

FUZZY DECISION TREES AS A DECISION-MAKING FRAMEWORK IN THE PUBLIC SECTOR

Jože BENČINA

*Faculty of Administration
University of Ljubljana, Ljubljana, Slovenia*
joze.bencina@fu.uni-lj.si

Received: October 2009 / Accepted: November 2011

Abstract: Systematic approaches to making decisions in the public sector are becoming very common. Most often, these approaches concern expert decision models. The expansion of the idea of the development of e-participation and e-democracy was influenced by the development of technology. All stakeholders are supposed to participate in decision making, so this brings a new feature to the decision-making process, in which amateurs and non-specialists are participating decision making instead of experts. To be able to understand the needs and wishes of stakeholders, it is not enough to vote for alternatives – it is important to participate in solution-finding and to express opinions about the important elements of these matters. The solution presented in this paper concerns fuzzy decision-making framework. This framework combines the advantages of the introduction of the decision-making problem in a tree structure and the possibilities offered by the flexibility of the fuzzy approach. The possibilities of implementation of the framework in practice are introduced by case studies of investment projects appraisal in a community and assessment of efficiency and effectiveness of public institutions.

Keywords: Decision tree, appraisal tree, fuzzy set, decision-making, public sector

MSC: 90B50.

1. INTRODUCTION

The making decisions in the public sector is a common subject of research; however, using systematic approaches is not common when making decisions. The public sector is supposed to act in public interest and consider the interests of all

stakeholders. It is obvious that a large number of diverse stakeholders have needs and wishes that must be considered when making decisions, which in the public sector can be clearly stated despite the different views of the definition of the term "the public interest". According to Bots and Lootsma, making decisions in the public sector differs from decision making in the private sector, but the stereotypes about the differences are not to be trusted. Nevertheless, as a result of the problem's scope, social diversity and dynamics, the stakeholder network will generally be more complex and less transparent, and the interests will be more diverse in public decision-making situations [4].

The systematic approach to decision-making in the public sector is more or less limited to expert decision-making. The systems for expert decision-making support comprise a small number of decision makers and assume that they have expert knowledge of issues under consideration. The above mentioned approach is quite appropriate for the selected environment; however, the challenge of broad public participation of stakeholders in decision-making processes [9] needs to be supported by adequately adjusted tools and methods. The public sector needs a well-designed solution for group multi-attribute decision-making, which will expand the systematic approach to decision-making beyond the limitations of expert systems approach to all kinds of public decision-making.

In general, the contribution of the research is the definition of the decision-making framework for the public sector, which comprises suitable methods and approaches within the general framework. The core of the solution is decision trees, which represent a common base of qualitative multi-attribute decision models. The use of the fuzzy approach enables the decision-makers to appraise the attributes of alternatives more easily and accurately [20]. Within the general definition, a comprehensive definition of the fuzzy appraisal tree is given. The main scientific contribution of the work is the definition of the fuzzy appraisal tree. Decision trees as well as fuzzy decision trees supporting the appraisal have not been formalised to the stage of classification and comparative trees yet, thus the definition of the fuzzy appraisal tree is an important contribution to the decision trees theory. The solution enables the use of any type of variable. The aggregation over the appraisal tree combines values of different types of variables without limitations. Furthermore, the solution exceeds the limitation of the number of vertices and their attributes of appraisal trees that use decision rules.

The first part of the article discusses the theoretical basis in three sections. In Section 2, the decision-making in the public sector is discussed and the argumentation for the design of the solution is given. Next, the review of different decision trees and their fuzzy implementations is provided. In Section 4, as much theory on fuzzy sets and fuzzy logic as needed for the understanding of the elements and structure of the fuzzy appraisal framework is covered. The general definition of the fuzzy appraisal framework and particularly the definition of the appraisal fuzzy tree are listed in Section 5. Section 6 presents two implementations of the general model into the appraisal of investment projects in municipality and into the balanced scorecard in the public sector. The final part of the article contains the conclusions.

2. DECISION MAKING IN THE PUBLIC SECTOR

In the application of a systematic approach when making decisions in the public sector it is important to consider the following points. Any negligence with respect to these points could possibly cause difficulties to the systematic approach to making decisions in the public sector [9]:

- a complex and less-transparent stakeholder network,
- many diverse interests,
- multiple problem perceptions and multiple preferences,
- a large set of appraisal criteria,
- aggregation of many and often divergent interests of society into such notions as "general welfare", which only masks the conflict.

The systematic approach to the decision-making process is based on systems for decision-making support that include methods, models and tools, and offer help with the quality of decision making. An approach such as this must suppress the causes for the slow application of this type of solution and must enable:

- the integration of numerous stakeholders and group formation,
- insight into multiple problem perceptions and multiple preferences and coordination,
- the handling of large sets of appraisal criteria,
- a simple and understandable introduction to the decision-making problem and the decisions,
- analysis of differences in preferences and the realisation of an opinion-reconciliation process and a stakeholder concordance search.

The use of decision making systems in the public sector is widely represented in the literature. The examples mainly refer to experts' work in the field. They refer to many different fields: medicine [2], [13], regional planning and ecology [8], [11], education [10]. Our statement is confirmed by a study of publications relating to applied analysis of decision making from the scientific literature, where we find most of the examples from the public sector related to making decisions in professional circles [17].

The decision-making development in the public sector needs to find an environment that will expand the systematic approach to decision-making from limited areas of expertise to all fields of decision-making in the public sector. The solution for an improved approach to making decisions is a decision making framework, based on aforementioned guidance, which includes:

- the use of a decision tree for the introduction of the decision-making problem,
- the use of fuzzy sets and fuzzy logic theories,
- the merging of tree-structure values (aggregation of forests),
- the definition and use of variability measure.

The choice of the decision trees does not need justification, due to its frequent use in solving problems of this type. In any case, the principal advantages of tree-based methods are [26]:

- clarity and conciseness,
- context sensitivity,
- flexibility.

The use of a fuzzy approach makes possible the modelling of cognitive uncertainties, defined as the vagueness or fuzziness and the ambiguity or non-specific nature of a possibility distribution. This modelling is an approximation of the human mind and its way of thinking.

The fuzzy approach is defined by the appraisal technique and the presentation of results, where the use of linguistic variables for making decisions is of great importance. Appraisal by descriptive values demands significantly less mental effort. This type of appraisal is easier and more precise. The same can be said for the presentation of the results. Metaxiotis exposed the expectations regarding the use of fuzzy logic for decision making [20]:

- Fuzzy logic users will feel more confident when dealing with the vagueness and fuzziness of real data than non-users.
- Fuzzy logic users will be more satisfied with the final results of the decision than non-users.
- Fuzzy logic users will consider a wider range of alternatives than non-users.

