

## ECONOMIC PRODUCTION QUANTITY CONCERNING LEARNING AND THE REWORKING OF IMPERFECT ITEMS

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**Abstract:** The classical economic production quantity (EPQ) model assumes that items produced are of perfect quality and the production rate is constant. However, production quality depends on the condition of the process. Due to process deterioration or other factors, the production process may shift and produce imperfect quality items. These imperfect quality items sometimes can be reworked and repaired; hence, overall production-inventory costs can be reduced significantly. In addition, it can be found in practice that the time or cost required to repetitively produce a unit of a product decreases when the number of units produced by a worker or a group of workers increases. Under this circumstance, the unit production cost cannot be regarded as constant and, therefore, cannot be ignored when taking account of the total cost. This paper incorporates the effects of learning and the reworking of defective items on the EPQ model since they were not considered in existing models. An optimal operation policy that minimizes the expected total cost per unit time is derived. A numerical example is provided to illustrate the proposed model. In addition, sensitivity analysis is performed and discussed.

**Keywords:** Rework, imperfect quality, learning effect, economic production quantity.

**MSC:** 90B05, 90B30.

## 1. INTRODUCTION

The classical EPQ/EOQ (economic production/order quantity) model has been widely used in practice because of its simplicity (Osteryoung *et al.*, 1986; Silver *et al.*, 1988; Zipkin, 2000). Over the past five decades, numerous research efforts have been undertaken to extend the basic EPQ model by relaxing various assumptions so that the model conforms more closely to real-world situations. Two major assumptions in the classical EPQ/EOQ model are that the output of the production facility is of perfect quality and that the production rate is predetermined and fixed in advance. However, in reality, product quality is not always perfect, but instead a function of the reliability of the production process employed to manufacture the products. Due to deterioration of process, non-perfect technology, human mistakes, or many other factors, generation of imperfect quality items is inevitable. Many researchers have extended the classical EPQ model by considering the effect of imperfect production processes (for example, Cheng, 1991; Freimer *et al.*, 2006; Hou, 2007; Jaber, 2006; Khouja and Mehrez, 1994; Kim and Hong, 1999; Lee and Rosenblatt, 1987; Rosenblatt and Lee, 1986). In practical production environments, the imperfect quality items sometimes can be reworked and repaired; hence, overall production-inventory costs can be reduced significantly (Chiu, 2003; Chiu *et al.*, 2007; Hayek and Salameh, 2001; Sarker *et al.*, 2008). Examples of rework include the paper industry, the semiconductor industry, the glass industry, the metal processing industry, and the plastic industry (Barketau, *et al.*, 2008; Buscher and Lindner, 2007; Chiu *et al.*, 2007). A few studies have been accomplished to address the EPQ model with rework. Porteus (1986) incorporated the effect of defective items into the classical EPQ model. These defective items can be reworked in the same cycle. He found that the optimal EPQ was smaller than the EPQ in the classical model, because smaller lots produce fewer defective items. Hayek and Salameh (2001) derived an optimal operating policy for the finite production model under the effect of reworking imperfect quality items, assuming that all of the defective items were reworked and repaired. Chiu (2003) considered a finite production model with random defective rate, scrapping, the reworking of repairable defective items, and backlogging. An optimal operating policy including lot-size and backordering level that minimizes overall inventory costs was derived. Jamal *et al.* (2004) presented an inventory model to determine the optimum batch quantity in a single-stage system in which rework was done under two different operational policies. Recently, Buscher and Lindner (2007) have presented an approach which allows the simultaneous determination of production as well as rework lot and batch sizes. Other related studies on lot sizing with imperfect production process and allowable rework can be found in Cardenas-Barron (2008), Chiu *et al.* (2007), Inderfurth *et al.* (2005, 2007), Sarker *et al.* (2008) and their references.

Apart from the assumption that all units produced are of good quality, the constant production rate assumption is also not valid whenever the operator begins production of a new product, restarts production after some delay, implements a new production technique, or changes to a new machine. In such situations, the learning effect cannot be ignored. The literature concerning effect of learning on determining optimal lot size can be found as early as the 1960's. Keachie and Fontana (1966) dealt with the effects of learning on the calculation of the optimal economic lot size in intermittent

production. Adler and Nanda (1974a, 1974b) analyzed the effects of learning on optimal lot size determination for the manufacturing cycles of single and multiple products. They derived optimal lot sizes for cases where either lot sizes or production intervals were equal. Salameh *et al.* (1993) presented a modified production-inventory model that incorporates the effect of learning on the inventory system total cost per unit time and on the optimal production quantity. Li and Cheng (1994) considered the effects of learning and forgetting on EPQ in batch production systems, and developed a dynamic programming method to determine optimal lot sizes. Urban (1998) investigated the learning effect of run length on product quality modeling the defect rate of the process as a function of the run length, and closed form solutions were derived. Jaber and Bonney (2003) studied the effects of learning and forgetting in setups and product quality on the economic lot-sizing problem. Chiu and Chen (2005) considered the problem of incorporating both learning and forgetting in setups and production into the dynamic lot-sizing model to obtain an optimal production policy that included the optimal number of production runs and the optimal production quantities during the finite period planning horizon. Jaber and Bonney (2007) investigated the effect of lot-size dependent learning and forgetting rates on the lot-size problem by incorporating the dual-phase learning–forgetting model (DPLFM) developed by Jaber and Kher (2002) into the EPQ model.

Recently, rework activities have attracted considerable attention because of the reduction of the natural resources and the rise of the cost of raw material. Rework activities play an important role in eliminating waste and effectively controlling the cost of manufacturing in a production system. Therefore, determining the optimal lot size in a system that allows rework is a worthwhile objective to minimize the total inventory cost. Besides the foregoing economical motive, rework activities were also supported by a growing environmental consciousness. Rework activities reduce energy use, reduce the need to landfill defective items, and save more natural resources for the future generations, so the companies are contributing to sustainable development. Although many researchers have studied the effect of learning or imperfect production processes on optimal lot size decisions, little attention has been paid to the area of investigating the joint effects of learning and the reworking of defective items on the optimal production quantity. Thus, this study investigates the learning effect of the unit production time for an imperfect production system. We assume that the percentage of the defective items is a random variable with a known probability density function. All of the defective items can be reworked to acceptable quality, and rework time is also considered in the model. Defective items are assumed to be reworked while regular production processes are completed. The expected total cost function is derived and a solution procedure is established to find the optimal lot size. A numerical example is given to illustrate the proposed model. Furthermore, sensitivity analysis is carried out in order to study the impact of different problem parameters on the behavior of the model. Finally, we conclude with a summary of the results and suggestions for future research.

