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# PROCUREMENT-DISTRIBUTION MODEL FOR PERISHABLE ITEMS WITH QUANTITY DISCOUNTS INCORPORATING FREIGHT POLICIES UNDER FUZZY ENVIRONMENT

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**Abstract**: A significant issue of the supply chain problem is how to integrate different entities. Managing supply chain is a difficult task because of complex integrations, especially when the products are perishable in nature. Little attention has been paid on ordering specific perishable products jointly in uncertain environment with multiple sources and multiple destinations. In this article, we propose a supply chain coordination model through quantity and freight discount policy for perishable products under uncertain cost and demand information. A case is provided to validate the procedure.

**Keywords**: Supply Chain Management, Perishable Products, Transportation Cost, Fuzzy Set Theory.

**MSC:** 90I305.

## 1. INTRODUCTION

One of the most tangible investments for any retail and manufacturing organization is applying smart supply chain management strategies; this kind of investment can not only help boost profit, but can also make difference between the business thriving or barely surviving. Procurement and distribution in supply chain are relatively more important issues when the demand is uncertain and products perishable in nature. This necessitates high inventory level, which worsens the situation. Many business owners donot realize the true cost of carrying excessive inventory, which can be 29 percent of the inventory's value when all the carrying costs (interest, storage, damage, obsolescence, etc.) are included. These costs come directly off the bottom-line profit. Therefore, it is required to find out the optimum levels of ordered quantity and carrying inventory so that total procurement and distribution cost can be minimized. This paper develops policies for supply chain model, which include procurement, holding inventory and transportation decisions in order to keep the total cost to its minimum.

Integrated procurement and distribution decisions for perishable products under fuzzy environment have been the least studied by researchers, though subjects have been studied separately extensively. Thus, this work is motivated to bridge the gap in the literature by proposing a supply chain coordination model through quantity and freight discount policy for perishable products under uncertain environment. Better coordination amongst the suppliers, distributors and retailers is the key to success for every supply chain. The authors in [19] developed a lot-for-lot discount pricing policy for deteriorating items with constant demand rate; in [20], an optimal quantity-discount pricing strategy in a collaborative system for deteriorating items with instantaneous replenishment rate is developed; in [3], integrated vendor-buyer cooperative inventory models with variant permissible delay in payments are thought out; in [17], optimal policy for decaying items with stock dependent demand under inflation in a supply chain is discussed; in [16], an optimal batch size for integrated production-inventory policy in a supply chain has been introduced; in [10], optimal order quantity when all units' quantity discounts are available on purchasing price and freight cost are determined; in [18], a constant demand rate is assumed, and a model with freight and price discounts, where freight discount structure is based on weight, is developed; in [5], a single stage multi incapacitated dynamic lot sizing problem (MILSP) with transportation cost is taken, and finite planning horizon with dynamic demand is assumed; he considered all unit inventory management models to formulate the problem with piece wise linear transportation cost function. In [14], an unconstrained integrated inventory-transportation model is developed to decide optimal order quantity for inventory system over a finite horizon.

In the crisp environment, all parameters in the total cost such as holding cost, set-up cost, purchasing price, rate of deterioration, demand rate, production rate, etc. are known and have definite value without ambiguity. Some of the business situations fit such conditions, but in most of the situations and in the day-by-day changing market scenario, the parameters and variables are highly uncertain or imprecise. For any particular problem in the crisp scenario, the aim is to maximize or minimize the objective function under the given constraints. But in many practical situations, the decision maker may not be in the position to specify the objective or the constraints precisely. In such situations, these parameters and variables are treated as fuzzy parameters. The fuzzification grants authenticity to the model; it allows vagueness in the whole setup, which brings it closer to reality. The fuzzy set theory was first introduced by [1]. Recently, the theory of fuzzy sets and fuzzy logic has found wide applications in operations management; in [8], we have carried out a detailed review. As a part of operations management, inventory control and supply chain management have also seen an exhaustive applications of fuzzy sets. A brief review of supply chain models based on fuzzy sets is discussed below. In [4], a fuzzy inventory model with backorder option is analyzed; in [9], two fuzzy models with fuzzy parameters are introduced, and the optimal production quantity is derived by using graded mean integration representation method and extended Lagrangian method. The author shows that a crisp model is a specific case of the fuzzy model. In [6], [11, 12], [13], [2], [21], different problems that consider inventory with backorder, inventory without backorder and production inventory in the fuzzy sense are discussed; in [22], an inventory model without backorder is considered, where total demand and holding costs are assumed to be fuzzy in nature, and the authors used different methods to derive total cost. In [15], an EOQ model with uncertain inventory cost under arithmetic operations of extension principle is developed, and trapezoidal fuzzy numbers to represent the inventory costs are used. Later, in [23], an EOQ model with fuzzy order quantity is developed, and shortages are considered.

