Yugoslav Journal of Operations Research
3 (1993), Number 1, 61–71

THE ECONOMIC DESIGN FOR CONTROLLING THE MEAN OF NON–NORMAL VARIABLE

Kun–Jen CHUNG

National Taiwan Institute of Technology, Department of Industrial Management, Taipei, Taiwan, R. O. C.

Abstract. In 1971, Y.Nagendra and G.Rai concerned with the economic design for controlling the mean of non–normal variables. Their procedure consists of two equations to get the economic design. In this article, our procedure only consists of solving an explicit equation for h (sampling interval) in terms of n (sample size) and k (control limit factor). We conclude that our model is not only more accurate, more applicable, and more flexible, but also simpler to solve than Y.Nagendra and G.Rai's model.

Key words: economic design, cost function, quality control.

1. INTRODUCTION

An important factor in control chart usage is the design of the control chart. By this, we mean the selection of the appropriate sample size n, control limits, the width of the control limits k and the time interval between samples h, n, k and h are three design parameters of the control chart.

In this article, we attempt to determine the economic design for detecting a certain shift in the process average of non–normal variables whose density function is represented by the first four terms of an Edgeworth series when the width of the control chart is fixed by specifying the factor k. In most of the studies concerned with the sensitivity of normal theory procedures the non–normal parent population is represented by the first four terms of an Edgeworth series, (see Hyrenius (1964)).

Nagendra and Rai's solution for the optimum values of these parameters is based on certain approximations and assumptions which lead to some limitations, and the scheme of finding the optimum solution is fairly complicated and involved. Nagendra and Rai's

---

Communicated by J.Petrić

procedure (1971) consists of solving two equations to get the approximate optimum values of n, k and h. However, for mathematical simplicity and practical convenience, Nagendra and Rai (1971) need to make some approximations and assumptions in the minimization procedure. In this paper, a very simple procedure is described for determining the optimum values of n, k and h based on Nagendra and Rai's model. The new procedure only consists of solving an explicit equation for h in terms of n and k and yields lower loss costs than those of the above model mentioned. The new procedure is free from those assumptions and approximations made on the above model mentioned. No terms are neglected for finding the optimum solution. Therefore it gives more accurate and more reliable optimum values for the design parameters.

2. COST FUNCTION

Following Nagendra and Rai (1971), we find that when the process average is shifted from μ to $\mu' = \mu + \delta\sigma$, because of the presence of an assignable cause, the probability of detecting the assignable cause for non-normal variables is given by

$$P' \cong 1 - \Phi(\xi) + \frac{r_1}{6\sqrt{n}}\phi^{(2)}(\xi) - \frac{r_2}{24n}\phi^{(3)}(\xi) - \frac{r_1^2}{72n}\phi^{(5)}(\xi) \qquad (1)$$

where $\xi = k - \delta\sqrt{n}$, $\delta > 0$,

$\phi(x)$ is the normal probability integral,

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2},$$

$$\phi^{(r)}(x) = \left(\frac{d}{dx}\right)^r \phi(x),$$

$$r_1 = \sqrt{\beta_1}, \qquad r_2 = \beta_2 - 3,$$

μ is process average, σ is process variation, and β_1 and β_2 are the moment coefficients of skewness and kurtosis. The values of non-normality parameter r_1, r_2 considered for the determination of the optimum sample size and sampling interval are assumed to lie within limits Barton and Dennis (1952).

The probability of wrongly indicating the presence of assignable cause

$$\alpha' = \alpha - \alpha_c,$$

where

$$\alpha_c = \frac{1}{36n}\left\{3r_2\phi^{(3)}(k) + r_1{}^2\phi^{(5)}(k)\right\}$$
$$\alpha = 2\Phi(-k). \qquad (2)$$

Nagendra and Rai (1971) show that an approximate function for the average net income per hour of using the control chart for mean of non–variables may be obtained as

$$\Gamma = V_0 - L'$$

where

$$L'(n,k,h) = M + \frac{\frac{\lambda T\alpha'}{e^{\lambda h}-1} + \lambda W - M}{1+\lambda B'} + \frac{b+cn}{h}, \qquad (3)$$

$$B' = \frac{h}{P'} - \frac{1}{\lambda} + \frac{h}{e^{\lambda h}-1} + en + D$$

We shall not repeat the mathematical argument here. The definitions of other parameters in equation (3) can be found in Nagendra and Rai (1971) and Duncan (1956). We shall not repeat them here as well.

