

TWO ECHELON PARTIAL TRADE CREDIT FINANCING IN A SUPPLY CHAIN DERIVED ALGEBRAICALLY

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Abstract: Trade credit financing has become a powerful tool to improve sales & profit in an industry. In general, a supplier/retailer frequently offers trade credit to its credit risk downstream member in order to stimulate their respective sales. This trade credit may either be full or partial depending upon the past profile of the downstream member. Partial trade credit may be offered by the supplier/retailer to their credit risk downstream member who must pay a portion of the purchase amount at the time of placing an order and then receives a permissible delay on the rest of the outstanding amount to avoid non-payment risks. The present study investigates the retailer's inventory problem under partial trade credit financing for two echelon supply chain where the supplier, as well as the retailer, offers partial trade credit to the subsequent downstream member. An algebraic approach has been applied for finding the retailer's optimal ordering policy under minimizing the annual total relevant cost. Results have been validated with the help of examples followed by comprehensive sensitivity analysis.

Keywords: EOQ, inventory, supply chain, partial trade credit, algebraic method.

MSC: 90B05.

1. INTRODUCTION

Mostly in business transactions, supplier allows a specified credit period to the retailer for the payment without any penalty to stimulate his demand. During this period the retailer can sell the product and earn interest on the revenue generated. A higher interest is charged if the payment is not settled at the end of the credit period offered. This is termed as one echelon trade credit financing. During past few years, many articles

dealing with a range of inventory models under one-echelon trade credit have appeared in various journals. Goyal [4] established a single-item inventory model under permissible delay in payments when selling price equals the purchase cost. Further, Aggarwal and Jaggi [1] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Jamal et.al [18] further generalized the model with shortages. Chung [3] developed an alternative approach to determine the economic order quantity under permissible delay in payments. Teng [20] amended Goyal's [4] model by considering the difference between selling price and purchasing cost.

In all the above mentioned papers, it is assumed that the supplier offers credit period to the retailer but the retailer does not provide it to its downstream supply chain member. In practice, this assumption is quite unrealistic. Huang [7] presented an inventory model assuming that the retailer also offers credit period to his/her customers. This is termed as two-echelon trade credit financing. Huang [7] assumed that the credit period offered by the retailer to his/her customers is shorter than the credit period offered by the supplier. Jaggi et. al [17] incorporated the concept of credit-linked demand for the retailer and determined the optimal credit as well as replenishment policy jointly.

Nowadays, to avoid non-payment risks, a supplier/ retailer frequently offers partial credit period to its credit risk downstream members who must pay a portion of the purchase amount at the time of placing an order and then receives a permissible delay on the rest of the outstanding amount. Huang [12] developed retailer's optimal ordering policy for one echelon partial trade credit where the supplier offers partial credit to the retailer. Further, Huang and Hsu [15] presented an EOQ model where the supplier provides full trade credit to the retailer but the retailer offers partial trade credit to his/her customers. Thangam [22] analyzed partial trade credit in a two echelon supply chain in an EPQ framework. Teng [21] explored optimal ordering policies for a retailer who offers distinct trade credits (i.e. full or partial) to its good and bad credit customers. However, in practice the partial trade credit can be considered in two echelon supply chain where a retailer as well as his customer may pay a portion of the purchase amount initially. The above research articles consider partial trade credit at one level where either a supplier or retailer offers partial credit financing to its subsequent downstream member. The present article incorporates the partial trade credit financing at two levels where a supplier as well as the retailer offers partial trade credit.

Moreover, in all the previous research, the optimal solution is derived by using the differential calculus which needs to satisfy the second order conditions. An alternative approach to finding optimal solution is deriving algebraically, which was first introduced by Grubbstrom [5] for deriving the optimal expressions for the basic EOQ model. Following this approach Grubbstrom and Erdem [6], Cardenas-Barron [2] derived the EOQ and EPQ model with backlogging. This approach provides an educational advantage for explaining the EOQ and EPQ concepts where one is not familiar with the knowledge of derivatives and the procedure to construct and examine Hessian matrix. This method provides an easy and fast solution for the complicated cost functions. Moreover, the method is known to everyone who has studied mathematics in higher classes. Huang [9] adopted the easy algebraic procedures to reinvestigate Goyal's model [4] and Teng's model [20] to find the optimal cycle time under permissible delay in payments. In a subsequent paper, Huang [10] also applied the mentioned algebraic approach to determine the retailer's optimal ordering policy under the condition where

trade credit is linked to order quantity. Huang [13] also used this method to solve the retailer's inventory replenishment problem under two levels of trade credit and limited storage space. Some other related references of algebraic approach are Huang [8,11,14,16] and Manna[19]. The present paper provides an algebraic method to investigate partial trade credit financing in a two echelon supply chain where the supplier as well as the retailer offers partial credit period to its subsequent downstream members. It is assumed that since the retailer offers its customers a permissible delay period N , hence, he will receive its revenue from N to $T + N$ and not from 0 to T . The retailer's inventory system has been developed as a cost minimization problem to determine his optimal ordering policies. A sensitivity analysis on different parameter has been performed.

