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QUANTILE ESTIMATION FOR THE GENERALIZED PARETO DISTRIBUTION WITH APPLICATION TO FINANCE

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Abstract: Generalized Pareto distributions (GPD) are widely used for modeling excesses over high thresholds (within the framework of the POT-approach to modeling extremes). The aim of the paper is to give the review of the classical techniques for estimating GPD quantiles, and to apply these methods in finance - to estimate the Value-at-Risk (*VaR*) parameter, and discuss certain difficulties related to this subject.

Keywords: Generalized Pareto distributions, excesses over high thresholds, quantiles of the distribution, value at risk.

MSC: 62P20.

1. INTRODUCTION

The two-parameter generalized Pareto distribution with the shape parameter γ and the scale parameter σ (denoted GPD (γ, σ)) is the distribution of the random variable $X = \sigma\left(1-e^{-\gamma Y}\right)/\gamma$ where Y is a random variable with the standard exponential distribution. GPD (γ, σ) has the distribution function

$$F_{\gamma,\sigma}(x) = \begin{cases} 1-\left(1-\dfrac{\gamma x}{\sigma}\right)^{\frac{1}{\gamma}}, & \gamma \neq 0, \quad \sigma > 0, \\ 1-\exp\left(-\dfrac{x}{\sigma}\right), & \gamma = 0, \quad \sigma > 0, \end{cases} \tag{1.1}$$

where

$$0 \le x < \infty, \gamma \le 0,$$

$$0 \le x \le \frac{\sigma}{\gamma}, \gamma > 0.$$

A number of important and commonly used probability distributions belong to GPD family:

1. For $\gamma = 0$, GPD reduces to the exponential distribution with mean σ.
2. For $\gamma = 1$, GPD reduces to the uniform $U\,[0, \sigma]$ distribution.
3. For $\gamma < 0$, GPD reduces to the Pareto distribution with parameter γ.

Generalized Pareto distributions were introduced by Pickands [20]. These are the only continuous distribution functions that are stable with respect to excess over threshold operations (POT-stable). Precisely, if a random variable X has a GPD (γ, σ) distribution, then the conditional distribution of X - u given $X > u$ is GPD (γ, σ-γu). This property has a key role in the POT-approach to modeling extremes.

POT-approach consists of fitting the GPD to the distribution of the excesses over a sufficiently high threshold, i.e. to the conditional distribution of X - u given $X > u$, when u tends to the right endpoint of the support of the distribution. This type of approximation is justified by the Theorem Balkema-de Haan (1974) [2].

Important issues concerning fitting the GPD to the data, that still attract a considerable attention, are estimating unknown parameters (shape or scale parameter, or both of them) and estimating quantiles of the distribution [1, 3, 5, 6, 10, 11, 13, 17, 19]. A recent review of the subject is given in [7].

The aim of this paper is to compare three different methods for estimating parameters and quantiles of the two-parameter GPD. These are: method of moments (MOM), method of probability weighted moments (PWM), and EPM (elemental percentile method). The three methods are compared in terms of their bias, root mean squared error and robustness, using a large number of simulated data sets.

GPD model, within the framework of POT-approach, has numerous applications in hydrology [5, 21], insurance, finance [8, 18, 21], waiting time problems [12], ecology climatology [5, 14, 16], and other fields. An example of applying the POT-approach to finance (estimating *VaR*-parameter) is given in this paper, with special emphasis to certain difficulties related to the problem.

The paper is organized as follows: definitions and main properties of the three estimation methods are given in Section 2, simulation results are given in Section 3, and the case study - an example of applying these methods to finance is given in Section 4. All computations in this work are performed in MATLAB.

2. ESTIMATING GPD PARAMETERS

The most traditional GPD estimation methods are MOM, PWM and maximum likelihood method (ML). In the last couple of decades, several modifications of these methods were developed, as well as some new procedures, mostly computationally intensive. Recent reviews of this problem are given in [7] and [17].

The present paper deals with the two simplest methods, MOM and PWM, and with the procedure developed by Castillo and Hadi [3]. The study is focused on small samples, so the ML method is not considered (it has been demonstrated in [11] that it is less reliable than the other estimation methods for samples of the size smaller than 500).

2.1. Method of moments

Let $x_1, x_2,\dots x_n$ be a random sample from GPD (γ, σ) and let x and s^2 be the sample mean and sample variance, respectively. Then the *method-of-moments* (MOM) estimates for the parameters γ and σ are given by

$$\gamma_{MOM} = \frac{1}{2}\left(\frac{x^2}{s^2} - 1\right), \quad \sigma_{MOM} = \frac{1}{2}x\left(\frac{x^2}{s^2} + 1\right). \tag{2.1}$$

MOM estimates are defined in [11]. According to [3] and [11], they are recommended in cases $0 < \gamma < 0.4$. Since they are very easy to compute, MOM estimates can also be used as the initial estimates in other estimation procedures (which require numerical techniques).