Most of the references cited in the above coordination models have considered models with crisp parameters only, and the authors, who developed the models with fuzzy parameter, considered only non-perishable items. There is hardly any study about perishable products in procurement-distribution supply chain under uncertainty. This particular study shows how retailers in a supply chain can use their resources for the best possible outcome.

As in [7], retailing in developing countries was observed, prior to the 1990s, and the predictions were that there would be no primary and extensive retail transformation in the near future. The ‘supermarket revolution’ in developing countries with its ‘take-off’ in the early mid-1990s flies in the face of these earlier predictions with presence of retail chains Reliance Fresh, Food bazaar, More, Spencers etc. In a current study, three retail stores (RS1, RS2, RS3) of a well established company is surveyed for its procurement and distribution policies for three months (periods). The stores procure food items (like grains, grocery, dairy, poultry, and etc.) from two warehouses (WH1, WH2) of a supplier, whose carrying cost is borne by the stores. In the study, a perishable food segment is considered, which requires regular inspection, with inspection cost of $2 per sack assuming perishability of 5% in a lot, and the weight per sack of grains is 6,7, 8 and 5 kg, respectively. As the companies rarely break contractual agreements, they are offered discounts on bulk purchase. Also, goods are transported from supplier to retail stores through various modes, i.e. truckload (TL), less than truckload (LTL) and combination of both. In TL transportation, the cost of one truck is fixed up to a given capacity. The capacity for each truck is 1,500kgs. However, in some cases the weighted quantity may not be large enough to substantiate the cost associated with a TL mode. In such a situation, a LTL mode may be used. LTL may be defined as a shipment of weighted quantity which does not fill a truck, and the transportation cost is taken on the basis of per unit weight. The cost of transporting each sack in this mode is $2. As the products’ are perishable, predicting a concrete demand is impossible and leads to uncertainty for procurement and distribution. Here, we are examining such situations where demand is uncertain and try to minimize the vagueness of total costs using fuzzy sets and membership functions.

The formulation and solution of the above enlightened model is discussed in the following sections. Section 2 presents the details of model’s assumptions, sets, and symbols. Section 3 provides the model formulation and its analysis. Section 4 discusses the conclusion with future prospects.

## 2. SETS AND SYMBOLS

### 2.1. Assumptions

The assumptions of this research are essentially the same as those of EOQ model except for the transportation cost. The section considers a single stage system with finite planning horizon. The demand is dynamic and fuzzy in nature. Shortages are not allowed. Lead times are assumed to be zero for both modes of transportation available, namely TL and LTL, i.e. supply is immediate. The initial inventory of each product is zero at the beginning of the planning horizon, and the holding cost is independent of the purchase price and any capital invested in transportation.

### 2.2. Sets

- Product set with cardinality $P$ and indexed by $i$.
- Period set with cardinality $T$ and indexed by $t$.
- Product discount break point set with cardinality $L$ and indexed by small $l$.
- Source set with cardinality $J$ and indexed by $j$.
- Destination set with cardinality $M$ and indexed by $m$.

### 2.3. Parameters

| | |
|---|---|
| $\tilde{C}$ | Fuzzy total cost |
| $C_0$ | Aspiration level of fuzzy total cost |
| $C_0^*$ | Tolerance level of fuzzy total cost |
| $c_t$ | Cost of unit weighted quantity of period $t$ |
| $\tilde{D}_{imt}$ | Fuzzy demand for product $i$ in period $t$ for $m^{th}$ destination |
| $\bar{D}_{imt}$ | Defuzzified demand for product $i$ in period $t$ for $m^{th}$ destination |
| $h_{ijmt}$ | Inventory holding cost per unit of item $i$ per period $t$ |
| $w_i$ | Per unit weight of item $i$ in kgs |
| $\phi_{ijmt}$ | Unit purchase cost for $i^{th}$ item in $t^{th}$ period |
| $\beta_{jmt}$ | Fixed freight cost for each TL |
| $d_{ijmlt}$ | It reflects the fraction of regular price that the buyer pays for purchased items. |
| $a_{ijmlt}$ | Limit beyond which a price break becomes valid in period $t$ for product $i$ for $l^{th}$ price break |
| $IN_i$ | Inventory level at the beginning of planning horizon for product $i$ |
| $d_f$ | Quantity discount factor |
| $\omega$ | Weight transported in each full truck |
| $s$ | Cost per kg of shipping in LTL mode |
| $\eta$ | Percentage defect in the lot |
| $m_i$ | Cost of per unit inspection of $i^{th}$ item |

.

