Yugoslav Journal of Operations Research
2 (1992), Number 2, 219-233

MODELLING OF UNSTEADY FLOW AS A SUPPORT TO THE OPERATIONAL CONTROL OF A WATER RESOURCES SYSTEM

Miloš BAOŠIĆ

"Jaroslav Černi" Institute, Belgrade, Yugoslavia

Branislav ĐORĐEVIĆ

University of Belgrade, Belgrade, Yugoslavia

Abstract. The multipurpose water resources system Danube-Tisza-Danube (WRS DTD) (flood control, drainage, irrigation, water supply, navigation, water quality control, etc.) is hydraulically very complex. WRS is controlled through a number of weirs which send the water in various directions, depending on the actual hydrologic or water resources conditions. To improve the operation of the system a control model has been developed. The complex development task is realized by a chain of gradually developed MM. First, WRS was decomposed: (a) spatially (divided into subsystems); (b) temporally (by time intervals — or the periods of quasy-steady and unsteady flow); (c) according to the hydraulic function (the weirs operation MM, network flow MM, inflow estimation MM, etc.); and (d) according to numerical aspects. The non-standard problems of unsteady flow with varying discharge throughout the WRS (tributaries, pumping stations for irrigation and drainage) are solved by using the latest advancements in the computational hydraulics theory (Preismann scheme, Cholesky scheme, etc.). A very operable unsteady-flow MM has been developed and its introduction to practice will improve the operation efficiency of the WRS.

Key words and phrases: water resources system, real-time control, modelling, unsteady flow

1. INTRODUCTION

The Danube-Tisza-Danube Water resources System (WRS DTD) in the Pannonian Plain is the largest and most important drainage and irrigation system in Yugoslavia. Particularly important and intrinsic, from the aspect of water management is its part east of the Tisza, in Banat region. This part is spatially separate and independent subsystem form the rest of the WRS DTD. The rivers of the DTD system (Tisza, Zlatica, the Old and Navigable Begej, Tamiš, Brzava, Moravica, Karaš, etc.), which flow from the Carpathians, have very unfavourable, nonuniform water regimes. The main canals receive water from all these intersecting rivers forming a water resource system which is expected to provide the most efficient flood control, successful drainage, irrigation, navigation, industrial (and municipal in the future) water supply and water quality control. An extension of the WRS use to the hydroelectric power generation is planned for the future, thus leading to an extremely complex WRS.

Each of the above mentioned purposes of the WRS is fulfilled essentially through the maintenance of the required water levels and discharges at specific WRS sections. These tasks are accomplished by a system of weirs and gates (Fig. 1) installed at inlets and/or outlets and along the course of the main canals. Discharges and levels are controlled by operating gates (the key control intervention in WRS) according to the requirements of the WRS. Realizations of various complex flow patterns (as indicated by arrows in Fig. 1) are feasible depending on hydrologic conditions and applied gate procedures.

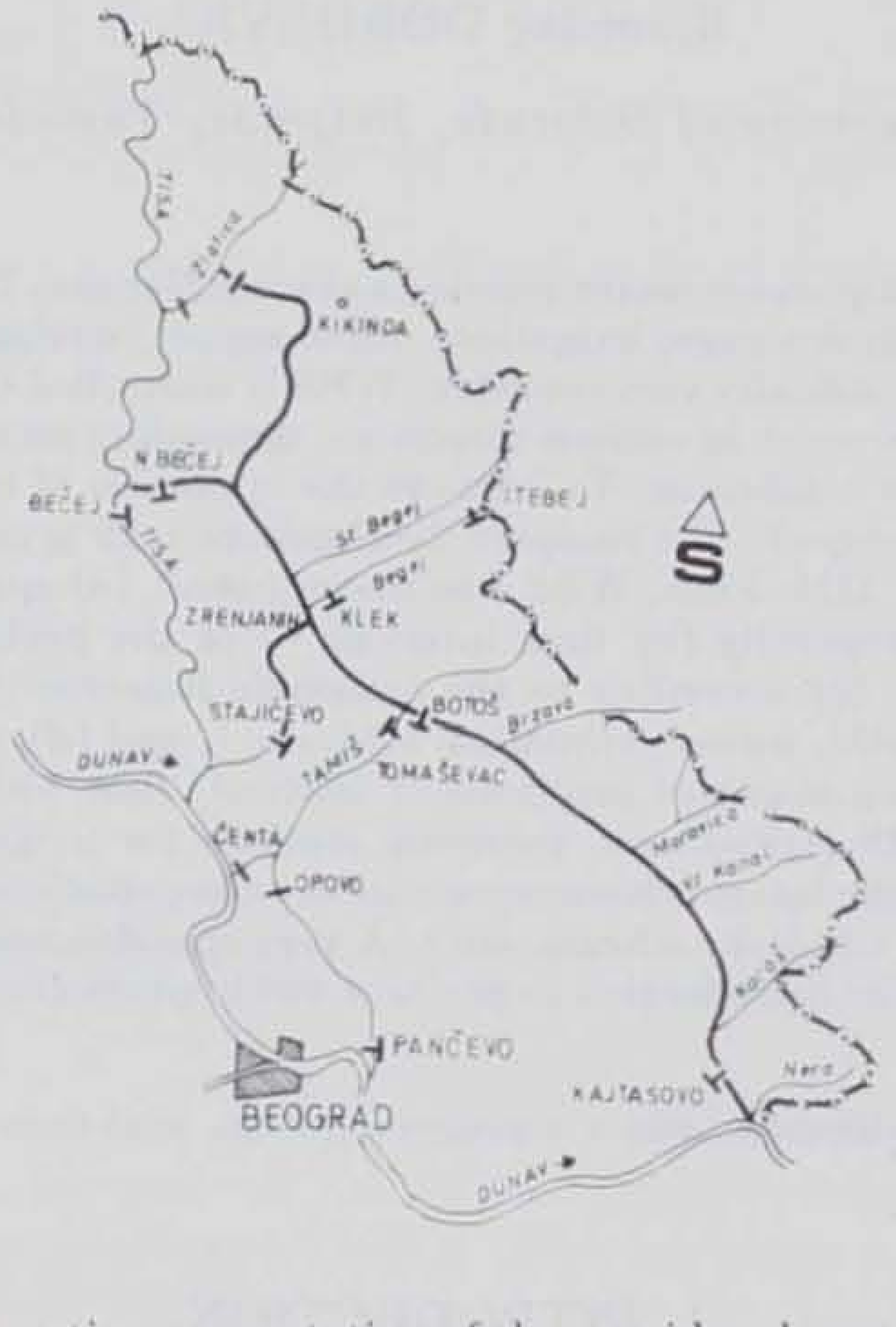

Figure 1. Schematic representation of the considered part of the Danube-Tisza-Danube WRS.