2.2. Method of probability weighted moments

Let $x_{1:n} \le x_{2:n} \le \cdots \le x_{n:n}$ be a sorted random sample from GPD (γ, σ) and let

$$\alpha_v = \frac{1}{n}\sum_{j=1}^{n}\left(1 - p_{j:n}\right)^v x_{j:n} \text{ and } p_{j:n} = \frac{j - 0.35}{n}$$

Then the *probability-weighted-moments* (PWM) estimates for γ and σ are given by

$$\gamma_{PWM} = \frac{\alpha_0}{\alpha_0 - 2\alpha_1} - 2, \quad \sigma_{PWM} = \frac{2\alpha_0\alpha_1}{\alpha_0 - 2\alpha_1} \tag{2.2}$$

PWM estimation method is defined in [10]. According to [3], the method is recommended for $-0.4 < \gamma < 0$.

2.3. Elemental percentile method

Elemental percentile method (EPM) is a procedure for fitting the GPD to data, defined in [3]. It is performed in two stages:

1. calculating certain number of initial estimates for parameters γ and σ;
2. obtaining overall estimates for γ and σ from the initial estimates.

1. Let $x_{1:n} \le x_{2:n} \le \cdots \le x_{n:n}$ be a sorted sample from GPD (γ, σ) and let

$$F_{\gamma,\sigma}\left(x_{i:n}\right) = p_{i:n}, \quad F_{\gamma,\sigma}\left(x_{j:n}\right) = p_{j:n}, \quad p_{i:n} = \frac{i - \alpha}{n + \beta}. \tag{2.3}$$

According to [3], the best results are obtained for $\alpha = 0$ and $\beta = 1$.

Putting $\frac{\sigma}{\gamma}=\delta$ in (2.3) and eliminating γ from the system yields

$$C_i \log\left(1-\frac{x_{j:n}}{\delta}\right)-C_j \log\left(1-\frac{x_{i:n}}{\delta}\right)=0, \tag{2.4}$$

where $C_i=\log\left(1-p_{i:n}\right)<0$.

It follows from the Theorem 1 in [3], that the equation (2.4), which is a function of only one variable δ, has a finite solution $\delta\ (i, j)$ that can be found using the bisection method. Substituting $\delta\ (i, j)$ into (2.3) gives estimates $\gamma\ (i, j)$ and $\sigma\ (i, j)$ for γ and σ.

2. When the estimates $\gamma\ (i, j)$ and $\sigma\ (i, j)$ are obtained for all possible combinations of indices (i, j), then the overall γ and σ estimates are given by

$$\begin{aligned}\gamma_{EPM}&=median\left(\gamma(1,2),\gamma(1,3),...,\gamma(n-1,n)\right),\\ \sigma_{EPM}&=median\left(\sigma(1,2),\sigma(1,3),...,\sigma(n-1,n)\right),\end{aligned} \tag{2.5}$$

with median $(a_1, a_2, ..., a_n)$ being the median of $\{a_1, a_2, ..., a_n\}$.
If n is large, the number of distinct pairs (i, j) is large, so as the number of necessary calculations. One possible way to overcome this difficulty is to consider only the cases $i = 1, 2,...\ n-1$ and $j = n$, instead of all possible pairs (i, j). This simplification is also applied in the present work.
Important advantage of the EPM method over the other estimation procedures is that EPM estimates exist for all values of parameters γ and σ. Although the EPM is computationally intensive, the numerical algorithm never shows convergence problems. According to [3], the method is recommended when γ lies outside the range [-0.4, 0.4].

2.4. Estimating GPD quantiles

The problem closely related to fitting the GPD to data is estimating quantiles of the distribution.
Quantiles of the GPD (γ, σ) are given in terms of the parameters by

$$x(F)=\begin{cases}\sigma\left(1-(1-F)^{\gamma}\right)/\gamma, & \gamma\neq 0,\\ -\sigma\log\left(1-F\right), & \gamma=0.\end{cases} \tag{2.6}$$

The problem is then reduced to obtaining estimates for the parameters γ and σ and their substitution into (2.6).

