

AN ORDER LEVEL INVENTORY MODEL UNDER TWO LEVEL STORAGE SYSTEM WITH FUZZY DEMAND

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Received: January 2013 / Accepted: April 2013

Abstract: Deterministic inventory model with two levels of storage has been studied by numerous authors. In this paper we developed a fuzzy inventory model with two ware houses (one is the existing storage known as own warehouse (OW) and the other is hired on rental basis known as rented warehouse (RW)). The model allows constant levels of item deterioration in both houses. The stock is transferred from RW to OW in continuous release pattern and the associated transportation cost is taken into account. To make the model more realistic in nature, fuzzy demand has been considered. Using α -cut for defuzzification, the total variable cost per unit time is derived. Therefore, the problem is reduced to crisp annual costs. The multi-objective model is solved by Global Criteria Method supported by GRG (Generalized Reduced Gradient) Technique, which is illustrated by a numerical example.

Keywords: Fuzzy demand, deterioration, defuzzification, warehouse

MSC: 90B05.

1. INTRODUCTION

The warehouse storage capacity is defined as the amount of storage space needed to accommodate the materials to be stored to meet a desired service level which specifies the degree of storage space availability. Assumption that stock items are delivered exactly when needed is impractical. Therefore, it is important to investigate the

influence of warehouse capacity in various inventory policy problems. In recent years, various researchers have discussed the problem of two warehouse inventory system. This kind of system was first discussed by Hartley [10]. Sarma[17] has developed an EOQ inventory model with two separate storage facilities, viz., the own warehouse(OW) and a rented warehouse(RW).

Nowadays, in important market places like super markets, municipality markets etc., it is almost impossible to have a big showroom / shop due to the scarcity of space and very high rents. Normally, moderate and large business houses operate through two warehouses-one smaller in size is in the heart of the market place and the other one, with large capacity is slightly little away from the market place. During the last two decades, two warehouse inventory models have been developed and resolved by many researchers. In general, when a supplier provides price discounts for bulk purchases, or when the item under consideration is a seasonal product like the output of harvest, the manager may purchase more goods than can be stored in his own warehouse (OW). Therefore, these excess quantities are stored in a rented warehouse (RW). Further, the inventory costs (including holding cost and deterioration cost) in RW are usually higher than those in OW due to additional cost of maintenance, material handling etc. RW generally provides better preserving facilities than the OW resulting in a lower deterioration rate of the goods. To reduce the inventory costs, it is economical to consume the RW before OW. As a result, the firm stores goods first in OW but clears the stock first in RW. This means that the stock is required to be transported in some optimum fashion from RW to OW so to empty RW first. Sarma [17] discussed a two storage model for a deterministic inventory situation of non-deteriorating items with infinite replenishment rate and without shortages, and called this procedure of transferring bulk size from RW to OW as a bulk release rule. Sarma [18] has considered further improvement in the working rule. Dave[5] reconsidered two separate storage facilities for both finite and infinite production rate with single, as well as the bulk release pattern of the stock without shortages. An extensive survey of literature concerning inventory models for deteriorating items was conducted by Rifaat,Wolfe and Eldin[15]. Goswami and Chowdhuri[6]developed bulk release pattern with linear trend of demand and infinite replenishment rate. Both patterns, ordinary release pattern and bulk release pattern of the stock from RW to OW are considered, where in the former pattern units are released singly. Pakkala and Achary [14] studied a deterministic inventory model for deteriorating items with two warehouses and finite replenishment rate. Banerjee and Agrawal [3] proposed a two-warehouse inventory model for items with three-parameter Weibull distribution deterioration, shortages and linear trend in demand. Chung et al. [4] developed a two-warehouse inventory model with imperfect quality production processes source. Gayen and Pal [8] provided a two-warehouse inventory model for deteriorating items with stock dependent demand rate and holding cost.

Rong, et al. [16] presented a two warehouse inventory model for a deteriorating item with partially/fully backlogged shortage and fuzzy lead time. Recently, researches related to this area are Niu and Xie [13], Kofjac et al. [11], Lee and Hsu [12], Yang [19], Zhou [20]. Since there are two separate storage facilities OW and RW, where OW has a limited storage capacity, it will be uneconomical to receive the total delivery at OW first and then to transfer the excess quantity to RW. The supplier then must be requested to deliver the ordered amount in two separate, appropriate consignments directly to OW and RW respectively.