## 2. NOTATIONS AND BASIC ASSUMPTIONS

The notations to be used throughout this paper are defined as follows:

$Q$             Production lot size for each cycle (decision variable);

|                                   |                                                                                            |
|-----------------------------------|--------------------------------------------------------------------------------------------|
| $r$                               | Demand rate in units per unit time;                                                        |
| $T_1$                             | Regular production time;                                                                   |
| $T_2$                             | Reworking time;                                                                            |
| $T_3$                             | Time required to deplete the inventory achieved by the end of time $T_2$ ;                 |
| $T$                               | Cycle time, $T=T_1+T_2+T_3=\frac{Q}{r}$ ;                                                  |
| $C_1$                             | Learning rate in regular production;                                                       |
| $C_2$                             | Learning rate in reworking production;                                                     |
| $b_1$                             | Learning coefficient associated with regular production, $b_1 = \frac{\log C_1}{\log 2}$ ; |
| $b_2$                             | Learning coefficient associated with reworking production,                                 |
| $b_2 = \frac{\log C_2}{\log 2}$ ; |                                                                                            |
| $a_1$                             | Time required to produce the first unit for each cycle;                                    |
| $a_2$                             | Time required to rework the first unit for each cycle;                                     |
| $t_1(x)$                          | Time required to produce the $x$ th unit in the regular production run,                    |
| $t_1(x) = a_1 x^{b_1}$ ;          |                                                                                            |
| $t_2(y)$                          | Time required to rework the $y$ th unit in the reworking production run,                   |
| $t_2(y) = a_2 y^{b_2}$ ;          |                                                                                            |
| $\beta$                           | Percentage of defective items in $Q$ ;                                                     |
| $f(\beta)$                        | Probability density function of $\beta$ ;                                                  |
| $I_1(t_1)$                        | Inventory level of non-defective items at time $t_1$ , $0 \leq t_1 \leq T_1$ ;             |
| $I_2(t_2)$                        | Inventory level of non-defective items at time $t_2$ , $0 \leq t_2 \leq T_2$ ;             |
| $I_{1max}$                        | Maximum inventory level of non-defective items, when the regular production process stops; |
| $I_{2max}$                        | Maximum inventory level of non-defective items, when the rework process stops;             |
| $C_s$                             | Setup cost for each production run;                                                        |
| $C_{L1}$                          | Labor production cost per unit time (inspection cost is included);                         |
| $C_{L2}$                          | Repair cost of imperfect quality items per unit time;                                      |
| $C_{h1}$                          | Holding cost for each perfect item (i.e., serviceable item) per unit time;                 |
| $C_{h2}$                          | Holding cost for each imperfect quality item being reworked per unit time;                 |
| $TC(Q)$                           | Total cost for each cycle;                                                                 |
| $TCU(Q)$                          | Total cost per unit time.                                                                  |

The following assumptions are made:

1. Only one product is considered in a single-stage production system.
2. Wright's formulation (1936) of the learning effect is utilized to characterize the learning phenomenon for the unit production time.
3. Throughout our present work, we assume  $-1 < b_1 \leq 0$  and  $-1 < b_2 \leq 0$ , since, for most practical situations, the value of learning rate is greater than 50% (Argote and Epple, 1990; Camm, 1985; Elmaghraby, 1990; Jaber and Guiffreda, 2004; Li and Cheng, 1994; Muth and Spremann, 1983).

4. The demand rate is constant.
5. All of the defective items can be reworked to acceptable quality. Moreover, no defective occurs during the rework process because of careful operation and special attention. The unit reworking (or repairing) cost is proportional to the unit reworking time.
6. The inventory holding costs include all produced items (i.e. defective and non-defective items). The holding cost of non-defective items is greater than or equal to that of defective items (i.e.  $C_{h1} \geq C_{h2}$ ).
7. All items are screened while producing, and the inspection cost is included in the labor production cost.
8. A fixed setup cost is charged for each cycle.
9. The production rate of perfect quality items must always be greater than or equal to the sum of the demand rate and the generation rate of defective items. Moreover, the rework rate must always be greater than the demand rate.
10. No shortages or stockouts are allowed.

### 3. MODEL FORMULATION

Consider a production system where the effect of learning is evident. A single product manufactured in batches will be produced at an increasing rate, but it will be consumed at a constant rate  $r$ , units per unit time. The production cycle begins with zero-inventory and starts at time  $t = 0$ . A batch quantity  $Q$  is produced for  $T_1$  time units. Because the production quality is not perfect, a percentage ' $\beta$ ' of imperfect quality is assumed to occur during the regular production process ( $T_1$ ). The amount of defective items produced in each cycle is  $\beta Q$ . The rework of these defective items is subsequently done for  $T_2$  time units, when the regular production process  $T_1$  ends. After the rework is completed, the production is terminated. From this point on, the on-hand inventories will be used to meet the demand. Another production run will be started when all on-hand inventory are depleted. The behaviors of the on-hand inventory level of non-defective items, and defective items are illustrated in Figures 1 and 2, respectively.

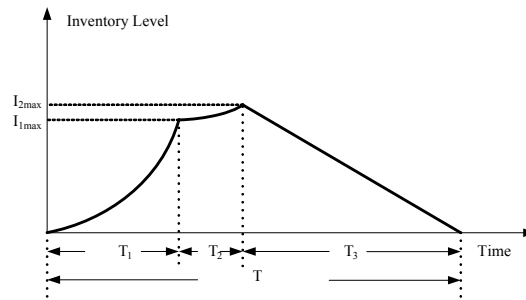


Figure 1. On-hand inventory level of non-defective items.

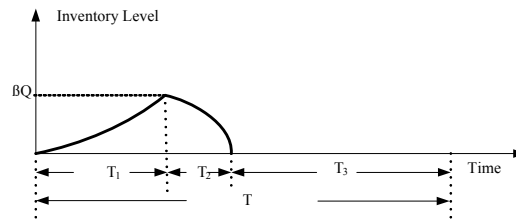


Figure 2. On-hand inventory level of defective items.

The total cost for the imperfect process discussed in this paper includes production setup cost ( $SC(Q)$ ), inventory holding cost for the non-defective items ( $HC_1(Q)$ ), inventory holding cost for the imperfect quality items being reworked ( $HC_2(Q)$ ), production cost ( $PC(Q)$ ), and reworking cost ( $RC(Q)$ ). The formulations of these costs are described in detail as follows.

### 3.1. Setup cost

Setup cost for each cycle is

$$SC(Q) = C_s. \quad (1)$$

### 3.2. Holding cost for the non-defective items

Under the assumptions and notations presented in the previous section, the cumulative time to produce  $Q$  units in regular production run,  $T_1$ , can be expressed as

$$\begin{aligned} T_1 &= t_1(1) + t_1(2) + t_1(3) + \dots + t_1(Q) \\ &= a_1 + a_1 2^{b_1} + a_1 3^{b_1} + \dots + a_1 Q^{b_1} \\ &= a_1 (1 + 2^{b_1} + 3^{b_1} + \dots + Q^{b_1}) \\ &= a_1 \sum_{x=1}^Q x^{b_1}. \end{aligned} \quad (2)$$

An approximation can be obtained by treating Eq. (2) as a continuous function rather than a discrete one (Camm *et al.*, 2004; Li and Cheng, 1994; Salameh *et al.*, 1993; Jaber and Guiffreda, 2004; Smunt and Meredith, 2000). With suitable limits, we have

$$\begin{aligned} T_1 &\approx \int_0^Q a_1 x^{b_1} dx \\ &= \frac{a_1 Q^{b_1+1}}{b_1+1}. \end{aligned} \quad (3)$$

Thus, the expression of  $Q$  can be found as

$$Q = \left[ \frac{(b_1+1)}{a_1} T_1 \right]^{\frac{1}{b_1+1}}. \quad (4)$$

For  $0 \leq t_I \leq T_1$ , the inventory level of non-defective items at time  $t_I$ ,  $I_I(t_I)$ , can be computed as

$$I_I(t_I) = (1-\beta) \left[ \frac{(b_1+1)}{a_1} t_I \right]^{\frac{1}{b_1+1}} - r t_I. \quad (5)$$