### 2.4. Decision Variables

$X_{ijmt}$ **A**mount of product $i$ ordered in period $t$ transported from $j^{th}$ source to $m^{th}$ destination ordered in period $t$.

$R_{ijmlt}$ If the $i^{th}$ ordered quantity from $j^{th}$ source to $m^{th}$ destination in $t^{th}$ period falls in $l^{th}$ price break then the variable takes value 1 otherwise zero.

$$R_{ijmlt} = \begin{cases} 1 & if \;\; X_{ijmt} \; falls \text{ in } l^{th} \; pricebreak \\ 0 & \text{otherwise} \end{cases}$$

$I_{ijmt}$ Inventory level for $i^{th}$ product at the end of period $t$ at $j^{th}$ source borne by $m^{th}$ destination at the end of period $t$.

$\delta_{jmt}$ Total weighted quantity transported from $j^{th}$ source to $m^{th}$ destination in period $t$.

$\alpha_{jmt}$ Total number of trucks from $j^{th}$ source to $m^{th}$ destination in $t^{th}$ period.

$y_{jmt}$ Amount in excess of TL capacity (in weights) from $j^{th}$ source to $m^{th}$ destination in $t^{th}$ period.

$u_{jmt}$ *(or, 1-*$u_{jmt}$*)* The variable reflects usage of policies, either both TL and LTL policies or only TL policy or only LTL.

$$u_{jmt} = \begin{cases} 1, if \; considering \; TL \,\&\, LTL \, or \, only \, LTL \; policy \\ 0, if \; considering \; only \, TL \; policy \end{cases}$$

## 3. FUZZY OPTIMIZATION FORMULATION

Most of our traditional tools of modeling are crisp, deterministic, and precise in character. But for many practical problems, there are incompleteness and unreliability of input information. This enforces us to use fuzzy optimization method with fuzzy parameters. Crisp mathematical programming approaches provide no such mechanism to quantify these uncertainties. Fuzzy optimization is a flexible approach that permits more adequate solutions of real problems in the presence of vague information, providing well defined mechanisms to quantify the uncertainties directly.

Therefore, we formulate fuzzy optimization model on vague aspiration levels on total cost and demand; the decision maker may decide his aspiration levels on the basis of his past experience and knowledge.

$$\begin{aligned} Min\,\tilde{C} = & \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{j=1}^{J}\sum_{i=1}^{P}\left\{ h_{ijmt} I_{ijmt} + m_i X_{ijmt} + \sum_{l=1}^{L} R_{ijmlt} d_{ijmlt} \phi_{ijmt} X_{ijmt} \right\} \\ & + \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{j=1}^{J}\left[ (sy_{jmt} + \alpha_{jmt}\beta_{jmt}) u_{jmt} + (\alpha_{jmt}+1)\beta_{jmt}(1-u_{jmt}) \right] \end{aligned} \quad (1)$$

$$\sum_{j=1}^{J} I_{ijmt} = \sum_{j=1}^{J} I_{ijmt-1} + \sum_{j=1}^{J} X_{ijmt} - \tilde{D}_{imt} - \eta \sum_{j=1}^{J} I_{ijmt} \quad where\; i = 1 \ldots P \;\;,\;\; t = 2 \ldots T \quad (2)$$

$$\sum_{j=1}^{J} I_{ijm1} = \sum_{j=1}^{J} I_{ijm1} + \sum_{j=1}^{J} X_{ijm1} - \tilde{D}_{im1} - \eta \sum_{j=1}^{J} I_{ijm1} \quad where\ i = 1...P \tag{3}$$

$$(1-\eta)\sum_{j=1}^{J}\sum_{t=1}^{T} I_{ijmt} + \sum_{j=1}^{J}\sum_{t=1}^{T} X_{ijmt} \geq \sum_{t=1}^{T} \tilde{D}_{imt}, where\ i = 1...P\ ;\ m = 1...M \tag{4}$$

$$X_{ijmt} \geq \sum_{l=1}^{L} a_{ijmlt} R_{ijmlt},\ \ i = 1...P;\ j = 1..J;\ m = 1...M;\ t = 1...T \tag{5}$$