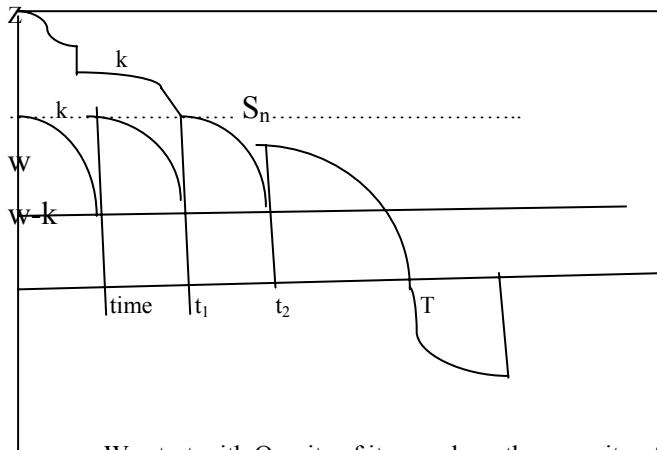
In the present paper, the cost of transporting a unit is assumed to be significant and the effect of releasing the stocks of RW in n shipments with a bulk size of K units per shipment instead of withdrawing an arbitrary quantity, is considered. Here, K is to be decided optimally and we call this as K -release rule. The problem is to decide the optimal values of Q and K which minimize the sum of ordering, holding and transport costs of the system. Here we have tried to develop a two storage inventory model with deteriorating items considering fuzzy demand.

2. ASSUMPTIONS AND NOTATIONS

1 Demand $R(t)$ is time dependent: $R(t) = \tilde{a}e^{bt}$ $a > 0, 1 > b > 0$

2. Lead time is zero.
3. Time horizon is T , which is to be determined.
4. The rate of deterioration is $\beta (< 1)$ and is constant.
5. Rs C_4 is the known cost of deteriorated unit.
6. Rs C_3 is the cost of deteriorated unit.
Rs C_2 is the shortage cost.
7. Rs. C_1 is the holding cost per unit per unit time in OW
Rs F is the holding cost per unit per unit time in RW where $F > C_1$.
8. The transportation cost of K units at a time is Rs C_t , which is constant.
9. There is no spoilage and wastage during the transportation.
10. Shortages are allowed and backlogged.

3. MODEL FORMULATION AND ANALYSIS



We start with Q units of items where the capacity of OW is W units ($Q > W$). The remaining part $Z = (Q - W)$ is kept in RW. Initially, the demand is met from OW until the stock level in OW reaches the level of $(W - K)$. Then K units are transferred from RW to

OW to reach the stock level W in OW. We repeat this process till the stock level in RW is exhausted. The stock in OW is exhausted partly to meet up the demand and partly for deterioration. Let the time taken to consume first K units in OW be t_k .

Own Ware House (OW)

The differential equation for inventory during $[0, T]$ in Own Warehouse (OW)

$$\frac{dI}{dt} + \beta I(t) = -\tilde{a}e^{bt} \quad (i-1)t_k \leq t \leq it_k, \quad i = 1, 2, \dots, (n-1) \quad (1)$$

With the boundary conditions $I(0) = I(it_k) = W$

$$\frac{dI}{dt} + \beta I(t) = -\tilde{a}e^{bt} \quad t_1 \leq t \leq t_2 \quad (2)$$