Section 2 presented general discussions on decision-making with the emphasis on the public sector and provided the motivation for the choice of including the fuzzy logic in our solution. In the next section, we discuss the types of decision trees in general and their combination with fuzzy logic.

3. DECISION TREES

The main idea of an oriented graph starting from a particular point (root) that diverges into a connected structure of nodes and ends in leaves is to use the graph for different purposes with numerous different approaches. In general, decision trees can be used for one of the following purposes:

- as a classification, i.e. scenario, where decisions in the root and in the nodes lead to many outcomes, i.e. the leaves of the decision tree;
- as a structure for evaluating alternatives, where the appraisal of the leaves with an aggregation from the nodes to the root results in an appraisal;
- as a comparative structure for finding differences or changes in the state of the structure.

The use of the classification trees to classify subjects into groups is very common. As an example, the prediction of predetermined vegetation types from environmental properties [8], and maternity risk grouping [13]. Classification trees for diagnosis are often used in medicine and other fields when making decisions about diagnoses, the knowledge search in design for outsourcing [6], and decision making in medicine [24].

The trees are used in many fields of human activity as applications of qualitative multi-attribute decision models in healthcare [2], finance [5], software development [6], personnel selection [16], recreation and tourism [25], software selection [27] and electrical and electronic equipment treatment system [31].

Approximate tree matching has applications in genetic sequence comparison, scene analysis, error recovery and correction in programming languages, and cluster analysis [28].

The fuzzy decision tree combines the theory of the decision tree and the theory of fuzzy sets and fuzzy logic. In this way the decision trees can be used for modelling vagueness and ambiguity. Often the decision trees are not mentioned explicitly when making decisions with a fuzzy approach, but inferential algorithms of fuzzy decision systems are correct tree structure [20]. On the other hand, there are hierarchical multi-attribute decision models for decision-problem solutions using tree structures and descriptive appraisal attributes [2] without mentioning fuzzy theory.

All types of decision trees can be made fuzzy, so that in compliance with the paragraph of decision trees introduced at the beginning of the section, the following can be discussed: fuzzy classification trees, fuzzy appraisal trees and fuzzy comparative trees.

With fuzzy classification it is common that special attention is assigned to inductive learning, fuzzy-decision-tree formation with the help of examples (learning data). The problem with the induction of classification decision trees is not a subject of discussion in this work; therefore, the problem is only illustrated with a list of a few sources of information [23], [32], [33]. Among those previously mentioned in this section, examples of the use of classification-decision trees, fuzzy classification trees address only the following [7], [24], [19]. There are multiple similar examples, including solutions not related to decision trees [20].

Examples that would combine explicitly discussed decision trees and the use of fuzzy logic, hierarchical multi-criteria or multi-attribute models were reworded by some authors with a fuzzy approach, e.g. [5], [6]; and by others with a descriptive appraisal and aggregation with some logic rules without mentioning fuzzy logic theory, e.g. [2], [16], [25], [27], [31].

A fuzzy comparison of decision structures is not very common. Among the examples previously mentioned at the beginning of the section, this kind of approach is used for the analysis of the state of a military formation at the time of combat [29].

If we take a closer look at the examples of appraisal trees and fuzzy appraisal trees, we can extract following main approaches:

non-fuzzy approaches:

- hierarchical decision model with if-then decision or aggregating rules, nominal variables, expert-oriented [2], [10], [16], [31],
- outranking method using the impact Matrix, ordinal variables, expert-oriented, [25],
- hierarchical appraisal tree, numeric variables, expert-oriented [27],

fuzzy approaches:

- fuzzy classification tree, fuzzy variables [5],
- fuzzy appraisal, aggregation of fuzzy intervals, fuzzy variables [6], [14], [15], [18].

All examples are more or less expert-oriented and do not consider non-expert use of the solution. With the exception of the outranking method, the use of all other approaches is restricted to a single type of variables. The solutions based on if-then aggregating rules have two main restrictions. First, the definition of aggregating rules takes up a lot of time, and second, due to the exponential growth of the number of aggregating rules, the number of attributes and the cardinality of the domain of variables have to be very moderate. The last approach mentioned above considers fuzzy appraisal with the aggregation of fuzzy variables, but does not consider real variables. The fuzzy appraisal tree is not explicitly defined, and the results are given as real numbers.

The list of literature on decision-making in the public sector ([2], [10], [14], [15], [18]) is not as extensive as for the private sector; however, researchers use very similar approaches to those used for the private sector. We have found some solutions for the public sector using the hierarchical fuzzy appraisal model with fuzzy variables [14], [15], [18]; however, no explicit definition of the fuzzy appraisal framework or fuzzy appraisal tree is provided nor are different types of variables used. In general, papers offer solutions for a specific expert decision-making problem.

In order to achieve the main objective of the research, a comprehensive solution for the non-expert appraisal of variants is to be built. It has to overcome the restrictions of available solutions and offer tools and guidelines for the appraisal support to non-expert appraisers. For this reason, as is described in Section 5, we combined different methods into a comprehensive model, ie. the fuzzy appraisal framework. It consists of the definition of the fuzzy appraisal tree and methods for fuzzy aggregation, as well as some other methods required.

The following section which includes the required theory of fuzzy sets and fuzzy logic is the preparation stage for the introduction of the decision-making framework.

4. FUZZY SETS AND FUZZY LOGIC

Fuzzy logic and approximate reasoning are parts of the framework with the definition of the linguistic variable. The review of fuzzy methods is completed with an introduction to the transformations between crisp and fuzzy and linguistic and fuzzy variables (fuzzyfication, defuzzyfication, linguistic variable to fuzzy number mapping and approximation).

The concept of a characteristic function of a (Cantorian or crisp) set was generalised by L. A. Zadeh [35] by replacing, in the co-domain, the two-element set $\{0,1\}$ by the unit interval $[0,1]$. Logically speaking, this is supposed to work in logic with a continuum of truth values (fuzzy logic) rather than in classical Boolean logic with two values, true and false, only.

