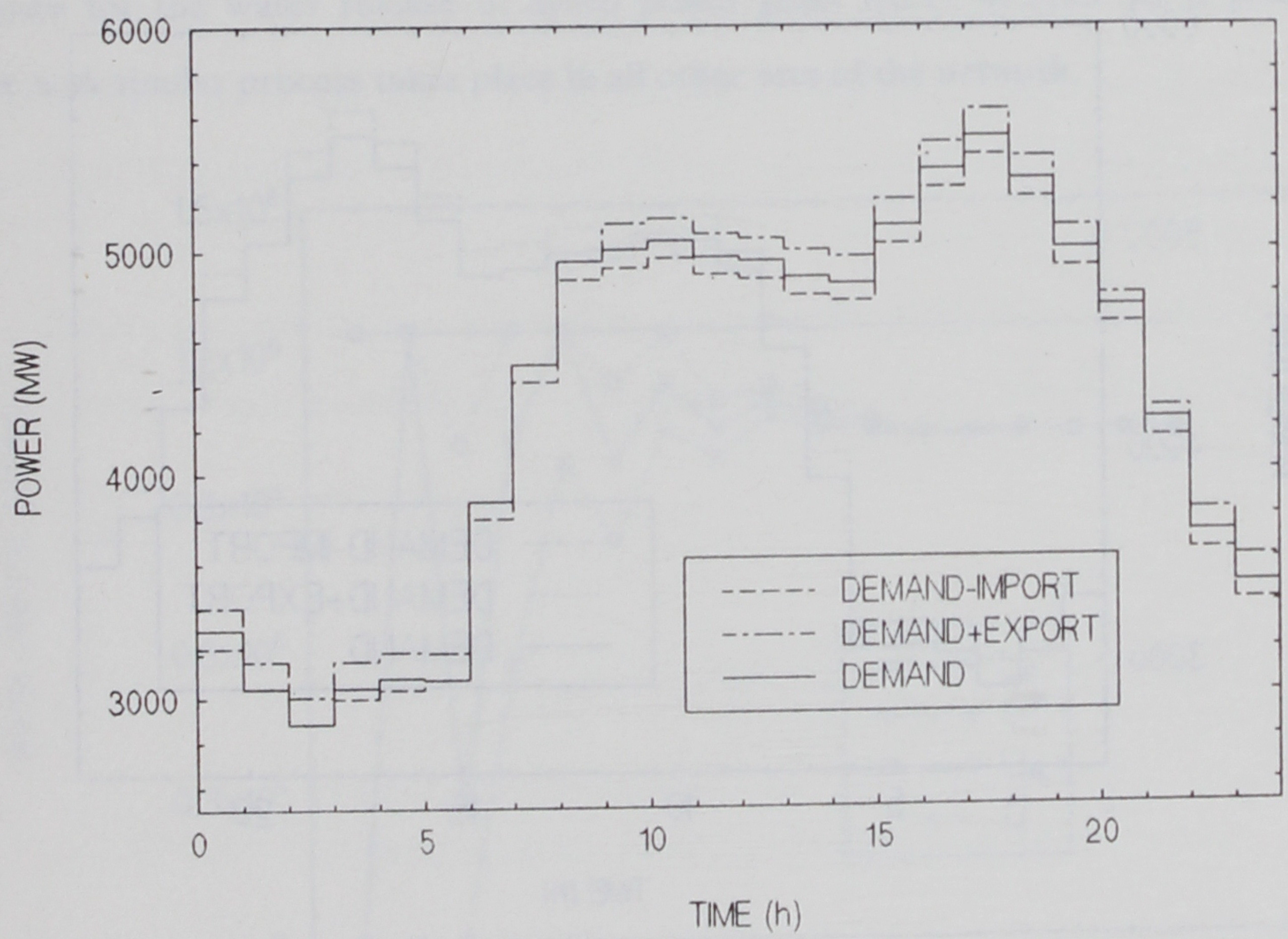

Figure 3. Load diagram and power exchange plan in the "low" inflow scenario

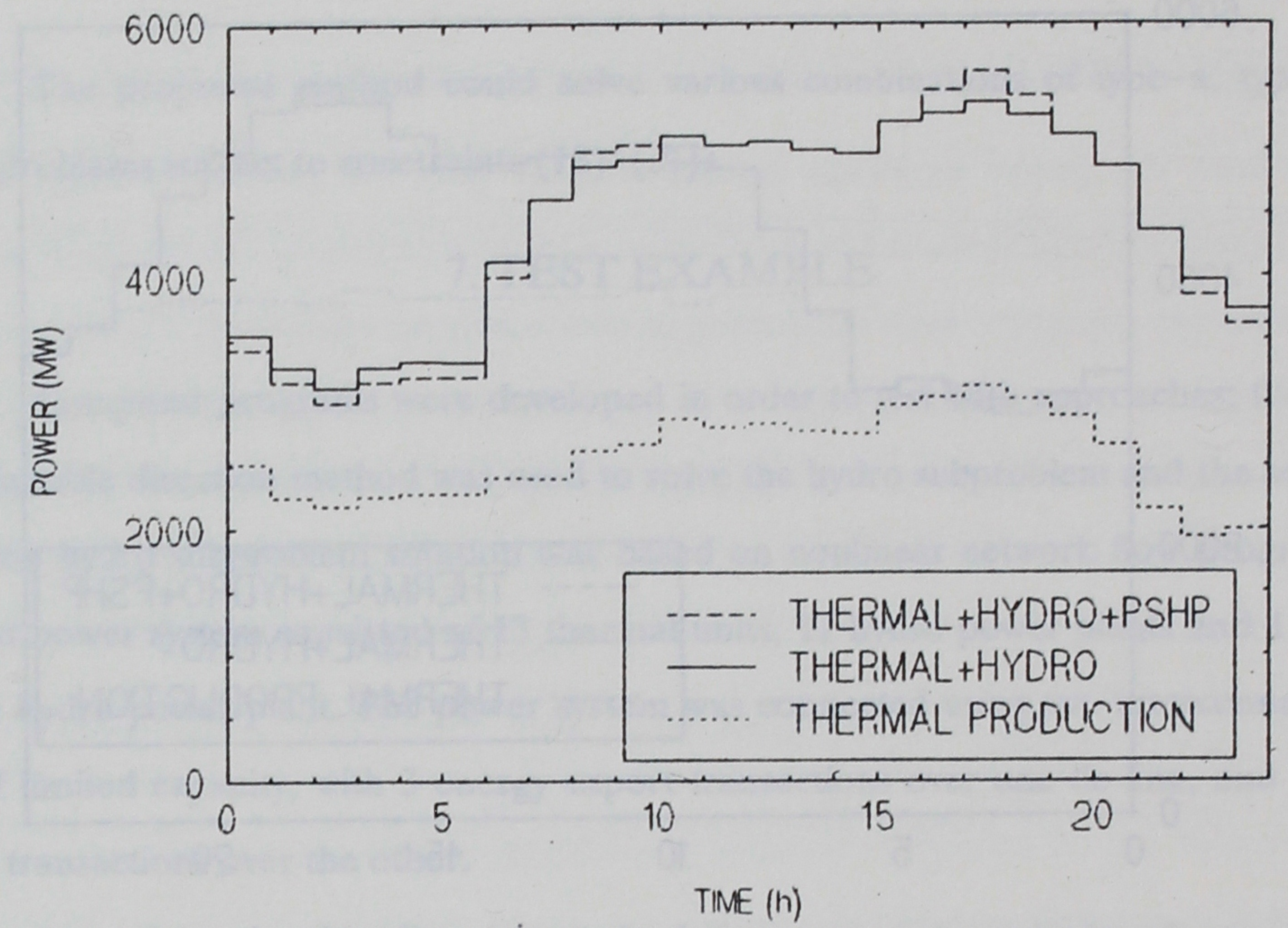

Figure 4. Operation plan of a power system in the "high" inflow scenario

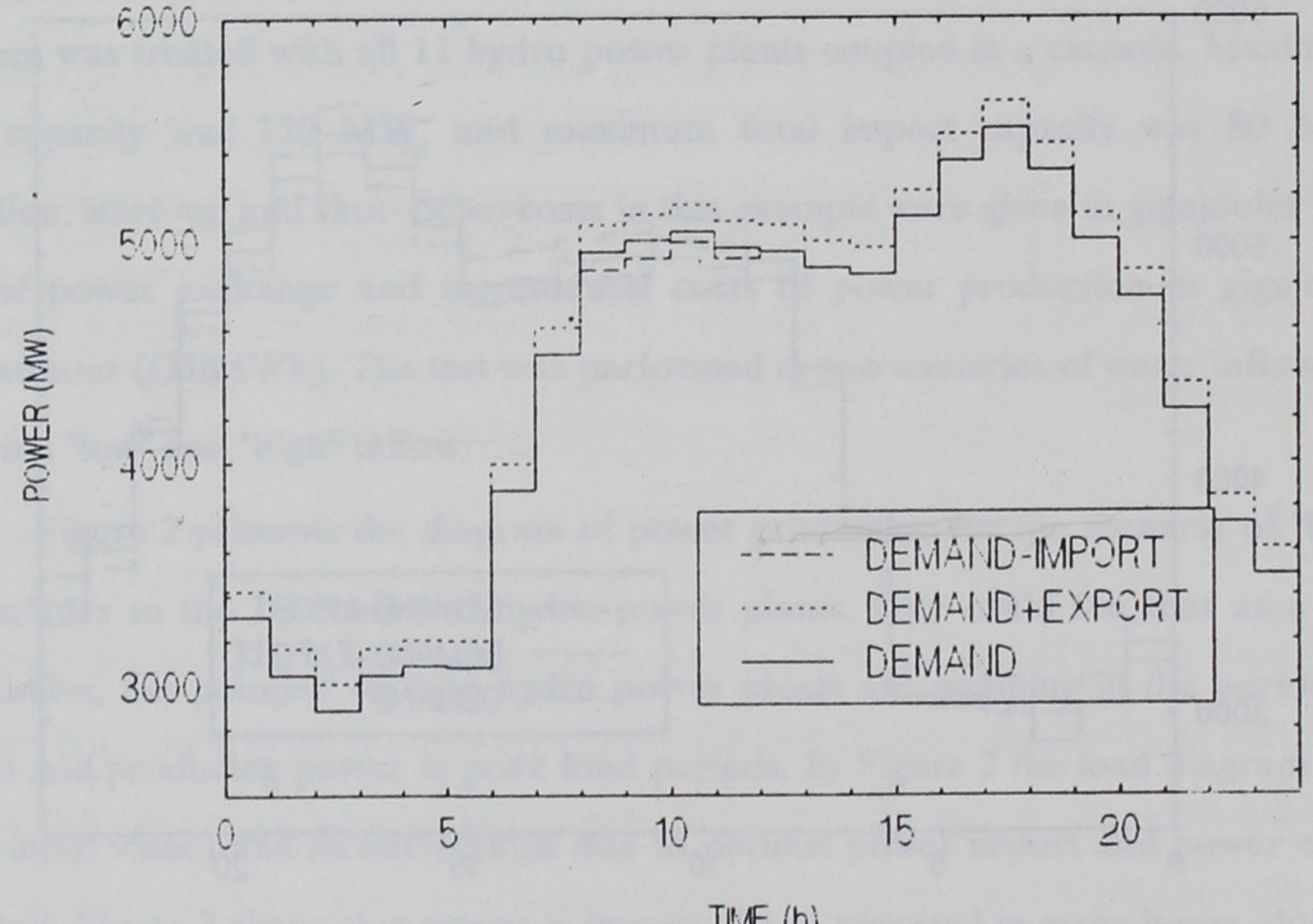

Figure 5. Load diagram and power exchange plan in the "high" inflow scenario

In Figure 4 the diagram of power production for "high" water inflow is presented. It is obvious that the algorithm tends to use all available water and to ensure that all thermal units are operating within the defined limits. So, pumped storage power plants are pumping in periods of low demand and producing power in the periods of high load. The diagram of demand and its corrections due to power exchange in this case is presented in Figure 5. One could see that in the "high" inflow scenario, optimal power exchange is scheduled so that the power is exported all over the planning period, and it is imported only in several hours of high load.

In the tests performed, the relative difference between the values of optimization criteria of the dual and primal problem solutions was less than 0.2%. The results could be obtained even with better accuracy, but there is no use of performing more computation for a small effect.

The hydro subproblems in the test example are solved using nonlinear network flow programming and linearization technique in the neighborhood of the last hydro solution, as shown in Appendix B. The change of the neighborhood boundaries through iterations for the water release of hydro power plant No.11 in hour 10 is presented in Figure 6. A similar process takes place in all other arcs of the network.