And

$$I(t_1) = W' = W - K + S_n$$

$$\frac{dI}{dt} = -\tilde{a}e^{bt}, \quad t_2 \leq t \leq T \quad (3)$$

Using α - cut to equation (1), we get

$$\frac{dI_1^+}{dt} + \beta I_1^+(t) = -a^- e^{bt} \quad (i-1)t_k \leq t \leq it_k, \quad i = 1, 2, \dots, (n-1)$$

$$\frac{dI_1^-}{dt} + \beta I_1^-(t) = -a^+ e^{bt} \quad (i-1)t_k \leq t \leq it_k, \quad i = 1, 2, \dots, (n-1)$$

$$\frac{dI_1^+}{dt} + \beta I_1^+(t) = -\{a_1 + \alpha(a_2 - a_1)\} e^{bt} \quad (i-1)t_k \leq t \leq it_k, \quad i = 1, 2, \dots, (n-1) \quad (4)$$

$$\frac{dI_1^-}{dt} + \beta I_1^-(t) = -\{a_3 + \alpha(a_3 - a_2)\} e^{bt} \quad (i-1)t_k \leq t \leq it_k, \quad i = 1, 2, \dots, (n-1) \quad (5)$$

Where $I(0) = I(it_k) = W$

Solving equation (4) and (5), we get

$$I_1^+(t) = W - \{a_1 + \alpha(a_2 - a_1)\} t - W\beta t \quad (6)$$

$$I_1^-(t) = W - \{a_3 + \alpha(a_3 - a_2)\} t - W\beta t \quad (7)$$

Taking $p^+ = W\beta + a^+$ and $p^- = W\beta + a^-$ equations (6) and (7) reduces to

$$I_1^+(t) = W - p^- t \quad 0 \leq t \leq t_k \quad (6)$$

$$I_1^-(t) = W - p^+ t \quad 0 \leq t \leq t_k \quad (7)$$

At $t = t_k$, inventory level in OW becomes W-K

Using (4) and (5) we get, $t_k = K / p^-$ and $t_k = K / p^+$

At time $t = t_k$, K units are transferred from RW to OW and the stock level of RW to $Z - K - \beta Z t_k$. In this model W, β are constant. Therefore, t_k depends on K only.

The stock level of OW attains the level of W after receiving K units from RW, it takes again t_k unit of time to reach the level of (W-K). Repeating this process (n-1) times let the stock level of RW to become S_n which is transferred to OW at $t_1 = n t_k = nK / p$

with the transportation cost of C_t' per unit and the stock level of OW becomes $W - K + S_n$ and ultimately reaches the level of zero at $t = t_2$. Then shortages occur. The new cycle begins at $t = T$. We optimize K in such a way that $W - K + S_n \leq W$, since the capacity of OW is W i.e. $S_n \leq K$.

Using α -cut in equation (2) we get,

$$\frac{dI_2^+(t)}{dt} + \beta I_2^+(t) = -a^- e^{bt} \quad t_1 \leq t \leq t_2$$

$$\frac{dI_2^-(t)}{dt} + \beta I_2^-(t) = -a^+ e^{bt} \quad t_1 \leq t \leq t_2$$

$$\frac{dI_2^+(t)}{dt} + \beta I_2^+(t) = -\{a_1 + \alpha(a_2 - a_1)\} e^{bt} \quad t_1 \leq t \leq t_2 \quad (8)$$

$$\frac{dI_2^-(t)}{dt} + \beta I_2^-(t) = -\{a_3 + \alpha(a_3 - a_2)\} e^{bt} \quad t_1 \leq t \leq t_2 \quad (9)$$

$$I(t_1) = W - K + S_n$$

Solving equation (8) and (9), we get

$$I_2^+(t) = -\{a_1 + \alpha(a_2 - a_1)\} \frac{e^{bt}}{(b + \beta)} + (1 - \beta t) \left[(W - K + S_n) + \frac{\{a_1 + \alpha(a_2 - a_1)\}}{(b + \beta)} + \left[\{a_1 + \alpha(a_2 - a_1)\} - \beta(W - K + S_n) \right] t_1 \right] \quad (10)$$

$$I_2^-(t) = -\{a_3 + \alpha(a_3 - a_2)\} \frac{e^{bt}}{(b + \beta)} + (1 - \beta t) \left[(W - K + S_n) + \frac{\{a_3 + \alpha(a_3 - a_2)\}}{(b + \beta)} + \left[\{a_3 + \alpha(a_3 - a_2)\} - \beta(W - K + S_n) \right] t_1 \right] \quad (11)$$