L' can be treated as the per hour cost of keeping the process under the surveillance of mean chart for controlling the process average of non–normal variables. The optimum n, h and k can be derived by minimizing L'.

3. THE EXPLICIT EQUATION FOR h

It can be shown that Theorems 1 and 2 hold.

THEOREM 1: $1 > \frac{1}{\lambda h} - \frac{1}{e^{\lambda h}-1} > 0$ for all $\lambda > 0$ and $h > 0$. .

THEOREM 2: $\lim_{x\to 0^+}\left(\frac{1}{x} - \frac{1}{e^x - 1}\right) = \frac{1}{2}.$

Theorem 1 tells us that the difference between $\frac{1}{\lambda h}$ and $\frac{1}{e^{\lambda h}-1}$ is within 0 and 1.

In Duncan (1956), it is shown the expected number of samples taken during the in–control

period is given by $\frac{1}{e^{\lambda h}-1}$. So, from the point of the expected number of samples taken during the in-control period, the number $\frac{1}{\lambda h}$, in general, is a good approximation to $\frac{1}{e^{\lambda h}-1}$. However, examination of the numerical model literature shows that with very few exceptions, cost/risk parameters leading to small λh (0.4 or less) are of the most practical interest. By Theorems 1 and 2, we take 0.5 as a correction number. Replacing $\frac{1}{e^{\lambda h}-1}$ in equation (3) by $\frac{1}{\lambda h}-\frac{1}{2}$, we shall get equation (4).

$$L_1(n,k,h) = M + \frac{\frac{T\alpha'}{h} - \frac{\lambda T\alpha'}{2} + \lambda W - M}{1+\lambda B_1} + \frac{b+cn}{h}, \tag{4}$$

where $B_1 = \left(\frac{1}{P'}-\frac{1}{2}\right)h + en + D$.

For a given (n, k), setting the partial derivative of L_1 with respect to h equals to zero yields

$$s_1 h^2 + s^2 \mathrm{h} + s_3 = 0$$

where

$$s_1 = \lambda\left(\frac{1}{P'}-\frac{1}{2}\right)\left(\lambda W - M - \frac{\lambda T\alpha'}{2}\right) + (b+cn)\left[\lambda\left(\frac{1}{P'}-\frac{1}{2}\right)\right]^2$$

$$s_2 = 2\left(\frac{1}{P'}-\frac{1}{2}\right)\lambda\left[T\alpha' + (b+cn)(\lambda en + \lambda D + 1)\right],$$

$$s_3 = (\lambda en + \lambda D + 1)\left[T\alpha' + (b+cn)(\lambda en + \lambda D + 1)\right].$$

So $h = \frac{-s_2 - \sqrt{{s_2}^2 - 4s_1 s_3}}{2s_1}$. (5)

4. THE PROCEDURE

In the process of obtaining the approximate optimum values n^*, k^* and h^*, we treat n and k as discrete variables and assume that the values of k are within the range between 0.00 and 6.00 (the unit length of k is 0.01), and the values of n are within the range between 0 and $\frac{40}{\delta^2}$. We employ a numerical procedure for obtaining the approximate optimum values n^*, k^* and h^* of the design variables n, k and h respectively as follows.

(i) Given a pair (n_0, k_0) where $0 \le n_0 \le \frac{40}{\delta^2}$ and $0 \le k_0 \le 6$.

(ii) Calculate α' and P' from equations (1) and (2).

(iii) Calculate h_0 from equation (5).

(iv) Calculate $L'(n_0, k_0, h_0)$ from equation (3).

(v) Find $L_{n_0}^{\bullet} = \underset{0 \le k_0 \le 6}{\text{minimum}} \left[L'(n_0, k_0, h_0) \right]$.

(vi) Calculate $L^{\bullet} = \underset{0 \le n_0 \le \frac{40}{\delta^2}}{\text{minimum}} \left(L_{n_0}^{\bullet} \right)$

$$= L'\left(n^{\bullet}, k^{\bullet}, h^{\bullet}\right) \qquad \text{(say)}.$$

The values $n^{\bullet}$, $k^{\bullet}$, $h^{\bullet}$ which give the overall approximate minimum loss–cost $L^{\bullet}$ are the approximate design values for the control chart.