The key goals of the WRS are:

— Efficient discharge of flood flows into the rivers with the aim of redistributing flows and providing the lowest levels in critical system sections during flood control periods.

— Effective drainage of irrigated areas (either by gravity or pumping), depending on the progress of hydrologic conditions. There are many outlets from the drainage systems, which causes difficulties for the development of the mathematical model (MM).

— Provision of adequate conditions for irrigation, by means of maintaining the necessary water levels in the WRS junctions and providing necessary flow rates.

— Provision of adequate water levels for navigation.

— Provision of conditions for safe discharging of municipal and industrial waste waters.

— Prevention of floods (which were frequent in Banat region) caused by the Tisza river in the WRS protected area.

— Maintenance of water levels in individual junctions of the WRS within the limits defined by the agreement between Yugoslavia and Romania, for a better drainage and irrigation in the two countries.

— Upon the construction of power plants on the Tisza and the canals, the maintenance of such water levels and flow rates which will maximize hydroelectric power generation.

The strategy of the drainage and flood control policy is based on an important hydrologic feature of the flood generation in the catchment area: flood peaks of the Tisza, as the major recipient, do not coincide with the floods of other rivers. The delay of these floods, provided good forecast and skilled manipulation of gates, allows for an efficient flood control and drainage. The water is sent to outlet of the WRS (to the Tisza and the Danube) thus maintaining low water levels at critical WRS sections. Being a very important system, which protects the granary of Yugoslavia and many towns and industries, its maximum efficiency and reliability is an imperative.

2. COMPLEX CHAIN OF MATHEMATICAL MODELS

The complex development task is realized by a chain of gradually developed MM. First, WRS was decomposed: (a) spatially (divided into subsystems); (b) temporally (by time intervals or the periods of quasi-steady and unsteady flow); (c) according to the hydraulic function (the weirs operation MM, network flow MM, inflow estimation MM, etc.); and (d) according to numerical aspects.

The chain of control models consists of the following groups:

(a) *Simulation models*: the quasi-steady flow MM; unsteady flow MM; and the weirs-operation MM. These models can be used in a real-time manner if appropriate information is added to the existing models.

(b) *Estimation models*: the MM for input estimation (forecasting); and MM for level estimation at recipients — the Danube and Tisza Rivers (which are the boundary conditions).

(c) *Optimization models*: MM with embedded criteria for evaluation of the WRS state variables.

(d) *Expert system* (model): the highest development level in the chain of MM.

Weir operation model structure. Simulation of the weir operation is based on the well-known hydraulic relations for any possible type of flow that may occur:

submerged outflow, free overflow, submerged overflow and broad-crested weir flow. The model is of the type

$$Q = f(z_k, z_r, z_{ui}, m, \sigma) \tag{1}$$

where z_k = the water level upstream of the weir; z_r = the water level downstream of the weir; z_{ui} = the water level of the i-th control weir; m = the coefficient of discharge/overflow derived from measurements at weirs; σ = the coefficient of submergence: $\sigma = (z_k, z_r, z_{uj})$; and Q = discharge. Adjustments to the coefficients and constants were done on the basis of the in situ measurements.

Canal flow model structure. Canal and river flow models were developed for steady and unsteady flow conditions for the Danube-Tisza-Danube Rivers case. The steady flow MM was based on the Manning equation. The model in the form of parametric functions determined the upstream water level for the given downstream water level. This model is numerically simpler than the unsteady flow MM. It is more expedient in simulation, which is important for operations management. Under the steady flow conditions the Manning equation is generally given as:

$$z = F(Q, A, R, n, \Delta x) \tag{2}$$

where A = the cross-section area; z = the water level; R = the hydraulic radius; Q = discharge; n = channel roughness; and Δx = the length of the channel section. Similarly to weir-operation equations, the coefficient (n) values were determined from the field measurements. Numerous measurements gave a good agreement with the model results. The weir operation and steady flow model were combined into one model which determines the system's behavior for any weir operation condition.

3. UNSTEADY FLOW MODEL

The unsteady flow MM is developed also for flood conditions in the WRS. The model is complex due to its specific control characteristics: (a) there are various control options for redirecting flood flows to different recipients, depending on the conditions at weirs; (b) the main canals are intersected by a numerous number of streams and irrigation canals with variable directions and quantities of flow; (c) there are strict constraints imposed on water levels at the certain points, some of them being based on international agreements; and (d) high efficiency of the model is required due to its frequent use in the system's operation. The St. Venant conservation of momentum and continuity equations, for the case of lateral inflow, are given in the matrix form

$$\begin{bmatrix} 1 & 0 & 0 & B \\ 2Q/A^2 & 1/A & g & 0 \end{bmatrix} \cdot \begin{bmatrix} \partial Q/\partial x \\ \partial Q/\partial t \\ \partial z/\partial x \\ \partial z/\partial t \end{bmatrix} = \begin{bmatrix} -q \\ -W \cdot Q \end{bmatrix} \tag{3}$$

and

$$W = -\frac{Q}{A^3} \cdot \frac{\partial A}{\partial x} + g \cdot \frac{|Q| n^2}{A^2 R^{4/3}} \tag{4}$$

where x and t are the independent spatial and temporal variables, z is the water surface elevation, Q is the discharge, q is the lateral inflow, A is the wetted cross-sectional area, R is the hydraulic radius, n is the Manning coefficient, and g is the gravitational constant.