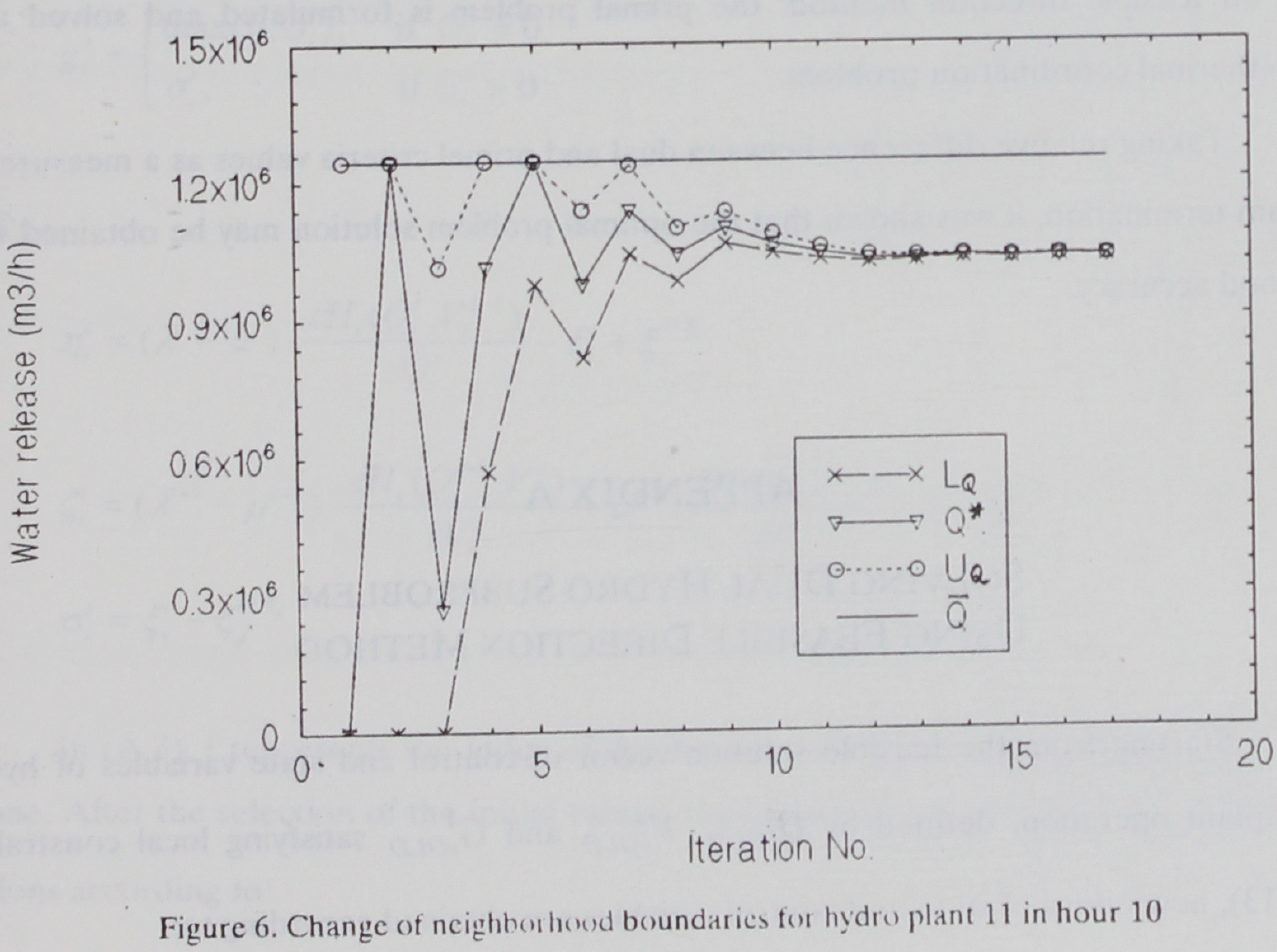

Figure 6. Change of neighborhood boundaries for hydro plant 11 in hour 10

8. CONCLUSION

The solution method of the short–term operation planning problem of hydro–thermal power systems and power exchange scheduling problem with other companies presented in this paper, gives the solution to the unit commitment problem, pumped storage hydro plant scheduling problem, interchange scheduling problem, as well as river basin optimization problem for two types of hydro subsystems: (a) with large numbers of cascade–coupled hydro power plants, and (b) with isolated hydro power plants predominantly. System constraints and all local constraints on the operation of power plants and constraints on power exchange schedules are taken into account. The solution method and computer program for power systems operation and power exchange planning are based on Lagrangian relaxation and decomposition technique.

In the cases with a great number of cascade–coupled hydro plants the solution procedure is based on linearization of the nonlinear criterion function and narrowing of hydro power plants operation feasibility bounds in each iteration. The linearized hydro subproblem is solved using linear network flow algorithm. The primal problem is formulated and solved as a thermal optimization problem.

In the cases with predominantly isolated hydro plants the solution procedure is based on feasible direction method: the primal problem is formulated and solved as a hydro–thermal coordination problem.

Taking relative difference between dual and primal criteria values as a measure for program termination, it was shown that the optimal problem solution may be obtained with very good accuracy.

APPENDIX A.

SOLVING DUAL HYDRO SUBPROBLEM USING FEASIBLE DIRECTION METHOD