Applying α - cut to equation (3) we get

$$\frac{dI_3^+}{dt} = -\{a_1 + \alpha(a_2 - a_1)\} e^{bt} \quad (12)$$

$$\frac{dI_3^-}{dt} = -\{a_3 + \alpha(a_3 - a_1)\} e^{bt} \quad (13)$$

Solving Equations (12) and (13), we get

$$I_3^+(t) = \frac{\{a_1 + \alpha(a_2 - a_1)\}}{b} (e^{-bt_2} - e^{-bt}) \quad (14)$$

$$I_3^-(t) = \frac{\{a_3 + \alpha(a_3 - a_1)\}}{b} (e^{-bt_2} - e^{-bt}) \quad (15)$$

Upper α - cut of total inventory in own warehouse

$$\begin{aligned} (P_3^+) &= \sum_{i=1}^n \int_{(i-1)t_k}^{it_k} I_1^+(t) dt + \int_{t_1}^{t_2} I_2^+(t) dt \\ &= \int_0^{t_k} I_1^+(t) dt + \int_{t_k}^{2t_k} I_1^+(t) dt + \int_{2t_k}^{3t_k} I_1^+(t) dt + \dots + \int_{(n-1)t_k}^{nt_k} I_1^+(t) dt + \int_{t_1}^{t_2} I_2^+(t) dt \\ &= nt_k \left[W - \{a_1 + \alpha(a_2 - a_1)\} \frac{1}{2} nt_k - \frac{Wn\beta t_k}{2} \right] - \frac{\{a_1 + \alpha(a_2 - a_1)\} (e^{-bt_2} - e^{-bt_1})}{(b + \beta)} \\ &+ \left[W' + \frac{a_1 + \alpha(a_2 - a_1)}{(b + \beta)} + \{a_1 + \alpha(a_2 - a_1)\} t_1 - \beta W' t_1 \right] (t_2^2 - t_1^2) \left(1 - \frac{\beta}{2}\right) \end{aligned} \quad (16)$$

Lower α -cut of total inventory in own warehouse

$$\begin{aligned}
 (P_3^-) &= \sum_{i=1}^n \int_{(i-1)t_k}^{it_k} t_k I_1^-(t) dt + \int_{t_1}^{t_2} I_2^-(t) dt \\
 &= \int_0^{t_k} I_1^-(t) dt + \int_{t_k}^{2t_k} I_1^-(t) dt + \int_{2t_k}^{3t_k} I_1^-(t) dt + \dots + \int_{(n-1)t_k}^{nt_k} I_1^-(t) dt + \int_{t_1}^{t_2} I_2^-(t) dt \quad (17) \\
 &= nt_k \left[W - \{a_3 + \alpha(a_3 - a_2)\} \frac{1}{2} nt_k - \frac{Wn\beta t_k}{2} \right] - \frac{\{a_3 + \alpha(a_3 - a_2)\} (e^{bt_2} - e^{bt_1})}{(b + \beta)} \\
 &+ \left[W' + \frac{a_3 + \alpha(a_3 - a_2)}{(b + \beta)} + \{a_3 + \alpha(a_3 - a_2)\} t_1 - \beta W' t_1 \right] (t_2^2 - t_1^2) (1 - \frac{\beta}{2})
 \end{aligned}$$

Total demand of inventory

$$\begin{aligned}
 &= \int_0^T \tilde{a} e^{bt} dt = \int_0^T f_1(t) f_2(t) f_3(t) dt
 \end{aligned}$$

Where

$$f_1(t) = a_1 e^{bt}, f_2(t) = a_2 e^{bt}, f_3(t) = a_3 e^{bt}$$

Therefore upper α - cut of total demand

$$\begin{aligned}
 (D^+) &= \int_0^T [f_3(t) + \alpha(f_3(t) - f_2(t))] dt \\
 &= \frac{\{a_3 + \alpha(a_3 - a_2)\} (e^{bT} - 1)}{b} \quad (18)
 \end{aligned}$$

Therefore lower α -cut of total demand

$$\begin{aligned}
 (D^-) &= \int_0^T [f_1(t) + \alpha(f_2(t) - f_1(t))] dt \\
 &= \frac{\{a_1 + \alpha(a_2 - a_1)\} (e^{bT} - 1)}{b} \quad (19)
 \end{aligned}$$