At time  $t_I = T_1$ , the inventory level of  $I_{I\max}$  can be easily obtained that

$$I_{I\max} = (1-\beta)Q - rT_1 \quad (6)$$

During the regular production run, the average inventory level of non-defective items in each cycle,  $AIL_I$ , can be computed as

$$\begin{aligned} AIL_I &= \int_0^{T_1} \left[ (1-\beta) \left( \frac{b_1+1}{a_1} t_I \right)^{\frac{1}{b_1+1}} - r t_I \right] dt_I \\ &= (1-\beta) \left( \frac{b_1+1}{a_1} \right)^{\frac{1}{b_1+1}} \frac{b_1+1}{b_1+2} T_1^{\frac{b_1+2}{b_1+1}} - \frac{r T_1^2}{2} \end{aligned} \quad (7)$$

Substituting  $\frac{a_1 Q^{b_1+1}}{b_1+1}$  for  $T_1$  in the first term of Eq. (7), we have

$$\begin{aligned} AIL_I &= (1-\beta) \left( \frac{b_1+1}{a_1} \right)^{\frac{1}{b_1+1}} \frac{b_1+1}{b_1+2} \left( \frac{a_1}{b_1+1} \right)^{\frac{b_1+2}{b_1+1}} Q^{b_1+2} - \frac{r T_1^2}{2} \\ &= (1-\beta) \left( \frac{a_1}{b_1+2} \right) Q^{b_1+2} - \frac{r T_1^2}{2} \end{aligned} \quad (8)$$

During time interval of reworking production (i.e.  $0 \leq t_2 \leq T_2$ ), the inventory level of non-defective items at time  $t_2$ ,  $I_2(t_2)$ , can be computed as

$$I_2(t_2) = (1-\beta)Q - rT_1 + \left[ \frac{(b_2+1)}{a_2} t_2 \right]^{\frac{1}{b_2+1}} - rt_2. \quad (9)$$

At time  $t_2 = T_2$ , the inventory level of  $I_{2max}$  can be easily obtained that

$$I_{2max} = Q - r(T_1 + T_2) \quad (10)$$

Since the amount of defective items that must be reworked in each cycle is  $\beta Q$ , the cumulative time to rework  $\beta Q$  units in  $T_2$  can be calculated as

$$\begin{aligned} T_2 &= \frac{a_2(\beta Q)^{b_2+1}}{b_2+1} \\ &= \frac{a_2 Q^{b_2+1} \beta^{b_2+1}}{b_2+1} \end{aligned} \quad (11)$$

During the reworking production run, the average inventory level of non-defective items in each cycle,  $AIL_2$ , can be computed as

$$\begin{aligned} AIL_2 &= \int_0^{T_2} [(1-\beta)Q - rT_1 + \left( \frac{b_2+1}{a_2} t_2 \right)^{\frac{1}{b_2+1}} - rt_2] dt_2 \\ &= (1-\beta)QT_2 - rT_1T_2 + \left( \frac{b_2+1}{a_2} \right)^{\frac{1}{b_2+1}} \frac{b_2+1}{b_2+2} T_2^{\frac{b_2+2}{b_2+1}} - \frac{rT_2^2}{2} \end{aligned} \quad (12)$$

Substituting  $\frac{a_2 Q^{b_2+1} \beta^{b_2+1}}{b_2+1}$  for  $T_2$  in the third term of Eq. (12), we have

$$AIL_2 = (1-\beta)QT_2 - rT_1T_2 + \left( \frac{a_2}{b_2+2} \right) \beta^{b_2+2} Q^{b_2+2} - \frac{rT_2^2}{2} \quad (13)$$

From Fig. 1,  $T_3$  can be computed as

$$T_3 = \frac{I_{2max}}{r} = \frac{Q - r(T_1 + T_2)}{r} \quad (14)$$

During time  $T_3$ , the average inventory level of non-defective items in each cycle,  $AIL_3$ , can be computed as

$$AIL_3 = \frac{[Q - r(T_1 + T_2)]^2}{2r} \quad (15)$$

The holding cost ( $HC_I(Q)$ ) of the non-defective items for each cycle is the total inventory level of non-defective items multiplied by the unit holding cost, and is given as  $HC_I(Q) = C_{hI} (AIL_1 + AIL_2 + AIL_3)$

$$\begin{aligned}
&= C_{hl} \left\{ (1-\beta) \left( \frac{a_1}{b_1+2} \right) Q^{b_1+2} - \frac{rT_1^2}{2} + (1-\beta)QT_2 - rT_1T_2 + \right. \\
&\quad \left. \left( \frac{a_2}{b_2+2} \right) \beta^{b_2+2} Q^{b_2+2} - \frac{rT_2^2}{2} + \frac{[Q - r(T_1 + T_2)]^2}{2r} \right\} \\
&= C_{hl} \left\{ (1-\beta) \left( \frac{a_1}{b_1+2} \right) Q^{b_1+2} - \frac{rT_1^2}{2} + (1-\beta)QT_2 - rT_1T_2 + \right. \\
&\quad \left. \left( \frac{a_2}{b_2+2} \right) \beta^{b_2+2} Q^{b_2+2} - \frac{rT_2^2}{2} + \right. \\
&\quad \left. \frac{Q^2}{2r} - Q(T_1 + T_2) + \frac{r(T_1 + T_2)^2}{2} \right\}
\end{aligned} \tag{16}$$

After some manipulations, Eq. (16) can be reduced to

$$HC_I(Q) = C_{hl} \left[ \frac{Q^2}{2r} + a_1 Q^{b_1+2} \left( \frac{1-\beta}{b_1+2} - \frac{1}{b_1+1} \right) - \frac{a_2 \beta^{b_2+2} Q^{b_2+2}}{(b_2+1)(b_2+2)} \right] \tag{17}$$

### 3.3. Holding cost for the imperfect quality items being reworked

During time interval of regular production (i.e.  $0 \leq t_1 \leq T_1$ ), the inventory level of imperfect quality items at time  $t_1$ ,  $I_4(t_1)$ , can be computed as

$$I_4(t_1) = \beta \left( \frac{b_1+1}{a_1} t_1 \right)^{\frac{1}{b_1+1}} \tag{18}$$

In each cycle, the average inventory level of imperfect quality items during the regular production run,  $AII_4$ , can be obtained as

$$\begin{aligned}
AII_4 &= \int_0^{T_1} \beta \left( \frac{b_1+1}{a_1} t_1 \right)^{\frac{1}{b_1+1}} dt_1 \\
&= \beta \left( \frac{a_1}{b_1+2} \right) Q^{b_1+2}
\end{aligned} \tag{19}$$

During time interval of reworking production (i.e.  $0 \leq t_2 \leq T_2$ ), the inventory level of imperfect quality items at time  $t_2$ ,  $I_5(t_2)$ , can be computed as

$$I_5(t_2) = \beta Q - \left( \frac{b_2 + 1}{a_2} t_2 \right)^{\frac{1}{b_2 + 1}} \quad (20)$$

In each cycle, the average inventory level of imperfect quality items during the reworking production run,  $AIL_5$ , can be obtained as

$$\begin{aligned} AIL_5 &= \int_0^{T_2} \left[ \beta Q - \left( \frac{b_2 + 1}{a_2} t_2 \right)^{\frac{1}{b_2 + 1}} \right] dt_2 \\ &= \beta Q T_2 - \frac{a_2}{b_2 + 2} (\beta Q)^{b_2 + 2} \end{aligned} \quad (21)$$