The above system can be summarized in the operator form

$$\begin{aligned}\Delta(Q, C_{Q_1}) + \Delta(z, C_{z_1}) = 0\\ \Delta(Q, C_{Q_2}) + \Delta(z, C_{z_2}) = 0\end{aligned} \tag{5}$$

where Δ is the differential operator in the form

$$\Delta(f, C) = C_1\frac{\partial f}{\partial x} + C_2\frac{\partial f}{\partial t} + C_3 f + C_4 \qquad C = [C_1, C_2, C_3, C_4] \tag{6}$$

where the parameters are defined in the matrix form

$$\begin{aligned}C_{Q_1} = [1, 0, 0, 0]; \qquad C_{Q_2} = [2Q/A^2, 1/A, W, 0]\\ C_{z_1} = [0, B, 0, q]; \qquad C_{z_2} = [g, 0, 0, 0].\end{aligned} \tag{7}$$

This system of partial equations is solved using the Preismann scheme (implicit finite difference method) [5], also known as the four-point scheme

$$f = \varphi[\theta \cdot f_{i+1}^{j+1} + (1-\theta)f_{i+1}^{j}] + (1-\varphi)[\theta f_i^{j+1} + (1-\theta)f_i^j]$$

$$\frac{\partial f}{\partial x} = \frac{\theta}{\Delta x}(f_{i+1}^{j+1} - f_i^{j+1}) + \frac{1-\theta}{\Delta x}(f_{i+1}^{j} - f_i^{j}) \tag{8}$$

$$\frac{\partial f}{\partial t} = \frac{\varphi}{\Delta t}(f_{i+1}^{j+1} - f_{i+1}^{j}) + \frac{1-\varphi}{\Delta t}(f_i^{j+1} - f_i^{j}). \tag{9}$$

The relation $\Delta(f, C)$ of Eq. (6) may be rewritten in the discretized form

$$\Delta(f, C) = D_i f_i^{j+1} + D_{i+1} f_{i+1}^{j+1} + E_i; \qquad i = \overline{1, n} \tag{9}$$

where expressions for D_i, D_{i+1} and E_i are rather complex and will not be given here.

Figure 2 gives the three-dimensional interpretation of "hydraulic surfaces" where the four-point scheme represents a projection of the hydraulic surface on the x-t plane. The x-t plane is discretized into rectangles, using the off center points M for each "hydraulic surface" in the space (x, t, f).

Using Eq. (9) for $\Delta(f, C)$, the system of Eqs. (5) can be rewritten in the developed form as a system of $2n - 2$ algebraic equations with $2n$ unknowns. Two additional relations come from the boundary conditions, whereas the values (of unknowns and coefficients at each x point), at the initial time step are determined from the initial condition. The steady-state conditions are assumed for the initial condition.

The resulting algebraic system of equations is

$$A \cdot X = \alpha \tag{10}$$

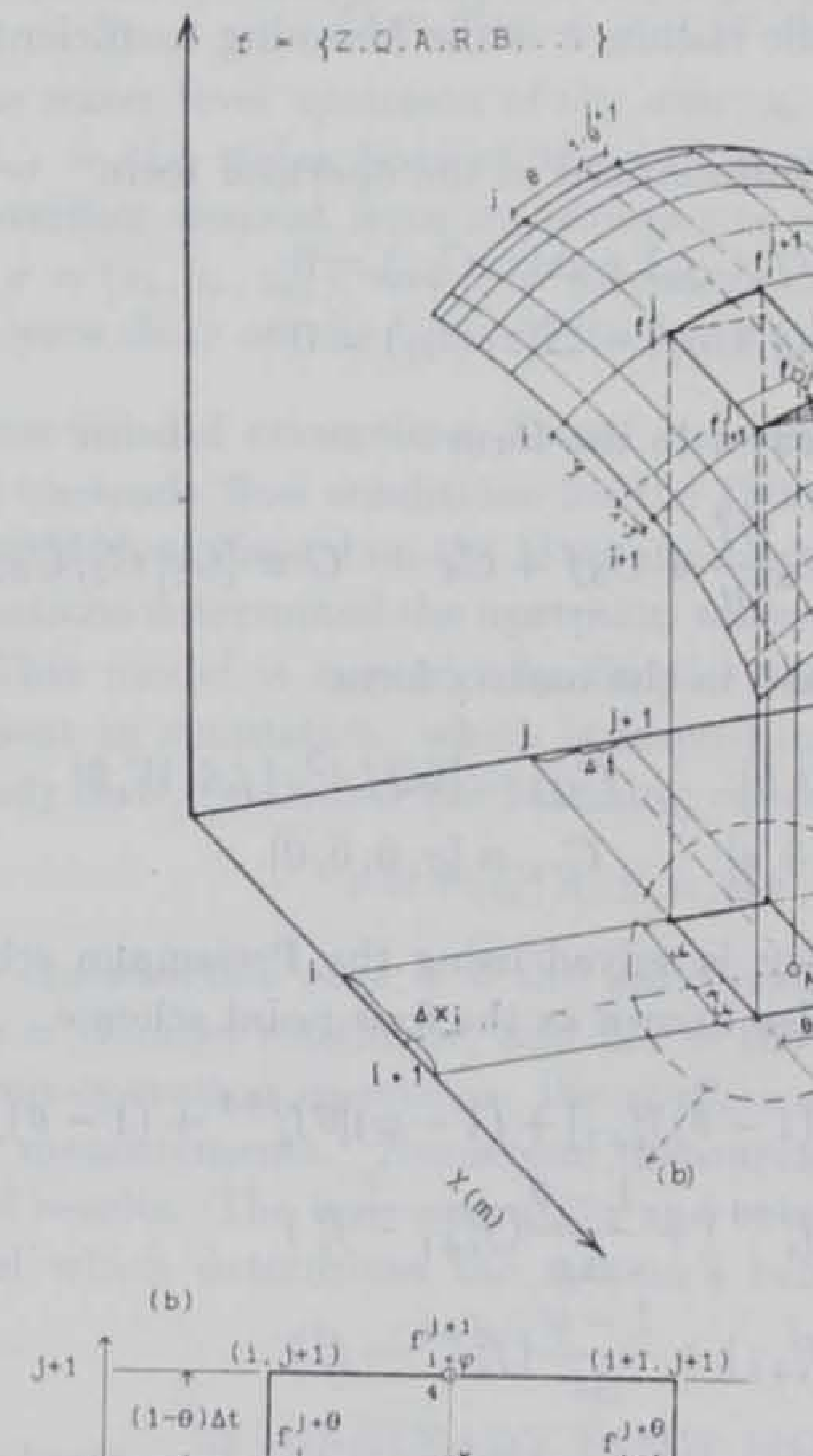

Figure 2. The three-dimensional discretization of the "hydraulic surfaces" (a), and detailed four-point scheme (b).

where A = the two-dimensionalnonlinear-coefficient (a_{ij}) matrix, X = the one-dimensional (column) matrix whose elements are Q and z values, and α = the one-dimensional matrix consisting of the free terms.