2. ASSUMPTIONS AND NOTATIONS

The mathematical model proposed in this paper is based on the following assumptions:

1. The present study considers single supplier, single retailer and multiple customers.
2. Shortages are not allowed.
3. Demand rate D , is known and constant.
4. The lead time is zero.
5. Time period is infinite.
6. Both the supplier and the retailer adopt partial trade credit policy. The supplier offers a partial credit of M periods to the retailer, i.e. initially the retailer has to make payment $c(1-\alpha)DT$ to the supplier and rest of the payment will be made at M . It is assumed that the retailer has financed the initial amount from the bank at the rate I_p which will be settled at $T + N$. When $T \geq M$, the account is settled to the supplier at M and the retailer starts paying for the interest charges on the items in stock with rate I_p .
7. The retailer also offers a partial credit of N periods to his customers. The customers have to make an initial payment on β units to the retailer at the time of purchasing and rest of the payment would be made at N . The retailer receives his revenue from N to $T + N$. The retailer earns interest on the β units of every purchases up to T . Also when $N \leq M$, the retailer can accumulate revenue and earn interest on rest of $(1-\beta)$ units at the rate I_e .
8. At the end of permissible delay period M , the retailer pays all of the purchasing cost to the supplier and incurs a capital opportunity cost at a rate of I_p for the items still in stock and for the items sold but have not been paid for, yet.
9. The interest rate charged by the bank is not necessarily higher than the retailer's investment rate.

In addition, following notations are used throughout the paper.

D Annual demand.

A Cost of placing an order.

c Unit purchasing price per item.

p Unit selling price per item, $p \geq c$.

h Unit stock holding cost per year per item excluding interest charge.

I_p Interest charged per \$ investment in inventory per year.

I_e Interest earned per \$ per year.

M Retailer's partial trade credit period offered by the supplier in years.

N Customer's partial trade credit period offered by the retailer in years.

T Cycle time in years.

$1-\alpha$ Percentage of permissible delay in payments for retailer, $0 \leq \alpha \leq 1$.

β Fraction of the purchase cost which the customer must pay the retailer at the time of placing an order, $0 \leq \beta \leq 1$.

$TRC(T)$ Annual total relevant cost, which is a function of $T > 0$.

T^* The optimal cycle time.

Q^* The optimal order quantity $= D T^*$.

3. MODEL ANALYSIS AND DESCRIPTION

The retailer wants to stimulate his sales by offering partial trade credit to his/her customer, while making himself eligible for the same by giving an initial payment on $(1-\alpha)DT$ units to the supplier. From the values of N and M , we have two potential cases: (i) $N \leq M$ and (ii) $N \geq M$.

3.1. Case1. $N \leq M$

The objective is to minimize total annual relevant cost which comprises of following elements:

$TRC(T) =$ a) Annual ordering cost + b) Annual stock holding cost + c) Annual

interest payable -d) Annual interest earned where

$$1) \text{ Annual ordering cost} = \frac{A}{T}.$$

$$2) \text{ Annual stock holding cost (excluding interest charges)} = \frac{hDT}{2}.$$

To calculate annual interest charged and annual interest earned (according to assumption 7 and 8), three sub-cases may arise: Sub-case 3.1.1: $M \leq T$ Sub-case 3.1.2: $T \leq M \leq T + N$ Sub-case 3.1.3: $T + N \leq M$

Sub-case 3.1.1. $M \leq T$

The retailer has to arrange finances from bank to make the initial payment to the supplier, so, an interest is being charged on the initial amount that will be settled at $T + N$. Also, the retailer needs to finance the rest of the inventory, which is divided into two parts viz. (i) all items sold after M for the portion of instant payment, and (ii) all items sold after $(M - N)$ for the portion of credit payment. Hence, the interest paid will be

$$cI_p \left(\frac{(1-\alpha)^2 DT^2}{2} + (1-\alpha)DT(T+N) + \frac{\beta D(T-M)^2}{2} + (1-\beta) \frac{D(T+N-M)^2}{2} \right).$$

Further, the retailer starts selling products at time 0, from which he gets an initial amount of $p\beta DT$ of every unit sold, and the rest of the payment will be received from N up-to M . Consequently, the retailer accumulates revenue in an account that earns interest starting from N through M at the rate I_e per dollar per year. Therefore, interest earned is

$$pI_e \left[\frac{\beta DM^2}{2} + (1-\beta) \frac{D(M-N)^2}{2} \right].$$

Thus, the retailer's annual total relevant cost per unit time is given by,

$$TRC_{11}(T) = \frac{A}{T} + \frac{hDT}{2} + \frac{cI_p}{T} \left(\frac{(1-\alpha)^2 DT^2}{2} + (1-\alpha)D(T+N) + \frac{\beta D(T-M)^2}{2} + (1-\beta) \frac{D(T+N-M)^2}{2} \right) - \frac{pI_e}{T} \left[\frac{\beta DM^2}{2} + (1-\beta) \frac{D(M-N)^2}{2} \right]. \quad (1)$$

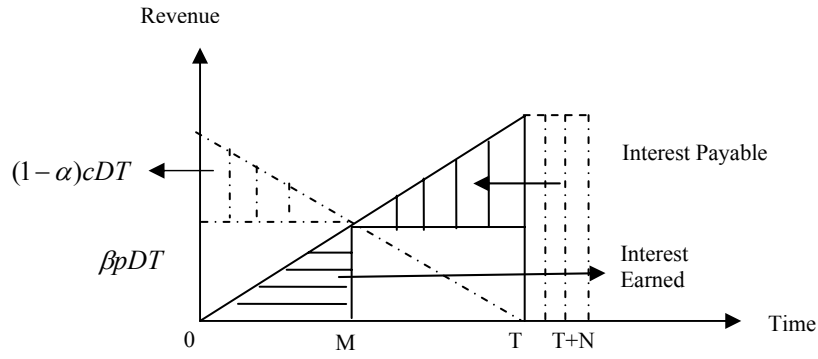


Figure 1. $N \leq M$ and $M \leq T$ (instant payment)