2.5. Estimating tail and quantiles of the distribution with GPD fitted to the excesses

Let $x_1, x_2,...\ x_n$ be a random sample from the distribution with unknown underlying distribution function F. The upper tail of F can be presented in the following form:

$$1-F\left(x+u\right)=\left(1-F^{(u)}\left(x\right)\right)\left(1-F\left(u\right)\right),\ x\geq 0, \tag{2.7}$$

with $F^{(u)}(x)$ being the conditional distribution of the excesses X - u, given $X > u$. Suppose that $F^{(u)}(x)$ can be approximated by GPD (γ, σ), and let N_u be the number of excesses of the threshold u in the given sample. Estimating the first term on the right hand side of (2.7) by $1 - F_{\gamma,\sigma}(x)$ and the second term by N_u/n , one can estimate the tail of F by

$$1 - F(x+u) = \frac{N_u}{n}\left(1 - F_{\gamma,\sigma}(x)\right). \tag{2.8}$$

Inverting the formula (2.8), one obtains the estimate for the p-quantile of F:

$$x(p) = \begin{cases} u + \dfrac{\sigma}{\gamma}\left(1 - \left(\dfrac{n}{N_u}(1-p)\right)^{\gamma}\right), \gamma \neq 0, \\ u - \sigma \log\left(\dfrac{n}{N_u}(1-p)\right), \qquad \gamma = 0. \end{cases} \tag{2.9}$$

For a fixed threshold, the term n/N_u is constant and the estimates (2.6) and (2.9) have similar properties.

3. SIMULATIONS

Robustness is an important issue in statistics, particularly in modeling extremes. Extreme observations are often contaminated (inaccurately measured, truncated in inappropriate way…). A way to deal with this type of problems is to fit a model which is robust, i.e. not affected too much by small departures from model assumptions.

The techniques considered here (MOM, PWM, EPM) are not robust, in general. However, if the focus is on a particular problem of estimating high quantiles (95% and 99% quantiles, that are commonly used in practical applications), results may be satisfactory.

In order to check the properties of the three methods for estimating 95% and 99% quantiles, according to [13] and [19], 1000 samples (sample sizes n = 15, 45) are generated from:

1. GPD (γ, 1), for $\gamma \in \{ -1, -0.6, -0.2, 0, 0.2, 0.6, 1 \}$;

2. GPD (γ, 1) slightly contaminated with GPD with different shape parameter (mixture 0.9 $F_{\gamma,1}$ + 0.1 $F_{2\gamma,1}$), for the same values of γ;

3. GPD (γ, 1) slightly contaminated with GPD with different scale parameter (mixture 0.9 $F_{\gamma,1}$ + 0.1 $F_{\gamma,2}$), for the same values of γ.

Since all three estimation methods are invariant to the value of σ, only the case $\sigma = 1$ is considered (as it was done in [3] and [11]). Results for contaminated samples and $n = 45$ are omitted from the text for space-saving purposes.

3.1. Simulation results

Simulation results summarized in Tables 2 – 5 show the bias and the root mean squared error (RMSE) of the 95% and 99% quantile estimators. Biases and RMSE's of the estimators were scaled by the true values of the quantiles being estimated. The true values of the quantiles are given in the Table 1.

Table 1: The true values of the quantiles being estimated

γ	95% quantile	99% quantile
-1	19.00	99.00
-0.6	8.39	24.75
-0.2	4.10	7.56
0	3.00	4.61
0.2	2.25	3.01
0.6	1.39	1.56
1	0.95	0.99

Table 2: Simulated bias (RMSE) for $n = 15$, no contamination

γ	MOM	PWM	EPM
95% quantile			
-1	1.12 (16)	0.03 (3.9)	16.78 (216.29)
-0.6	0.01 (1.21)	-0.11 (0.57)	2.04 (10.03)
-0.2	-0.08 (0.36)	-0.06 (0.36)	0.47 (1.17)
0	-0.06 (0.28)	-0.04 (0.29)	0.25 (0.59)
0.2	-0.04 (0.22)	-0.03 (0.23)	0.13 (0.24)
0.6	-0.02 (0.15)	-0.01 (0.16)	0.03 (0.14)
1	-0.01 (0.13)	0.01 (0.12)	0 (0.07)
99% quantile			
-1	-0.03 (7.62)	-0.25 (3.64)	5608.99 (127343.78)
-0.6	-0.31 (1.07)	-0.2 (0.83)	45.76 (592.01)
-0.2	-0.16 (0.46)	-0.05 (0.56)	2.1 (7.68)
0	-0.09 (0.36)	-0.01 (0.45)	0.82 (2.18)
0.2	-0.04 (0.29)	0.01 (0.36)	0.37 (0.94)
0.6	0.01 (0.21)	0.04 (0.26)	0.1 (0.29)
1	0.03 (0.18)	0.05 (0.2)	0.03 (0.13)