Definition 1: *Fuzzy set [35]*

Given a (crisp) universe of discourse X , the fuzzy set \tilde{A} (more precisely, the fuzzy subset \tilde{A} of X) is given by its membership function $\mu_{\tilde{A}}(x): X \rightarrow [0,1]$, and the value $\mu_{\tilde{A}}(x)$ is interpreted as the degree of membership of x in the fuzzy set \tilde{A} . The group of all fuzzy subsets of X is denoted as $F(X)$.

Definition 2: *Fuzzy number [37]*

A fuzzy number \tilde{A} is a convex normalised ($\sup_x \mu_{\tilde{A}}(x) = 1$) fuzzy set over the real numbers with a continuous membership function having only one mean value $x_0 \in \mathbb{R}, \mu_{\tilde{A}}(x_0) = 1$.

If the mean value covers a subinterval $[a,b] \subseteq [0,1]$ then we are talking about a fuzzy interval. If the membership function of a fuzzy number or interval is constructed of linear functions, the first are triangular fuzzy numbers and the later are trapezoidal fuzzy numbers.

Definition 3: Trapezoidal fuzzy number

A trapezoidal fuzzy number is expressed as $\tilde{A} = (a, b, \alpha, \beta)$ and defined by the linear membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{a-x}{\alpha} & \text{if } a - \alpha \leq x \leq a \\ 1 & \text{if } a \leq x \leq b \\ 1 - \frac{x-b}{\beta} & \text{if } b \leq x \leq b + \beta \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

A triangular fuzzy number is a degenerated trapezoidal fuzzy number ($a = b$). For this reason, from this point the term fuzzy number will be used for fuzzy interval (trapezoidal fuzzy number), as well as for fuzzy number (triangular fuzzy number). As a short break, have a look at a graph of a fuzzy number (more precisely, a fuzzy interval or trapezoidal fuzzy number):

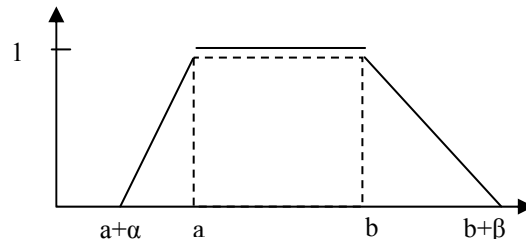


Figure 1: Graph of a fuzzy interval

For fuzzy numbers, the computation necessary for algebraic operations are considerably simplified. The calculations within the decision-making framework are only done with positive fuzzy numbers ($\mu_{\tilde{A}}(x) = 0, \forall x < 0$), and therefore only the arithmetic for positive fuzzy numbers is introduced (the definitions comprise the fuzzy numbers $\tilde{A} = (a, b, \alpha, \beta)$ and $\tilde{B} = (c, d, \gamma, \delta)$):

Table 1: Arithmetic operations for trapezoidal fuzzy numbers [3]

Operation	Result	
$\frac{1}{\tilde{A}}$	$(\frac{1}{b}, \frac{1}{a}, \frac{\beta}{b(b+\beta)}, \frac{\alpha}{a(a-\alpha)})$	(2)
$\tilde{A} + \tilde{B}$	$(a+c, b+d, \alpha+\gamma, \beta+\delta)$	(3)
$\tilde{A} - \tilde{B}$	$(a-d, b-c, \alpha+\delta, \beta+\gamma)$	(4)
$\tilde{A} \cdot \tilde{B}$	$(ac, bd, a\gamma+c\alpha-\alpha\gamma, b\delta+d\beta-\beta\delta)$	(5)
$\frac{\tilde{A}}{\tilde{B}}$	$(\frac{a}{d}, \frac{b}{c}, \frac{a\delta+d\alpha}{d(d+\delta)}, \frac{b\gamma+c\beta}{c(c-\gamma)})$	(6)

Zadeh introduced mapping between linguistic variables and fuzzy sets by the definition of a linguistic variable.

Definition 4: *Linguistic variable* [36]

A linguistic variable is defined by a quintuple $(\kappa, T(\kappa), U, G, \tilde{M})$ in which κ is the name of the variable; $T(\kappa)$ (or simply T) is the term set of κ , that is, the set of names for linguistic values κ , with each value being a fuzzy variable denoted generically by X and ranging over a universe of discourse U which is associated with the base variable u ; G is a syntactic rule (which usually has the form of grammar) for generating names X of values of κ ; and \tilde{M} is a semantic rule for associating each X with its meaning $\tilde{M}(X)$, which is a fuzzy subset of U . A particular X , that is, a name generated by G is called a term. A term consisting of a word or words which function as a unit (i.e. always occur together) is called an atomic term. A concatenation of components of a composite term is a sub-term.

An example of a term set is:

$$T = \{\text{Reject, Lowest, Very Low, Low, Middle, High, Very High, Highest, Must Be}\} \quad (7)$$

The modelling of linguistic variables with trapezoidal fuzzy numbers was proposed by Bonissone and Decker [3]. A choice of the cardinality of the term set depends on the characteristics of the problem in this case, and the same is true for the membership functions of the corresponding fuzzy numbers. Any kind of term set can be considered without any major changes, and in that respect the framework is flexible.

A metric of the fuzzy sets is required as a definition of all the mappings between crisp values (real numbers), fuzzy numbers and linguistic values (Definition 5). The Tran-Duckstein distance takes into account the fuzziness of the fuzzy sets and is confirmed in practice in an environmental-vulnerability assessment [30]. We have, therefore, decided to choose it for our framework. For trapezoidal fuzzy numbers the general definition is simplified as:

Definition 5: Tran-Duckstein distance for trapezoidal fuzzy numbers $(f(\alpha) = \alpha)$ [30]:

$$\begin{aligned}
 D_T^2(\tilde{A}, \tilde{B}, \alpha) &= \left(\frac{a+b}{2} - \frac{c+d}{2}\right)^2 + \frac{1}{3} \left(\frac{a+b}{2} - \frac{c+d}{2}\right) \\
 &[\beta - \alpha - \delta + \lambda] + \frac{2}{3} \left(\frac{b-a}{2}\right)^2 + \frac{1}{9} \left(\frac{b-a}{2}\right) \\
 &[\beta + \alpha] + \frac{2}{3} \left(\frac{d-c}{2}\right)^2 + \frac{1}{9} \left(\frac{d-c}{2}\right) \\
 &[\delta + \gamma] + \frac{1}{18} [\beta^2 + \alpha^2 + \delta^2 + \gamma^2] - \frac{1}{18} \\
 &[\alpha\beta + \gamma\delta] + \frac{1}{12} [\beta\gamma + \alpha\delta + \beta\delta + \alpha\gamma]
 \end{aligned} \tag{8}$$

The proposed framework introduces parallel use of three types of variables, the real number (crisp value), the fuzzy number and the linguistic variable. For this reason the transformations between them are needed (Figure 2).