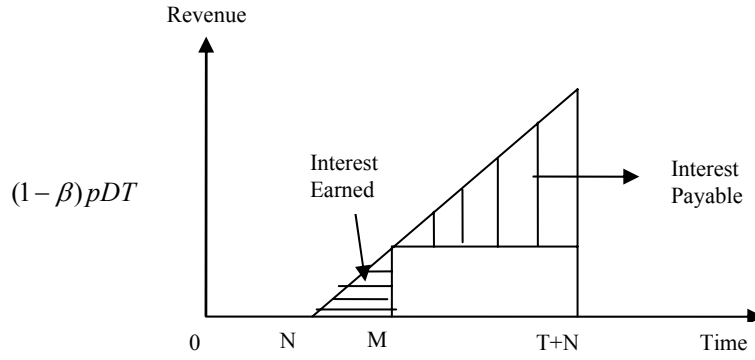


Figure 2. $N \leq M$ and $M \leq T$ (credit payment)

Sub-case 3.1.2. $T \leq M \leq T + N$

Again, the retailer has to finance all items sold after $(M - N)$ and for the initial payment made to the supplier. Consequently, the interest paid will be

$$cI_p \left(\frac{(1-\alpha)^2 DT^2}{2} + (1-\alpha)D(T+N) + \frac{(1-\beta)D(T+N-M)^2}{2} \right).$$

The retailer can accumulate revenue from two resources, i.e. (i) from the portion of instant payment in the interval $[0, M]$ and (ii) from the credit payment in the interval $[N, M]$. Therefore, the interest earned will be

$$pI_e \left(\frac{\beta DT^2}{2} + \beta DT(M-T) + \frac{(1-\beta)D(M-N)^2}{2} \right).$$

Hence, the retailer's annual total relevant cost per unit time is given by

$$TRC_{12}(T) = \frac{A}{T} + \frac{hDT}{2} + \frac{cI_p}{T} \left(\frac{(1-\alpha)^2 DT^2}{2} + (1-\alpha)D(T+N) + \frac{(1-\beta)D(T+N-M)^2}{2} \right) - \frac{pI_e}{T} \left(\frac{\beta DT^2}{2} + \beta DT(M-T) + \frac{(1-\beta)D(M-N)^2}{2} \right). \quad (2)$$

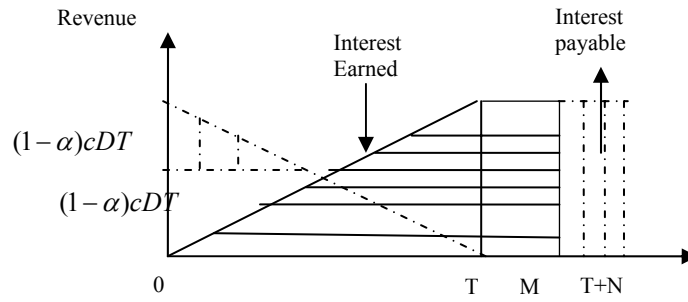


Figure 3. $N \leq M$ and $T \leq M \leq T+N$ (instant payment)

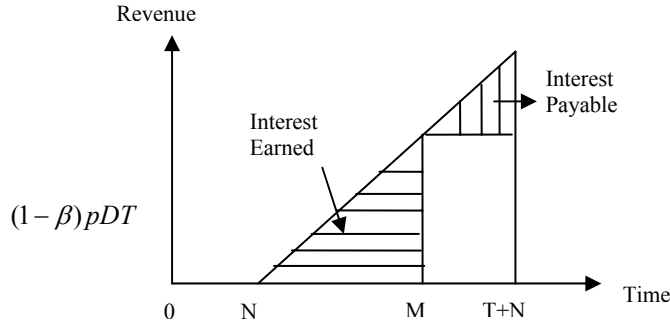


Figure 4. $N \leq M$ and $T \leq M \leq T+N$ (credit payment)

Sub-case 3.1.3. $T+N \leq M$

For this case, the interest paid will be only for the amount that is financed for the payment to the supplier initially. Therefore, the interest payable is given by

$$cI_p \left(\frac{(1-\alpha)^2 DT^2}{2} + (1-\alpha)D(T+N) \right).$$

Also, the interest earned for the retailer will be (i) for the portion of instant payment in the interval $[0, M]$ and (ii) from the credit payment in the interval $[N, M]$. Thus the interest earned will be

$$pI_e \left(\frac{\beta DT^2}{2} + \beta DT(M-T) + (1-\beta)DT(M-T-N) \right).$$

Consequently, the retailer's annual total relevant cost per unit time is given by

$$\begin{aligned} TRC_{13}(T) = & \frac{A}{T} + \frac{hDT}{2} + \frac{cI_p}{T} \left(\frac{(1-\alpha)^2 DT^2}{2} + (1-\alpha)DTN \right) \\ & - \frac{pI_e}{T} \left(\frac{DT^2}{2} + \beta DT(M-T) + (1-\beta)DT(M-T-N) \right). \end{aligned} \quad (3)$$

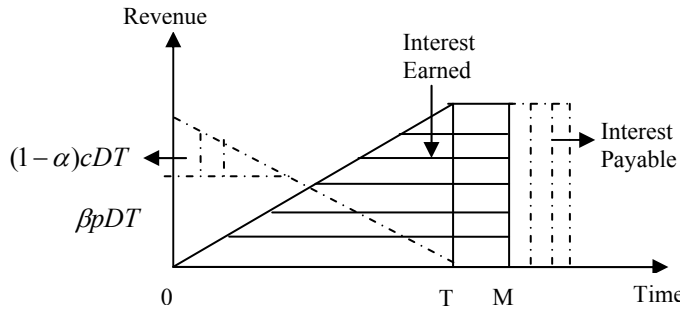


Figure 5. $N \leq M$ and $T + N \leq M$ (instant payment)