5. COMPARISONS WITH NAGENDRA AND RAI'S RESULTS

Optimum designs for 144 examples from the table in Nagendra and Rai's paper (1971, p.640) were obtained by using the new procedure. We get Table 1 to compare them with Nagendra and Rai's solutions. All value of R in Table 1 are within the range between 1.0000 and 1.3219.

Table 1. Comparison between Two Procedures

Example No.	Cost Parameters Values			(λ = 0.01, M = 100, W = 25, e = 0.05) (b = 0.5, c = 0.1, T = 50, D = 2) Our Solution					Nagendra and Rai's Solution					Comparison
				n	h	k	L'	P'	n	h	k	L'	P'	R
	δ	r_1	r_2	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(9) / (4)
1	0.5	-0.5	-0.5	24	2.2043	2.1	7.5485	0.6417	66	3.2871	3.0	9.4134	0.8554	1.2471
2	0.5	0.0	-0.5	24	2.1942	2.1	7.5795	0.6363	67	3.3253	3.0	9.4555	0.8626	1.2475
3	0.5	0.5	-0.5	25	2.2498	2.1	7.6004	0.6496	68	3.3651	3.0	9.4977	0.8697	1.2496
4	0.5	-0.5	0.0	24	2.2078	2.1	7.5517	0.6420	66	3.2991	3.0	9.4126	0.8555	1.2464
5	0.5	0.0	0.0	24	2.1977	2.1	7.5827	0.6366	67	3.3394	3.0	9.4544	0.8628	1.2468
6	0.5	0.5	0.0	25	2.2532	2.1	7.6033	0.6500	68	3.3782	3.0	9.4967	0.8698	1.2490
7	0.5	-0.5	1.0	24	2.2148	2.1	7.5582	0.6426	67	3.3451	3.0	9.4562	0.8625	1.2511
8	0.5	0.0	1.0	25	2.2767	2.1	7.5886	0.6561	68	3.3894	3.0	9.4980	0.8696	1.2516
9	0.5	0.5	1.0	25	2.2601	2.1	7.6090	0.6507	68	3.4034	3.0	9.4950	0.8701	1.2479
10	0.5	-0.5	2.0	24	2.2218	2.1	7.5646	0.6433	67	3.3676	3.0	9.4548	0.8628	1.2499
11	0.5	0.0	2.0	25	2.2836	2.1	7.5942	0.6568	68	3.4051	3.0	9.4971	0.8698	1.2506
12	0.5	0.5	2.0	25	2.2670	2.1	7.6147	0.6514	69	3.4483	3.0	9.5393	0.8767	1.2527
13	0.5	-0.5	-0.5	24	2.2043	2.1	7.5485	0.6417	50	2.9196	2.5	8.4028	0.8493	1.1132
14	0.5	0.0	-0.5	24	2.1942	2.1	7.5795	0.6363	50	2.9963	2.5	8.3952	0.8496	1.1076
15	0.5	0.5	-0.5	25	2.2498	2.1	7.6004	0.6496	50	2.9761	2.5	8.3957	0.8497	1.1046
16	0.5	-0.5	0.0	24	2.2078	2.1	7.5517	0.6420	50	2.9794	2.5	8.3980	0.8495	1.1121
17	0.5	0.0	0.0	24	2.1977	2.1	7.5827	0.6366	51	3.0576	2.5	8.4392	0.8579	1.1130
18	0.5	0.5	0.0	25	2.2532	2.1	7.6033	0.6500	52	3.0984	2.5	8.4820	0.8660	1.1156
19	0.5	-0.5	1.0	24	2.2148	2.1	7.5582	0.6426	52	3.2241	2.5	8.4817	0.8653	1.1222
20	0.5	0.0	1.0	25	2.2767	2.1	7.5886	0.6561	52	3.1466	2.5	8.4834	0.8659	1.1179
21	0.5	0.5	1.0	25	2.2601	2.1	7.6090	0.6507	52	3.1306	2.5	8.4819	0.8664	1.1147
22	0.5	-0.5	2.0	24	2.2218	2.1	7.5646	0.6433	53	3.4152	2.5	8.5292	0.8727	1.1275
23	0.5	0.0	2.0	25	2.2836	2.1	7.5942	0.6568	53	3.2297	2.5	8.5293	0.8735	1.1231
24	0.5	0.5	2.0	25	2.2670	2.1	7.6147	0.6514	53	3.2160	2.5	8.5268	0.8742	1.1198
25	0.5	-0.5	-0.5	24	2.2043	2.1	7.5485	0.6417	41	3.2858	2.0	8.0525	0.8839	1.0668
26	0.5	0.0	-0.5	24	2.1942	2.1	7.5795	0.6363	42	3.3288	2.0	8.0948	0.8924	1.0680
27	0.5	0.5	-0.5	25	2.2498	2.1	7.6004	0.6496	43	3.3897	2.0	8.1318	0.9008	1.0699
28	0.5	-0.5	0.0	24	2.2078	2.1	7.5517	0.6420	42	3.3334	2.0	8.0980	0.8913	1.0723
29	0.5	0.0	0.0	24	2.1977	2.1	7.5827	0.6366	42	3.3565	2.0	8.0939	0.8926	1.0674
30	0.5	0.5	0.0	25	2.2532	2.1	7.6033	0.6500	43	3.3927	2.0	8.1325	0.9009	1.0696
31	0.5	-0.5	1.0	24	2.2148	2.1	7.5582	0.6426	43	3.4173	2.0	8.1434	0.8984	1.0774
32	0.5	0.0	1.0	25	2.2767	2.1	7.5886	0.6561	43	3.4325	2.0	8.1393	0.8998	1.0726
33	0.5	0.5	1.0	25	2.2601	2.1	7.6090	0.6507	43	3.4440	2.0	8.1315	0.9012	1.0687
34	0.5	-0.5	2.0	24	2.2218	2.1	7.5646	0.6433	43	3.4644	2.0	8.1430	0.8987	1.0765
35	0.5	0.0	2.0	25	2.2836	2.1	7.5942	0.6568	44	3.5057	2.0	8.1863	0.9065	1.0780