Upper α - cut of deteriorated items

$$\begin{aligned}
 (DC^+) &= P_1 - P_3^+ - D^- \\
 &= Znt_k - \frac{Zn(n-1)\beta t_k^2}{2} + (2-n)\frac{K}{\beta} - Kt_k + nt_k \left[W - \{a_1 + \alpha(a_2 - a_1)\} \frac{nt_k}{2} - \frac{Wn\beta t_k}{2} \right] \\
 &\quad - \frac{\{a_1 + \alpha(a_2 - a_1)\} (e^{bt_2} - e^{bt_1})}{(b+\beta)} + \left[W' + \frac{\{a_1 + \alpha(a_2 - a_1)\}}{(b+\beta)} + \{a_1 + \alpha(a_2 - a_1)\} t_1 - \beta W' t_1 \right] \\
 &\quad (t_2^2 - t_1^2) \left(1 - \frac{\beta}{2}\right) - \frac{\{a_1 + \alpha(a_2 - a_1)\} (e^{bT} - 1)}{b}
 \end{aligned} \tag{20}$$

Lower α - cut of deteriorated items

$$\begin{aligned}
 (DC^-) &= P_1 - P_3^- - D^+ \\
 &= Znt_k - \frac{Zn(n-1)\beta t_k^2}{2} + (2-n)\frac{K}{\beta} - Kt_k + nt_k \left[W - \{a_3 + \alpha(a_3 - a_2)\} \frac{nt_k}{2} - \frac{Wn\beta t_k}{2} \right] \\
 &\quad - \frac{\{a_3 + \alpha(a_3 - a_2)\} (e^{bt_2} - e^{bt_1})}{(b+\beta)} + \left[W' + \frac{\{a_3 + \alpha(a_3 - a_2)\}}{(b+\beta)} + \{a_3 + \alpha(a_3 - a_2)\} t_1 - \beta W' t_1 \right] \\
 &\quad (t_2^2 - t_1^2) \left(1 - \frac{\beta}{2}\right) - \frac{\{a_3 + \alpha(a_3 - a_2)\} (e^{bT} - 1)}{b}
 \end{aligned} \tag{21}$$

Total amount of shortages

$$S = \int_{t_2}^T \tilde{I}(t) dt$$

Upper α - cut of total shortages

$$\begin{aligned}
 S^+ &= \int_{t_2}^T I_3^+(t) dt \\
 &= \frac{\{a_1 + \alpha(a_2 - a_1)\} e^{bt_2} (T - t_2)}{b} - \frac{\{a_1 + \alpha(a_2 - a_1)\} (e^{bT} - e^{bt_2})}{b^2}
 \end{aligned} \tag{22}$$

Lower α -cut of total shortages

$$\begin{aligned}
 S^- &= \int_{t_2}^T I_3^-(t) dt \\
 &= \frac{\{a_3 + \alpha(a_3 - a_2)\} e^{bt_2} (T - t_2)}{b} - \frac{\{a_3 + \alpha(a_3 - a_2)\} (e^{bT} - e^{bt_2})}{b^2}
 \end{aligned} \tag{23}$$

Rented Warehouse

During the period $[0, t_k]$ the stock level of RW can be described in the following way:

At $t = 0$, the stock level $S_0 = Z$

$t = t_k$, the stock level $S_1 = Z(1 - \beta t_k) - K$

$t = 2t_k$, the stock level $S_2 = Z(1 - \beta t_k)^2 - K(1 - \beta t_k) - K$

.....