The total inventory cost of imperfect quality items for each cycle can be expressed as follows:

$$\begin{aligned} HC_2(Q) &= C_{h2} (AIL_4 + AIL_5) \\ &= C_{h2} \left\{ \beta \left( \frac{a_1}{b_1 + 2} \right) Q^{b_1 + 2} + \beta Q T_2 - \frac{a_2}{b_2 + 2} (\beta Q)^{b_2 + 2} \right\} \end{aligned} \quad (22)$$

Similarly, after some manipulations, Eq. (22) can be reduced to

$$HC_2(Q) = C_{h2} \left[ \frac{a_1 \beta Q^{b_1 + 2}}{b_1 + 2} + \frac{a_2 \beta^{b_2 + 2} Q^{b_2 + 2}}{(b_2 + 1)(b_2 + 2)} \right] \quad (23)$$

### 3.4 Production and reworking costs

The production cost ( $PC(Q)$ ) and reworking cost ( $RC(Q)$ ) per cycle are calculated as follows:

$$PC(Q) + RC(Q) = C_{L1} T_1 + C_{L2} T_2 = C_{L1} \frac{a_1 Q^{b_1 + 1}}{b_1 + 1} + C_{L2} \frac{a_2 Q^{b_2 + 1} \beta^{b_2 + 1}}{b_2 + 1} \quad (24)$$

### 3.5 Total cost

Summing the setup cost, the inventory holding cost for the non-defective items, the inventory holding cost for the imperfect quality items being reworked, the production cost, and the reworking cost, the total cost for each cycle can be obtained as

$$\begin{aligned} TC(Q) &= C_s + C_{h1} \left[ \frac{Q^2}{2r} + a_1 Q^{b_1 + 2} \left( \frac{1 - \beta}{b_1 + 2} - \frac{1}{b_1 + 1} \right) - \frac{a_2 \beta^{b_2 + 2} Q^{b_2 + 2}}{(b_2 + 1)(b_2 + 2)} \right] + \\ &C_{h2} \left[ \frac{a_1 \beta Q^{b_1 + 2}}{b_1 + 2} + \frac{a_2 \beta^{b_2 + 2} Q^{b_2 + 2}}{(b_2 + 1)(b_2 + 2)} \right] + C_{L1} \frac{a_1 Q^{b_1 + 1}}{b_1 + 1} + \\ &C_{L2} \frac{a_2 Q^{b_2 + 1} \beta^{b_2 + 1}}{b_2 + 1} \end{aligned} \quad (25)$$

The total cost per unit time,  $TCU(Q)$ , is determined by  $TC(Q)/T$ . Since the cycle time  $T=Q/r$ , one has

$$TCU(Q) = C_s \frac{r}{Q} + C_{h1} \left[ \frac{Q}{2} + a_1 r Q^{b_1+1} \left( \frac{1-\beta}{b_1+2} - \frac{1}{b_1+1} \right) - \frac{a_2 r \beta^{b_2+2} Q^{b_2+1}}{(b_2+1)(b_2+2)} \right] +$$

$$C_{h2} \left[ \frac{a_1 r \beta Q^{b_1+1}}{b_1+2} + \frac{a_2 r \beta^{b_2+2} Q^{b_2+1}}{(b_2+1)(b_2+2)} \right] + C_{L1} \frac{a_1 r Q^{b_1}}{b_1+1} +$$

$$C_{L2} \frac{a_2 r Q^{b_2} \beta^{b_2+1}}{b_2+1} \quad (26)$$

The expected value of  $TCU(Q)$  is  $E[TCU(Q)]$ , where

$$E[TCU(Q)] = C_s \frac{r}{Q} + C_{h1} \left[ \frac{Q}{2} + a_1 r Q^{b_1+1} \left( \frac{1-E(\beta)}{b_1+2} - \frac{1}{b_1+1} \right) - \frac{a_2 r Q^{b_2+1} E(\beta^{b_2+2})}{(b_2+1)(b_2+2)} \right] +$$

$$C_{h2} \left[ \frac{a_1 r Q^{b_1+1} E(\beta)}{b_1+2} + \frac{a_2 r Q^{b_2+1} E(\beta^{b_2+2})}{(b_2+1)(b_2+2)} \right] +$$

$$C_{L1} \frac{a_1 r Q^{b_1}}{b_1+1} + C_{L2} \frac{a_2 r Q^{b_2} E(\beta^{b_2+1})}{b_2+1} \quad (27)$$

Our objective is to minimize the expected total cost per unit time. Thus, by taking the first derivative of  $E[TCU(Q)]$  with respect to  $Q$  and setting the result to zero, one has

$$\frac{dE[TCU(Q)]}{dQ} = -C_s \frac{r}{Q^2} +$$

$$C_{h1} \left[ \frac{1}{2} + a_1 r (b_1+1) Q^{b_1} \left( \frac{1-E(\beta)}{b_1+2} - \frac{1}{b_1+1} \right) - \frac{a_2 r Q^{b_2} E(\beta^{b_2+2})}{(b_2+2)} \right] +$$

$$C_{h2} \left[ \frac{a_1 (b_1+1) r Q^{b_1} E(\beta)}{b_1+2} + \frac{a_2 r Q^{b_2} E(\beta^{b_2+2})}{(b_2+2)} \right] +$$

$$C_{L1} \frac{a_1 b_1 r Q^{b_1-1}}{b_1+1} + C_{L2} \frac{a_2 b_2 r Q^{b_2-1} E(\beta^{b_2+1})}{b_2+1} = 0 \quad (28)$$

Taking the second derivative, one has

$$\begin{aligned}
\frac{d^2 E[TCU(Q)]}{dQ^2} &= 2C_s \frac{r}{Q^3} + C_{h1} \left[ \frac{a_1 b_1 r (b_1 + 1) Q^{b_1 - 1} \left( \frac{1 - E(\beta)}{b_1 + 2} - \frac{1}{b_1 + 1} \right)}{-\frac{a_2 b_2 r Q^{b_2 - 1} E(\beta^{b_2 + 2})}{(b_2 + 2)}} \right] + \\
&C_{h2} \left[ \frac{a_1 b_1 r (b_1 + 1) Q^{b_1 - 1} E(\beta)}{b_1 + 2} + \frac{a_2 b_2 r Q^{b_2 - 1} E(\beta^{b_2 + 2})}{(b_2 + 2)} \right] + \\
&C_{L1} \frac{a_1 b_1 (b_1 - 1) r Q^{b_1 - 2}}{b_1 + 1} + C_{L2} \frac{a_2 b_2 (b_2 - 1) r Q^{b_2 - 2} E(\beta^{b_2 + 1})}{b_2 + 1} \\
&= 2 \frac{r}{Q^3} C_s + \frac{a_1 b_1 (b_1 + 1) r Q^{b_1 - 1} E(\beta)}{b_1 + 2} (C_{h2} - C_{h1}) + \\
&\frac{a_2 b_2 r Q^{b_2 - 1} E(\beta^{b_2 + 2})}{b_2 + 2} (C_{h2} - C_{h1}) + \\
&\frac{-a_1 b_1 (b_1 + 1) r Q^{b_1 - 1}}{(b_1 + 1)(b_1 + 2)} C_{h1} + \frac{a_1 b_1 (b_1 - 1) r Q^{b_1 - 2}}{b_1 + 1} C_{L1} + \\
&\frac{a_2 b_2 (b_2 - 1) r Q^{b_2 - 2} E(\beta^{b_2 + 1})}{b_2 + 1} C_{L2} > 0
\end{aligned} \tag{29}$$

The result of Equation (29) is positive, because  $0 \leq b_1 < -1$ ,  $0 \leq b_2 < -1$ , and  $Ch_1 \geq Ch_2$ . Hence,  $E[TCU(Q)]$  is a strictly convex function for all values of  $Q$ .  $E[TCU(Q)]$  is therefore a unimodal function with its minimum at  $Q=Q^*$ , where  $\frac{dE[TCU(Q^*)]}{dQ^*} = 0$ .