$$\sum_{l=1}^{L} R_{ijmlt} = 1\ ,\ i = 1...P;\ j = 1...J,\ m = 1...M,\ t = 1...T \tag{6}$$

$$\delta_{jmt} = \sum_{i=1}^{P}\left[ w_i X_{ijmt} \sum_{l=1}^{L} R_{ijmlt} \right],\ \ j = 1...J;\ m = 1....M;\ t = 1...T \tag{7}$$

$$\delta_{jmt} \leq (y_{jmt} + \alpha_{jmt}\omega)u_{jmt} + (\alpha_{jmt} + 1)\omega(1 - u_{jmt}) \tag{8}$$

$$t = 1,...,T;\ \ j = 1,...,J;\ \ m = 1,....,M$$

$$\delta_{jmt} = (y_{jmt} + \alpha_{jmt}\omega)\ \ j = 1....J;\ m = 1...M;\ t = 1...T \tag{9}$$

$$i=1,...,P;\ t=1,...,T;\ l=1,...,L;\ \ j=1,......,J;\ m=1,.....,M \tag{10}$$

$$X_{ijmt}, I_{ijmt}, \delta_{jmt}, y_{jmt}, \alpha_{jmt} \geq 0\ and\ \text{integers};$$

$$R_{ijmlt}, u_{jmt} \in \{0,1\}$$

Constraint (1) represents a fuzzy objective function which minimizes the total cost borne by the firm for the duration of the planning horizon. The ordering cost is a fixed cost not affected by the ordering quantities and therefore, it is not the part of the objective function. The components of the total cost reflected by the first tern of constraint (1) are; inventory carrying cost at the source, inspection cost (with constant inspection reate for all the products) and purchase cost. The second term of the objective function represents the total transportation cost from various sources and to different destinations. Constraints (2) – (4) are called balancing constraints, where constraint (2) calculates the ending inventory of $i^{th}$ product in $t^{th}$ period by deducting the cumulative of fuzzy demand and fraction of perished inventory from the sum of remaining inventory of the previous period and ordered quantity of the $i^{th}$ product in $t^{th}$ period. In a similar manner, constraint (3) evaluates the total ending inventory of $i^{th}$ product in the first period by subtracting the cumulative of fuzzy demand (of all the destinations) and fraction of perished inventory (of the same period) from the sum of initial inventory of the planning horizon and ordered quantity of first period. Constraint (4) shows; the total fuzzy demand of all periods from all the destinations is; less than or equal to the sum of ending inventory and ordered quantity; at all the sources in all the periods, i.e. there are no shortages. Constraint (5) finds out the order quantity of all products in $t^{th}$ period, which may exceed the quantity break threshold, and hence, avails discount on purchase cost at exactly one quantity discount level. Constraint (6) restricts the activation at exactly one level, either discount or no discount state. The integrator for procurement and distribution is constraint (7), which calculates transported quantity according to product weight. Constraint (8) gives the minimum weighted quantity transported and further, constraint (9) measures the overhead units from TL capacity in weights.

### 3.1. Price Breaks and Freight Breaks

As stated in section 2.4, variable $R_{ijmlt}$ specifies the fact that; when the order size in $t^{th}$ period is larger than threshold $a_{ijmlt}$, it results in discounted prices. Here, let L be the number of levels corresponding to the changes in fraction of the regular price( $d_{ijmlt}$ )and the threshold quantity $a_{ijmlt}$, then the price breaks are defined as:

$$d_f = \begin{cases} d_{ijmlt} & a_{ijmlt} \le X_{ijmt} \le a_{ijm(l+1)t} \\ d_{ijmLt} & X_{it} \ge a_{ijmLt} \end{cases}$$

$$i = 1,...,P;\ j = 1,...,J; m = 1,...M;\ \ l = 1,...,L;\ t = 1,...,T$$

### 3.2. Solution Algorithm

Following algorithm [24] specifies the sequential steps to solve the fuzzy mathematical programming problems.

1. Compute the crisp equivalent of the fuzzy parameters using a defuzzification function. The same defuzzification function is to be used for each of the parameters. Here, we use the defuzzification function of the type $F_2(A) = \dfrac{\left(a^1 + 2a^2 + a^3\right)}{4}$, where $a^1, a^2, a^3$ are triangular fuzzy numbers.

Here, let $\bar{D}_{imt}$ be the defuzzified value of $\tilde{D}_{imt}$ and $(D_{imt}{}^1, D_{imt}{}^2 \text{ and } D_{imt}{}^3)$ be triangular fuzzy numbers then, $\bar{D}_{imt} = \dfrac{D^1_{imt} + 2D^2_{imt} + D^3_{imt}}{4}$ *where* $i = 1...P; t = 1...T$ are defuzzified aspiration levels of the model's demand.