Starting from the feasible solution vector of control and state variables of hydro power plant operation, defined by Q^t_{iOLD}, V^t_{iOLD} and G^t_{iOLD} satisfying local constraints (10)–(13), new values of state and control variables are obtained according to:

$$Q_{iNEW}^t = Q_{iOLD}^t + \alpha_m \cdot q_i^t$$

$$V_{iNEW}^t = V_{iOLD}^t + \alpha_m \cdot v_i^t, \qquad i \in \Omega_m, t=1,...,T \tag{A.1}$$

$$G_{iNEW}^t = G_{iOLD}^t + \alpha_m \cdot g_i^t$$

where in (A.1) q_i^t, v_i^t and g_i^t are vectors of directions of change of variables Q_i^t, V_i^t and G_i^t respectively, so as to satisfy local constraints (10)–(13) and α_m is the step size unique for river basin m.

If constraints (13) are relaxed and introduced into (33) using Lagrangian multipliers ξ_i^t, the direction of change of control and state variables are obtained according to the following expressions:

$$q_i^t = \begin{cases} \max(\eta_i^t, 0), & \text{if } Q_i^t = \underline{Q}_i \\ \eta_i^t, & \text{if } \underline{Q}_i < Q_i^t < \overline{Q}_i \\ \min(\eta_i^t, 0), & \text{if } Q_i^t = \overline{Q}_i \end{cases}$$

$$v_i^t = \begin{cases} \max(\zeta_i^t, 0), & \text{if } V_i^t = \underline{V}_i \\ \zeta_i^t, & \text{if } \underline{V}_i < V_i^t < \overline{V}_i \\ \min(\zeta_i^t, 0), & \text{if } V_i^t = \overline{V}_i \end{cases} \tag{A.2}$$

$$g_i^t = \begin{cases} \max(0, \sigma_i^t), & \text{if } G_i^t = 0 \\ \sigma_i^t, & \text{if } G_i^t > 0 \end{cases}$$

where

$$\eta_i^t = (\lambda^t - \mu^t) \cdot \frac{\partial H_i(Q_i^t, V_i^{t-1})}{\partial Q_i^t} - \xi_i^t + \xi_j^{t+\theta_{ij}}$$

$$\zeta_i^t = (\lambda^{t+1} - \mu^{t+1}) \cdot \frac{\partial H_i(Q_i^{t+1}, V_i^t)}{\partial V_i^t} + \mu^{t+1} \cdot \frac{\partial H_i(\overline{Q}_i, V_i^t)}{\partial V_i^t} - \xi_i^t + \xi_i^{t+1}, \tag{A.3}$$