36	0.5	0.5	2.0	25	2.2670	2.1	7.6147	0.6514	44	3.5146	2.0	8.1782	0.9081	1.0740
37	1.0	-0.5	-0.5	11	1.6468	2.6	5.1996	0.7652	17	1.8675	3.0	5.3957	0.8675	1.0377
38	1.0	0.0	-0.5	12	1.7638	2.6	5.2079	0.8053	18	1.9237	3.0	5.4333	0.8926	1.0433
39	1.0	0.5	-0.5	12	1.7569	2.6	5.2097	0.8031	18	1.9413	3.0	5.4285	0.8945	1.0420
40	1.0	-0.5	0.0	11	1.6587	2.6	5.2114	0.7663	18	1.9911	3.0	5.4383	0.8910	1.0435
41	1.0	0.0	0.0	12	1.7749	2.6	5.2181	0.8062	18	2.0026	3.0	5.4324	0.8930	1.0411
42	1.0	0.5	0.0	12	1.7680	2.6	5.2199	0.8040	19	2.0605	3.0	5.4745	0.9155	1.0488
43	1.0	-0.5	1.0	12	1.7962	2.6	5.2315	0.8092	20	2.2002	3.0	5.5440	0.9270	1.0597
44	1.0	0.0	1.0	12	1.7970	2.6	5.2381	0.8081	20	2.2088	3.0	5.5366	0.9299	1.0570
45	1.0	0.5	1.0	12	1.7900	2.6	5.2400	0.8059	20	2.2198	3.0	5.5300	0.9329	1.0553
46	1.0	-0.5	2.0	12	1.8182	2.6	5.2512	0.8111	21	2.3422	3.0	5.6078	0.9407	1.0679
47	1.0	0.0	2.0	12	1.8189	2.6	5.2577	0.8099	21	2.3498	3.0	5.6002	0.9437	1.0651
48	1.0	0.5	2.0	12	1.8120	2.6	5.2597	0.8077	21	2.3590	3.0	5.5930	0.9469	1.0634
49	1.0	-0.5	-0.5	11	1.6468	2.6	5.1996	0.7652	13	1.8235	2.5	5.2457	0.8635	1.0089
50	1.0	0.0	-0.5	12	1.7638	2.6	5.2079	0.8053	14	1.8983	2.5	5.2792	0.8923	1.0137
51	1.0	0.5	-0.5	12	1.7569	2.6	5.2097	0.8031	14	1.9128	2.5	5.2684	0.8944	1.0113
52	1.0	-0.5	0.0	11	1.6587	2.6	5.2114	0.7663	15	2.0998	2.5	5.3219	0.9122	1.0212
53	1.0	0.0	0.0	12	1.7749	2.6	5.2181	0.8062	15	2.1142	2.5	5.3185	0.9151	1.0192
54	1.0	0.5	0.0	12	1.7680	2.6	5.2199	0.8040	15	2.3446	2.5	5.3129	0.9181	1.0178
55	1.0	-0.5	1.0	12	1.7962	2.6	5.2315	0.8092	17	2.4131	2.5	5.4401	0.9445	1.0399
56	1.0	0.0	1.0	12	1.7970	2.6	5.2381	0.8081	16	2.3146	2.5	5.3803	0.9336	1.0271
57	1.0	0.5	1.0	12	1.7900	2.6	5.2400	0.8059	17	2.4306	2.5	5.4232	0.9516	1.0350
58	1.0	-0.5	2.0	12	1.8182	2.6	5.2512	0.8111	19	2.6497	2.5	5.5799	0.9651	1.0626
59	1.0	0.0	2.0	12	1.8189	2.6	5.2577	0.8099	18	2.5763	2.5	5.5104	0.9593	1.0481
60	1.0	0.5	2.0	12	1.8120	2.6	5.2597	0.8077	19	2.6664	2.5	5.5637	0.9717	1.0578
61	1.0	-0.5	-0.5	11	1.6468	2.6	5.1996	0.7652	13	2.6164	2.0	5.6985	0.9419	1.0959
62	1.0	0.0	-0.5	12	1.7638	2.6	5.2079	0.8053	13	2.6288	2.0	5.6969	0.9457	1.0939
63	1.0	0.5	-0.5	12	1.7569	2.6	5.2097	0.8031	13	2.6395	2.0	5.6757	0.9499	1.0894
64	1.0	-0.5	0.0	11	1.6587	2.6	5.2114	0.7663	14	2.7513	2.0	5.7449	0.9555	1.1024
65	1.0	0.0	0.0	12	1.7749	2.6	5.2181	0.8062	14	2.7709	2.0	5.7422	0.9592	1.1004
66	1.0	0.5	0.0	12	1.7680	2.6	5.2199	0.8040	14	2.7706	2.0	5.7226	0.9634	1.0963
67	1.0	-0.5	1.0	12	1.7962	2.6	5.2315	0.8092	15	2.9405	2.0	5.8047	0.9658	1.1096
68	1.0	0.0	1.0	12	1.7970	2.6	5.2381	0.8081	15	2.9574	2.0	5.8020	0.9693	1.1077
69	1.0	0.5	1.0	12	1.7900	2.6	5.2400	0.8059	15	2.9575	2.0	5.7836	0.9732	1.1037
70	1.0	-0.5	2.0	12	1.8182	2.6	5.2512	0.8111	17	3.1234	2.0	5.9321	0.9797	1.1297
71	1.0	0.0	2.0	12	1.8189	2.6	5.2577	0.8099	16	3.1076	2.0	5.8700	0.9767	1.1165
72	1.0	0.5	2.0	12	1.8120	2.6	5.2597	0.8077	16	3.1104	2.0	5.8531	0.9803	1.1128
73	1.5	-0.5	-0.5	7	1.4720	2.9	4.3926	0.8545	7	1.2554	3.0	4.4182	0.8319	1.0058
74	1.5	0.0	-0.5	7	1.4737	2.9	4.3887	0.8561	8	1.3371	3.0	4.4274	0.8922	1.0088
75	1.5	0.5	-0.5	7	1.5430	2.8	4.3849	0.8795	8	1.3787	3.0	4.4177	0.8949	1.0075
76	1.5	-0.5	0.0	7	1.4889	2.9	4.4081	0.8558	9	1.6312	3.0	4.4628	0.9289	1.0124
77	1.5	0.0	0.0	7	1.4905	2.9	4.4042	0.8574	9	1.6383	3.0	4.4531	0.9332	1.0111
78	1.5	0.5	0.0	7	1.5618	2.8	4.4026	0.8807	9	1.6496	3.0	4.4463	0.9379	1.0099
79	1.5	-0.5	1.0	7	1.5222	2.9	4.4386	0.8585	11	1.9877	3.0	4.6065	0.9718	1.0378
80	1.5	0.0	1.0	7	1.5239	2.9	4.4346	0.8601	11	1.9935	3.0	4.5981	0.9756	1.0369