Although the optimal lot size  $Q^*$  cannot be expressed in a closed form, it can be obtained through the use of numerical methods. We propose the following simple search procedure to find the optimal lot size.

*Step1: Let*

$$\begin{aligned}
f(q) &= -C_s \frac{r}{q^2} + C_{h1} \times \left[ \frac{1}{2} + a_1 r (b_1 + 1) q^{b_1} \left( \frac{1 - E(\beta)}{b_1 + 2} - \frac{1}{b_1 + 1} \right) - \frac{a_2 r q^{b_2} E(\beta^{b_2 + 2})}{(b_2 + 2)} \right] + \\
&C_{h2} \left[ \frac{a_1 (b_1 + 1) r q^{b_1} E(\beta)}{b_1 + 2} + \frac{a_2 r q^{b_2} E(\beta^{b_2 + 2})}{(b_2 + 2)} \right] + C_{L1} \frac{a_1 b_1 r q^{b_1 - 1}}{b_1 + 1} + \\
&C_{L2} \frac{a_2 b_2 r q^{b_2 - 1} E(\beta^{b_2 + 1})}{b_2 + 1}.
\end{aligned}$$

Set a pre-specified relative error tolerance,  $\varepsilon$ , and  $\varepsilon > 0$ . Choose  $q_\ell$  and  $q_u$  as two guesses for the root such that  $f(q_\ell) f(q_u) < 0$ .

$$\text{Step 2: Set } q_{opt} = \frac{q_\ell + q_u}{2}.$$

Step 3: Check the following

- a. If  $f(q_\ell)f(q_{opt}) < 0$ , set  $q_u = q_{opt}$ . Then go to Step 4.
- b. If  $f(q_\ell)f(q_{opt}) > 0$ , set  $q_\ell = q_{opt}$ . Then go to Step 4.
- c. If  $f(q_\ell)f(q_{opt}) = 0$ ; then the  $Q^*$  is  $q_{opt}$ . Stop the algorithm.

Step 4: Find the new estimate of the root  $q_{opt} = \frac{q_\ell + q_u}{2}$ . Calculate the absolute approximate relative error ( $\varepsilon_a$ ) as

$$\varepsilon_a = \left| \frac{q_{opt}^{new} - q_{opt}^{old}}{q_{opt}^{new}} \right| \times 100\%$$

where

$q_{opt}^{new}$  = estimated root from present iteration

$q_{opt}^{old}$  = estimated root from previous iteration.

Step 5: If  $\varepsilon_a > \varepsilon$ , then go to Step 3. Otherwise, the root is  $q_{opt}$ , and the algorithm is stopped.

#### 4. MODEL VERIFICATION

Suppose that no defective items are produced, i.e.  $\beta=0$ . Then Eq. (28) becomes

$$\frac{dE[TCU(Q)]}{dQ} = -C_s \frac{r}{Q^2} + C_{hl} \left( \frac{1}{2} - \frac{ra_1 Q^{b_1}}{(b_1 + 2)} \right) + C_{Ll} r \left( \frac{a_1 b_1 Q^{b_1-1}}{b_1 + 1} \right) = 0. \quad (30)$$

This yields the same result as derived by Salameh *et al.* (1993). Therefore, the result described in Salameh *et al.* (1993) is a special case of our model.

Further, suppose that all of the defective items produced can be reworked and 100% repaired, but without considering the learning effects and the holding cost of the imperfect quality items (i.e.,  $b_1=0$ ,  $b_2=0$ , and  $C_{h2}=0$ ). Eq. (28) is reduced to the following equation:

$$\frac{dE[TCU(Q)]}{dQ} = -C_s \frac{r}{Q^2} + C_{hl} \left[ \frac{1}{2} - a_1 r \frac{1 + E(\beta)}{2} - \frac{a_2 r E(\beta^2)}{2} \right] = 0 \quad (31)$$

Let  $a_1 = a_2 = \frac{1}{p}$ , and solving Eq. (31). The optimal value of  $Q^*$  is obtained as

$$Q^* = \sqrt{\frac{2prC_s}{Ch_1[p - (1 + E(\beta) + E(\beta^2))r]}} \quad (32)$$

Eq. (32) becomes the same result of immediate rework process model as derived by Jamal *et al.* (2004). Therefore, the result described in Jamal *et al.* (2004) is also a special case of our model.

Furthermore, suppose that all items produced in the regular production run are of perfect quality and without considering the learning effects, i.e.  $b_1=0$ ,  $b_2=0$ , and  $\beta=0$ , Eq. (28) is reduced to the following equation:

$$\frac{dE[TCU(Q)]}{dQ} = -C_s \frac{r}{Q^2} + C_h \left( \frac{1 - ra_1}{2} \right) = 0. \quad (33)$$

Hence, the EPQ can be found from  $\frac{dE[TCU(Q)]}{dQ} = 0$ , which yields

$$Q_{trad}^* = \sqrt{\frac{2C_s r}{C_{h1} \left( 1 - \frac{r}{p} \right)}} \quad (34)$$

This is the same equation as that given by the traditional EPQ model, where  $p$  is the production rate, and  $p = \frac{1}{a_1}$ .

## 5. NUMERICAL EXAMPLE

A manufactured product has a constant demand rate of 60 units/day. The setup cost is \$20000 per production run. The holding costs for the non-defective and defective items are \$20/unit/day and \$8/unit/day, respectively. The labor production cost and rework cost are \$1000/day and \$400/day, respectively. The production rate of defective items is uniformly distributed over the interval  $[0, 0.4]$ . The time to produce the first unit ( $a_1$ ) is 0.01 day, and the time to rework the first unit ( $a_2$ ) is 0.008 day. The learning rates in the regular production run and in the rework production run are 94% and 91%, respectively. That is,

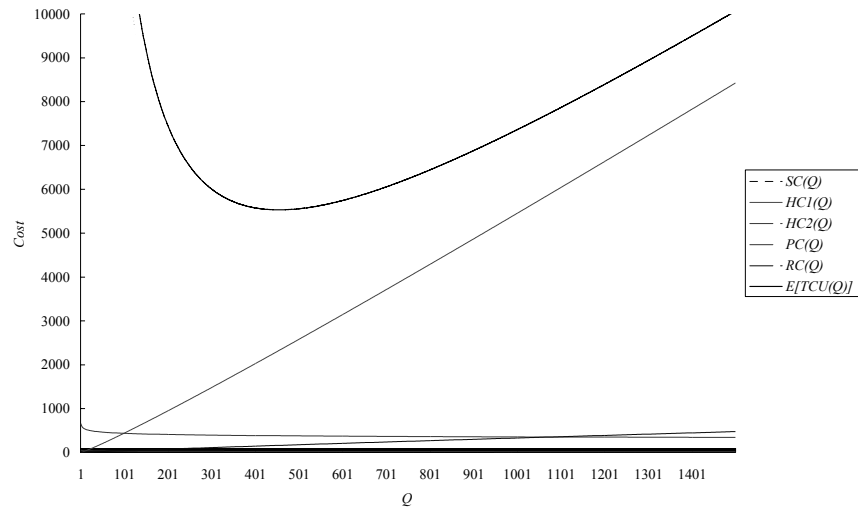
$$\begin{aligned} r &= 60 \text{ units/day}, C_s = \$20000, C_{h1} = \$20/\text{unit/day}, C_{h2} = \$8/\text{unit/day}, C_{L1} = \$1000/\text{day}, \\ C_{L2} &= \$400/\text{day}, \beta = \text{uniformly distributed over the interval } [0, 0.4], a_1 = 0.01 \text{ day}, \\ a_2 &= 0.008 \text{ day}, \alpha_1 = 94\%, \alpha_2 = 91\%. \end{aligned}$$

Because  $\beta$  is uniformly distributed over the range  $[0, 0.4]$ , then the probability density function  $f(\beta)$  is

$$f(\beta) = \begin{cases} 2.5 & \text{for } 0 \leq \beta \leq 0.4 \\ 0 & \text{otherwise} \end{cases}$$

Therefore,  $E(\beta)=0.2$ ,  $E(\beta^{b_2+1})=0.2431$ , and  $E(\beta^{b_2+2})=0.06329$ .