$$\sigma_i^t = \xi_i^t - \xi_j^{t+\theta_{ij}}$$

In (A.3), j represents the index of the downstream hydro plant neighboring to the i–th one. After the selection of the initial values, multipliers ξ_i^t are modified in a number of iterations according to:

$$\xi_i^{t(n+1)} = \xi_i^{t(n)} + k_i^{t(n)} \cdot \left[v_i^t + q_i^t + g_i^t - v_i^{t-1} - \sum_{j\in\Omega_{ui}} (q_j^{t-\theta_{ji}} + g_j^{t-\theta_{ji}}) \right]^{(n)} \quad (A.4)$$

where n is the iteration index and Ω_{ui} is the set of indices of upstream hydro plants neighboring to the i–th one. When the modification of ξ_i^t parameters is completed, new values of ξ_i^t parameters are obtained so as to guarantee the satisfaction of constraint (13) for each river basin.

After the best directions of change of control and state variables are found, the optimal value of the step size is:

$$\frac{d}{d\alpha_m^{opt}} \mathcal{L}_{Hm}(Q_i^t + \alpha_m^{opt} \cdot q_i^t, V_i^t + \alpha_m^{opt} \cdot v_i^t) = 0. \quad (A.5)$$

Since the optimal step size could cause the violation of some constraints, the actual step size which does not violate any of local constraints (10)–(12) is determined:

$$\alpha_m = \min(\alpha_m^{opt}, \alpha_{UQ}, \alpha_{LQ}, \alpha_{UV}, \alpha_{LV}, \alpha_{LG}) \quad (A.6)$$

where α_{UQ} and α_{LQ} are the boundary step sizes which do not violate the upper and lower bounds on water releases of any hydro power plant respectively, α_{UV} and α_{LV} are the boundary step sizes which do not violate the upper and lower bounds on the content of any water reservoir in the river basin and α_{LG} is the boundary step size which does not violate the constraints on nonnegative spilling.

APPENDIX B.

SOLVING DUAL HYDRO SUBPROBLEM USING LINEAR NETWORK FLOW ALGORITHM

RELAXT [1] is a FORTRAN code based on relaxation method for solving linear cost ordinary network flow problem in the form:

$$\min_{x_1,\dots,x_n} \sum_{j=1}^{n} c_j \cdot x_j \quad (B.1)$$

subject to

$$\sum_{j \text{ with } t(j)=i} x_j - \sum_{j \text{ with } h(j)=i} x_j = -b_j \qquad i = 1,\ldots,n \tag{B.2}$$

$$0 \le x_j \le u_j, \qquad j = 1,\ldots,m \tag{B.3}$$

where

n	– is the number of nodes in the network
m	– is the number of arcs in the network,
$t(j)$	– is the starting node of the j–th arc,
$h(j)$	– is the ending node of the j–th arc,
c_j	– is the cost of the j–th arc,
u_j	– is the upper bound of flow through the j–th arc,
b_i	– is the flow demand at the i–th node.

A special feature of RELAXT is the balance of all flow demands and external inflows to the nodes:

$$\sum_{i=1}^{n} b_i = 0. \tag{B.4}$$

The criterion function of the dual hydro subproblem is defined by (32). If the hydro subproblem is linearized, the criterion function assumes the following linear form

$$\mathcal{L}'_{Hm} = -\sum_{t=1}^{T} \sum_{i\in\Omega_m} (C^t_{Qi} \cdot Q^t_i + C^t_{Vi} \cdot V^{t-1}_i) \tag{B.5}$$

where the linear cost coefficients

$$\begin{aligned} C^t_{Qi} &= (\lambda^t - \mu^t) \cdot \frac{\partial H_i(Q^t_i, V^{t-1}_i)}{\partial Q^t_i}, \\ C^t_{Vi} &= (\lambda^t - \mu^t) \cdot \frac{\partial H_i(Q^t_i, V^{t-1}_i)}{\partial V^{t-1}_i} + \mu^t \cdot \frac{\partial H_i(\overline{Q}_i, V^{t-1}_i)}{\partial V^{t-1}_i} \end{aligned} \tag{B.6}$$

are obtained by linearization of $\mathcal{L}_{Hm}$ in the neighborhood of some previous feasible solution $\{Q^t_i, V^t_i, G^t_i\}$ satisfying constraints on upper and lower bounds of all variables and water balance constraints (3), while the cost coefficient of water spill is zero since $\dfrac{\partial H_i}{\partial G^t_i} = 0$.

In order to avoid oscillations in the solution of nonlinear problem using linearization technique and iterative procedure, in every iteration additional constraints on water releases and on reservoir contents are introduced, based on the feasible solution in the previous iteration. Implementation of additional constraints in each iteration is presented in Figure 7.