81	1.5	0.5	1.0	7	1.5253	2.9	4.4347	0.8605	10	1.9558	3.0	4.5234	0.9644	1.0200
82	1.5	-0.5	2.0	7	1.5551	2.9	4.4682	0.8611	12	2.1972	3.0	4.7061	0.9821	1.0532
83	1.5	0.0	2.0	8	1.6234	3.0	4.4611	0.8965	12	2.2040	3.0	4.6993	0.9852	1.0534
84	1.5	0.5	2.0	8	1.6297	3.0	4.4579	0.8993	12	2.2144	3.0	4.6938	0.9887	1.0529
85	1.5	-0.5	-0.5	7	1.4720	2.9	4.3926	0.8545	6	1.4123	2.5	4.5049	0.8755	1.0256
86	1.5	0.0	-0.5	7	1.4737	2.9	4.3887	0.8561	7	1.4363	2.5	4.5480	0.9285	1.0363
87	1.5	0.5	-0.5	7	1.5430	2.8	4.3849	0.8795	6	1.4592	2.5	4.4868	0.8807	1.0232
88	1.5	-0.5	0.0	7	1.4889	2.9	4.4081	0.8558	8	1.8510	2.5	4.5611	0.9545	1.0347
89	1.5	0.0	0.0	7	1.4905	2.9	4.4042	0.8574	8	1.8706	2.5	4.5595	0.9593	1.0353
90	1.5	0.5	0.0	7	1.5618	2.8	4.4026	0.8807	8	1.8699	2.5	4.5407	0.9650	1.0314
91	1.5	-0.5	1.0	7	1.5222	2.9	4.4386	0.8585	11	2.3728	2.5	4.8088	0.9906	1.0834
92	1.5	0.0	1.0	7	1.5239	2.9	4.4346	0.8601	10	2.3443	2.5	4.7327	0.9870	1.0672
93	1.5	0.5	1.0	7	1.5253	2.9	4.4347	0.8605	10	2.3504	2.5	4.7199	0.9907	1.0643
94	1.5	-0.5	2.0	7	1.5551	2.9	4.4682	0.8611	13	2.6435	2.5	5.0071	0.9967	1.1206
95	1.5	0.0	2.0	8	1.6234	3.0	4.4611	0.8965	13	2.6537	2.5	5.0093	0.9977	1.1229
96	1.5	0.5	2.0	8	1.6297	3.0	4.4579	0.8993	12	2.6228	2.5	4.9215	0.9974	1.1040
97	1.5	-0.5	-0.5	7	1.4720	2.9	4.3926	0.8545	6	2.2632	2.0	5.0869	0.9473	1.1581
98	1.5	0.0	-0.5	7	1.4737	2.9	4.3887	0.8561	7	2.4533	2.0	5.1162	0.9758	1.1658
99	1.5	0.5	-0.5	7	1.5430	2.8	4.3849	0.8795	6	3.2907	2.0	5.1336	0.9594	1.1707
100	1.5	-0.5	0.0	7	1.4889	2.9	4.4081	0.8558	7	2.5494	2.0	5.1177	0.9708	1.1610
101	1.5	0.0	0.0	7	1.4905	2.9	4.4042	0.8574	7	2.5808	2.0	5.1231	0.9755	1.1632
102	1.5	0.5	0.0	7	1.5618	2.8	4.4026	0.8807	7	2.5662	2.0	5.0916	0.9812	1.1565
103	1.5	-0.5	1.0	7	1.5222	2.9	4.4386	0.8585	9	2.8829	2.0	5.2559	0.9907	1.1841