As the main canals of WRS are intersected by numerous natural water courses, irrigation canals and intakes, the flow-regime is very complex. The computational scheme shown in Fig. 3 considers two adjacent cross sections between which the discharge changes for the amount Q_{pr}. As the two cross sections (i) and (i+1) are very close, the solution of the problem is based on the continuity equation and the equal water surface elevations

$$Q_{i+1}^{j+1} = Q_i^{j+1} + Q_{pr}^{j+1}, \qquad z_{i+1}^{j+1} = z_i^{j+1}. \tag{11}$$

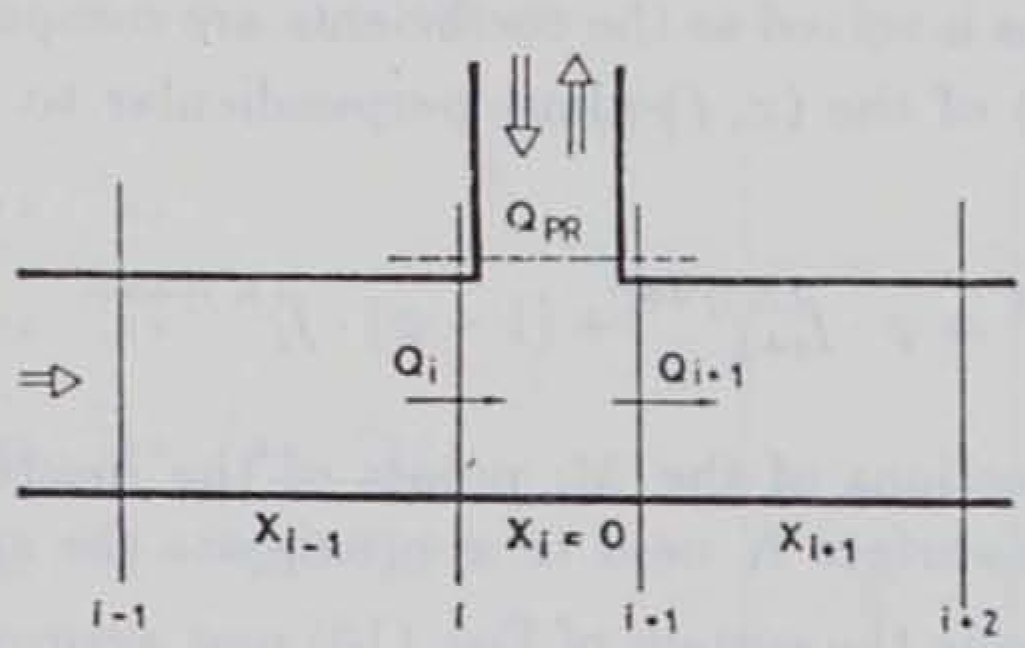

Figure 3. Inflow/outflow scheme at a confluence.

This is used at each location of flow variation: locations of river inflows into the main canals ($Q_{pr} > 0$), locations of drainage-canal inflows ($Q_{pr} > 0$), and locations of irrigation turnouts ($Q_{pr} < 0$). The problem is solved numerically by introducing the two quadruplets of coefficients $[-1\ 0\ 1\ 0]$ and $[0\ -1\ 0\ 1]$ into the matrix A, and the respective pair of values $(Q_{pr}, 0)$ into the matrix of the system of Eqs. (10). This is done for all locations with flow division/confluence. Obviously such numerical approach leads to an increased number of equations.

Consequently, the algebraic system of Eqs. (10) as augmented by using Eq. (11) may be represented in the matrix form of Fig. 4.

$$
\begin{bmatrix}
\times & \times & \times & & & & & 0 & 0 \\
\times & \times & \times & & & & & & 0 \\
& \times & \times & \times & \times & & & & \\
& \times & \times & \times & \times & & & & \\
& & -1 & 0 & 1 & 0 & & & \\
& & 0 & -1 & 0 & 1 & & & \\
& & & & \times & \times & \times & \times & \\
& & & & \times & \times & \times & \times & \\
& & & & & \cdots & & & \\
0 & & & & & & \times & \times & \times \\
0 & 0 & & & & & \times & \times & \times
\end{bmatrix}
\cdot
\begin{bmatrix}
\times \\ \times \\ \times \\ Q_l \\ z_l \\ Q_{l+1} \\ z_{l+1} \\ \cdots \\ \cdots \\ \times \\ \times
\end{bmatrix}
=
\begin{bmatrix}
\times \\ \times \\ \times \\ \times \\ Q_{pr} \\ 0 \\ \times \\ \times \\ \cdots \\ \times \\ \times
\end{bmatrix}
$$

Figure 4. The matrix form of the system of equations.

The iterative algorithm applied here was an original generalization of the Verwey's variant of the Preissmann's scheme [2]. According to the Verwey's procedure, a symmetric scheme is to be used ($\varphi = \theta = 1/2$). The authors applied a generalized procedure, allowing various values for the weighing coefficients, φ and θ, to be used. Iterations are computed in the (t, f) planes, perpendicular to the x-axis (Fig. 2) at points x_i, $i = 1, 2, \ldots, N$ (N is the total number of cross sections along the reach). The following equation is used

$$f_i^{(K)j+\theta} = \theta \cdot f_i(z_i^{(K-1)j+1}) + (1-\theta) \cdot f_i^j \tag{12}$$

where, K = the index of iteration i, $f_i^j = f_i(z_i^j)$.

The system of equations is solved as the coefficients are computed at the points $M_i^{(K)}$ $(i = 1, 2, \ldots, N-1)$ of the (x, f)-plane perpendicular to the t-axis at the points $(j+\theta)\cdot\Delta t$

$$f_{i+\varphi}^{(K)j+\theta} = \varphi \cdot f_{i+1}^{(K)j+\theta} + (1-\varphi)\cdot f_i^{(K)j+\theta}. \tag{13}$$

The $M_i^{(K)}$ points are projections of the M_i points of the discretized "hydraulic surfaces" f on the iteration surface K used to approximate the area f.

Using Eq. (13) and solving the system of Eqs. (10) new approximate values are obtained $z_i^{(K),j+1}$ and $Q_i^{(K),j+1}$ for all points (i) of the time interval $(j+1)\Delta t$ and the procedure is repeated until the required accuracy is achieved, i.e., until

$$\max\left|z_i^{(K)j+1} - z_i^{(K-1)j+1}\right| \le \delta \tag{14}$$

where $1 < i < N$, and δ = the required model accuracy (e.g., $\delta = 1$ cm).