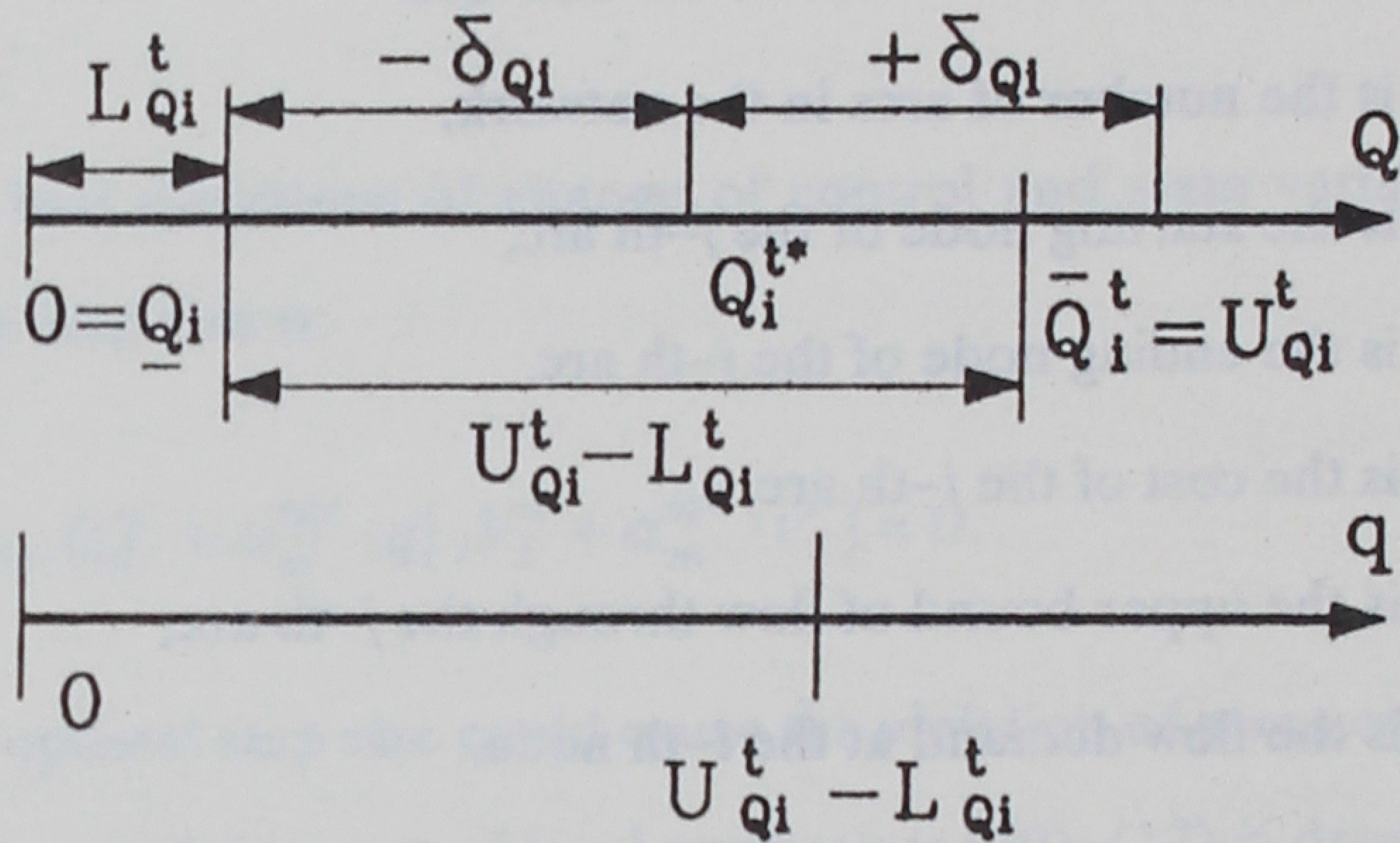

Figure 7. Implementation of additional constraints and variable transformation

Let us denote by $\{Q_i^t, V_i^t, G_i^t\}$ the solution vector obtained in iteration $(n-1)$, satisfying all local constraints. The solution vector in the next iteration is then obtained through the following steps:

1. Determine the upper and lower bounds of the region where the next feasible solution is searched for:

$$
\begin{aligned}
U^t_{Qi} &= \min\left\{Q^t_i + \delta^{(n)}_{Qi}, \overline{Q}_i\right\}, \qquad U^t_{Vi} = \min\left\{V^t_i + \delta^{(n)}_{Vi}, \overline{V}_i\right\} \\
L^t_{Qi} &= \max\left\{Q^t_i - \delta^{(n)}_{Qi}, \underline{Q}_i\right\}, \qquad L^t_{Vi} = \max\left\{V^t_i - \delta^{(n)}_{Vi}, \underline{V}_i\right\}
\end{aligned}
\tag{B.7}
$$

   where $U^t_{Qi}, U^t_{Vi}, L^t_{Qi}$ and L^t_{Vi} are upper and lower bounds of water releases and reservoir contents of the i–th hydro power plant, respectively, and $\delta^{(n)}_{Qi}$ and $\delta^{(n)}_{Vi}$ determine the size of the neighborhood of Q^t_i and V^t_i in which the solution is searched for in the next iteration.

2. Determine cost coefficients C^t_{Qi} and C^t_{Vi} according to (B.6).

3. For each node update node injection by adding lower bounds of all variables corresponding to incoming arcs and subtracting the lower bounds of all variables corresponding to outcoming arcs, as presented in Figure 8.