104	1.5	0.0	1.0	7	1.5239	2.9	4.4346	0.8601	9	2.9023	2.0	5.2624	0.9931	1.1867
105	1.5	0.5	1.0	7	1.5253	2.9	4.4347	0.8605	9	2.9016	2.0	5.2429	0.9958	1.1822
106	1.5	-0.5	2.0	7	1.5551	2.9	4.4682	0.8611	12	3.0469	2.0	5.4906	0.9984	1.2288
107	1.5	0.0	2.0	8	1.6234	3.0	4.4611	0.8965	11	3.1064	2.0	5.4275	0.9979	1.2166
108	1.5	0.5	2.0	8	1.6297	3.0	4.4579	0.8993	11	3.0939	2.0	5.4148	0.9988	1.2147
109	2.0	-0.5	-0.5	5	1.4209	3.0	4.0075	0.9233	3	1.0032	3.0	4.1423	0.6865	1.0336
110	2.0	0.0	-0.5	5	1.4253	3.0	3.9950	0.9289	3	1.1150	3.0	4.1709	0.6755	1.0440
111	2.0	0.5	-0.5	5	1.4377	3.0	3.9893	0.9350	3	1.1563	3.0	4.2272	0.6595	1.0596
112	2.0	-0.5	0.0	5	1.4005	3.1	4.0266	0.9100	5	1.4340	3.0	4.0288	0.9240	1.0005
113	2.0	0.0	0.0	5	1.4027	3.1	4.0131	0.9151	5	1.4336	3.0	4.0163	0.9296	1.0008
114	2.0	0.5	0.0	5	1.4598	3.0	4.0103	0.9357	5	1.4543	3.0	4.0103	0.9357	1.0000
115	2.0	-0.5	1.0	5	1.3939	3.2	4.0612	0.8967	8	1.9632	3.0	4.2773	0.9938	1.0532
116	2.0	0.0	1.0	5	1.3943	3.2	4.0478	0.9010	7	1.9192	3.0	4.1861	0.9884	1.0342
117	2.0	0.5	1.0	5	1.4527	3.1	4.0449	0.9222	7	1.9317	3.0	4.1817	0.9926	1.0338
118	2.0	-0.5	2.0	5	1.4256	3.2	4.0884	0.8993	9	2.1800	3.0	4.4041	0.9976	1.0772
119	2.0	0.0	2.0	5	1.4261	3.2	4.0751	0.9036	9	2.1845	3.0	4.4013	0.9987	1.0800
120	2.0	0.5	2.0	5	1.4373	3.2	4.0750	0.9075	9	2.1931	3.0	4.4019	0.9996	1.0802
121	2.0	-0.5	-0.5	5	1.4209	3.0	4.0075	0.9233	3	1.3023	2.5	4.1932	0.8280	1.0463
122	2.0	0.0	-0.5	5	1.4253	3.0	3.9950	0.9289	3	1.3215	2.5	4.2213	0.8290	1.0566
123	2.0	0.5	-0.5	5	1.4377	3.0	3.9893	0.9350	3	1.3753	2.5	4.1933	0.8263	1.0511
124	2.0	-0.5	0.0	5	1.4005	3.1	4.0266	0.9100	5	1.7212	2.5	4.2246	0.9703	1.0492
125	2.0	0.0	0.0	5	1.4027	3.1	4.0131	0.9151	5	1.7468	2.5	4.2286	0.9757	1.0537