2. Define appropriate membership functions for each fuzzy inequality and a constraint corresponding to the objective function.

The membership function for the fuzzy cost is given as:

$$\mu_C(X) = \begin{cases} 1 & ;\ \mathrm{C}(X) \le C_0 \\ \dfrac{C_0^* - C(X)}{C_0^* - C_0} & ;\ C_0 \le C(X) < C_0^* \\ 0 & ;\ \mathrm{C}(X) > C_0^* \end{cases}$$

where $C_0$ is the restriction level and $C_0^*$ the tolerance level to the fuzzy total cost.

3. Employ extension principle to identify the fuzzy decision, which results in a crisp mathematical programming problem given by

Maximize $\theta$ Subject to $\mu_C(X) \geq \theta$,

where $\theta$ represents the degree up to which the aspiration of the decision-maker is met. The above problem can be solved by the standard crisp mathematical programming algorithms. On substituting the values for $\tilde{D}_{imt}$ as $\bar{D}_{imt}$ and $\mu_C(x)$, the problem becomes

Maximize $\theta$

subject to: $\mu_C(X) \geq \theta$,

$$I_{ijmt} = I_{ijmt-1} + X_{ijmt} - \sum_{m=1}^{M} \tilde{D}_{imt} - \eta\, I_{ijmt}$$

$$where\ i = 1...P,\ j=1...J,\ m=1...M,\ t = 2...T \quad ...(1)$$

$$I_{ijm1} = IN_i + X_{ijm1} - \sum_{m=1}^{M} \tilde{D}_{im1} - \eta I_{ijm1} \quad where\ i = 1...P...(2)$$

$$(1-\eta)\sum_{t=1}^{T} I_{ijmt} + \sum_{t=1}^{T} X_{ijmt} \geq \sum_{t=1}^{T}\sum_{m=1}^{M} \tilde{D}_{imt,}\ where\ i = 1...P \quad ...(3)$$

$$X \in S = \{X_{ijmt} I_{ijmt}, \delta_{jmt}, \alpha_{jmt}, y_{jmt} \geq 0\ and\ integer;$$
$$R_{ijmlt} \cdot u_{jmt} \in \{0,1\}\ /\ satisfying\ eq\ (4)\ to\ (8);$$
$$i = 1...P;\ j = 1...J;\ l = 1...L;\ m = 1...M;\ t = 1...T\};\ \theta \in [0,1]$$

can be solved by the standard crisp mathematical programming algorithm.

Now, the main challenge is to optimize various cost components viz. purchase, transportation, inspection cost and holding cost in order to gain maximum benefits. The next section provides the analysis of the solution.

### 3.3. Solution Analysis

The crucial objectives of any firm are to determine the amount of the quantity to order, and the way to minimize the total cost. A case study together with formulated crisp model in procurement-distribution scenario of a supply chain illustrates the way we answered to these objectives. The solution of the optimization problem is obtained by programming it into Lingo 13.0 software.

LINGO is a comprehensive software tool designed to provide solutions to linear, nonlinear (convex & non-convex/Global), quadratic and integer optimization models in a fast and efficient manner. The required data sets and parameters pertaining to quantity demanded, various costs, initial inventory, weights per product, quantity thresholds ,discounts etc. are tabulated in Appendix A (data is changed due to cutting edge competition and cannot be revealed but the model is applicable in same scenarios in complex data) and are fed in the lingo program to generate the solution. In particular, we have considered 3 periods, 4 products, 2 sources, 4 destinations, 3 price breaks in the current scenario. In general, we can incorporate any number of products, periods, sources, destinations and price breaks to obtain the solution of the desired problem.

After solving the problem, we find that one of the possible minimum optimal total costs incurred by the company is $2,256,314 with 75% minimization of vagueness. The solution presented in Appendix B reflects that, during the first period, order quantity of four grains from WH1 to RS1 are 200, 0, 100 and 0 sacks with discount on purchase cost of 20%, 0%, 30% and 0%, respectively. The ending inventories are 100, 560, 0 and 0 sacks of Rice, Sorghum, Wheat and Maize, respectively. Weighted quantity of grains from WH1 to RS1 during the first period is 500 kgs. The firm employs only TL mode during the same period. For rest of the periods; procurement and distribution strategies are shown in Appendix B.