To start the iterative procedure for the non-linear functions (for the first iteration) it is assumed that

$$f_i^{(1)j+\theta} = f_i^j; \qquad f_{i+\theta}^{(1)j+\varphi} = \varphi\cdot f_{i+1}^j + (1-\varphi)\cdot f_i^j. \tag{15}$$

Numerical aspects. Having in mind the purpose of the model, which had to determine the optimal strategy of weir management for flood control purposes, the MM operability was expressed by the speed of iterative computations. Cholesky scheme, suitable for large sparse systems of equations, is applied. However, an original algorithm for compacting matrices and changing their indices is developed within this particular MM. Instead of the matrix of size $(2n-2)\times(2n-2)$ a $(2n-2)\times 4$ matrix is used. The upper and lower auxiliary triangular matrices, from the Cholesky scheme, of the dimension $(2n-2)\times(2n-2)$ are also compressed and re-indexed with new dimensions $(2n-2)\times 3$. This procedure shortened the computations while preserving the same accuracy. Thus, the total number of the matrix elements is significantly reduced, from $12\times(n-1)\times(n-1)$ elements originally, to $20\times(n-1)$ after the transformations (a total reduction of $0.6\times(n-1)$ elements). To illustrate how large the reduction is, let us assume that the number of computation points (i) was 101; the actual number of matrix elements would be reduced sixty times after applying the transformations. It sets free the computer memory from the storage of large square matrices with a large number of zero elements, and eliminates operations with these elements, leading to a radical improvement in MM operability.

The essence of this original procedure is illustrated in Fig. 5 on the example of an eight-dimensional coefficient matrix A, which, after the compacting and change of indices, becomes A. Similarly, both the lower triangular-matrix B and the upper triangular matrix C after transformations become B and C respectively. In the new form, coefficient matrices A, B and C are used for further computations.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & & & & 0 & 0 \\ a_{21} & a_{22} & a_{23} & & & & & 0 \\ & a_{32} & a_{33} & a_{34} & a_{35} & & & \\ & a_{42} & a_{43} & a_{44} & a_{45} & & & \\ & & & a_{54} & a_{55} & a_{56} & a_{57} & \\ & & & a_{64} & a_{65} & a_{66} & a_{67} & \\ 0 & & & & & a_{76} & a_{77} & a_{78} \\ 0 & 0 & & & & a_{86} & a_{87} & a_{88} \end{bmatrix} \qquad \overline{A} = \begin{bmatrix} 0 & a^{12}_{11} & a^{13}_{12} & a^{14}_{13} \\ 0 & a^{22}_{21} & a^{23}_{22} & a^{24}_{23} \\ a^{31}_{32} & a^{32}_{33} & a^{33}_{34} & a^{34}_{35} \\ a^{41}_{42} & a^{42}_{43} & a^{43}_{44} & a^{44}_{45} \\ a^{51}_{54} & a^{52}_{55} & a^{53}_{56} & a^{54}_{57} \\ a^{61}_{64} & a^{62}_{65} & a^{63}_{66} & a^{64}_{67} \\ a^{71}_{76} & a^{72}_{77} & a^{73}_{78} & 0 \\ a^{81}_{86} & a^{82}_{87} & a^{83}_{88} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & & & & & & & \\ b_{21} & b_{22} & & & & & & \\ & b_{32} & b_{33} & & & & & \\ & b_{42} & b_{43} & b_{44} & & & & \\ & & & b_{54} & b_{55} & & & \\ & & & b_{64} & b_{65} & b_{66} & & \\ 0 & & & & & b_{76} & b_{77} & \\ 0 & 0 & & & & b_{86} & b_{87} & b_{88} \end{bmatrix} \qquad \overline{B} = \begin{bmatrix} 0 & 0 & b^{13}_{11} \\ 0 & b^{22}_{21} & b^{23}_{22} \\ 0 & b^{32}_{32} & b^{33}_{33} \\ b^{41}_{42} & b^{42}_{43} & b^{43}_{44} \\ 0 & b^{52}_{54} & b^{53}_{55} \\ b^{61}_{64} & b^{62}_{65} & b^{63}_{66} \\ 0 & b^{72}_{76} & b^{73}_{77} \\ b^{81}_{86} & b^{82}_{87} & b^{83}_{88} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & c_{12} & c_{13} & & & & 0 & 0 \\ & 1 & c_{23} & & & & & 0 \\ & & 1 & c_{34} & c_{35} & & & \\ & & & 1 & c_{45} & & & \\ & & & & 1 & c_{56} & c_{57} & \\ & & & & & 1 & c_{67} & \\ 0 & & & & & & 1 & c_{78} \\ 0 & 0 & & & & & & 1 \end{bmatrix} \qquad \overline{C} = \begin{bmatrix} 1 & c^{12}_{12} & c^{13}_{13} \\ 1 & c^{22}_{23} & 0 \\ 1 & c^{32}_{34} & c^{33}_{35} \\ 1 & c^{42}_{45} & 0 \\ 1 & c^{52}_{56} & c^{53}_{57} \\ 1 & c^{62}_{67} & 0 \\ 1 & c^{72}_{78} & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Figure 5. Matrices A, B, and C and "compacted matrices" $\overline{A}$, $\overline{B}$ and $\overline{C}$.

Solving the system of Eqs. (10) gives a direct solution (x_d) for the given error term ΔX. With the proposed algorithm, a correct solution (X_t) is obtained as

$$X_t = X_d + \Delta X. \tag{16}$$

Correction ΔX is obtained by solving the following system of equations

$$A \cdot \Delta X = \Delta \alpha \tag{17}$$

using the same direct procedure, where

$$(18) \qquad \Delta \alpha = \alpha - A \cdot X_d.$$

The original procedure leads to the solution of the system of four-diagonal equations.

The introduced modification have proven to be very useful: the computation procedure is correct and effective even for the real-time control of the system. For a general picture, a computation for a specified hydrologic input for the very complex southern subsystem of the Danube-Tisza-Danube systemtakes only 20 seconds.

4. USE OF SIMULATION MODELS

Introduced in operation control, these simulation models provide the answers to any kind of questions important for the control process, such as:

(a) What will happen in WRS if, for any hydrologic situation, specific weir management instructions are initiated?

(b) How to manage the weirs to establish some required states of WRS in order to meet different demands for various purposes?

(c) How to redistribute flows in WRS in flood periods, respecting the assigned constraints at each critical section?

(d) How to operate the system in hypothetical emergency situations (weir gates are not operational, dike breach, etc.)?

(e) How sensitive is WRS to a control error or an error in flood forecast?

To summarize, a simulation MM provides for the operating strategy for the normal or emergency operation. The model answers to possible questions are prompt and clear.

Three graphs, representing the relationship $Q = f(x, t)$ for the three control situations of the southern subsystem are given in Fig. 6–8. The first case, (a), represents a flood control period, when the water is intensively released from WRS even through the inlet weirs of the subsystem ($Q_{ul} < 0$) (Fig. 6); the second case, (b), is the one when the inlet weir is closed (this weir separates two subsystems, ($Q_{ul} = 0$) (Fig. 7); the third case, (c), is opposite from the case (a), the flood control problem in a recipient is alleviated by directing water from one river towards another river, i.e., the outlet recipient ($Q_{ul} > 0$) (Fig. 8).