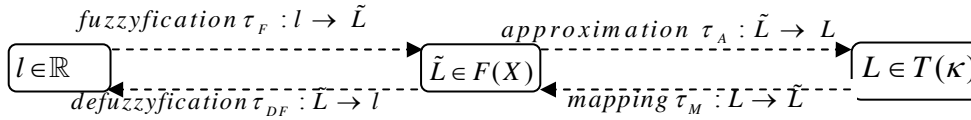


Figure 2: Crisp \leftrightarrow fuzzy \leftrightarrow linguistic transformation diagram

Definition 6: Real number \leftrightarrow fuzzy number \leftrightarrow linguistic variable transformations

fuzzyfication $\tau_F : l \rightarrow \tilde{L}$

Fuzzyfication makes the transformation from normalised real numbers $l \in \mathbb{R}$ to fuzzy sets $\tilde{L} \in F(X)$ (in our case, fuzzy numbers) using membership functions. It is carried out in two steps:

mapping $\tau_{M_i} : L \rightarrow \tilde{L}$ of the real number $l \in \mathbb{R}$ to the fuzzy set $\tilde{L} \in F(X)$, where in the case of multiple corresponding fuzzy sets the weighted average operator is used:

$$\tilde{L}_F = \frac{1}{\sum_k \tilde{\mu}_k(l)} \sum_k \tilde{\mu}_k(l) \cdot \tilde{L}_k; k = 1, N;$$

N is number of fuzzy sets tuched by l , \tilde{L}_k are the fuzzy sets tuched by l and $\tilde{\mu}_k(x)$ are the membership functions of the fuzzy sets \tilde{L}_k ;

translation $\tau_T : \tilde{L} \rightarrow \tilde{L}_1$ of the fuzzy set $\tilde{L} \in F(X)$ so that the result of defuzzyfication of fuzzy set \tilde{L}_1 , $\tau_{DF} : \tilde{L}_1 \rightarrow x$ is equal to the input real number $l \in \mathbb{R}$.

defuzzyfication $\tau_{DF} : \tilde{L} \rightarrow l$

Defuzzyfication makes the transformation from fuzzy sets $\tilde{L} \in F(X)$ to real numbers $l \in \mathbb{R}$. A "centre-of-gravity" method was chosen for all the possible transformations of

fuzzy sets into crisp values. The method is the most trivial weighted average and has a distinct geometrical meaning:

$$x_{COG} = \frac{\int_x x \cdot \mu(x) dx}{\int_x \mu(x) dx} \quad (9)$$

A simple calculation for a fuzzy number $\tilde{A} = (a, b, \alpha, \beta)$ gives the simple formula:

$$x_{COG} = \frac{-a^2 + b^2 + a\alpha + b\beta - \frac{\alpha^2}{3} + \frac{\beta^2}{3}}{-2a + 2b + \alpha + \beta} \quad (10)$$

Linguistic variable $L \in T(\kappa)$ to fuzzy variable $\tilde{L} \in F(X)$ mapping $\tau_M : L \rightarrow \tilde{L}$

The mapping of linguistic values into fuzzy numbers is part of linguistic-variable definition, where suitable parameters are defined:

- the name of the linguistic variable,
- the cardinality of the term set and the terms, the elements of the term set,
- for each term the corresponding fuzzy number (mapping function).

The linguistic variable "Appraisal", with nine values and names was used for this study:

Table 2: Linguistic variable "Appraisal" mapping function

Reject	Lowest	Very Low	Low	Medium	High	Very High	Highest	Must Be
0	.01	.10	.22	.41	.63	.78	.98	1
0	.02	.18	.36	.58	.80	.92	.99	1
0	.01	.06	.05	.09	.05	.06	.05	0
0	.05	.05	.06	.07	.06	.05	.01	0

Fuzzy set $\tilde{L} \in F(X)$ to linguistic value $L \in T(\kappa)$ approximation $\tau_A : \tilde{L} \rightarrow L$

The fuzzy number \tilde{A} is approximated to a linguistic value \tilde{L}_{approx} so that the closest fuzzy number \tilde{L} , representative of the nearest linguistic value, is found:

$$L_{approx} = L : D_T(\tilde{A}, \tilde{L}, \alpha) = \min D_T(\tilde{A}, \tilde{L}_i, \alpha); i = 1, \dots, n. \quad (11)$$

For higher granularity of the end results we introduced the approximation deviation. This is defined as the relative number of the difference in distance of the approximated fuzzy number and the fuzzy number image of the linguistic approximation and the difference between two adjacent linguistic values [1]:

$$\tilde{A} \quad (12)$$

The approximation with the deviation is then labelled as:

$$\begin{aligned}
&\leftarrow L_{approx}, \text{ if } Dev\% < -25\% \\
&L_{approx}, \text{ if } -25\% \leq Dev\% \leq 25\% \\
&L_{approx} \rightarrow, \text{ if } Dev\% > 25\%
\end{aligned} \tag{13}$$

At this point, we are well equipped with all that is needed to define the proposed model. We know that in order to perform the appraisal, an appraisal tree should be constructed and that in the public sector it is very suitable to perform an appraisal with the help of fuzzy variables and fuzzy aggregation. Therefore, in a comprehensive definition of the fuzzy appraisal framework and within it, the definition of the fuzzy appraisal tree is presented in Section 5.

4. FUZZY APPRAISAL FRAMEWORK

The suggested appraisal framework resulted from the problem when solving the group multi-attribute decision making in the public sector. An investigation of the problem and the development of the solution lead to a general appraisal framework combining the advantages of the introduction of a tree structure and the use of a fuzzy approach for the appraisal of attributes or indicators, as well as a comparison of the criteria and perspectives.

The entire fuzzy appraisal framework includes the definition of the fuzzy appraisal tree, averaging operators for the calculation of the average value of forests (groups of trees, with respect to groups of evaluators, groups of alternatives, organisation units of the same kind, etc.), methods for tree comparison, and tree classification (regarding the root, regarding the individual nodes, regarding the structure, etc.), methods for the analysis of tree variability (regarding the root, regarding the individual nodes, regarding the structure, etc.) and methods for tree optimisation (efficiency, information, entropy, etc.).