126	2.0	0.5	0.0	5	1.4598	3.0	4.0103	0.9357	5	1.7377	2.5	4.2031	0.9825	1.0481
127	2.0	-0.5	1.0	5	1.3939	3.2	4.0612	0.8967	9	2.3776	2.5	4.6160	0.9997	1.1366
128	2.0	0.0	1.0	5	1.3943	3.2	4.0478	0.9010	9	2.3856	2.5	4.6219	1.0000	1.1418
129	2.0	0.5	1.0	5	1.4527	3.1	4.0449	0.9222	8	2.3448	2.5	4.5263	0.9998	1.1190
130	2.0	-0.5	2.0	5	1.4256	3.2	4.0884	0.8993	11	2.5312	2.5	4.8210	1.0008	1.1792
131	2.0	0.0	2.0	5	1.4261	3.2	4.0751	0.9036	11	2.5463	2.5	4.8269	1.0008	1.1845
132	2.0	0.5	2.0	5	1.4373	3.2	4.0750	0.9075	11	2.5634	2.5	4.8244	1.0009	1.1839
133	2.0	-0.5	-0.5	5	1.4209	3.0	4.0075	0.9233	4	2.2733	2.0	4.8298	0.9719	1.2052
134	2.0	0.0	-0.5	5	1.4253	3.0	3.9950	0.9289	2	2.3256	2.0	5.1644	0.7906	1.2927
135	2.0	0.5	-0.5	5	1.4377	3.0	3.9893	0.9350	2	2.3526	2.0	5.1320	0.7819	1.2864
136	2.0	-0.5	0.0	5	1.4005	3.1	4.0266	0.9100	4	2.4460	2.0	4.8461	0.9713	1.2035
137	2.0	0.0	0.0	5	1.4027	3.1	4.0131	0.9151	4	2.4898	2.0	4.8640	0.9772	1.2120
138	2.0	0.5	0.0	5	1.4598	3.0	4.0103	0.9357	4	2.4569	2.0	4.8140	0.9848	1.2004
139	2.0	-0.5	1.0	5	1.3939	3.2	4.0612	0.8967	6	2.7167	2.0	4.9947	0.9960	1.2299
140	2.0	0.0	1.0	5	1.3943	3.2	4.0478	0.9010	8	2.9294	2.0	5.1674	0.9998	1.2766
141	2.0	0.5	1.0	5	1.4527	3.1	4.0449	0.9222	7	2.8916	2.0	5.0723	0.9997	1.2540
142	2.0	-0.5	2.0	5	1.4256	3.2	4.0884	0.8993	11	2.9623	2.0	5.4046	1.0004	1.3219
143	2.0	0.0	2.0	5	1.4261	3.2	4.0751	0.9036	10	3.1026	2.0	5.3453	1.0002	1.3117
144	2.0	0.5	2.0	5	1.4373	3.2	4.0750	0.9075	10	3.0960	2.0	5.3345	1.0002	1.3091