Due to its operability, the simulation model enables a very efficient support to the decision-making as presented in Fig. 9. Three different gate maneuvers within

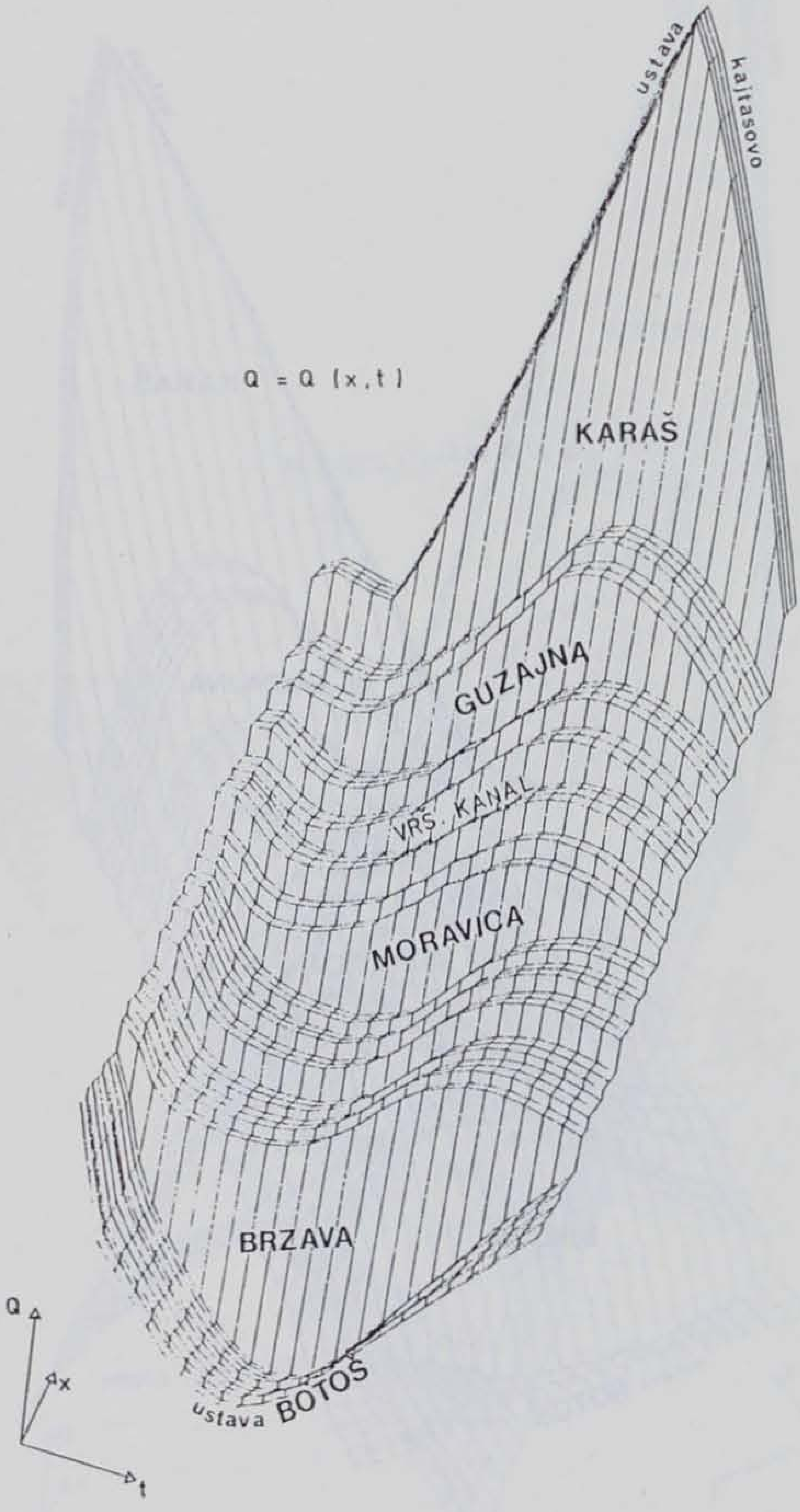

Figure 6. Graphical representation of the results: The water is intensively released from WRS.