Definition 7: *Fuzzy appraisal framework*

Fuzzy appraisal frame constitute a forest with fuzzy appraisal trees over which the following is defined:

- *fuzzy appraisal trees (Definition 8),*
- *averaging operators $O_{Avg} : (\tilde{T}_1, \dots, \tilde{T}_n) \rightarrow \tilde{T}_{Avg}$ for the calculation of average tree values in chosen sub-forests (Definition 9),*
- *methods for fuzzy tree comparison and fuzzy tree classification,*
- *variability measures (Definition 10), and*
- *methods for fuzzy tree optimisation.*

With a given framework, it is possible to use three types of variables – real, fuzzy and linguistic, the values of which represent an equivalent appraisal of an attribute, criteria, indicator or perspective represented by the nodes.

Ingoing values (in the leaves) and calculated values (in the nodes) are recalculated from the ingoing type into the other two – real number, fuzzy number, linguistic variable (Figure 2). All the necessary transformations (Definition 6) are defined in each node and proceed during the recalculations. The values in the inner nodes are filled from the aggregation functions over fuzzy numbers. The aggregation functions over linguistic values are not considered (simplicity, distinction from existing systems based

on system rules). For special cases, the aggregation function is defined over real variables. Ingoing variables for aggregation operators are defined by the connections from the successors. Like the nodes, the connections, which also represent the weights are evaluated with all three types of variables and equipped by transformations to transform one into the other.

Definition 8: *Fuzzy appraisal tree*

A fuzzy tree $\tilde{T} = (\tilde{V}, \tilde{E})$ consists of a finite, nonempty set of fuzzy nodes (or vertices) \tilde{V} and a set of fuzzy edges \tilde{E} .

A fuzzy vertex \tilde{V} (Figure 3) consists of:

Three variables, $l \in \mathbb{R}, \tilde{L} \in M(X), L \in L(\kappa)$; (crisp variable l , fuzzy number \tilde{L} and linguistic variable L), four transformations between them,

$$\begin{aligned} & \text{fuzzyfication } \tau_F : l \rightarrow \tilde{L}, \text{ defuzzyfication } \tau_{DF} : \tilde{L} \rightarrow l, \\ & \text{approximation } \tau_A : \tilde{L} \rightarrow l, \text{ and mapping } \tau_M : L \rightarrow \tilde{L} \end{aligned}$$

A fuzzy aggregation operator over the fuzzy variables of children (for internal nodes) $f : (\tilde{L}_{i+1,j,1}, \dots, \tilde{L}_{i+1,j,K_{ij}}) \rightarrow \tilde{L}_{ij}$ where i is the level of the node, j is the position of the node at the level i , and $K_{i,j}$ is the number of children of the node.

A fuzzy edge $\tilde{E}_{i,j} = (\tilde{V}_{i,j}, \tilde{V}_{i+1,j,k})$ consists of a path from the parent to a child and of the weight $\tilde{W}_{i,j,k}$ which consists of three variables (2/a) and four transformations between them (2/b).

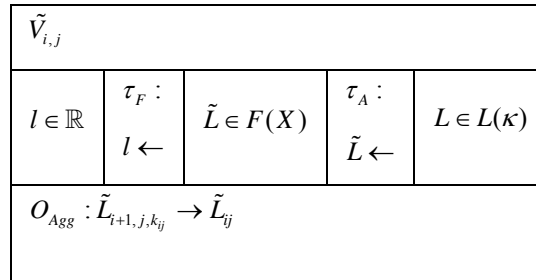


Figure 3: The structure of the fuzzy vertex \tilde{V} .

For a fuzzy appraisal framework to work, averaging operators to drive aggregation functions and to calculate averages of fuzzy forests are needed. Because of the simplicity principle, we have opted, among the many averaging operators [37], for generalised operators of the weighted mean of fuzzy numbers expressed by the formula in Definition 9.

Definition 9 [37]: *Generalised operators of the weighted mean of fuzzy numbers are:*

$$h_\alpha^w(a_1, \dots, a_n) = \left(\sum_{i=1}^n w_i a_i^\alpha \right)^{\frac{1}{\alpha}}, a_i \in [0, 1], i \in \mathbb{N}_n, \alpha \in \mathbb{R} (\alpha \neq 0) \tag{14}$$

where for the vector $\bar{w} = (w_1, \dots, w_n)$ it holds $\sum_{i=1}^n w_i = 1, w_i \geq 0 \forall_i \in \mathbb{N}_n$. The vector \bar{w} is termed the weighted vector, and its components w_i the weights. In the simplest version (equal weights $w_i = \frac{1}{n}$ and $\alpha = 1$), it is simply the arithmetic mean.

Comparison and classification is based on the comparison of calculated average values approximated into linguistic values (Definition 6/4).

The proximity measure and consensus measure are chosen for the analysis of the variability in a forest of appraisal results.

Definition 10: Proximity and consensus measure

The proximity measure is calculated as an opposite value in the uniform interval $I = [0, 1]$ of distance between the fuzzy value of an individual alternative of the individual estimator and a suitable group value:

$$p_{dm_n}(\tilde{A}_{i,j}) = 1 - D_T(\tilde{A}_{i,j, dm_n}, \tilde{A}_{i,j,G}) \leq 1, n = 1, \dots, |G| \tag{15}$$

$D = \{dm_1, \dots, dm_n\}$ is the set of all estimators, N (cardinality of set D) is the number of evaluators, $G \in P(D)$ is an element of a power set of D , and $|G| \cdot |G|$ (cardinality of subset $G \subseteq D$) is the number of evaluators in the group.

All the proximity measures for a given group (sub-forest) combine the consensus measure (average deviation of the proximity measures in the group):

$$C_G(\tilde{A}_{i,j}) = \frac{\sum_G P(\tilde{A}_{i,j})}{|G|} \tag{16}$$

The consensus and proximity measures are crisp numbers; therefore, any kind of analysis can be used, from simple ordering to cluster analysis.

Methods for the optimisation of decision trees are well defined for classification trees but not for appraisal trees. These methods require some adjustment. The two main approaches are: analysis of tree efficiency and analysis of information quality or entropy. Because this approach is still being introduced and has not been confirmed for larger amounts of data, consideration of this segment of the frameworks remains one of the most important tasks for future research.

The main idea of the research was to build a model that will be a useful support to systematic decision-making in the public sector. We have to follow the main guidelines for the solution to be useful and widely used in the selected environment. For this reason, we have to prepare a customised solution for every specific case. Thus, after the definition of elements of the fuzzy appraisal framework, we move to the presentation of two specific examples of the application of the appraisal framework.