By applying the proposed search procedure to solve this example, the convexity of the expected cost function is displayed in Figure 3, and the optimal solutions are summarized in Table 1. In addition, to see the effects of learning and imperfect production process, we also list the results of the Salameh *et al.* (1993) model, and the traditional EPQ model, respectively, in the same table. Let  $Q_o^*$  and  $Q_s^*$  denote the EPQ values obtained by applying our model and the Salameh *et al.* (1993) model, respectively. It can be seen from Table 1 that  $Q_s^* < Q_o^* < Q_{trad}^*$ . It can be found that the effects of learning and imperfect production process are evident when comparing  $Q_o^*$  and  $Q_{trad}^*$ . This comparison reveals a reduction in the optimal production quantity of about 16.97%  $[(1 - \frac{455}{548}) \times 100\%]$ . Furthermore, it can also be found that the proposed EPQ model has a larger expected total cost than that of the traditional EPQ model. From these comparisons, we note that if the effects of learning and imperfect production process are ignored in the problem formulation, then it may lead to a significant impact on the results of the proposed problem. In Table 1, it can also be found that the proposed EPQ model has a larger  $Q^*$  than does the EPQ model without considering the reworking of defective items (i.e. the Salameh *et al.* (1993) model) in order to ensure that the restriction of full demand satisfaction is always met.



**Figure 3.** Convexity of the cost function  $E[TCU(Q)]$ .

**Table 1:** Results for the numerical example

|                                                             | $Q^*$            | $E[TCU(Q^*)]$        | $T_1$  | $T_2$  | $T_3$  | $T$    |
|-------------------------------------------------------------|------------------|----------------------|--------|--------|--------|--------|
| Our model                                                   | 455 <sup>#</sup> | 5532.11              | 2.8930 | 0.4561 | 4.2342 | 7.5833 |
| Salameh <i>et al.</i> model<br>(i.e., $\beta=0$ )           | 437              | 5747.56 <sup>†</sup> | 2.7886 | 0      | 4.4948 | 7.2833 |
| Classical EPQ model<br>(i.e., $b_1=b_2=0$ , and $\beta=0$ ) | 548              | 4981.78 <sup>†</sup> | 5.4800 | 0      | 3.6533 | 9.1333 |

<sup>#</sup>  $Q^*$  is rounded down to the integer with the minimum total cost.

<sup>†</sup> Without including the reworking cost.

Using the Wright's learning curve equation, the time required to produce the first unit in the second cycle, unit number 456, can be calculated as

$$a_1 = 0.01 (456)^{\frac{\log 0.94}{\log 2}} = 0.0058 \text{ (days)}.$$

Similarly, the time required to rework the first unit ( $a_2$ ) in the second cycle is found to be 0.0043 days. The values of  $a_1$  and  $a_2$  are then used to determine the optimal quantity for cycle 2. This procedure is continued for all cycles, from 1 through 10. Table 2 shows the economic production quantities,  $Q^*$ , and the cycle time,  $T$ , for all cycles. The tabulated results indicate that the EPQ reaches a plateau of 389 units. In addition, the  $T$  values decrease with each additional cycle the same way the optimal production quantities decrease. The decreases in the optimal production quantity and cycle time are more drastic between the first and the second cycle than in the later ones. It can also be realized that the effects of learning and imperfect production process are most evident in the early stages of production, rather than at later stages.

**Table 2:** Summary of the optimal solutions for cycles 1-10

| Cycle | $Q^*$ | $T$    |
|-------|-------|--------|
| 1     | 455   | 7.5833 |
| 2     | 399   | 6.6500 |
| 3     | 396   | 6.6000 |
| 4     | 394   | 6.5667 |
| 5     | 392   | 6.5333 |
| 6     | 391   | 6.5167 |
| 7     | 390   | 6.5000 |
| 8     | 390   | 6.5000 |
| 9     | 389   | 6.4833 |
| 10    | 389   | 6.4833 |

## 6. SENSITIVITY ANALYSIS

In order to explore the effects of learning and imperfect production process on the EPQ, we extend a wide range of values for the important problem parameters:

$C_s=8000, 14000, 20000, 26000, 32000$ ;

$C_{hl}=8, 14, 20, 26, 32$ ;

$C_{LI}=400, 700, 1000, 1300, 1600$ ;

$\alpha_l=90\%, 92\%, 94\%, 96\%, 98\%$ ;

$r=40, 50, 60, 70, 80$ .

$E(\beta)=0, 0.1, 0.2, 0.3, 0.4$ .

The remaining parameters are assigned the same values as presented in the previous section.

Accounting for learning and imperfect quality items alters the manufactured lot size. The percentage change in the EPQ,  $PCQ$ , is given as

$$PCQ = \left( \frac{Q_{trad.}^* - Q_o^*}{Q_{trad.}^*} \right) \times 100\% \quad (35)$$

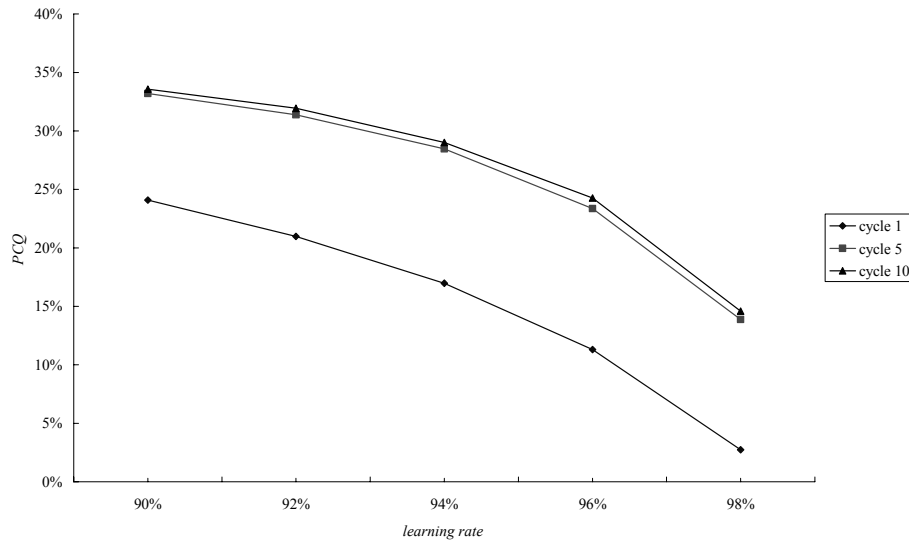
to demonstrate the difference between  $Q_o^*$  and  $Q_{trad.}^*$ .