$t = (n - 1)t_k$, the stock level $S_{n-1} = Z(1 - \beta t_k)^{n-1} - K(1 - \beta t_k)^{n-2} - \dots - K(1 - \beta t_k) - K$

and $S_n = Z - (n - 1)K - Zn\beta t_k$

Total inventory in RW, $P_1 = P_2 \times t_k$

Where

$$\begin{aligned}
 P_2 &= [S_0 + S_1 + S_2 + \dots + S_{n-1}] \\
 &= \{Z + Z(1 - \beta t_k) + \dots + Z(1 - \beta t_k)^{n-1}\} - (n-1)K - (n-2)K(1 - \beta t_k) - (n-3)K(1 - \beta t_k)^2 \\
 &\quad - \dots - 2K(1 - \beta t_k)^{n-3} - K(1 - \beta t_k)^{n-2} \\
 &= Z\{1 + (1 - \beta t_k) + \dots + (1 - \beta t_k)^{n-1}\} - (n-1)K - (n-1)K(1 - \beta t_k) - K(n-1-2)(1 - \beta t_k)^2 \\
 &\quad - \dots - \{(n-1) - (n-3)\}K(1 - \beta t_k)^{n-3} - K\{(n-1) - (n-2)\}(1 - \beta t_k)^{n-2} \\
 &= (Z/\beta t_k)\{1 - (1 - \beta t_k)^n\} - (n-1)K - (n-1)\{(1 - \beta t_k) + (1 - \beta t_k)^2 + (1 - \beta t_k)^3 + \dots + (1 - \beta t_k)^{n-3} + (1 - \beta t_k)^{n-2}\}K \\
 &\quad + (1 - \beta t_k)K + 2K(1 - \beta t_k)^2 + \dots + (n-2)(1 - \beta t_k)^{n-2}K \\
 &= (Z/\beta t_k)\{1 - (1 - \beta t_k)^n\} - (n-1)K - (n-1)K(1 - \beta t_k)\{1 + (1 - \beta t_k) + \dots + (1 - \beta t_k)^{n-3}\} + P_n
 \end{aligned} \tag{24}$$

Where

$$\begin{aligned}
 P_n &= K\{(1 - \beta t_k) + 2(1 - \beta t_k)^2 + \dots + (n - 2)(1 - \beta t_k)^{n-2}\} \\
 P_n &= K\{x + 2x^2 + 3x^3 + \dots + (n - 2)x^{n-2}\}, \quad \text{where } x = (1 - \alpha t_k) \\
 xP_n &= K\{x^2 + 2x^3 + 3x^4 + \dots + (n - 3)x^{n-2} + (n - 2)x^{n-1}\}, \\
 P_n(1 - x) &= K(x + x^2 + x^3 + \dots + x^{n-2} - (n - 2)x^{n-1}) \\
 P_n &= xK(1 + x + x^2 + \dots + x^{n-3}) / (1 - x) - (n - 2)Kx^{n-1} / (1 - x) \\
 &= Kx(1 - x^{n-2}) / (1 - x) - K(n - 2)x^{n-1} / (1 - x)
 \end{aligned}$$

Therefore putting the value of P_n in equation (24)

$$\begin{aligned}
 P_2 &= \frac{Z\{1-(1-\beta t_k)^n\}}{\beta t_k} - (n-1)K - \frac{(n-1)(1-\beta t_k)\{1-(1-\beta t_k)^{n-2}\}K}{\beta t_k} \\
 &+ \frac{(1-\beta t_k)\{1-(1-\beta t_k)^{n-2}\}K}{\beta t_k} - \frac{(n-2)(1-\beta t_k)^{n-1}K}{\beta t_k} \\
 P_1 &= P_2 \times t_k = \frac{Z\{1-(1-\beta t_k)^n\}}{\beta} - (n-1)K t_k - (n-1)(1-\beta t_k) \frac{K}{\beta} \\
 &+ (n-1)(1-\beta t_k)^{n-1} \frac{K}{\beta} + \frac{(1-\beta t_k)K}{\beta} \\
 &- (1-\beta t_k)^{n-1} \frac{K}{\beta} - (n-2)(1-\beta t_k)^{n-1} \frac{K}{\beta} \\
 &= \frac{Z\{1-(1-\beta t_k)^n\}}{\beta} + \frac{(n-1)\{-\beta K t_k - K + \beta K t_k\}}{\beta} + \frac{(1-\beta t_k)K}{\beta} \\
 &= Z n t_k - \frac{Z n (n-1) \beta t_k^2}{2} + (2-n) \frac{K}{\beta} - K t_k
 \end{aligned} \tag{25}$$

$$\text{Holding Cost in } RW = F * P_1 \tag{26}$$

Upper α - cut of total average cost

$$(TVC^+) = \left[C_4 + F P_1 + C_1 P_3^+ + C_3 (DC)^+ + C_2 S^+ + (n-1)C_i + S_n C_i' \right] \left(\frac{1}{T} \right) \tag{27}$$