Table 3: Simulated bias (RMSE) for n = 15, contamination with $F_{2\gamma,1}$

γ	MOM	PWM	EPM
95% quantile			
-1	275.26 (7961.39)	66.05 (1914.74)	6533.39 (194002.23)
-0.6	1.35 (27.59)	0.28 (6.7)	17.16 (344.15)
-0.2	-0.04 (0.41)	-0.03 (0.39)	0.6 (1.71)
0	-0.06 (0.28)	-0.04 (0.29)	0.25 (0.59)
0.2	-0.06 (0.22)	-0.05 (0.23)	0.11 (0.34)
0.6	-0.05 (0.16)	-0.04 (0.16)	0.02 (0.15)
1	-0.03 (0.13)	-0.01 (0.13)	0 (0.08)
99% quantile			
-1	130.79 (3799.35)	61.11 (1779.95)	---------[1]
-0.6	0.83 (23.25)	0.43 (11.01)	8711.38 (254249.93)
-0.2	-0.12 (0.53)	0.01 (0.66)	3.18 (16.54)
0	-0.09 (0.36)	-0.01 (0.45)	0.82 (2.18)
0.2	-0.06 (0.29)	-0.02 (0.35)	0.34 (0.91)
0.6	-0.02 (0.21)	0.01 (0.25)	0.09 (0.3)
1	0.01 (0.18)	0.03 (0.2)	0.04 (0.14)

Table 4: Simulated bias (RMSE) for n = 15, contamination with $F_{\gamma,2}$

γ	MOM	PWM	EPM
95% quantile			
-1	1.31 (16.61)	0.12 (4.05)	18.37 (222.04)
-0.6	0.09 (1.35)	-0.03 (0.61)	2.4 (11.09)
-0.2	0.02 (0.4)	0.04 (0.4)	0.69 (1.51)
0	0.05 (0.33)	0.07 (0.34)	0.46 (0.89)
0.2	0.08 (0.3)	0.1 (0.31)	0.36 (0.66)
0.6	0.15 (0.3)	0.15 (0.29)	0.34 (0.56)
1	0.2 (0.33)	0.19 (0.31)	0.39 (0.59)
99% quantile			
-1	0.06 (7.9)	-0.19 (3.77)	5809.77 (127620.56)
-0.6	-0.24 (1.16)	-0.12 (0.89)	51.42 (611.72)
-0.2	-0.06 (0.5)	0.08 (0.66)	2.93 (10.26)
0	0.05 (0.45)	0.15 (0.58)	1.44 (3.69)
0.2	0.14 (0.45)	0.2 (0.54)	0.95 (2.13)
0.6	0.26 (0.49)	0.27 (0.52)	0.71 (1.33)
1	0.33 (0.53)	0.31 (0.5)	0.7 (1.14)

[1] In this case bias and RMSE are larger than 10^8

Table 5: Simulated bias (RMSE) for n = 45, no contamination

γ	MOM	PWM	EPM
95% quantile			
-1	0.87 (6.32)	-0.16 (0.67)	1.73 (6.36)
-0.6	0 (0.6)	-0.08 (0.31)	0.57 (1.46)
-0.2	-0.04 (0.21)	-0.02 (0.21)	0.18 (0.43)
0	-0.02 (0.16)	-0.01 (0.16)	0.1 (0.25)
0.2	-0.01 (0.12)	-0.01 (0.13)	0.05 (0.15)
0.6	-0.01 (0.08)	0 (0.09)	0.01 (0.06)
1	0 (0.07)	0 (0.07)	0 (0.03)
99% quantile			
-1	-0.12 (3.08)	-0.39 (0.76)	20.77 (185.22)
-0.6	-0.26 (0.6)	-0.13 (0.5)	2.59 (10.26)
-0.2	-0.09 (0.3)	-0.01 (0.35)	0.49 (1.17)
0	-0.03 (0.22)	0 (0.26)	0.23 (0.52)
0.2	-0.01 (0.17)	0.01 (0.21)	0.1 (0.25)
0.6	0.01 (0.12)	0.02 (0.14)	0.02 (0.07)
1	0.01 (0.1)	0.02 (0.11)	0 (0.02)

3.2. Conclusions

Results of the simulated experiments can be summarized as follows:

- Bias and RMSE of both quantile estimates decrease when the sample size increases, which indicates that all of these estimators are consistent. There are a few exceptions in cases $\gamma \in \{-1, -0.6\}$, but it may be due to the random number generation procedure used for simulating GPD data, especially with contamination.
- EPM method performs extremely bad in cases $\gamma \in \{-1, -0.6\}$, which is in agreement with the results reported in [17].
- With a few exceptions, MOM and PWM method have negative bias in cases with no contamination and with contamination of the first type. In cases with the contamination of the second type, bias is positive. EPM method has strong positive bias in all cases (with or without contamination).
- MOM and PWM method perform similarly when estimating both quantiles. This is not the case with EPM method, which shows the worse performance, both in terms of bias and RMSE, when estimating 99% quantile. EPM performs particularly bad in cases $\gamma \in \{-1, -0.6, -0.2\}$.
- For both sample sizes and $\gamma \in \{-1, -0.6\}$, all the three methods (EPM, MOM, PWM) respond much better to contamination of the second type than to the contamination of the first type (bias and RMSE do not change significantly). PWM can be recommended for these cases, since it is less sensitive to contamination and performs better than the other methods for all combinations of parameters and sample sizes.