The distribution of R in Table 1 is summarized as Table 2. From Table 2, we find that in all 144 examples considered the loss-costs the new procedure yields are not higher than those of Nagendra and Rai's model, the true optimum lost costs of 65 examples (45.14%) (or 103 examples (71.53%)) among all 144 examples are at least 10% (or 5%) lower than those of Nagendra and Rai's solutions. The mean of R is 1.1083.

Table 2. The Distribution of R in Table 1

Class Boundaries	Total	Percentage (%)	Cumulative Percentage (%)
R=1.00	1	0.69	0.69
1.00<R≤1.05	40	27.78	28.47
1.05<R≤1.10	38	26.39	54.86
1.10<R≤1.15	26	18.06	72.92
1.15<R≤1.20	12	8.33	81.25
1.20<R≤1.25	16	11.11	92.36
1.25<R≤1.30	8	5.56	97.92
1.30<R≤1.35	3	2.08	100.00
Total	144	100.00	$\bar{R}$ =1.1083

On the other hand, fix the pair (r_1, r_2) , comparing the powers from our solutions with the corresponding powers from Nagendra and Rai's solutions, we find that our solutions have the property that the larger the number δ, the larger the power, but Nagendra and Rai's solutions don't have that property. So our solutions confirm the sentence that the Shewhart chart detects the larger shift faster than the small shift but Nagendra and Rai's solutions don't. To sum up, our model is significantly better than Nagendra and Rai's model. Consequently, we conclude that our model is not only more accurate, more applicable, and more flexible, but also simpler to solve than Nagendra and Rai's model.

REFERENCES

Barton,D.E., and Dennis,K.E., (1952), "The Condition under which Gram Charlier and Edgeworth Curves are Positive Definite and Unimodal", *Biometrika*, 39, December 1952, 452.

Cowden,D.J., (1957), *Statistical Methods in Quality Control*, Prentice–Hall, New York.

Duncan,A.J., (1956), "The Economic Design of $\bar{X}$–Charts Used to Maintain Current Control of a Process", *Journal of the American Statistical Association*, 51, June 1956, 228.

Goel,A.L., Jain,S.C., and Wu,S.M., (1968), "An Algorithm for the Determination of the Economic Design of $\bar{X}$–Charts Based on Duncan's Model", *Journal of the American Statistical Association*, 63 March 1968, 304.

Hyrenins,H., Adolfsson,I., et al. (1964), *Selected Bibliography of Non–normality*, Stockholm: Almquist and Wicksoll.

Moore,P.G., (1957), "Non–normality in Quality Control Charts", *Applied Statistics*, 6, November 1957.

Nagendra,Y., and G.Rai,G., (1971), "Optimum Sample Size and Sampling Interval for Controlling the Mean of Non–Normal Variables", *Journal of the American Statistical Association*, 66 September, 1971, 637.