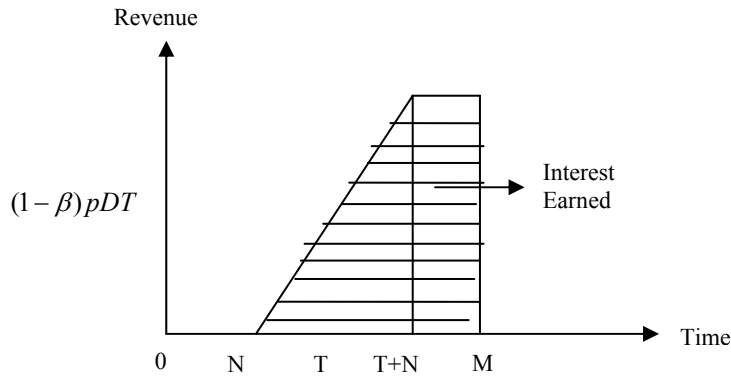


Figure 6. $N \leq M$ and $T + N \leq M$ (credit payment)

From the above arguments, the retailer's annual total relevant cost per unit time can be expressed as

$$TRC(T) = \begin{cases} TRC_{11}(T) & \text{if } M \leq T \\ TRC_{12}(T) & \text{if } M \leq T \leq T + N \\ TRC_{13}(T) & \text{if } T + N \leq M \end{cases} \quad (4)$$

Since $TRC_{11}(M) = TRC_{12}(M)$, $TRC_{12}(M - N) = TRC_{13}(M - N)$, $TRC_{ij}(T)$, $i = 1, j = 1, 2, 3$ are well defined on $T > 0$.

Now, Equation (1) can be rewritten as

$$\begin{aligned} TRC_{11}(T) &= \frac{2A + D(cI_p - pI_e)((M - N)^2 - (N^2 - 2MN)\beta)}{2T} \\ &\quad + \frac{DT(h + cI_p(\alpha - 2)^2)}{2} - cI_p D(M - (1 - \beta)N). \\ &= \frac{D(h + cI_p(\alpha - 2)^2)}{2T} \left[T - \sqrt{\frac{2A + D(cI_p - pI_e)((M - N)^2 - (N^2 - 2MN)\beta)}{D(h + cI_p(\alpha - 2)^2)}} \right]^2 \\ &\quad + \left[\sqrt{D(h + cI_p(\alpha - 2)^2)(2A + D(cI_p - pI_e)((M - N)^2 - (N^2 - 2MN)\beta))} \right. \\ &\quad \left. - cI_p D(M - (1 - \beta)N) \right]. \end{aligned} \quad (5)$$

Equation (5) implies that the minimum of $TRC_{11}(T)$ can be obtained when the quadratic non-negative term, depending on T , is equated to zero. Therefore, the optimum value T_{11}^* is given by

$$T_{11}^* = \sqrt{\frac{2A + D((cI_p - pI_e)((M - N)^2 - (N^2 - 2MN)\beta))}{D(h + cI_p(\alpha - 2)^2)}}. \quad (6)$$

Accordingly, equation (5) has a minimum value of $TRC_{11}(T)$ for the optimal value of T_{11}^* given by

$$TRC_{11}(T^*) = \left[\sqrt{\frac{D(h + cI_p(\alpha - 2)^2)(2A + D(cI_p - pI_e))}{((M - N)^2 - (N^2 - 2MN)\beta)}} \right. \\ \left. - cI_p D(M - (1 - \beta)N) \right] \quad (7)$$

Similarly, $TRC_{12}(T)$ can be derived algebraically as

$$\begin{aligned}
TRC_{12}(T) &= \frac{2A + D(M - N)^2(1 - \beta)(cI_p - pI_e)}{2T} + \frac{DT(h + cI_p((\alpha - 2)^2 - \beta) + pI_e\beta)}{2} \\
&\quad - DM(cI_p - \beta(cI_p - pI_e)) + cI_p DN(2 - \alpha - \beta). \\
&= \frac{D(h + cI_p((\alpha - 2)^2 - \beta) + pI_e\beta)}{2T} \left[T - \sqrt{\frac{2A + D(M - N)^2(1 - \beta)(cI_p - pI_e)}{D(h + cI_p((\alpha - 2)^2 - \beta) + pI_e\beta)}} \right]^2 \\
&\quad + \left[\sqrt{D(h + cI_p((\alpha - 2)^2 - \beta) + pI_e\beta)(2A + D(M - N)^2(1 - \beta)(cI_p - pI_e))} \right. \\
&\quad \left. - DM(cI_p - \beta(cI_p - pI_e)) + cI_p DN(2 - \alpha - \beta) \right]. \tag{8}
\end{aligned}$$

Equation (8) represents that the minimum of $TRC_{12}(T)$ can be obtained by equating to zero the quadratic non-negative term depending on T . Therefore, the optimum value T_{12}^* is given by

$$T_{12}^* = \sqrt{\frac{2A + D(1 - \beta)(M - N)^2(cI_p - pI_e)}{D(h + cI_p(\alpha - 2)^2 - \beta(cI_p - pI_e))}}. \tag{9}$$