## 4. CONCLUSION

Procurement and distribution decisions play a major role in supply chain as two key forces. Therefore, in this paper, we have formulated an optimization model for multiple perishable products ordered from multiple sources to fulfill the demand of multiple retail outlets, to minimize the overall total cost of procurement and distribution under uncertain environment. Applicability of the model is demonstrated by using a leading Indian retail firm as the sample. The problem is solved by using mathematical programming approach on Lingo 13.0 software. The approach followed in the paper gives several useful results for the procurement-distribution supply chain strategies. The model can be applied to many real life situations as it can include as many products, sources, and destinations as desired. Hence, we can conclude from our present research that integration of various functions of different entities is possible, in order to minimize the aggregate cost of purchasing and transportation activities. In fact, the results of this study open several opportunities for further research and improvements. For future research, different extensions to the proposed model can be considered. For instance, models that include backlogging and stochastic demand could be developed. Other realistic dimensions that can be incorporated into the model are multi-stage systems in different environments and different lead times.

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## APPENDIX A

**Table 1:** UNIT PURCHASE COST (IN INR) OF GRAINS

| | Rice | | | Sorghum | | |
|---|---|---|---|---|---|---|
| Periods | ***1*** | ***2*** | ***3*** | ***1*** | ***2*** | ***3*** |
| WH1-RS1 | 130 | 175 | 140 | 155 | 180 | 170 |
| WH1-RS2 | 190 | 175 | 140 | 155 | 150 | 285 |
| WH1-RS3 | 190 | 150 | 190 | 355 | 200 | 310 |
| WH2-RS1 | 130 | 175 | 140 | 130 | 175 | 140 |
| WH2-RS2 | 190 | 175 | 140 | 190 | 175 | 140 |
| WH2-RS3 | 190 | 150 | 190 | 190 | 150 | 190 |
| | Wheat | | | Maize | | |
| Periods | 1 | 2 | 3 | 1 | 2 | 3 |
| WH1-RS1 | 175 | 140 | 190 | 130 | 160 | 180 |
| WH1-RS2 | 175 | 140 | 190 | 170 | 130 | 175 |
| WH1-RS3 | 150 | 190 | 170 | 140 | 190 | 175 |
| WH2-RS1 | 180 | 170 | 155 | 130 | 160 | 180 |
| WH2-RS2 | 150 | 130 | 175 | 170 | 155 | 180 |
| WH2-RS3 | 140 | 190 | 175 | 170 | 155 | 150 |

**Table 2:** HOLDING COST(IN INR) OF GRAINS

| | Rice | | | Sorghum | | |
|---|---|---|---|---|---|---|
| Periods | 1 | 2 | 3 | 1 | 2 | 3 |
| WH1-RS1 | 13 | 17 | 14 | 15 | 18 | 17 |
| WH1-RS2 | 19 | 17 | 14 | 15 | 15 | 13 |
| WH1-RS3 | 19 | 15 | 19 | 17 | 14 | 19 |
| WH2-RS1 | 13 | 17 | 14 | 30 | 13 | 16 |
| WH2-RS2 | 19 | 17 | 14 | 18 | 17 | 13 |
| WH2-RS3 | 19 | 15 | 19 | 17 | 14 | 19 |
| | Wheat | | | Maize | | |
| Periods | 1 | 2 | 3 | 1 | 2 | 3 |
| WH1-RS1 | 13 | 16 | 18 | 25 | 30 | 28 |
| WH1-RS2 | 17 | 15 | 18 | 25 | 30 | 13 |
| WH1-RS3 | 17 | 15 | 15 | 16 | 18 | 17 |
| WH2-RS1 | 25 | 30 | 28 | 25 | 45 | 15 |
| WH2-RS2 | 25 | 30 | 13 | 30 | 28 | 25 |
| WH2-RS3 | 16 | 18 | 17 | 30 | 28 | 25 |

**Table 3:** QUANTITY DISCOUNT OF GRAINS

| Rice | | | | Sorghum | | | |
|---|---|---|---|---|---|---|---|
| From WH1 to all RS | | From WH2 to all RS | | From WH1 to all RS | | From WH2 to all RS | |
| Quantity Thresholds | Discount Factor | Quantity Thresholds | Discount Factor | Quantity Thresholds | Discount Factor | Quantity Thresholds | Discount Factor |
| $0 \le X_{ijmt} < 100$ | 1 | $0 \le X_{ijmt} < 150$ | 1 | $0 \le X_{ijmt} < 200$ | 1 | $0 \le X_{ijmt} < 250$ | 1 |
| $100 \le X_{ijmt} < 200$ | 0.90 | $150 \le X_{ijmt} < 250$ | 0.90 | $200 \le X_{ijmt} < 400$ | 0.95 | $250 \le X_{ijmt} < 450$ | 0.95 |
| $200 \le X_{ijmt}$ | 0.80 | $250 \le X_{ijmt}$ | 0.80 | $400 \le X_{ijmt}$ | 0.90 | $450 \le X_{ijmt}$ | 0.90 |