5. APPLICATION EXAMPLES

The specific definition of a fuzzy appraisal framework is in general a choice of system elements according to the needs and possibilities of a specific problem. In this chapter the implementation of the fuzzy appraisal framework for two cases is presented.

The first one, the optimal selection of community investment projects, was the environment where the idea of the fuzzy appraisal framework was born. The second one is the project Balanced scorecard as an assessment and benchmarking tool in public sector running at Faculty of administration where the first implementation after definition of framework is going on.

5.1. Selection of investment projects in a municipality

The case is focused on the question of the optimum choice of investment projects in a local community burdened by various circumstances that could result in the municipality's inopportune investment orientation **Error! Reference source not found.** Decision making in municipalities takes place successively with two groups of participants. Professional services assess the investment projects and merge them into investment options according to professional criteria. The proposals are then revised and approved by the mayor and forwarded to the municipal council, which then decides independently and autonomously. The decision makers are confronted with various difficulties resulting from an unsystematic approach. Political decision makers are reluctant to take professional arguments into consideration, while professionals tend to disregard the political circumstances; however, an optimum decision is achieved only if all opinions and comments are dealt with in the decision-making process.

We have therefore been seeking a solution to the issue of making optimal decisions on investments in local government, in the phase of preparing the investments as well as in the phase of initiating their realisation and financing. The solution would have to establish a process that allow confrontation and coordination of diverse opinions and interests on the professional and political levels, in professional-political, as well as in professional-professional and political-political relations.

Based on the previous discussions, the fuzzy appraisal framework presented in Section 5 represents an appropriate approach to the solution of the given problem. The decision tree contains knowledge on the structure of the values that determine to what extent an individual alternative is suitable for inclusion in the budget. We have determined the structure of the appraisal tree, taking into account the framework of deciding on capital investments in the public sector [12], legally prescribed definitions and the analysis of the method of decision making in local communities in Slovenia.

The appraisal model for investment projects in local communities was defined according to the needs and possibilities, based on the general definition of the fuzzy appraisal framework (Definition 8) with adaptations as follows:

Definition 11: *Fuzzy appraisal model for selection of investment projects in a municipality*

- 1) *Fuzzy appraisal model for selection of investment projects in a municipality is a fuzzy appraisal framework (Definition 8).*
- 2) *The input values are linguistic variables (among the transformations in Definition 6 point 1 Fuzzyfication $\tau_F : l \rightarrow \tilde{L}$ is not needed).*
- 3) *The fuzzy aggregation operator over the fuzzy variables of children (for internal nodes) $O_{Agg} : (\tilde{L}_{t+1,j,1}, \dots, \tilde{L}_{t+1,j,K_{ij}}) \rightarrow \tilde{L}_{ij}$, is derived from (14), where $\alpha = 1$ and equal*

weights $w_i = \frac{1}{n}$ for all edges are chosen:

$$\tilde{A}_{ij} = \frac{1}{K_{ij}} \sum_k \tilde{A}_{i+1,j,k}; i = I - 1, \dots, 1; j = 1, \dots, J_i, k = 1, \dots, K_{ij} \quad (17)$$

where I is the number of levels of the tree, i is the current level of the tree, J_i is the branching of the tree, j is the position of the node at the i -th level, K_{ij} is the number of children of the parent in question at the level $i + 1$, and k is the position of the child of the parent in question.

4) The averaging operator $O_{Avg} : (\tilde{T}_1, \dots, \tilde{T}_n) \rightarrow \tilde{T}_{Avg}$ for the average tree value calculation in chosen sub-forests is derived (14), where $\alpha = 1$ and equal weights

$$w_i = \frac{1}{n} \text{ for all edges are chosen:}$$

$$\tilde{A}_{ij} = \frac{1}{|G|} \sum_G \tilde{A}_{i,j}; i = I - 1, \dots, 1; j = 1, \dots, J_i; \quad (18)$$

where G is the set of appraisers.

5) Variability measures are the proximity and consensus measure over the set of appraisers G (Definition 8).

The appraisal tree included three nodes (project contribution, feasibility and risk and cost/benefit appraisal), where the first two nodes each included three leaves and the third node included only two leaves[1]. The model was tested in three Slovenian municipalities. The sets of appraised projects include from seven to nine investment projects. Two types of appraisers were invited, representatives of municipal government and municipal councilors. Due to the reluctance of municipal councilors, the appraisal groups were rather small, comprising from nine to fifteen appraisers. We analysed the results represented with linguistic values and prepared a qualitative representation of results, where we considered the differences between projects and appraisal groups. The proposed solution attracted great interest, since the problem is of everyone's concern . It has been proven that the chosen method of appraisal is suitable for the chosen environments **Error! Reference source not found.** An interview was performed after each case study concerning the usefulness and suitability of the suggested approach for decision making in a chosen environment. The results proved the approach to be suitable due to the evaluators having no problems during the appraisal. The content of the appraisal was a bigger problem due to the evaluators not being introduced to it and/or the importance of the project was underestimated, also financially. This is a matter of preparation and organisation of appraisal processes, in which case the fuzzy appraisal framework can contribute to but not solve the problem.

5.2. Balanced scorecard as an assessment and benchmarking tool in the public sector

The fuzzy approach can be also effectively used when solving the problem of how to measure the successfulness of organisations with balanced scorecard. The balanced scorecard joins success indicators into four business perspectives – customers, finance, processes and learning and growth. In the profit sector the final result is measured with the financial perspective. It is enabled by the other three perspectives, which indicate successfulness of the organisation in the near future [22]. Organisations in non-profit sector are on the other hand focused on customers, so the main goal is represented by the customer perspective. The results of perspectives come out of indicators, chosen by the organisation according to its strategic goals.

An organisation is tree structured, where leaves are single employees or small departments and nodes combine subordinate units. The result of a unit is given by indicators defined for the unit, where some of them are calculated from equal indicators of subordinate units, and the others carry the results of the unit in question. The indicators of a unit are leaves of the appraisal tree of the unit. The nodes at the first level of the tree represent four perspectives of successfulness. The indicators of such an appraisal tree are defined over different variables which are hard to aggregate into joint value. The situation is the natural environment of the fuzzy appraisal framework which offers somehow simple solutions for quite difficult problems.