Therefore, equation (8) has a minimum value for the optimal value of T_{12}^* reducing $TRC_{12}(T)$ to

$$TRC_{12}(T_{12}^*) = \left[\sqrt{\frac{D(h + cI_p((\alpha - 2)^2 - \beta) + pI_e\beta)}{(2A + D(M - N)^2(1 - \beta)(cI_p - pI_e))}} \right. \\ \left. - DM(cI_p - \beta(cI_p - pI_e)) + cI_p DN(2 - \alpha - \beta) \right]. \tag{10}$$

Likewise, $TRC_{13}(T)$ can be derived without using derivatives as

$$\begin{aligned}
TRC_{13}(T) &= \frac{A}{T} + \frac{hDT}{2} + \frac{cI_p DT(1 - \alpha)(3 - \alpha)}{2} + \frac{pI_e DT}{2} \\
&\quad + (cI_p(1 - \alpha) + pI_e(1 - \beta))DN - pI_e DM. \\
&= \frac{D(h + pI_e + cI_p(3 - \alpha)(1 - \alpha))}{2T} \\
&\quad \left[T - \sqrt{\frac{2A}{D(h + pI_e + cI_p(3 - \alpha)(1 - \alpha))}} \right]^2 \\
&\quad + \left[\sqrt{2AD(h + pI_e + cI_p(\alpha - 3)(\alpha - 1))} \right. \\
&\quad \left. + (cI_p(1 - \alpha) + pI_e(1 - \beta))DN - pI_e DM \right]. \tag{11}
\end{aligned}$$

Again, equation (11) corresponds to the optimum value T_{13}^* which can be obtained by equating to zero the quadratic non-negative term of $TRC_{13}(T)$, depending on T . Thus,

$$T_{13}^* = \sqrt{\frac{2A}{D(h + pI_e + cI_p(\alpha - 3)(\alpha - 1))}}. \quad (12)$$

and the minimum value of $TRC_{13}(T)$. for optimum T_{13}^* is given by

$$TRC_{13}(T_{13}^*) = \left[\sqrt{2AD(h + pI_e + cI_p(\alpha - 3)(\alpha - 1))} + (cI_p(1 - \alpha) + pI_e(1 - \beta)) - pI_eDM \right]. \quad (13)$$

3.2. Case 2. $N \geq M$

Similarly, the annual total relevant cost for the retailer can be expressed as

$TRC(T)$ = a) Annual ordering cost + b) Annual stock holding cost + c) Annual interest payable - d) Annual interest earned

To calculate annual interest charged and annual interest earned (according to assumption 7 and 8), two sub-cases may arise: Sub-Case 3.2.1: $M \leq T$ and Sub-Case 3.2.2: $M \geq T$

Sub-Case 3.2.1. $M \leq T$

Since $N \geq M$, the retailer must finance (i) the initial payment made to the supplier, (ii) all the items sold after M for the portion of instant payment, and (3) the entire amount of the credit payment at the end of the trade credit period for the interval $[M, T + N]$. So, the interest charged will be

$$cI_p \left(\frac{(1 - \alpha)^2 DT^2}{2} + (1 - \alpha)DT(T + N) + \frac{\beta D(T - M)^2}{2} + (1 - \beta)D\left(\frac{T^2}{2} + T(N - M)\right) \right).$$

Also, the retailer has to settle the account to the supplier at M , therefore he can accumulate interest for the portion of instant payment till M at the rate I_e per dollar per year. Therefore, the interest earned will be

$$pI_e \left[\frac{\beta DM^2}{2} \right].$$

Hence, the annual total relevant cost for the retailer is given by

$$TRC_{21}(T) = \frac{A}{T} + \frac{hDT}{2} + \frac{cI_p}{T} \left(\frac{(1-\alpha)^2 DT^2}{2} + (1-\alpha)DT(T+N) + \frac{\beta D(T-M)^2}{2} + (1-\beta)D \left(\frac{T^2}{2} + T(N-M) \right) \right) - \frac{pI_e}{T} \left[\frac{\beta DM^2}{2} \right] \quad (14)$$

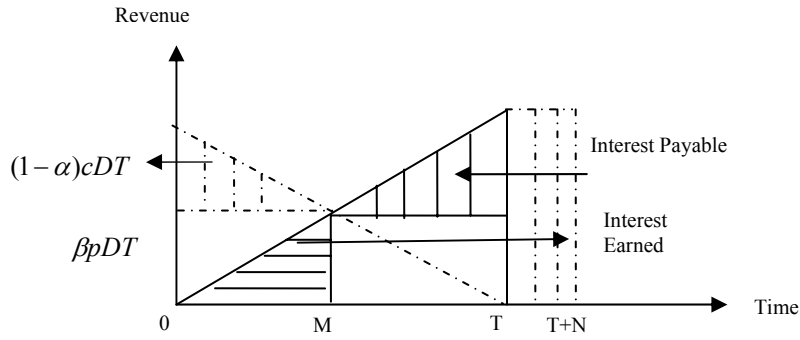


Figure 7. $N \geq M$ and $M \leq T$ (instant payment)

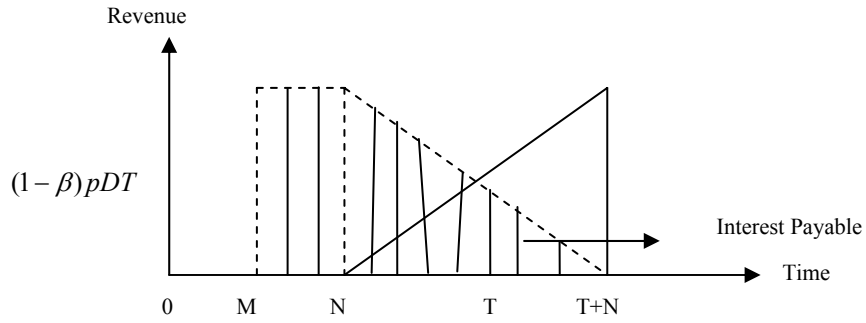


Figure 8. $N \geq M$ and $M \leq T$ (credit payment)