Lower α - cut of total average cost

$$(TVC^-) = \left[C_4 + F P_1 + C_1 P_3^- + C_3 (DC)^- + C_2 S^- + (n-1)C_i + S_n C_i' \right] \left(\frac{1}{T} \right) \tag{28}$$

The objective in this paper is to find an optimal cycle time to minimize the total variable cost per unit time. Therefore this model mathematically can be written as

$$\text{Minimize } \{TVC^+, TVC^-\} \tag{29}$$

Subject to $0 \leq \alpha \leq 1$

Therefore, the problem is a multiobjective optimization problem. To convert it to a single objective optimization problem, we use global criteria (GC) method.

Then, the above problem is reduced to

$$\text{Minimize } GC \tag{30}$$

Subject to $0 \leq \alpha \leq 1$

4. GLOBAL CRITERIA METHOD

The model presented by (29) is a multi-objective model which is solved by Global Criteria (GC) Method helped by Generalized Reduced Gradient Technique.

The Multi-Objective Non –linear Integer Programming(MONLIP) problems are solved by Global Criteria Method converting it to a single objective optimization problem .The solution procedure is as follows:

Step-1: Solve the multi-objective programming problem (29) as a single objective problem using only one objective at a time ignoring other.

Step-2: From the results of Step-1, determine the ideal objective vector, say $(TVC^{+\min}, TVC^{-\min})$ and the corresponding values of $(TVC^{+\max}, TVC^{-\max})$.Here, the ideal objective vector is used as a reference point. The problem is then to solve the following auxiliary problem:

$$Min(GC) = Minimize \left\{ \left(\frac{TVC^+ - TVC^{+\min}}{TVC^{+\max} - TVC^{+\min}} \right)^r + \left(\frac{TVC^- - TVC^{-\min}}{TVC^{-\max} - TVC^{-\min}} \right)^r \right\}^{1/r}$$

Where $1 \leq r < \infty$. This method is also sometimes called Compromise Programming.

5. NUMERICAL EXAMPLE

We now consider a numerical example showing the utility of the model from the practical point of view. According to the developed solution procedure of the proposed inventory system, the optimal solution has been obtained with the help of well known generalized reduced gradient method(GRG).To illustrate the developed model, an example with the following data has been considered:

Let $a_1 = 3$ units/month, $a_2 = 5$ units/month, $a_3 = 2$ units/month, $C_1 = \$0.1$ per unit, $F = \$0.4$ per unit, $C_2 = \$0.6$ per unit, $C_3 = \$0.4$ per unit, $C_4 = \$3000$ per order, $C'_1 = 7, C'_2 = 50, w = 20, \alpha = 25, \beta = 1.9, t_2 = 6, T = 8hrs$.

Substituting above parameters, the compromise solutions are $TVC^+ = \$428.92, TVC^- = \427.45

6. CONCLUSION

In this paper, an attempt is made to incorporate the different preservation facilities into an own and a rented warehouse for deteriorating items with fuzzy demand. This type of demand is applicable to a newly launched product in the market. The demand of a new product is always fuzzy in its nature .

The reasons for the adaptation of this model are as follows:

1. It is very difficult to define different parameters of an inventory problem precisely- especially the demand which is normally fuzzy in nature .This phenomenon is incorporated in the model.
2. At present, there is a crisis of having larger space in the market places. In most of the literature, two-warehouse models with one own warehouse (OW) at the market place and another rented warehouse (RW) situated little farther from the centre of the city are dealt with. The holding cost at OW is assumed to be less than the one at RW. Such a realistic situation has been considered in this model.
3. Due to the preserving condition of warehouses, items gradually lose their utility, i.e., deterioration takes place. This realistic phenomenon is incorporated in this model.
4. The bulk-release rule which is advantageous as the holding cost of RW is higher and the transportation cost per unit becomes higher than the cost of transporting per unit in case of bulk-release.

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