As a result of the research studying the problem of the implementation of the Balanced Scorecard into the public sector organisations we introduce the structure of the fuzzy appraisal framework for balanced scorecard as follows:

- each organisational unit is the carrier of indicators, which represent the results of the unit in question and joint indicators of the results of subordinate units,
- the indicators which are measuring the same results are by definition equal indicators,
- the equal indicators of units at the chosen level are aggregated into the equal indicators of the unit at the upper level of the organisational tree,
- the indicators of an organisational unit are linked into the appraisal tree of the unit, at the top of which four perspective nodes are defined, and the root of the tree represents the general appraisal of a unit,
- the root of the organisational tree is "the organisation", which links all the indicators defined for the subunits into joint appraisal tree.

Definition of the appraisal model for the balanced scorecard is based on the general definition of the fuzzy appraisal framework (Definition 8) with adaptations as follows:

Definition 12: *Fuzzy appraisal model of the balanced scorecard*

- 1) *Fuzzy appraisal model for selection of investment projects in a municipality is a fuzzy appraisal framework (Definition 8).*
- 2) *Each node is evaluated with three variables (crisp, fuzzy and linguistic), where one of them is the input variable.*

The fuzzy aggregation operator over the fuzzy variables of children
 $O_{\text{Agg}} : (\tilde{L}_{i+1,j,1}, \dots, \tilde{L}_{i+1,j,K_j}) \rightarrow \tilde{L}_{ij}$, *is derived from (14), where $\alpha = 1$:*

$$\tilde{A}_{ij} = \sum_k \tilde{W}_{i+1,j,k} \cdot \tilde{A}_{i+1,j,k}; \sum_k \tilde{W}_{i+1,j,k} = 1; i = I-1, \dots, 1; j = 1, \dots, J_i; k = 1, \dots, K_{ij} \quad (19)$$

where I is the number of levels of the tree, i is the current level of the tree, J_i is the branching of the tree, j is the position of the node at the i -th level, K_{ij} is the number of children of the parent in question at the level $i+1$, and k is the position of the child of the parent in question. The weights regulate the contribution of the children to the aggregation value of the parent.

- 3) The model of balanced scorecard comprises single organisational tree structure, so the averaging operator over forests of trees is not needed.
- 4) Variability is not a greater issue in the balanced scorecard model, but in any case the measure of proximity and consensus measure are available (Definition 8).

The case was realised in an administrative unit of the Slovenian state administration. The unit includes 5 departments with nearly one hundred employees. We defined twenty indicators in four groups (customers, internal processes, learning and growth and finance). We succeeded in building the appraisal tree and testing the calculations. The appraisal model for the balanced scorecard was implemented as a prototype in a web application as the support to the case studies of balanced scorecard in the Slovenian government and local administration. Unfortunately we have not managed to complete the above mentioned case, nor have we succeeded in persuading any other organisations to join the project. The main obstacle is the application, whereas the user interface is sufficiently user-friendly.

6. CONCLUSIONS

The structure of a fuzzy appraisal framework in the public sector is presented in this paper. The purpose of the framework is to develop solutions with properties adjusted specially for use in the public sector. The methods and approaches that lead to the satisfactory conclusion were systematically combined in the framework. This overcame insufficiencies in the systems for appraisal support that are reported in the scientific and technical literature.

The results of literature review indicated that these types of solutions very often originate from specific problems with expert decision making. This is true with an environment in the public sector environment, and for that reason the cases of numerous stakeholders co-decision making are rare and the problem is less well known. An initiative for increased participation in decision making and the development of e-democracy provide new challenges, including the quest for the solution of decision-making problems for a large number of non-specialist stakeholders.

The basic required solution properties suggest a combination of two approaches: the theory of decision trees and the theory of fuzzy sets and fuzzy logic. This led to incorporating the desired properties [26], [34]:

- clarity and conciseness, context sensitivity, flexibility;
- allowing the representation of cognitive uncertainties in decision making, providing more information to the decision maker, using linguistic terms

with soft boundaries to accommodate vagueness and ambiguity in human thinking and perception, into a framework.

The appraisal framework defines general elements of the system and gives guidelines to form concrete solutions. The approach was realised through its use in practice. Two cases were observed:

- the selection of investment projects in a municipality,
- the assessment of the performance of organisations with indicators – balanced scorecard.

Case studies proved the framework to be a suitable basis for implementing solutions of different decision-making problems in public sector.

However, the accommodation of the framework to the specific environment of the public sector is not a restriction but a generalisation. It incorporates more flexibility in the appraisal, which makes the solution easier to use. The framework gives practitioners and researchers a chance to broaden their research methods and tools, designed to make their appraisal applications better and more user-friendly for all kinds of use, both public and non-public.

However, new research challenges and motivation are perhaps even more important than a contribution to solving the specific problem in practice. The most important research issues for the future are:

- the definition of suitable membership functions of fuzzy sets and fuzzy numbers,
- the modelling of linguistic variables with fuzzy sets in accordance with the operators' comprehension and understanding,
- the definition of adequate functions and operators over a fuzzy set (aggregating and averaging operators, distances, etc.),
- the discussions of data variability defined with fuzzy tree structures,
- the methods for fuzzy tree optimisation.

REFERENCES

- [1] Benčina, J., "The use of fuzzy logic in coordinating investment projects in the public sector", *The Proceedings of Rijeka Faculty of Economics – Journal of Economics and Business*, 25 (1) (2007) 113-136.
- [2] Bohanec, M., Zupan, B., and Rajkovič, V., "Applications of qualitative multi-attribute decision models in health care", *International Journal of Medical Informatics*, 58-59 (2000) 191–205.
- [3] Bonissone, P. P., and Decker, K. S., "Selecting uncertainty calculi and granularity: an experiment in trading-off precision and complexity", in: Kanal, L. N., Lemmer, J. F. (eds.) *Uncertainty in Artificial Intelligence, Machine Intelligence and Pattern Recognition*, Elsevier Science Publisher B.V., Amsterdam 4 (1986) 217–247.
- [4] Bots, P. W. G., and Lootsma, F. A., "Decision support in the public sector", *Journal of Multi-Criteria Decision Analysis*, 9 (1-3) (2000) 1-6, <<http://ejournals.ebsco.com/direct.asp?ArticleID=Y0FUDUQFG2K4BH84K1X2L>>, accessed: May 2007.
- [5] Chen, L. H., and Chiou, T. W., "A fuzzy credit-rating approach for commercial loans - a Taiwan case", *Omega, International Journal of Management Science*, 27 (1999) 407–419.
- [6] Chen, S. M.: "Fuzzy group decision making for evaluating the rate of aggregative risk in software development", *Fuzzy Sets and Systems*, 118 (2001) 75–88.