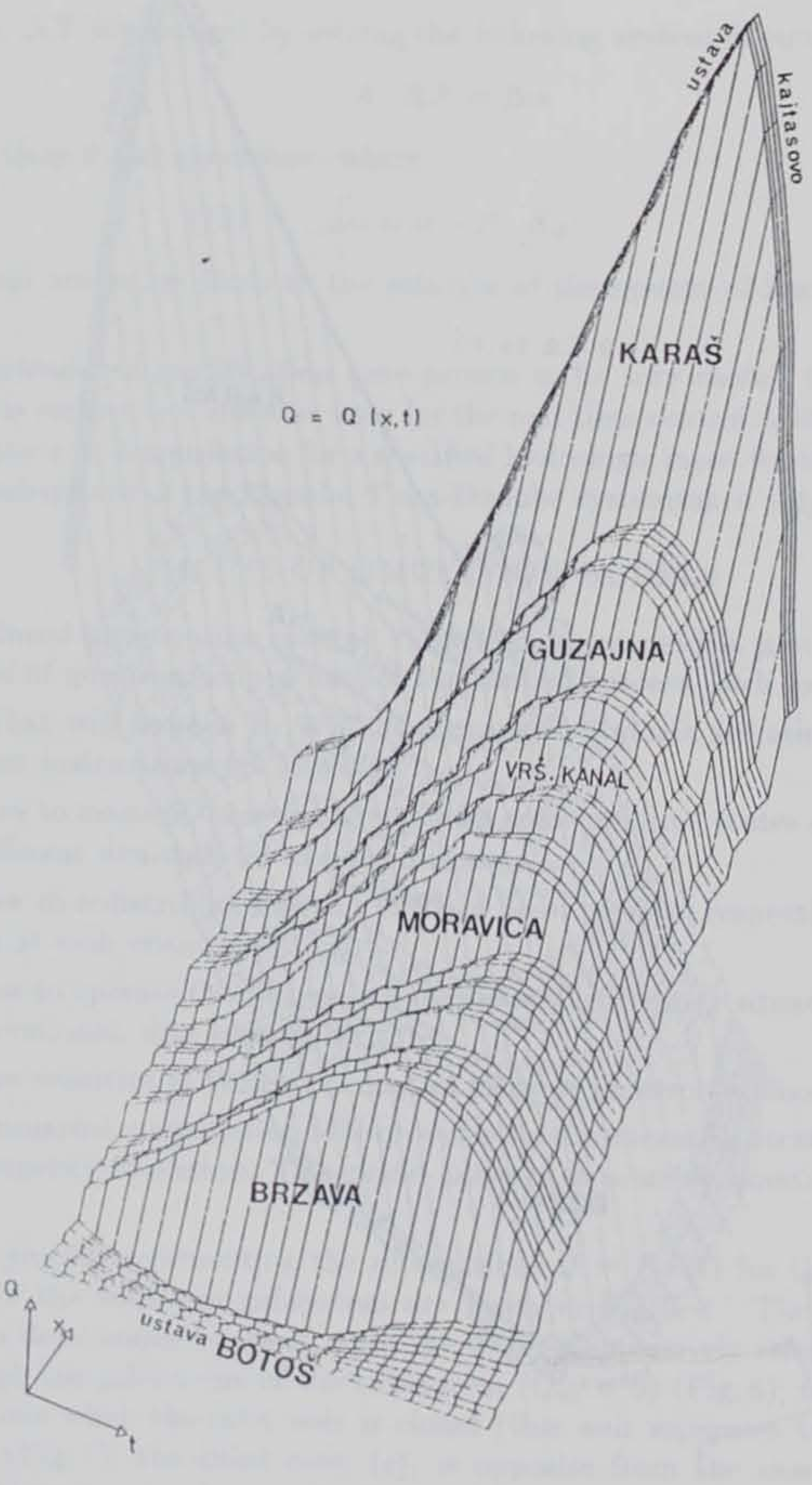

Figure 7. Graphical representation of the results: The inlet weir is closed.

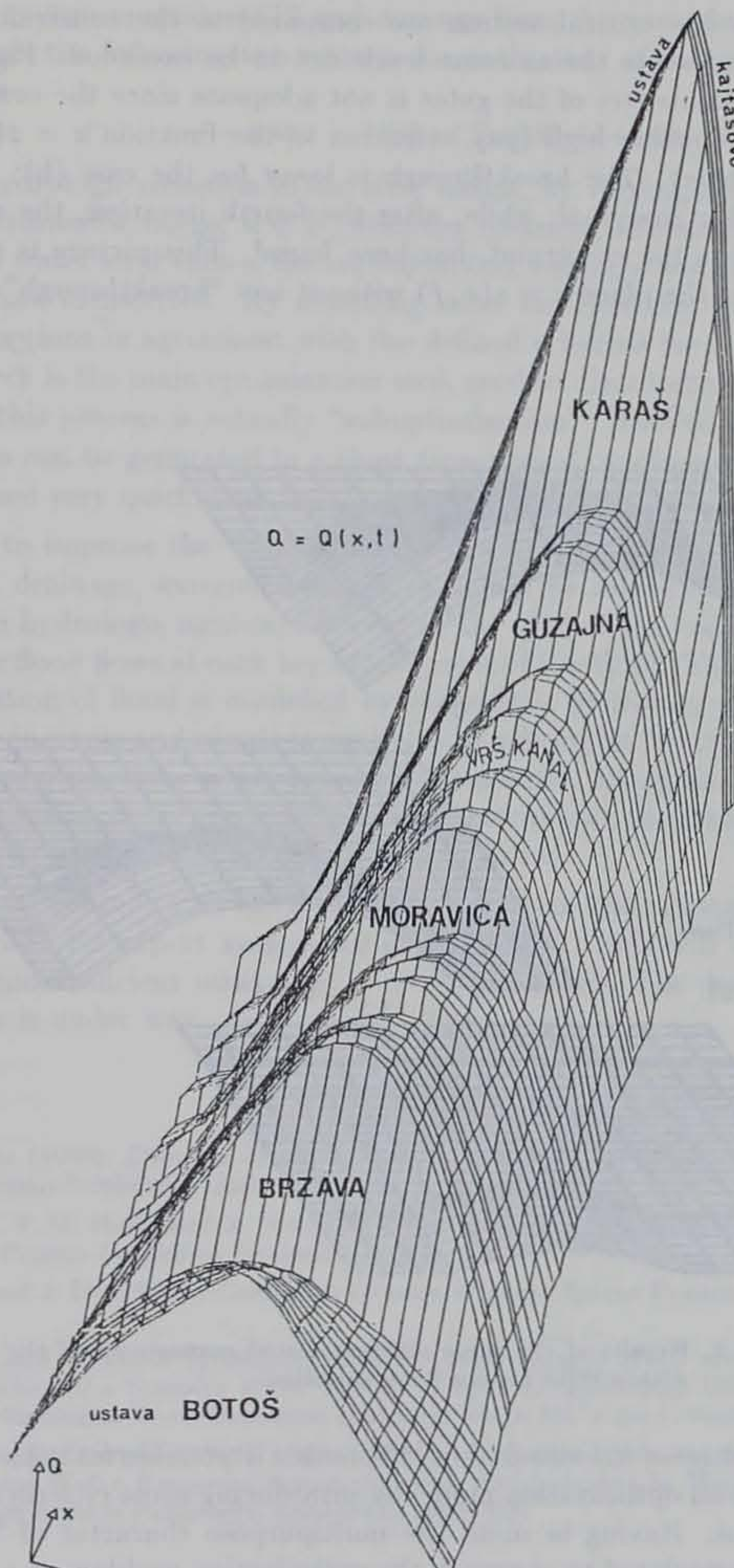

Figure 8. Graphical representation of the results: The outlet recipient $Q_{ul} > 0$.

the WRS are simulated for the flood conditions. The states in the Danube-Tisza-Danube WRS in one critical section are compared to the constraint defined by a plane which represents the extreme levels not to be exceeded. Figure 9a shows that the tested manoeuvre of the gates is not adequate since the constraint plane "breakthrough" is rather high (pay attention to the function $z = z(x, f)$ beyond the constraint plane). The breakthrough is lower for the case (b); while for the case (c) it is rather marginal, while, after the fourth iteration, the control which completely respects the constraint, has been found. This picture is not presented since it represents the plane $z = z(x, f)$ without any "breakthrough".