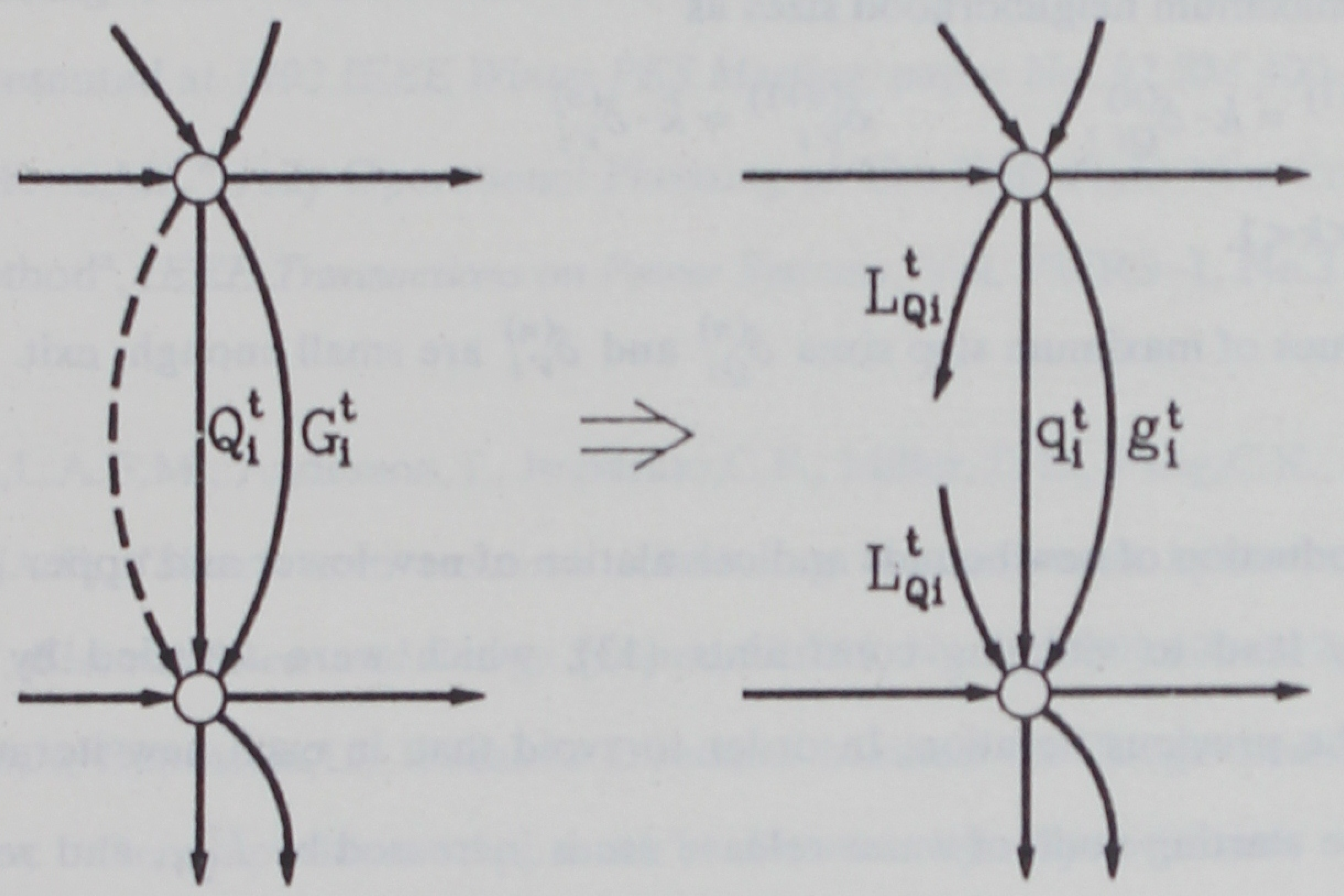

Figure 8. Preservation of feasibility by node injection modification

4. Define new set of variables q_i^t, v_i^t and g_i^t as

$$q_i^t = Q_i^t - L_{Qi}^t$$
$$v_i^t = V_i^t - L_{Vi}^t \tag{B.8}$$
$$g_i^t = G_i^t$$

Define a new linear network flow problem with the criterion function

$$\mathcal{L}'_{Hm} = -\sum_{t=1}^{T} \sum_{i \in \Omega_m} (C_{Qi}^t \cdot q_i^t + C_{Vi}^t \cdot v_i^t), \tag{B.9}$$

updated node injections as defined in item 3, and lower and upper bounds of variables:

$$0 \le q_i^t \le U_{Qi}^t - L_{Qi}^t$$
$$0 \le v_i^t \le U_{Vi}^t - L_{Vi}^t \tag{B.10}$$
$$0 \le g_i^t$$

Find the optimal solution $\{q_i^{t*}, v_i^{t*}, g_i^{t*}\}$ using RELAXT.

5. Update values of water releases, reservoir contents and spilling as:

$$Q_i^t = L_{Qi}^t + q_i^{t*}$$
$$V_i^t = L_{Vi}^t + v_i^{t*} \qquad \text{(B.11)}$$
$$G_i^t = g_i^{t*}$$

6. Reduce maximum neighborhood sizes as

$$\delta_{Qi}^{(n+1)} = k \cdot \delta_{Qi}^{(n)}, \qquad \delta_{Vi}^{(n+1)} = k \cdot \delta_{Vi}^{(n)} \qquad \text{(B.12)}$$

where $0<k<1$.