- For both sample sizes and $\gamma \in \{0.6, 1\}$, all the methods respond much better to contamination of the first type than to the contamination of the second type. EPM method can be recommended if there is some certainty about the value of the scale parameter (i.e. there is no contamination of the second type), in other cases MOM or PWM would be better choices.

- For both sample sizes and $\gamma \in \{-0.2, 0, 0.2\}$, all the three methods perform well under both types of contamination. MOM and PWM can be recommended in these cases since they always outperform the EPM method.

- Results obtained for cases with no contamination are in agreement with the simulation results given in [11] and [17]. Although the simulation study presented in [17] is related to estimating higher GPD quantile (99.9%) behavior of MOM, PWM and EPM methods leads to similar conclusions to the ones presented here.

4. APPLICATION TO FINANCE

4.1. Estimating *VaR* parameter

Banks, insurance companies, financial institutions (in general, anyone who takes part in financial transactions) need to know the approximate level of risk involved in their actions. Commonly used measure for that risk is the Value at Risk parameter (*VaR*). *VaR* is the maximum possible loss of a given speculative asset or a portfolio for a given time period, at a given confidence level. A way to obtain *VaR* estimates, in case of a single asset, according to [18] and [21], is explained below.

Let $I_1, I_2,... I_t$ be the prices of a single speculative asset given at discrete times t = 0, 1, 2... (period is usually day, week or month). Then, return within the period T (relative change of the value of the speculative asset, within the period T) is given by $\left(I_T - I_0\right)/I_0$ Return within the period T = 1 (daily, weekly, monthly return…) can be given in two forms:

$$s_t = \frac{I_t - I_{t-1}}{I_{t-1}}, \tag{4.1}$$

which is known as *arithmetic return*, or

$$r_t = \log I_t - \log I_{t-1}, \tag{4.2}$$

which is known as *log-return*.

These quantities are close to each other if the ratio I_t / I_{t-1} is close to 1, but r_t is more convenient for statistical modeling.

Let I_0 be the market value of a speculative asset at the beginning of the period and $r_1, r_2,...,r_t$ be log-returns within the period T. Then, the value I_T at the end of the period T is

$$I_T = I_0 \exp \sum_{t \le T} r_t, \tag{4.3}$$

The T-day (week, months) loss G_T is expressed in terms of returns as

$$G_T = \sum_{t \le T} (-r_t). \tag{4.4}$$

Let β be the given probability (usually 95% or 99%). It follows that VaR $(T; \beta)$ (the maximal loss within the period T, at the probability β), is given by

$$F_T = P\{G_T \le VaR(T;\beta)\} = \beta, \tag{4.5}$$

which means that VaR $(T; \beta)$ is the β-quantile of the loss distribution, F_T. The next steps are fitting the GPD model to the loss data over a suitable threshold and estimating the β-quantile of the loss distribution using formula (2.9).

A well known difficulty that occurs when estimating *VaR* parameter is the *weekend effect* in financial markets, which is the difference in returns between Mondays and other days of the week. Precisely, stock returns on Monday are often significantly lower than those of the immediately preceding Friday and may be unreliable.

Some of the earliest works devoted to the weekend effect are [9] and [15]. The subject still attracts the attention in economic literature (see, for example, [22]).
The following techniques for dealing with the weekend effect are suggested in [21]:

1. *returns with respect to trading days*: taking the prices for the given trading days and computing the returns;
2. *omitting Monday returns*: omitting the days for which the prices are not recorded, including the consecutive day (Monday);
3. *distributing Monday returns*: the return registered on Monday (after a gap on weekend) is equally distributed over Saturday, Sunday and Monday.

For dealing with this and other calendar effects, it is plausible to fit a robust model.

4.2. Example: estimating *VaR* parameter

As an example for the *VaR* estimation technique, historical stock prices were considered for the company Tigar a. d. Pirot, in the period May 31, 2005 – December 31, 2009. The data set consists of 1157 stock prices at consecutive trading days. The goal was to estimate parameters $VaR(1;0.95)$ and $VaR(1;0.99)$. Data were obtained from www.belex.rs. Daily returns were calculated as the first difference of the logarithm of daily prices. The loss data were obtained by multiplying the daily returns by -1.