Sub-Case 3.2.2. $M \geq T$

The retailer must arrange finances for (i) the initial payment made to the supplier, (ii) the entire amount of the credit payment at the end of the trade credit period for the interval $[M, T + N]$. Thus, the resultant interest charged will be

$$cI_p \left(\frac{(1-\alpha)^2 DT^2}{2} + (1-\alpha)DT(T+N) + (1-\beta)D \left(T(N-M) + \frac{T^2}{2} \right) \right).$$

In the interval $[0, M]$, the retailer accumulates his revenue in interest bearing account at the rate I_e per dollar per year. As a result, the interest earned for the retailer is given by

$$pI_e \left(\frac{\beta DT^2}{2} + \beta DT(M-T) \right).$$

Consequently, the annual total relevant cost for the retailer is given by

$$TRC_{22}(T) = \frac{A}{T} + \frac{hDT}{2} + \frac{cI_p}{T} \left(\frac{(1-\alpha)^2 DT^2}{2} + (1-\alpha)DT(T+N) + (1-\beta)D \left(\frac{T^2}{2} + T(N-M) \right) \right) - \frac{pI_e}{T} \left[\frac{\beta DT^2}{2} + \beta DT(M-T) \right]. \quad (15)$$

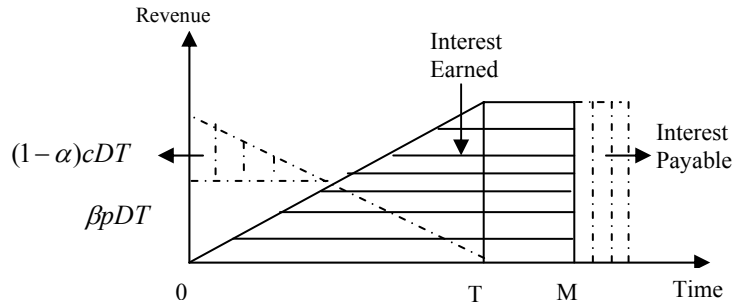


Figure 9. $N \geq M$ and $M \geq T$ (instant payment)

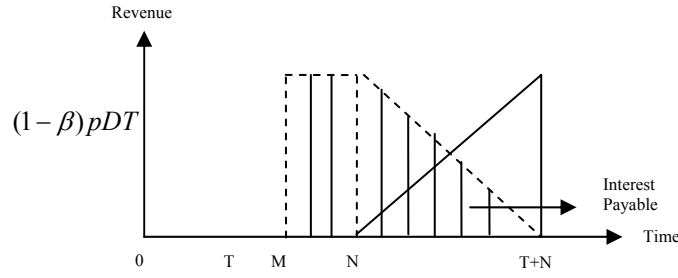


Figure 10. $N \geq M$ and $M \geq T$ (credit payment)

From the above arguments, the retailer's annual total relevant cost per unit time can be expressed as

$$TRC(T) = \begin{cases} TRC_{21}(T) & \text{if } M \leq T \\ TRC_{22}(T) & \text{if } M \geq T \end{cases} \quad (16)$$

Since $TRC_{21}(M) = TRC_{22}(M)$, all $TRC_{ij}(T), i = 2, j = 1, 2$ are continuous and well defined on $T > 0$.

Again, equation (14) can be derived algebraically as

$$TRC_{21}(T) = \frac{2A + M^2 D \beta (cI_p - pI_e)}{2T} + \frac{DT(h + cI_p(\alpha - 2)^2)}{2}.$$

$$TRC_{21}(T) = \frac{D(h + cI_p(\alpha - 2)^2)}{2T} \left[T - \sqrt{\frac{2A + M^2 D \beta (cI_p - pI_e)}{D(h + cI_p(\alpha - 2)^2)}} \right]^2$$

$$+ \left[\sqrt{D(h + cI_p(\alpha - 2)^2)(2A + M^2 D \beta (cI_p - pI_e))} + cI_p D(N(2 - \alpha - \beta) - M(1 - \beta)) \right]. \quad (17)$$

Equation (17) implies that the minimum of $TRC_{21}(T)$ can be obtained when the quadratic non-negative term, depending on T , is equated to zero. Therefore, the optimum value T_{21}^* is given by

$$T_{21}^* = \sqrt{\frac{2A + DM^2 \beta (cI_p - pI_e)}{D(h + cI_p(\alpha - 2)^2)}}. \quad (18)$$

Therefore, equation (17) has a minimum value for the optimal value of T_{21}^* reducing $TRC_{21}(T)$ to

$$TRC_{21}(T_{21}^*) = \left[\sqrt{D(h + cI_p(\alpha - 2)^2)(2A + M^2 D\beta(cI_p - pI_e))} + cI_p D(N(2 - \alpha - \beta) - M(1 - \beta)) \right] \quad (19)$$