7. If the values of maximum step sizes $\delta_{Qi}^{(n)}$ and $\delta_{Vi}^{(n)}$ are small enough, exit. Otherwise go to step 1.

Introduction of new bounds and calculation of new lower and upper bounds for all variables may lead to violating constraints (13), which were satisfied by the solution obtained in the previous iteration. In order to avoid that, in each new iteration, the flow demand in the starting node of water release arc is increased by L_{Qi}^t, and reduced by the same amount in the ending node, and the flow demand is increased by L_{Vi}^t in the starting node of the reservoir content arc and decreased for the same amount in the ending node of the same arc. This procedure enables that in every iteration: (a) additional constraints are introduced, (b) new constraints of type (B.3), i.e. (B.10) are calculated thus reducing the feasibility region and (c) once achieved, balance of water (13), is preserved through subsequent iterations.

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REFERENCES

[1] Bertsekas,D.P., Tseng,P., "Relaxation Method for Minimum Cost Ordinary Network Flow Problems", *Operations Research*, Vol.36, No.1 (1988) 93–114.

[2] van den Bosh,P.P.J., Honderd,G., "A Solution of the Unit Commitment Problem via Decomposition and Dynamic Programming", *IEEE Transactions on Power Apparatus and Systems*, Volume PAS–104, No.7 (1985), 1684–1690.

[3] Chao–an Li, Streiffert,D.L., "Incremental Network Flow Programming for Short–Term Hydrothermal Scheduling", *Proceedings of the* 10^{th} *PSCC* (1990), 1123–1128.

[4] Chao–an Li, Jap,P.J., and Streiffert,D.L., "Implementation of Network Flow Programming to the Hydrothermal Coordination in an Energy Management System", paper presented at *1992 IEEE Winter PES Meeting*, paper No. 92 SM 400–2 PWRS.

[5] Ea,K., Monti,M., "Daily Operational Planning of The EdF Plant Mix: Proposal for a New Method", *IEEE Transactions on Power Systems*, Vol. PWRS–1, No.3 (1986) 251–257

[6] Ferreira,L.A.F.M., Anderson,T., Imparato,C.F., Miller,T. E., Pang,C.K., Svoboda,A., Vojdani,A.F., "Short–term resource scheduling in multi–area hydrothermal power systems", *Electric Power and Energy Systems*, Vol.11, No.3 (1989) 200–212.

[7] Geromel,J., Baptistela,L., "Feasible Direction Method for Large–Scale Nonconvex Programs: Decomposition Approach", *Journal of Optimization Theory and Applications*, Vol.35, No.2 (1981) 231–249.

[8] Habibollahzadeh,H., Brannlund,H., Bubenko,T., Sjelvgren,D., Anderson,N., "Optimal Short–Term Planning of Hydro–Thermal Power System–Part I: Modeling", *Proceedings of PSCC Conference*, Helsinki, Finland, (1984).

[9] Habibollahzadeh,H., Brannlund,H., Bubenko,T., Sjelvgren,D., Anderson,N., "Optimal Short–Term Planning of Hydro–Thermal Power System–Part II: Solution Techniques", *Proceedings of PSCC Conference*, Helsinki, Finland, (1984).

[10] Lasdon,L., *Optimization Theory for Large Systems*, The Macmillan Company, London, 1970.

[11] Merlin,A., Sandrin,P., "A New Method for Unit Commitment at Electricite de France", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS–102, No.5 (1983) 1283–1225.

[12] Monti,M., Sandrin,P., Gonot,J.P., "The Pumped Storage Unit: Peak Generation or Operating Reserve? The Demeter Model Issues an Arbitrage", *Water Power & Dam Construction* (1984) 20–23.

[13] Rakić,M.V., Rakić,R., Vlaškalić,S., "Short-Term Operation Planning of Large Scale Hydrothermal Power System Using Decomposition Technique", *Energy and Environmental Systems*, Nice (1984).

[14] Rakić,M., Marković,Z., "The Solution of Hydro-Thermal Coordination Problem by Application of Nonlinear Programming Method and Decomposition Principle", *Proceedings of the XVI Symposium on Operations Research SYM-OP-IS '89* (1989) 151-154. (in Serbian)

[15] Rakić,M., Marković,M., Marković,Z., "Short-Term Operation Planning of Combined Power Systems: A Decomposition Technique", *Proceedings of the 10th PSCC* (1990) 1155-1162.

[16] Rakić,M., Marković,Z., "Short-Term Operation and Power Exchange Planning of Hydro-Thermal Power Systems", presented on the *1993 IEEE Winter PES Meeting*, paper No. 93 WM 243-6 PWRS.

[17] Soares,S., Lyra,C., Tavares,H., "Optimal Generation Scheduling of Hydrothermal Power Systems", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-99 (1980) 1107-1118.

[18] Wu,R.N., Lee,T.H., Hill,E.F., "Effect of Interchange on Short-Term Hydro-Thermal Scheduling", *IEEE Transactions on Power Systems*, Vol.6, No.3 (1991) 1217-1223.