The original series of daily prices show a linear trend, while the daily return series seem to be stationary (Figure 1). The threshold value necessary for fitting the GPD model is obtained by visual inspection of the mean excess function of the loss data. It was noticed that this function is approximately linear starting from u = 0.04 (Figure 2), therefore, this value is taken for a threshold (according to [18]).

From POT-stability property follows that if the GPD model is consistent with a set of data for a given threshold u, then it is also consistent with the data for all thresholds $u_1 > u$. This fact is applied in the present work, and threshold values u = 0.04, 0.06, 0.08 are considered as well.

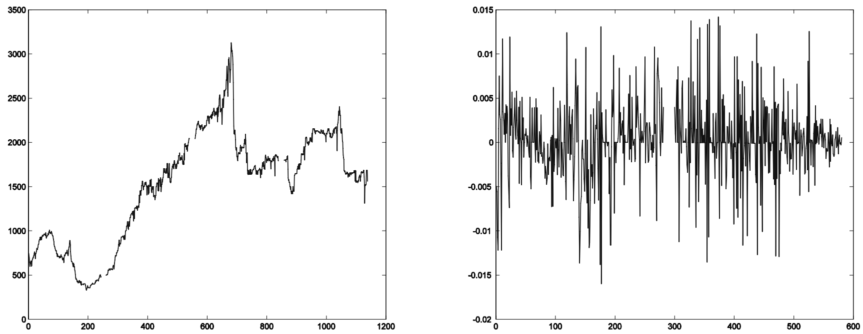

Figure 1: Daily stock prices for the company Tigar a. d. Pirot (original data) and consecutive daily returns (with respect to trading days)

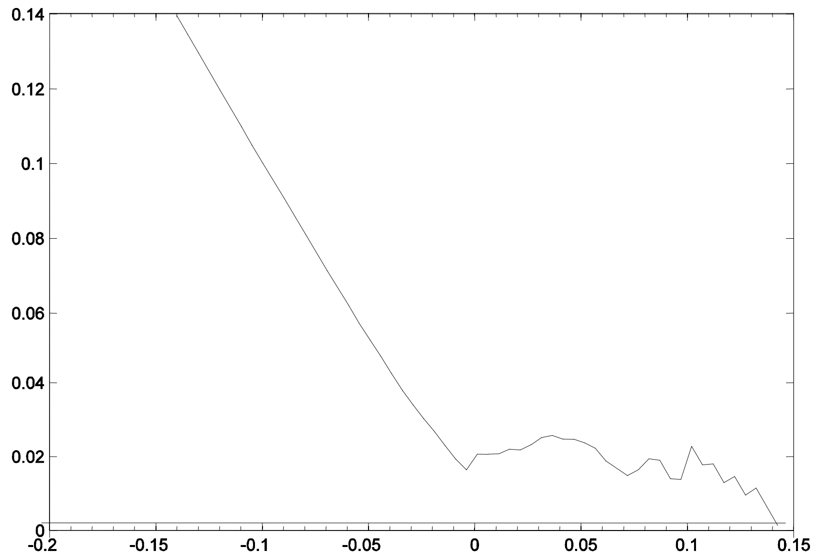

Figure 2: Mean excess function for the loss data (trading days)

The estimates for the $VaR(1;0.95)$ and $VaR(1;0.99)$ obtained using the daily returns, Monday-omitted daily returns and Monday-distributed daily returns are summarized in Tables 6 – 8 (V_1 stands for VaR (1;0.95) estimate, V_2 stands for VaR (1;0.99) estimate).

Table 6: Results obtained for returns with respect to trading days

u	N_u	Method	γ	σ	V_1	V_2
0.04	75	EPM	0.0984	0.0292	0.0475	0.0899
		MOM	0.2448	0.0328	0.0483	0.0893
		PWM	0.2549	0.0331	0.0484	0.0893
0.06	43	EPM	0.0020	0.0220	0.0535	0.0889
		MOM	0.1495	0.0236	0.0529	0.0882
		PWM	0.2380	0.0254	0.0522	0.0887
0.08	14	EPM	0.0431	0.0247	0.0440	0.0848
		MOM	0.2158	0.0256	0.0378	0.0849
		PWM	0.2868	0.0271	0.0329	0.0852