Figures 4-9 show the graphs when the values of  $PCQ$  are plotted against those of important parameters. The numerical results are summarized in Table 3. They are explained as follows:

1. Figure 4 and Table 3 show that the  $PCQ$  increases substantially with the decrease of the value of learning rates  $\alpha_l$ . For example, in cycle 1, as  $\alpha_l$  goes from 98% to 90%, the  $PCQ$  increases from 2.74% to 24.09%. This phenomenon implies that once the learning effect had occurred, the EPQ

drops rapidly. The above results reveal again that the effect of learning cannot be ignored. From Figure 4 and Table 3, it can also be found that if  $\alpha_l$  is fixed, then the  $PCQ$  increases as the production system proceeds. The main reason for this is the unit production time decrease as the number of units produced increases because of learning.

2. Figure 5 shows the behavior of  $PCQ$  for different demand rates,  $r$ . When demand rates increase, the values of  $PCQ$  increases substantially. The  $PCQ$  is highly sensitive to changes in  $r$ . For instance, in cycle 1, as  $r$  goes from 40 to 80, the  $PCQ$  increases drastically from 7.95% to 33.67%.
3. Figures 6 and 7 indicate that the  $PCQ$  decreases with increased values of  $E(\beta)$  or  $C_{hl}$ . It can also be found that the effects of  $E(\beta)$  and  $C_{hl}$  on the  $PCQ$  are most evident at the early stages of production (i.e. cycle 1). This reveals that production quality and the holding cost of the non-defective items have a considerable impact on optimal production lot size.
4. Figures 8 and 9 show that changing  $C_s$  or  $C_{LI}$  has little effect on the  $PCQ$  in the same production cycle. However, the  $PCQ$  value remains stable for higher levels.
5. In Table 3, most of the  $PCQ$  values are more than 15%. This reveals the effects of learning and imperfect production process on the lot size quantity are significant.
6. In summary, the  $PCQ$  is highly sensitive to changes in  $r$  and  $\alpha_l$ , whereas it is insensitive to changes in  $C_s$  and  $C_{LI}$ .



**Figure 4.** The behavior of  $PCQ$  with respect to learning rate  $\alpha_l$ .

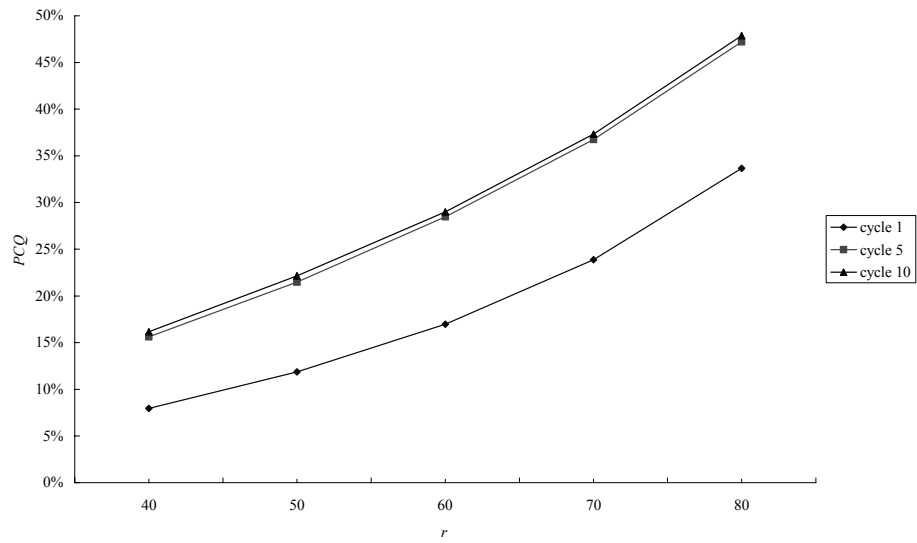


Figure 5. The behavior of  $PCQ$  with respect to demand rate  $r$ .

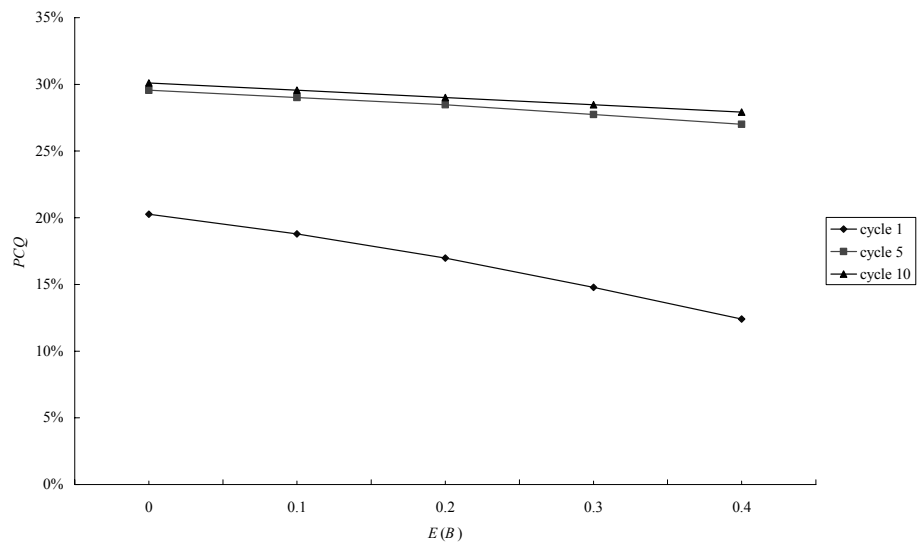
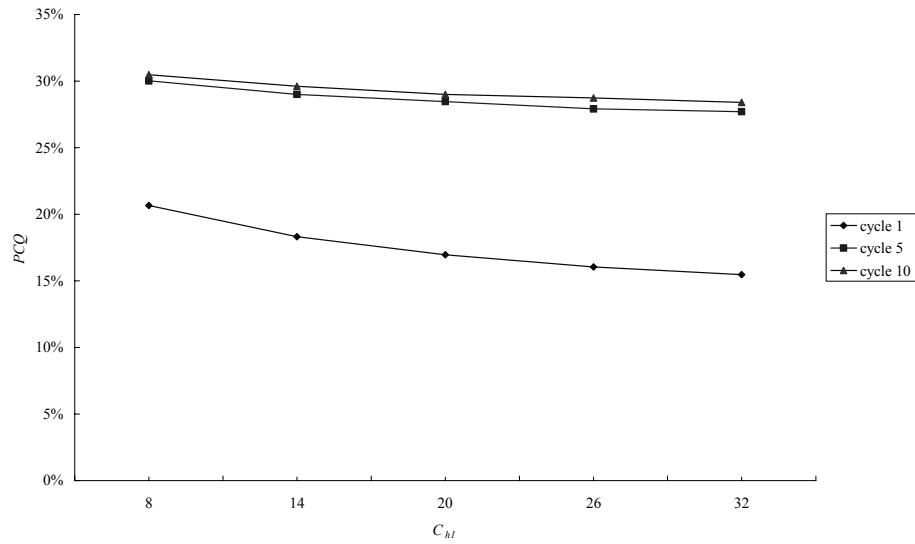
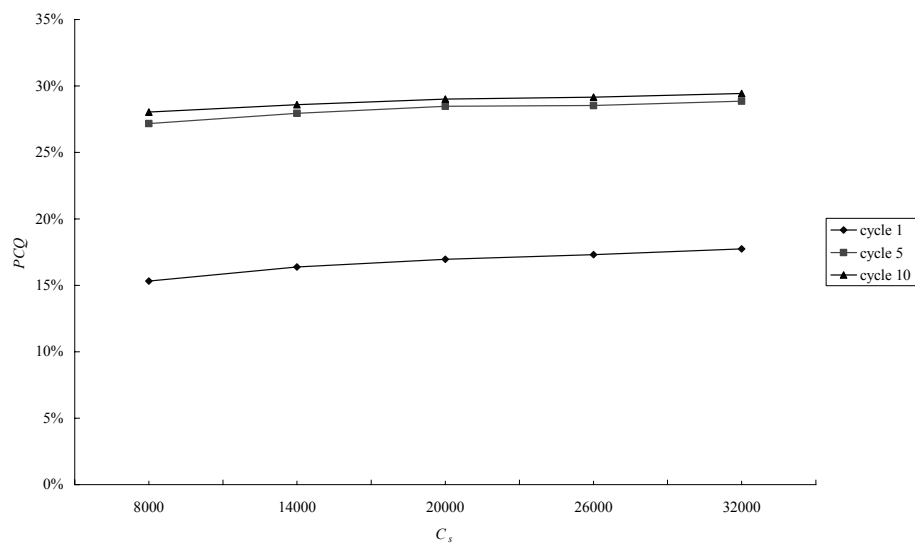