| Wheat | | | | Maize | | | |
|---|---|---|---|---|---|---|---|
| From WH1 to all RS | | From WH1 to all RS | | From WH1 to all RS | | From WH2 to all RS | |
| Quantity Thresholds | Discount Factor | Quantity Thresholds | Discount Factor | Quantity Thresholds | Discount Factor | Quantity Thresholds | Discount Factor |
| $0 \le X_{ijmt} < 50$ | 1 | $0 \le X_{ijmt} < 80$ | 1 | $0 \le X_{ijmt} < 300$ | 1 | $0 \le X_{ijmt} < 350$ | 1 |
| $50 \le X_{ijmt} < 100$ | 0.75 | $80 \le X_{ijmt} < 160$ | 0.75 | $300 \le X_{ijmt} < 600$ | 0.90 | $350 \le X_{ijmt} < 650$ | 0.90 |
| $100 \le X_{ijmt}$ | 0.70 | $160 \le X_{ijmt}$ | 0.70 | $600 \le X_{ijmt}$ | 0.80 | $650 \le X_{ijmt}$ | 0.80 |

**Table 4:** FIXED FREIGHT COST(IN INR) FOR EACH TRUCK

| Periods | 1 | 2 | 3 |
|---|---|---|---|
| WH1-RS1 | 1000 | 1051 | 1100 |
| WH1-RS2 | 1100 | 1000 | 1000 |
| WH1-RS3 | 1000 | 1050 | 1100 |
| WH2-RS1 | 1000 | 1051 | 1100 |
| WH2-RS2 | 1100 | 1000 | 1000 |
| WH2-RS3 | 1000 | 1050 | 1100 |

**Table 5:** DEMAND OF THE RETAIL STORES

| | Rice | | | Sorghum | | |
|---|---|---|---|---|---|---|
| Periods | 1 | 2 | 3 | 1 | 2 | 3 |
| RS1 | 230 | 175 | 140 | 155 | 280 | 170 |
| RS2 | 190 | 275 | 140 | 155 | 250 | 285 |
| RS3 | 190 | 150 | 290 | 355 | 200 | 310 |
| | Wheat | | | Maize | | |
| | 1 | 2 | 3 | 1 | 2 | 3 |
| RS1 | 130 | 175 | 140 | 355 | 200 | 310 |
| RS2 | 190 | 175 | 140 | 155 | 305 | 360 |
| RS3 | 290 | 150 | 190 | 305 | 270 | 360 |