Table 7: Results obtained for Monday-omitted and Monday-distributed returns (γ and σ estimates are the same in both cases)

u	N_u	Method	γ	σ	V_1	V_2	V_1	V_2
					Monday-omitted returns		*Monday-distributed returns*	
0.04	57	EPM	0.0396	0.0275	0.0457	0.0882	0.0304	0.0738
		MOM	0.1922	0.0310	0.0463	0.0876	0.0288	0.0747
		PWM	0.1915	0.0310	0.0463	0.0876	0.0288	0.0747
0.06	32	EPM	-0.0824	0.0205	0.0526	0.0868	0.0417	0.0744
		MOM	0.0993	0.0227	0.0515	0.0866	0.0380	0.0750
		PWM	0.1556	0.0239	0.0509	0.0870	0.0362	0.0755
0.08	11	EPM	-0.1335	0.0214	0.0521	0.0839	0.0426	0.0721
		MOM	0.0572	0.0216	0.0479	0.0839	0.0347	0.0718
		PWM	0.0689	0.0218	0.0473	0.0839	0.0336	0.0717

The results given in Tables 6 – 7 can be interpreted as follows: for example, 0.0475 in the sixth column of Table 6 means that the maximum possible loss that may happen in the period of one day, with the probability greater that 5% is equal to 4.75%.

Table 8: Goodness of fit (fitting the GPD to the excesses over the threshold $u = 0.04$)

Method	Coefficient of determination	Residual sum of squares
Using returns with respect to trading days		
EPM	0.9820	0.1136
MOM	0.9868	0.0858
PWM	0.9869	0.0852
Using Monday-omitted returns		
EPM	0.9851	0.0722
MOM	0.9883	0.0588
PWM	0.9883	0.0588

4.3. Comparison with Belgrade Stock Exchange indices

The reason for creating Belgrade Stock Exchange indices was to improve the information process, transparency and comparison of the market data.

In order to compare risk estimates obtained for the company Tigar a.d. Pirot with the maximal and the average risk for investing on Belgrade Stock Exchange, the following indices were considered:

1. BELEX15: leading index of the Belgrade Stock Exchange (describes the movement of prices for the 15 most liquid Serbian companies, which is calculated in real time);
2. BELEXline: general share index of Belgrade Stock Exchange (calculated at the end of a trading day).

Historical data for both indices in the period May 31, 2005 – December 31, 2009 are obtained from www.belex.rs. Results of risk estimation are summarized in Table 9.

Table 9: Risk estimates for BELEX15 and BELEXline

	$VaR(1;0.95)$ estimate	$VaR(1;0.99)$ estimate
BELEX15	0.0246	0.0484
BELEXline	0.0176	0.0304

Since $VaR(1;0.95)$ and $VaR(1;0.99)$ estimates obtained for the company Tigar a.d. Pirot are approximately equal to 0.048 and 0.09, respectively (estimates obtained for the largest sample), one can conclude that the risk involved in investing in this company's shares is approximately two times greater than the risk for investing in the most liquid shares, and approximately three times greater than the general risk involved in investing on Belgrade Stock Exchange. The information obtained in this way is meant to help the investor to compare liquidity of different companies and make a good decision regarding the investment.

4.4. Conclusions

The results obtained for estimating risk parameters for the company Tigar a. d. Pirot lead to the following conclusions:

- Estimates obtained for $VaR(1;0.95)$ and $VaR(1;0.99)$ using the Monday-omitted returns are not significantly different from those obtained using the returns with respect to trading days. It means that stock prices registered on Mondays were not too different from the prices registered on other trading days, i.e. the weekend effect did not have a significant impact in this situation.
- Estimates obtained for $VaR(1;0.95)$ and $VaR(1;0.99)$ using the Monday-distributed returns are significantly lower than those obtained for returns with respect to all trading days and Monday-omitted returns, although the estimates obtained for parameters γ and σ are exactly the same, and so is the number of excesses. This is due to the fact that sample size is greater in these cases, which affects the formula (2.9).

- It was noticed that the γ estimates change significantly with the sample size (i.e. number of excesses above the threshold). It is true for all the three estimation methods. However, both *VaR* estimates ($VaR(1;0.95)$ and $VaR(1;0.99)$) are still very stable in these cases. The only exceptions are values obtained for the threshold value $u = 0.09$. However, these estimates are obtained from very small samples and are less reliable. It is in agreement with the simulation results given in Section 3.2, precisely, with the fact that for values of γ close to 0, all the three estimation methods show good performance (small bias and RMSE) and react well to the contamination of both types. Table 8 indicates that the goodness of fit of GPD to excesses over the threshold $u = 0.04$ is very good with all the three estimation methods. Therefore, *VaR* estimates obtained with all the three estimation methods, MOM, PWM and EPM, are reliable.
- Another way of checking the adequacy of *VaR* estimates is to compute *failure rate* (the proportion of the number of times the observations exceed the forecasted *VaR* to the number of all observations). *VaR* estimate is adequate if the failure rate is close to pre-specified *VaR* level, 1- β. For this example, failure rates are approximately 0.052 for $VaR(1;0.95)$ estimates and 0.011 for $VaR(1;0.99)$ estimates. Results are satisfactory in both cases.