Likewise, $TRC_{22}(T)$ can be derived without using derivatives as

$$TRC_{22}(T) = \frac{D(h + cI_p(\alpha - 2)^2 - \beta(cI_p - pI_e))}{2T} \left[T - \sqrt{\frac{2A}{D(h + cI_p(\alpha - 2)^2 - \beta(cI_p - pI_e))}} \right]^2 + \sqrt{2AD(h + cI_p(\alpha - 2)^2 - \beta(cI_p - pI_e))} + cI_p DN(2 - \alpha - \beta) + MD\beta(cI_p - pI_e). \quad (20)$$

$$T_{22}^* = \sqrt{\frac{2A}{D(h + cI_p(\alpha - 2)^2 - \beta(cI_p - pI_e))}}. \quad (21)$$

$$TRC_{21}(T_{21}^*) = \left[\sqrt{2AD(h + cI_p(\alpha - 2)^2 - \beta(cI_p - pI_e))} + cI_p DN(2 - \alpha - \beta) + MD\beta(cI_p - pI_e) \right] \quad (22)$$

3.3. Decision rule for finding the optimal cycle time T^* .

3.3.1. Case1. $N \leq M$

Equation (6) implies that

$$\text{If } 2A + D((M - N)^2 - (N^2 - 2MN)\beta)(cI_p - pI_e) \geq 0$$

then, the optimal value of T when $M \leq T$ can be obtained by substituting equation (6)

in $M \leq T_{11}^*$, i.e. if and only if

$$\Delta_1 \equiv 2A - M^2 D(h + cI_p(\alpha - 1)(\alpha - 3) + pI_e) + (N^2 - 2MN)D(1 - \beta)(cI_p - pI_e) \geq 0. \quad (23)$$

Similarly, Equation (9) implies that the optimal value of T , when $T \leq M \leq T + N$ can be obtained by substituting T_{12}^* in $T \leq M \leq T + N$. Thus, we get

$$\Delta_1 \equiv 2A - M^2 D(h + cI_p(\alpha - 1)(\alpha - 3) + pI_e) + (N^2 - 2MN)D(1 - \beta)(cI_p - pI_e) \leq 0. \quad (24)$$

and

$$\Delta_2 \equiv 2A - D(M - N)^2(h + pI_e + cI_p(\alpha - 3)(\alpha - 1)) \geq 0. \quad (25)$$

Finally, by substituting Equation (12) in $T + N \leq M$, we get

$$\Delta_2 \equiv 2A - D(M - N)^2(h + pI_e + cI_p(\alpha - 3)(\alpha - 1)) \leq 0. \quad (26)$$

$$\Delta_1 - \Delta_2 = (N^2 - 2MN)D(h + cI_p(\alpha - 2)^2 - \beta(cI_p - pI_e)) \leq 0. \quad (27)$$

From the above arguments and the fact of $\Delta_1 < \Delta_2$, we summarize the above results in the following theorem:

Theorem 1. For $N \leq M$,

- A. If $\Delta_1 \geq 0$, then $T^* = T_{11}^*$.
- B. If $\Delta_2 \geq 0$ and $\Delta_1 \leq 0$, then $T^* = T_{12}^*$.
- C. If $\Delta_2 \leq 0$, then $T^* = T_{13}^*$.

Proof. It immediately follows from (23), (24), (25), (26) and (27).

3.3.2. Case2. $N \geq M$

Equation (18) implies that if $cI_p > pI_e$, then the optimal value of T when $M \leq T$ can be obtained by substituting equation (6) in $M \leq T_{21}^*$, i.e. if and only if

$$\Delta_3 \equiv 2A - M^2D(h + cI_p(\alpha - 2)^2 - \beta(cI_p - pI_e)) \geq 0. \quad (28)$$

Similarly, equation (21) implies that the optimal value of T , when $T \leq M$ can be obtained by substituting T_{22}^* in $T \leq M$. Thus, we get

$$\Delta_3 \equiv 2A - M^2D(h + cI_p(\alpha - 2)^2 - \beta(cI_p - pI_e)) \leq 0. \quad (29)$$

From the above arguments, we summarize the above results in the following theorem:

Theorem 2. For $N \geq M$,

- A. If $\Delta_3 \geq 0$, then $T^* = T_{21}^*$.
- B. If $\Delta_3 \leq 0$, then $T^* = T_{22}^*$.

Proof. It follows from (27) and (29).

4. NUMERICAL ANALYSIS

To illustrate all the results obtained in the present study, following numerical examples have been solved by the proposed method.

4.1. Case 1. $N \leq M$

Example1. Let $A = \$100/\text{order}$, $D = 2500\text{units}/\text{year}$, $c = \$10/\text{unit}$, $p = \$30/\text{unit}$, $h = \$5/\text{unit}/\text{year}$,

$I_p = \$0.15/\text{\$/year}$, $I_e = \$0.13/\text{\$/year}$, $M = 0.1\text{year}$, $N = 0.07\text{year}$, $\beta = 0.2$ and $(1 - \alpha) = 0.1$.

Here $\Delta_1 = 13.3 > 0$, $\Delta_2 = 179.26 > 0$. Using Theorem 1 (A), we get $T^* = T_{11}^* = 0.1038\text{ year}$.

The optimal order quantity is 260 units. $TRC(T_{11}^*) = \$1604.025$.

Example2. Let $A = \$80/\text{order}$, $D = 2000\text{units}/\text{year}$, $c = \$10/\text{unit}$, $p = \$30/\text{unit}$, $h = \$7/\text{unit}/\text{year}$,

$I_p = \$0.15/\text{\$/year}$, $I_e = \$0.13/\text{\$/year}$, $M = 0.1\text{year}$, $N = 0.07\text{year}$, $\beta = 0.2$ and $(1 - \alpha) = 0.1$.

Here $\Delta_1 = -29.356 < 0$, $\Delta_2 = 139.813 > 0$. Using Theorem 1 (B), we get $T^* = T_{12}^* = 0.0917\text{year}$. The optimal order quantity is 184 units. $TRC(T_{12}^*) = \$1120.92$.