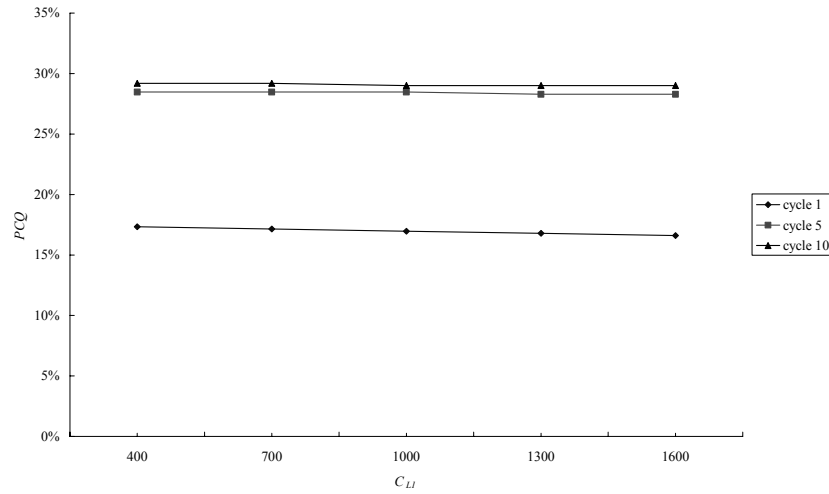
Figure 6. The behavior of  $PCQ$  with respect to expected defect rate  $E(\beta)$ .



**Figure 7.** The behavior of  $PCQ$  with respect to holding cost  $C_{hI}$ .



**Figure 8.** The behavior of  $PCQ$  with respect to setup cost  $C_s$ .



**Figure 9.** The behavior of  $PCQ$  with respect to labor cost  $C_{LI}$ .

**Table 3:** The  $PCQ$  values for various values of  $a_I$ ,  $r$ ,  $E(\beta)$ ,  $C_{hl}$ ,  $C_S$ , and  $C_{LI}$

| $a_I =$      | 90%    | 92%    | 94%    | 96%    | 98%    |
|--------------|--------|--------|--------|--------|--------|
| cycle 1      | 24.09% | 20.99% | 16.97% | 11.31% | 2.74%  |
| cycle 5      | 33.21% | 31.39% | 28.47% | 23.36% | 13.87% |
| cycle 10     | 33.58% | 31.93% | 29.01% | 24.27% | 14.60% |
| $r =$        | 40     | 50     | 60     | 70     | 80     |
| cycle 1      | 7.95%  | 11.86% | 16.97% | 23.87% | 33.67% |
| cycle 5      | 15.62% | 21.48% | 28.47% | 36.75% | 47.20% |
| cycle 10     | 16.16% | 22.15% | 29.01% | 37.34% | 47.87% |
| $E(\beta) =$ | 0      | 0.1    | 0.2    | 0.3    | 0.4    |
| cycle 1      | 20.26% | 18.80% | 16.97% | 14.78% | 12.41% |
| cycle 5      | 29.56% | 29.01% | 28.47% | 27.74% | 27.01% |
| cycle 10     | 30.11% | 29.56% | 29.01% | 28.47% | 27.92% |
| $C_{hl} =$   | 8      | 14     | 20     | 26     | 32     |
| cycle 1      | 20.67% | 18.32% | 16.97% | 16.04% | 15.47% |
| cycle 5      | 30.02% | 29.01% | 28.47% | 27.92% | 27.71% |
| cycle 10     | 30.48% | 29.62% | 29.01% | 28.75% | 28.41% |
| $C_S =$      | 8000   | 14000  | 20000  | 26000  | 32000  |
| cycle 1      | 15.32% | 16.38% | 16.97% | 17.31% | 17.75% |
| cycle 5      | 27.17% | 27.95% | 28.47% | 28.53% | 28.86% |
| cycle 10     | 28.03% | 28.60% | 29.01% | 29.17% | 29.44% |
| $C_{LI} =$   | 400    | 700    | 1000   | 1300   | 1600   |
| cycle 1      | 17.34% | 17.15% | 16.97% | 16.79% | 16.61% |
| cycle 5      | 28.47% | 28.47% | 28.47% | 28.28% | 28.28% |
| cycle 10     | 29.20% | 29.20% | 29.01% | 29.01% | 29.01% |

## 7. CONCLUSION

Reworking of defective products has received increasing attention over the last decade. Not only a growing environmental concern and enforced legislation in many countries but also economic incentives are driving factors behind this development. The classical EPQ model is not appropriate when produced lots have some defective items and the effects of learning are considered. Therefore, new models are required for more realistic solutions in real-life problems. In this paper, we study the effects of learning and the reworking of imperfect quality items produced on the production-inventory model. The unit production time decreases with the increase of total units produced as a result of learning. When regular production stops, all defective items are assumed to be reworked. An optimal production policy that minimizes the expected total cost per unit time for the production model is derived. We have shown that the traditional EPQ model, the Salameh *et al.* (1993) model, and the immediate rework process model proposed by Jamal *et al.* (2004) are special cases of our model. A numerical example has been used to illustrate the proposed methodology. The results indicate that ignoring the effects of learning and imperfect production process may result in production lot-size decisions with high percentage errors. It is also found that, in the presence of learning and imperfect production process, the proposed EPQ model has a smaller production lot size and a larger expected total cost than that of the traditional EPQ model. In addition, the effects of learning and imperfect production process on the lot size quantity are more influential than that on the expected total cost. Finally, a sensitivity analysis is performed to study the impact of different problem parameters on the behavior of the model. It is found that the difference between  $Q_o^*$  and  $Q_{no\ learning}^*$  is highly sensitive to changes in  $r$  and  $\alpha_l$ , whereas it is insensitive to changes in  $C_s$  and  $C_{LI}$ .

Our model can provide guidelines for managerial decisions in actual manufacturing situations and can easily be extended to a situation where some of the defectives are not reworkable. Another extension could be to investigate the effect of forgetting on the proposed model.

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