5. CONCLUDING REMARKS

The purpose of this paper was to compare different methods for estimating 95% and 99% quantiles of the generalized Pareto distribution. It was done through a Monte Carlo simulation study. Results indicate that, in most cases, it is possible to choose beetween MOM, PWM and EPM method in such a way to obtain acceptable estimates, which are not affected too much by small changes in model assumptions.

The three methods are successfully applied to the real data example, estimating *VaR* parameter for the company Tigar a.d. Pirot, taking into account some difficulties characteristic for that type of problems, such as threshold selection and weekend-effect.

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REFERENCES

[1] Ashkar, F., and Tatsambon, C.N., “Revisiting some estimation methods for the generalized Pareto distribution”, *Journal of Hydrology*, 346 (2007) 136-143.

[2] Balkema, A., and de Haan, L., “Residual life time at great age”, *Annals of Probability*, 2 (1974) 792-804.

[3] Castillo, E., and Hadi, A.S., “Fitting the generalized Pareto distribution to data”, *Journal of the American Statistical Association,* 92 (440) (1997) 1609-1620.

[4] Connolly, R.A., “An examination of the robustness of the weekend effect”, *Journal of Financial and Quantitative Analysis*, 24 (2) (1989) 133-169.

[5] Davison, A.C., and Smith, R.L., “Models for exceedances over high thresholds”, *Journal of the Royal Statistical Society*, B 52 (3) (1990) 393-442.

[6] del Castillo, J., and Daoudi, J., "Estimation of the generalized Pareto distribution", *Statistics and Probability Letters,* 79 (2009) 684-688.

[7] de Zea Bermudez, P., and Kotz, S., "Parameter estimation of the generalized Pareto distribution – Parts I and II", *Journal of Statistical Planning and Inference*, 140 (2010) 1353-1388.

[8] Embrechts, P., Kluppelberg, C., and Mikosch, T., *Modelling Extremal Events*, *4th edition,* Springer, Verlag, Berlin, 2003.

[9] French, K.R., "Stock returns and the weekend effect", *Journal of Financial Economics*, 8 (1980) 55-69.

[10] Greenwood, J.A., Landwehr, J.M., Matalas, N.C., and Wallis, R. "Probability weighted moments: definition and relation to parameters of several distributions, expressible in inverse form", *Water Resources Research*, 15 (5) (1979) 1049-1054.

[11] Hosking, J.R.M., and Wallis, J.R., "Parameter and quantile estimation for the generalized Pareto distribution", *Technometrics*, 29 (3) (1987) 339-349.

[12] Jocković, J., and Mladenović, P., "Coupon collector's problem and generalized Pareto distributions", *Journal of Statistical Planning and Inference*, 141 (2011) 2348–2352.

[13] Juárez, S.F., and Schucany, W.R., "Robust and efficient estimation for the generalized Pareto distribution", *EXTREMES*, 7 (2004) 237-251.

[14] Katz, R.W., Brush, G.S., and Parlange, M.B., "Statistics of extremes: Modeling ecological disturbances", *Ecology*, 86 (5) (2005) 1124-1134.

[15] Keim, D. and Stambaugh, R., "A further investigation of the weekend effect in stock returns", *Journal of Finance*, 39 (1984) 819-35.

[16] Kyselý, J., Picek, J., and Beranová, R., "Estimating extremes in climate change simulations using the peaks-over-threshold method with a non-stationary threshold", *Global and Planetary Change*, 72 (2010) 55–68.

[17] Mackay, E.B.L., and Challenor, P.G., "A comparison of estimators for the generalized Pareto distribution", *Ocean Engineering*, 38 (2011) 1338-1346.

[18] Mladenović, Z., and Mladenović, P., "Praktični problemi ocene rizika u analizi dnevnih vremenskih serija", *Zbornik SYM-OP-IS 2007* , 2007, 117-120.

[19] Peng, L., and Welsh, A.H., "Robust estimation of the generalized Pareto distribution", *EXTREMES*, 4 (1) (2001) 53-65.

[20] Pickands, J., "Statistical inference using extreme order statistics", *The Annals of Statistics*, 3 (1975) 119-131.

[21] Reiss, R.-D., and Thomas, M., *Statistical Analysis of Extreme Values From Insurance, Finance, Hydrology and Other Fields*, Birkhauser Basel, 2001.

[22] Ulussever, T., Guran Yumusak, I., and Kar, M, "The Day of the week effect in the Saudi stock exchange: a non linear Garch analysis", *Journal of Economic and Social Studies,* 1 (1) (2011) 10-23.