- [7] Chen, R. Y., Sheu, D. D., and Liu, C. M., "Vague knowledge search in the design for outsourcing using fuzzy decision tree", *Computers & Operations Research*, 34 (2007) 3628–3637.
- [8] Dale, M. B., Dale, P. E. R., and Tan, P., "Supervised clustering using decision trees and decision graphs: An ecological comparison", *Ecological Modelling*, 204 (1–2) (2007) 70–78.
- [9] Gammack, J., and Barker M., *E-Democracy and Public Participation: A Global Overview of Policy and Activity*, School of Management, Griffith University Queensland, 2003, <http://www.e-democracy.gov.uk/documents/retrieve.asp?pk_document=66&pagepath=http://www.e-democracy.gov.uk:80/knowledgepool/>, accessed: July 2007.
- [10] Gasar, S., Bohanec, M., and Rajkovič, V., "Combined data mining and decision support approach to the prediction of academic achievement", in: Bohanec, M. et. all.(eds.) *ECML/PKDD'02 workshop on Integrating Aspects of Data Mining, Decision Support and Meta-Learning*, University of Helsinki, 2002, 41–52.
- [11] Geissena, V., Kampichlerb, C., López-de Llergo-Juárez, J. J., and Galindo-Acántara, A., "Superficial and subterranean soil erosion in Tabasco, tropical Mexico: Development of a decision tree modeling approach", *Geoderma*, 139 (3–4) (2007) 277–287.
- [12] GAO/AIMD-98-110, *Leading Practices in Capital Decision-Making*, Washington:U.S. General Accounting Office, 1998. <<http://www.gao.gov/special.pubs/ai99032.pdf>>, accessed May, 2004.
- [13] Harper, P. R., and Winslett, D. J., "Classification trees: A possible method for maternity risk grouping", *European Journal of Operational Research*, 169 (1) (2006) 146–156.
- [14] Hsieh, T.Y., Lu, S.T., and Tzeng, G.-H., "Fuzzy MCDM approach for planning and design tenders selection in public office buildings", *International Journal of Project Management*, 22(7) (2004) 573-584.
- [15] Huang, W.C., Teng, J.Y. & Lin, M.C., "Application of fuzzy multiple criteria decision making in the selection of infrastructure projects", IN: *Fifth International Conference on Fuzzy Systems and Knowledge Discovery*, 2008 IEEE, 159-163.
- [16] Jereb, E., Rajkovič, U., and Rajkovič, V., "A hierarchical multi-attribute system approach to personnel selection", *International Journal of Selection and Assessment*, 13 (3) (2005) 198–205.
- [17] Keefer, D. L., Kirkwood, C. W., and Corner, J. L., "Summary of decision analysis applications in the operations research literature", 1990–2001, Technical Report, Department of Supply Chain Management, Arizona State University, Tempe, Arizona, 2002.
- [18] Lai, W. H., Chang, P. L., and Chou, Y. C., "Fuzzy MCDM approach to R&D project evaluation in Taiwan's public sectors", *Journal of Technology Management in China*, 5 (2010) 84-101.
- [19] Lo, K. L., Abidin, H. I. H. Z., "The fuzzy decision tree application to a power system problem", *The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, 23 (2) (2004) 436–451.
- [20] Metaxiotis, K., Psarras J., and Samouilidis E., "Integrating fuzzy logic into decision support systems: current research and future prospects", *Information Management & Computer Security*, 11 (2) (2003) 53–59.
- [21] Niven, P.R., *Balanced Scorecard Step-by-step for Government and Nonprofit Agencies*, Wiley, Hoboken, New Jersey, 2003.
- [22] Kaplan, R.S., and Norton, D.P., *The balanced scorecard: translating strategy into action*, Harvard Business School Press, Boston, 1996.
- [23] Olaru, C., and Wehenkel, L., "A complete fuzzy decision tree technique", *Fuzzy Sets and Systems*, 138 (2) (2003) 221–254.
- [24] Podgorelec, V., Kokol, P., Štiglic, B., and Rozman, I., "Decision trees: an overview and their use in medicine", *Journal of Medical Systems*, 26 (5) (2002) 445–463.

- [25] Proctor, W., and Drechsler, M., "Deliberative multi-criteria evaluation: A case study of recreation and tourism options in Victoria Australia", *European Society for Ecological Economics, Frontiers 2 Conference*, 2003.
- [26] Quinlan, J. R., "Decision trees and decision-making", *IEEE Transactions on Systems, Man and Cybernetics*, 20 (2) (1990) 339-346.
- [27] Sahay, B. S., and Gupta A. K., "Development of software selection criteria for supply chain solutions", *Industrial Management & Data Systems*, 103 (2) (2003) 97–110.
- [28] Sastry, R., and Ranganathan, N., "A VLSI architecture for approximate tree matching", *IEEE Transactions on Computers*, 47 (3) (1998) 346–352
- [29] Savšek, T., Vežjak M., and Pavešić, N., "Fuzzy trees in decision support systems", *European Journal of Operational Research*, 174 (1) (2006) 293–310.
- [30] Tran, L., and Duckstein, L., "Comparison of fuzzy numbers using a fuzzy distance measure", *Fuzzy Sets and Systems*, 130 (2002) 331–341.
- [31] Verdev, M., Bohanec, M., and Džeroski, S., *Decision Support for a Waste Electrical and Electronic Equipment Treatment System*, 2006 <http://www-ai.ijs.si/MarkoBohanec/pub/IS2006_WEEE.pdf>, accessed: 15.05.2007.
- [32] Wang, X., and Borgelt, C., "Information measures in fuzzy decision trees," in: *Proceedings of 2004 IEEE International Conference on Fuzzy Systems*, 1 (2004) 85–90.
- [33] Yanikow, C. Z., "Fuzzy Decision Trees: Issues and Methods", *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, 28 (1) (1998) 1–14.
- [34] Yuan, Y., and Shaw, M. J., "Induction of fuzzy decision trees", *Fuzzy Sets and Systems*, 69 (2) (1995), 125–139.
- [35] Zadeh, L. A., "Fuzzy sets", *Information and Control*, 8 (1965) 338–353.
- [36] Zadeh, L. A., "The concept of a linguistic variable and its application to approximate reasoning", *Information Sciences*, 8 (1975) 301–357.
- [37] Zimmerman, H. J., *Fuzzy Set Theory and Its Applications*, Kluwer Academic Publishers, Boston/Dordrecht/London, 2001.