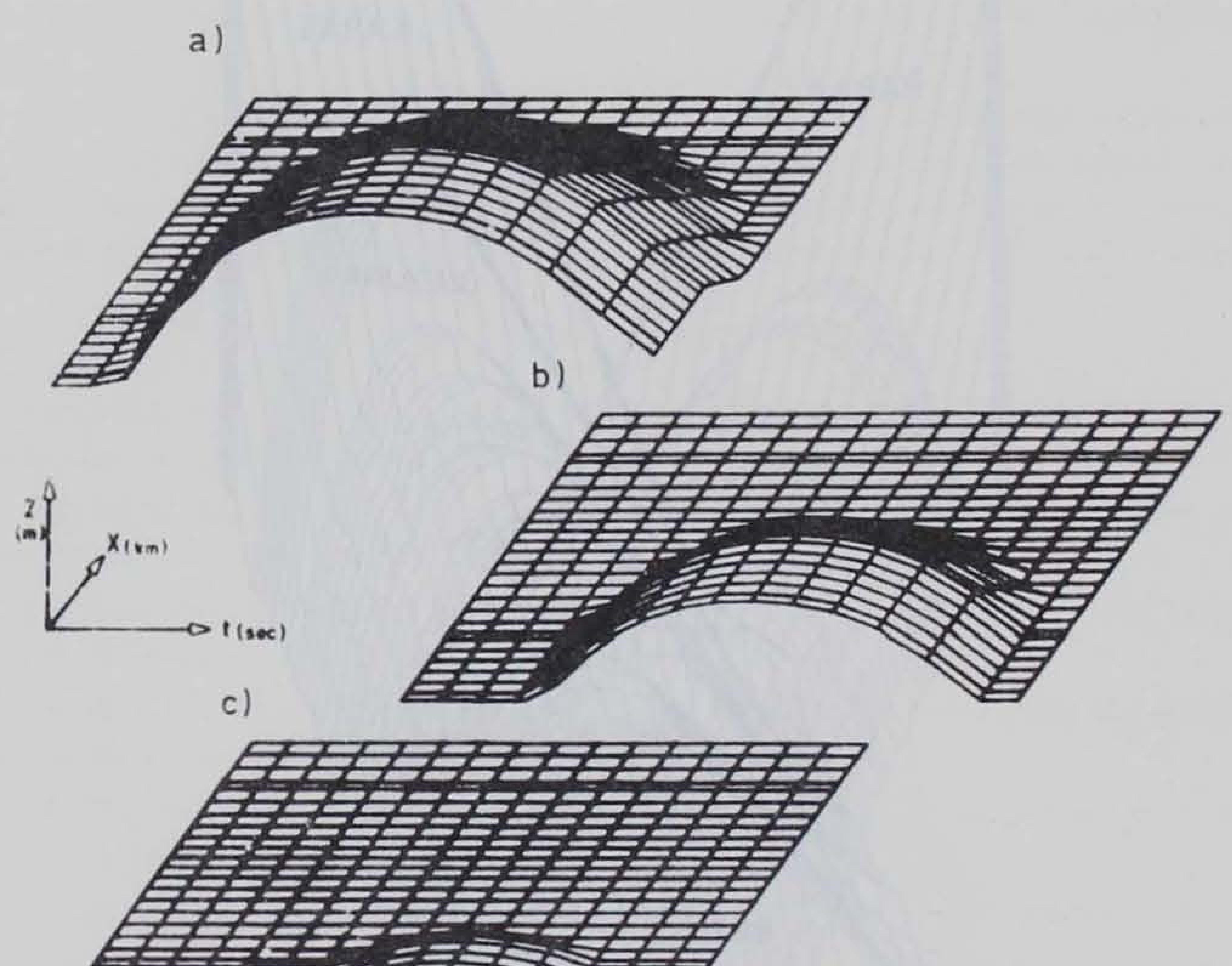

Figure 9. Results of the three different tested manoeuvres of the gates within WRS for one flood situation.

The very nature of the simulation MM makes it possible for it to be very easily transformed into an optimization model by introducing some criteria for evaluation of control actions. Having in mind the multipurpose character of WRS, various criteria can be formulated to approach the optimization problem as a multi-criteria optimization task. During the period of flood protection and intensive drainage, the WRS goals are reduced to: (a) demand for the weir control that would minimize the levels ($z_{\max}$) at the critical WRS sections; and (b) to minimize the cost of the drainage system operation (T) and damage from the excessive floods (R). For these two goals the following two criteria for evaluation of control actions can be formulated as

$$\text{(a)} \quad z \longrightarrow \min; \qquad \text{(b)} \quad (T+R) \longrightarrow \min. \tag{19}$$

For a given hydrologic situation in the river basins, by varying the control decision within the permissible range, $u \in U$, different matrices, containing the maximum discharge and water level values, the corresponding damages and pumping costs can be obtained and memorized. By searching these matrices one can determine the best control options in agreement with the defined criterion for control evaluation. Since the search is the main optimization tool, used to discriminate among different alternatives, this process is actually "suboptimization". However, a large numbers of alternatives can be generated in a short time period, so the suboptimal solution can be obtained very quickly, and with the required accuracy.

In order to improve the WRS efficiency during the period of flood conditions and intensive drainage, external estimators should be introduced into the analysis. These are hydrologic models, external to the MM of the considered WRS, for predicting the flood flows at each key inflow point of the WRS. The models are combined: generation of flood is modelled by parametric hydrology methods, whereas unsteady flow in rivers and canals is modelled by solving St. Venant equations. The estimation model structure is not presently considered. Using the game theory one can show that the introduction of external estimators into the management process will significantly improve the quality of management.

Finally, integration of all the mentioned models in one management algorithm will make a complex expert system for management. This will lead to the most detailed and most efficient utilization of all its potentials. The development of the expert system is under way.

REFERENCES

[1] M.B. Abbott (1980): *Elements of the Theory of Free Surface Flows — Computational Hydraulics*, Pitman Publishing Limited, London.

[2] J.A. Cunge, F.M. Holly and A. Verwey (1980): *Practical Aspects of Computational River Hydraulics*, Pitman Publishing Limited, London.

[3] A. George and J. Liu (1981): *Computer Solution of Large Sparse Positive Definite Systems*, New Jersey.

[4] B. Đorđević and M. Baošić (1986): *The Use of a Mathematical Model with the Estimator for the Optimization of a Complex Water Resources System*, Proceedings UNESCO Symposium on Decision Making in Water Resources Planning, Vol. 2, Ed. Arne J. Carlsen, Oslo, 519–539.

[5] M. Baošić and B. Đorđević (1988): *Numerical Aspects of Simulation and Optimization Models for a Complex Water Resources System Control*, Developments in Water Science, Ed. M. Celia, Elsevier Science Publishers, Amsterdam, 393–398.