Example3. Let $A = \$80/\text{order}$, $D = 2000\text{units}/\text{year}$, $c = \$10/\text{unit}$, $p = \$30/\text{unit}$, $h = \$7/\text{unit}/\text{year}$,

$I_p = \$0.15/\text{\$/year}$, $I_e = \$0.13/\text{\$/year}$, $M = 0.1\text{year}$, $N = 0.01\text{year}$, $\beta = 0.2$ and $(1 - \alpha) = 0.1$.

$\Delta_1 = -57 < 0$, $\Delta_2 = -21.683 < 0$. Using Theorem 1 (C), we get $T^* = T_{13}^* = 0.0834\text{year}$. The optimal order quantity is 169 units. $TRC(T_{13}^*) = \$1179.813$.

4.2. Case 2. $N \geq M$

Example4. Let $A = \$80/\text{order}$, $D = 5000\text{units}/\text{year}$, $c = \$10/\text{unit}$, $p = \$30/\text{unit}$, $h = \$10/\text{unit}/\text{year}$, $I_p = \$0.15/\text{\$/year}$, $I_e = \$0.13/\text{\$/year}$, $M = 0.05\text{year}$, $N = 0.06\text{year}$, $\beta = 0.2$ and $(1 - \alpha) = 0.1$

$\Delta_3 = 6.3125 > 0$. Using Theorem 2 (A), we get $T^* = T_{21}^* = 0.0514\text{ year}$. The optimal order quantity is 255 units. $TRC(T_{21}^*) = \$3121.215$.

4.3. Sensitivity Analysis

To discuss the influence of key model parameters on the optimal solutions, the sensitivity analysis on different parameters has been presented by considering the following data.

Example 5. Let $A = \$500/\text{order}$, $D = 3000\text{units}/\text{year}$, $c = \$100/\text{unit}$, $p = \$150/\text{unit}$, $h = \$0.136/\text{unit}/\text{year}$,
 $I_p = \$0.15/\text{\$/year}$, $I_e = \$0.13/\text{\$/year}$, $M = 0.1\text{year}$, $N = 0.07\text{year}$, $\beta = 0.2$ and $(1 - \alpha) = 0.1$.

Following inferences can be made from Table 1:

Table 1. Sensitivity on different parameters

% Changes in Parameter		% Changes in		Inference
		T^*	$TRC(T)$	
A	-30	-16.51	-16.90	Highly Sensitive
	-20	-10.66	-10.92	
	20	9.63	9.86	
	30	14.14	14.47	
M	-30	0.489	2.93	Moderate Sensitive
	-20	0.435	2.07	
	20	-0.876	-2.53	
	30	-1.483	-3.97	
I_p	-30	13.90	10.776	Highly Sensitive
	-20	8.616	6.643	
	20	-6.765	-5.129	
	30	-9.643	-7.284	
I_e	-30	3.85	3.95	Moderate Sensitive
	-20	2.53	2.59	
	20	-2.39	-2.25	
	30	-3.53	-3.62	
β	-30	0.674	1.324	Moderate Sensitive
	-20	0.447	0.88	
	20	-0.44	-0.868	
	30	-0.659	-1.29	
N	-30	-0.9319	-2.684	Moderate Sensitive
	-20	-0.566	-1.733	
	20	0.350	1.5125	
	30	0.445	2.186	

- Sensitivity on A reveals that the optimal cycle length and the total annual cost is highly sensitive w.r.t. ordering quantity.
- As M increases, there is a decrease in cycle time and total annual cost. Both parameters are moderately sensitive w.r.t. M .
- As the rate of interest charged increases, there is a decrease in the cycle length and of the total annual cost. Also, if I_p decreases, the total cost increases very sharply. Therefore, a retailer will always prefer a less interest charged rate to get minimum total cost. This is quite practical since a retailer may never wish to pay high interest charges and therefore procure less order quantity.
- As the rate of interest earned increases, there is a decrease in the cycle length and of the total annual cost.

- As the percentage of initial payment offered by the retailer to its customer i.e β decreases, there is a minor increase in the optimal cycle length and in the total annual cost.
- As N increases, there is a minor increase in the optimal cycle length and in the total annual cost. Since cycle length increases, there is an increase in the order quantity for the retailer. This reveals the fact that the credit period offered to the customer has a positive impact on the unrealized demand. Therefore, a retailer should offer more credit period to its customer.

5. CONCLUSION

The present study extends the assumptions of two echelon trade credit policy in the previously published results to reflect the realistic situations by incorporating partial trade credit. It is assumed that the supplier, as well as the retailer, adopts partial trade credit financing to stimulate their demand to get optimal inventory policy. Also, since the retailer offers its customers a permissible delay period N , hence, he will receive its revenue from N to $T + N$ instead of from 0 to T . An algebraic approach has been used to get the optimal results. The approach gives an advantage to the researchers not familiar with the calculus method and tedious Hessian matrix when explaining EOQ/EPQ concepts. The theorems help the retailer to yield the optimal order quantity efficiently. Comprehensive sensitivity analysis on different parameters has been performed. The results reveal that the credit period offered to the customer has a positive impact on the unrealized demand. Further, it shows that if a supplier/retailer decreases his portion of initial payment, then the subsequent downstream member will benefit, which eventually increases the order quantity for the retailer. The managerial significance of the model is that it helps to increase the total supply chain profit with the inception of partial trade credit.

For future research the model can be extended with deterioration, fuzzy environment, price dependent demand, etc.

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