## APPENDIX B

PROCUREMENT-DISTRIBUTION POLICY OF FIRM

| Product | $X_{ijmt}$ | Discount $(1-d_{ijmlt})$ | $I_{ijmt}$ | $\delta_{jmt}$ | $\alpha_{jmt}$ | $y_{jmt}$ | $u_{jmt}$ |
|---|---|---|---|---|---|---|---|
| | | | IN PERIOD 1 FROM WH1 TO RS1 | | | | |
| 1 | 200 | 20% | 100 | 2000 | 1 | 500 | TL |
| 2 | 0 | 0% | 560 | | | | |
| 3 | 100 | 30% | 0 | | | | |
| 4 | 0 | 0% | 0 | | | | |
| | | | In period 2 from WH1 to RS1 | | | | |
| 1 | 0 | 0% | 0 | 4800 | 3 | 300 | TL & LTL |
| 2 | 0 | 0% | 0 | | | | |
| 3 | 350 | 30% | 0 | | | | |
| 4 | 400 | 10% | 0 | | | | |
| | | | In period 3 from WH1 to RS1 | | | | |
| 1 | 0 | 0% | 0 | 400 | 0 | 400 | LTL |
| 2 | 0 | 0% | 0 | | | | |
| 3 | 50 | 25% | 0 | | | | |
| 4 | 0 | 0% | 0 | | | | |
| | | | In period 1 from WH1 to RS2 | | | | |
| 1 | 130 | 10% | 0 | 4500 | 3 | 0 | TL |
| 2 | 310 | 5% | 0 | | | | |
| 3 | 0 | 0% | 0 | | | | |
| 4 | 310 | 10% | 0 | | | | |
| | | | In period 2 from WH1 to RS2 | | | | |
| 1 | 200 | 20% | 0 | 10552 | 7 | 52 | TL & LTL |
| 2 | 500 | 10% | 0 | | | | |
| 3 | 279 | 30% | 340 | | | | |
| 4 | 724 | 20% | 0 | | | | |
| | | | In period 3 from WH1 to RS2 | | | | |
| 1 | 0 | 0% | 0 | 3400 | 2 | 400 | TL & LTL |
| 2 | 0 | 0% | 0 | | | | |
| 3 | 50 | 25% | 195 | | | | |
| 4 | 600 | 20% | 580 | | | | |
| | | | In period 1 from WH1 to RS3 | | | | |
| 1 | 130 | 10% | 0 | 3830 | 2 | 830 | TL |
| 2 | 0 | 0% | 0 | | | | |
| 3 | 0 | 0% | 0 | | | | |
| 4 | 610 | 20% | 0 | | | | |
| | | | In period 2 from WH1 to RS3 | | | | |
| 1 | 0 | 0% | 580 | 800 | 0 | 800 | TL |
| 2 | 0 | 0% | 620 | | | | |
| 3 | 0 | 0% | 0 | | | | |
| 4 | 0 | 0% | 120 | | | | |
| | | | In period 3 from WH1 to RS3 | | | | |
| 1 | 0 | 0% | 0 | 3040 | 2 | 40 | TL & LTL |
| 2 | 0 | 0% | 0 | | | | |
| 3 | 380 | 30% | 0 | | | | |
| 4 | 710 | 20% | 3 | | | | |
| | | | In period 1 from WH2 to RS1 | | | | |
| 1 | 365 | 20% | 0 | 13306 | 8 | 1306 | TL |
| 2 | 898 | 10% | 0 | | | | |
| 3 | 160 | 30% | 0 | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 4 | 0 | 0% | 0 | | | | |
| In period 2 from WH2 to RS1 | | | | | | | |
| 1 | 250 | 20% | 0 | 1500 | 1 | 0 | TL |
| 2 | 0 | 0% | 0 | | | | |
| 3 | 0 | 0% | 0 | | | | |
| 4 | 620 | 10% | 0 | | | | |
| In period 3 from WH2 to RS1 | | | | | | | |
| 1 | 280 | 20% | 0 | 9000 | 6 | 0 | TL |
| 2 | 340 | 5% | 0 | | | | |
| 3 | 230 | 30% | 0 | | | | |
| 4 | 0 | 0% | 0 | | | | |
| In period 1 from WH2 to RS2 | | | | | | | |
| 1 | 250 | 20% | 0 | 4540 | 3 | 40 | TL & LTL |
| 2 | 0 | 0% | 0 | | | | |
| 3 | 380 | 30% | 0 | | | | |
| 4 | 0 | 0% | 0 | | | | |
| In period 2 from WH2 to RS2 | | | | | | | |
| 1 | 350 | 20% | 0 | 5252 | 3 | 752 | TL |
| 2 | 0 | 0% | 0 | | | | |
| 3 | 394 | 30% | 0 | | | | |
| 4 | 0 | 0% | 120 | | | | |
| In period 3 from WH2 to RS2 | | | | | | | |
| 1 | 280 | 20% | 0 | 6310 | 4 | 310 | TL & LTL |
| 2 | 570 | 10% | 0 | | | | |
| 3 | 80 | 25% | 5 | | | | |
| 4 | 0 | 0% | 0 | | | | |
| In period 1 from WH2 to RS3 | | | | | | | |
| 1 | 250 | 20% | 0 | 11446 | 7 | 946 | TL |
| 2 | 710 | 10% | 0 | | | | |
| 3 | 622 | 30% | 40 | | | | |
| 4 | 0 | 0% | 0 | | | | |
| In period 2 from WH2 to RS3 | | | | | | | |
| 1 | 851 | 20% | 0 | 16579 | 11 | 79 | TL & LTL |
| 2 | 989 | 10% | 0 | | | | |
| 3 | 160 | 30% | 0 | | | | |
| 4 | 654 | 20% | 0 | | | | |
| In period 3 from WH2 to RS3 | | | | | | | |
| 1 | 0 | 0% | 0 | 3285 | 2 | 285 | TL & LTL |
| 2 | 0 | 0% | 0 | | | | |
| 3 | 0 | 0% | 0 | | | | |
| 4 | 657 | 20% | 57 | | | | |